Why England? Demand, Growth and Inequality During the Industrial Revolution

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Abstract:

Why was England first? And why Europe? We present a probabilistic model that builds on big-push models by Murphy, Shleifer and Vishny (1989), combined with hierarchical preferences. The interaction of exogenous demographic factors (in particular the English low-pressure variant of the European marriage pattern) and redistributive institutions – such as the “old Poor Law” – combined to make an Industrial Revolution more likely. Essentially, industrialization is the result of having a critical mass of consumers that is “rich enough” to afford (potentially) mass-produced goods. Our model is then calibrated to match the main characteristics of the English economy in 1750 and the observed transition until 1850. This allows us to address explicitly one of the key features of the British Industrial Revolution unearthed by economic historians over the last three decades – the slowness of productivity and output change. In our calibration, we find that the probability of Britain industrializing is 5 times larger than France’s. Contrary to the recent argument by Pomeranz, China in the 18th century had essentially no chance to industrialize at all. This difference is decomposed into a demographic and a policy component, with the former being far more important than the latter.
1 Introduction

After millennia of stagnation, living standards and productivity began to increase rapidly after 1750. Britain was the first country to break free from Malthusian constraints, with population size and living standards beginning to grow in tandem [Crafts (1985), Wrigley (1983)]. Over the following 250 years, more and more countries have industrialized, first in Europe and North America in the nineteenth century, and in the rest of the world since the middle of 20th century. Following Romer (1990) and Rebelo (1991), a large literature has evolved explaining long-term growth patterns. Recently, a new set of theoretical papers has modelled the transition from Malthusian stagnation to sustained growth.1 The question that has received much less attention is the cross-sectional variation in the timing of the transition to sustained growth. What explains these differences? Why did some countries industrialize so early, with major economic and political consequences that are still felt today? In this paper, we attempt to answer this question by modelling the transition from traditional production to advanced techniques in one country, and then calibrate the model with historical data.

We focus on the interplay of random events and structural factors that facilitated industrialization. This is in sharp contrast to recent models of the Industrial Revolution. The transition from stagnant living standards to self-sustaining growth is often inevitable, given the properties of the pre-industrial world. Hansen and Prescott (2002), for example, assume that exogenously driven technological change in the ‘modern’ sector will eventually lead to a shift in production techniques. For much of the late medieval and early modern period, growth occurred repeatedly during sustained expansions in many countries [Braudel #]. Most of these episodes sooner or later ground to a halt. Some highly advanced economies went into decline – with the Italian Republics the most prominent case – while others like Holland stagnated at a high level of income. What explains these stops and starts? Taking our cue from earlier work by Crafts (1977) and by Acemoglu and Zilibotti (1997), our model takes the probabilistic nature of the transition to self-sustaining growth into account. Crucially, we calibrate our model using historical data to obtain quantitative estimates of the industrialization probabilities for various countries.

Instead of assuming exogenously given or population-driven innovations, we emphasize structural change.2 Starting with the observation that many new techniques and manufacturing procedures had been available for some time, we are argue that adoption, not new inventions drove the industrialization process for most of its early period, from 1750-1850. It is change during this crucial period that we are attempting to explain.

We employ a “big push” model, and combine it with hierarchical preferences amongst consumers in the tradition of Murphy et al. (1989a) . Our model consists of two groups

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1 Prescott and Moav (2002); Galor and Weil (2000); Jones (2001).
2 In this sense, our paper pursues a modelling approach that is the logical opposite of Hansen and Prescott (2002).
of consumers – rich and poor – and three production sectors – agriculture, manufacturing and intermediate products. Initially, poor consumers cannot afford manufactured goods; they consume solely food products, while the rich have access to manufactured goods and food. Random, positive shocks to agricultural productivity can raise mass incomes and permit the poor to consume manufactured goods – the higher their starting incomes are, the greater the likelihood. The key element for adoption of the constant-returns-to-scale (“Solow”) technology with relatively high fixed costs is sufficient demand. The increased demand for manufactured products then makes it profitable for manufacturing firms to sink the up-front costs necessary for industrialization. We add an intermediate goods sector to capture some of the salient features of the Industrial Revolution as it occurred in England. Eventually, with enough manufacturing firms using advanced techniques, the intermediate products sector also industrializes. This allows us to explain the fact that steam engines (intermediate inputs) were widely adopted in England only a century after their invention, whereas innovations such as the spinning jenny and the Arkwright frame (used in manufacturing) had a much shorter adoption time span (Crafts 2004#).

One key determinant of effective demand was inequality. We follow the lead by Zweimüller (2000); Murphy et al. (1989a) and Foellmi and Zweimüller (2004), who used hierarchical preferences and inequality in growth models. The limiting factor in our model is the purchasing power of the poor rather than overall size of the market.England had higher consumption levels because of its favorable demographic regime and unusually generous welfare system. These factors made industrialization more likely but didn’t determine the outcome. Our approach allows us to model the problem of why England? in an explicitly probabilistic setting (Crafts 1977). Country-specific factors determine whether the switch from advanced techniques occurs at all, and with what likelihood.

This setup allows us to make sense of a number of peculiarities that have often been raised in the context of England’s early transition to self-sustaining growth. The finding of slow growth after 1750 implies that living standards must have already been relatively high already at this point in time. This was largely a result of a favorable demographic regime. Europeans in general enjoyed higher per capita living standards during the early modern period than their Asian, African and Latin American peers. The key reason was the ‘European marriage pattern’ – the fact that West of a line from St. Petersburg to Triest, age at first marriage for women was determined by socioeconomic conditions, not age at first menarche. England in particular was characterized by a low-pressure demographic regime – negative shocks to income were mainly absorbed by falls in fertility rather than increases in mortality [Wrigley and Schofield (1981); Wrigley et al. (1997)]. This stabilized per capita living standards and avoided the waste of resources and human lives that come from the operation of the ‘positive’ Malthusian check, when population declines because of widespread starvation.

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3In our model, consumption by the rich is not sufficient for industrialization to get under way.

4Pomeranz 2000 has recently argued that, for the Yankzi region, living standards were broadly similar with the most advanced regions in Europe. We nonetheless assume that European living standards were higher – China as a whole could probably not rival the European average (Maddison 2003). Also, recent work by Broadberry and Gupta (2005) has shown that Pomeranz’s claims, even for the Yankzi area, are probably exaggerated.
In addition, eighteenth-century England had a relatively generous early form of welfare. The Old Poor Law (or the ‘Speenhamland system’, as it was known) provided relatively generous outdoor relief for the rural poor, helping to stabilize their incomes. High per-capita income matters because our model uses hierarchical preferences. Since Europeans had higher incomes (and within Europe, Englishmen had the highest per capita living standard), they could afford a greater number of non-agricultural goods. This helps us to sidestep the potential problem for demand-based endogenous growth models of the Industrial Revolution noted by Crafts (1995) – that the cross-sectional evidence seems at variance with the timing of the transition, with France having a much larger GDP than the UK but a markedly later start. In our model, countries need to have sufficient demand for one or more higher-ranked goods to make adoption of the new technology feasible. In this way, there is a trade-off between higher per capita income and total GDP. Put differently – England may have been first because its per capita income was close to Dutch levels (where population was markedly smaller), while being much richer than the French (with a much greater population).

The papers that are closest in spirit to ours are Murphy et al. (1989b) and Acemoglu and Zilibotti (1997). They focus on project indivisibilities and the problem of diversifying away risks at an early stage of development. In the beginning, with low per capita living standards, the range of feasible projects in their model is severely limited. Chance in their model is crucial because a run of “good years” increases the probability of switching to high-productivity projects. Our model differs in terms of the mechanism that allows the switch to new technologies. We also emphasize indivisibilities, but focus on minimum efficient size considerations which require mass consumption for profitable adoption of advanced techniques. We also assume that financing is freely available.

Another important literature that relates closely to our work was pioneered by Gilboy (1932), who argued that a surge in additional demand in the eighteenth century contributed to the British Industrial Revolution. This work has been severely criticized by Mokyr (1977) and Crafts (1985), and been defended by Ben-Shachar (1984). Our model gives a rigorous formulation to the link between demand and industrialization without relying on any of the questionable assumptions that Gilboy made about the timing of changes in demand. We also do not need to resort to the idea that exogenous changes in taste altered the labor-leisure trade-off, as suggested by Jones (2001).

We proceed as follows. Section II discusses some of the historical evidence and offers a broader motivation for the paper. Section III presents our model and derives the basic properties of the pre-industrial and the industrialized equilibrium. In an exercise similar in spirit to Stokey (2001), we calibrate our model with the available data from Britain in Section III. Section IV presents some alternative specifications for other parts of the globe – namely China and France – in an attempt to derive predictions about the relative probabilities of industrialization. Section V concludes.

2 Motivation and Historical Evidence

This paper is an attempt to bridge the gap between two literatures. The Industrial Revolution, as modelled in a string of recent theoretical papers, shares few similarities with
the one uncovered by economic historians over the last two decades [Voith (2003)].

The former stress the discontinuous nature of economic change as a result of the transition from ‘Malthus to Solow’ [Hansen and Prescott (2002); Jones (2001)], compared to centuries of stagnation beforehand. Time is commonly measured in millennia, and economic development occurs in the world as a whole [Jones (2001), Galor and Moav (2002)]. Viewed from the perspective of the last few millennia of world history, the Industrial Revolution is indeed a sharp discontinuity with immediate consequences, affecting most parts of the globe relatively quickly.

The new orthodoxy in economic history, in contrast, has emphasized the slow, gradual nature of change. With every set of revisions over the last 20 years, growth rates of output and productivity have been reduced (Table 2). They are now anything but impressive even compared to earlier periods of economic history. Instead of a rapid break with Malthusian fetters, the most ‘revolutionary’ aspect of the Industrial Revolution lies in structural change (Crafts and Harley 1992). What is also remarkable about the period after 1750 is not growth or TFP performance as such, but the fact that accelerated population growth coincided with stagnant or slowly growing wages and output per head [Mokyr (1999)].

Table 1: Output and productivity growth during the Industrial Revolution

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<tbody>
<tr>
<td>1760-1800</td>
<td>1.1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1801-1831</td>
<td>2.7</td>
<td>2</td>
<td>1.9</td>
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<tr>
<td>1831-60</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
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</thead>
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<tr>
<td>1760-1800</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.27</td>
</tr>
<tr>
<td>1801-1831</td>
<td>1.3</td>
<td>0.7</td>
<td>0.35</td>
<td>0.54</td>
</tr>
<tr>
<td>1831-1860</td>
<td>0.8</td>
<td>1</td>
<td>0.8</td>
<td>0.33</td>
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</table>

A number of important stylized facts are not well accounted-for in current models of the Industrial Revolution. First, the nature of pre-industrial growth is often ill-defined. Under Malthusian conditions, increases in technology should not lead to long-run changes in living standards. Many authors therefore assume that an accumulation of positive and unexplained shocks will eventually push living standards beyond a threshold, fertility falls and growth in per capita terms takes off as a result. Second, the probabilistic nature of the Industrial Revolution has been given scant regard. Most models assume that, given the rate of knowledge accumulation, growth in human capital or technological change, the transition from ‘Malthus to Solow’ had to occur sooner or later. We present a model in which industrialization may occur, but need never happen at all. Connected with this, there should be scope for periods of episodic growth before the transition to rapid per capita income gains. The history of economic growth before 1750 suggests numerous periods of ‘false starts’, when some areas showed proto-industrialization and a move into sectors outside agriculture at a time of temporarily higher living standards – but without providing the basis for a sustained period of growth. This also permits us to understand why some areas, despite
showing signs of promise for some of their history, failed to industrialize. Fourth, the issue of changes in sectoral composition needs to be separated from the rate of technological change. While most papers analyzing the Industrial Revolution in a long-run perspective implicitly assume that the adoption of new techniques is synonymous with a shift in employment, the history of the British Industrial Revolution suggests the opposite. Growth in employment outside agriculture was not synonymous with broad-based adoption of revolutionary new techniques. Fifth, we should have something to say about processes occurring over decades, and not only centuries. While the abrupt nature of the Industrial Revolution is undisputed when viewed in the context of the last ten millennia, we aim to provide a theoretical structure that helps us understand change over the medium term.

Britain was an unequal society, even at a relatively early stage of development [Lindert and Williamson (1982), Lindert (2000)]. Nonetheless, average British standards of consumption were relatively high compared to the French, with a much higher minimum level of consumption. Fogel (1993) estimated that as a result of higher inequality and lower per capita output, the bottom 20-30 percent of the French population did not receive enough food to work productively. While the vast majority of industrial goods was purchased by the English well-to-do, the poor continued to have access to a significant share of industrial output (Table 2). Even during the 1790s, when food prices were high, up to 30% of working class budgets continued to be spent on non-food items (with 6% going on clothing). With most of the goods produced by the nascent modern sector having high income elasticities of demand (in excess of 2.3), even modest gains in real wages translated into markedly higher purchases of luxury goods. One factor that facilitated the non-food expenditure of poorer groups of society was redistributive policies. The Old Poor Law was an unusually generous form of redistribution. At its peak, transfers amounted to 2.5% of British GDP, and more 11% of the population received some form of relief. This may also have had indirect effects for the wages of those who were not recipients, by reducing competition in the labor market (Boyer 1990). Finally, because of the large absolute value of the own-price elasticity of non-food spending (of −1.8 amongst the English poor), productivity increases and subsequent price reductions facilitated the growth of the modern sector [Horrell (1996)].

Table 2: Domestic demand in the UK for manufactured goods

<table>
<thead>
<tr>
<th></th>
<th>1700</th>
<th>1760</th>
<th>1780</th>
<th>1801</th>
<th>1831</th>
<th>1851</th>
</tr>
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<tbody>
<tr>
<td><strong>Absolute values</strong> (mio. sterling)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>capitalists</td>
<td>6.9</td>
<td>7.1</td>
<td>18.1</td>
<td>26.5</td>
<td>84.8</td>
<td>121.7</td>
</tr>
<tr>
<td>workers</td>
<td>3</td>
<td>4.8</td>
<td>6.5</td>
<td>13</td>
<td>18.7</td>
<td>30.9</td>
</tr>
<tr>
<td><strong>Proportions</strong> (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>capitalists</td>
<td>70</td>
<td>60</td>
<td>74</td>
<td>67</td>
<td>82</td>
<td>80</td>
</tr>
<tr>
<td>workers</td>
<td>30</td>
<td>40</td>
<td>26</td>
<td>33</td>
<td>18</td>
<td>20</td>
</tr>
</tbody>
</table>

Source: Crafts (1985), p. 136
3 The Model

Consider a continuum of infinitely-lived agents \( i \in [0, N] \), where \( N \) indicates population size. Each consumer is endowed with one unit of labor that he supplies inelastically in each period. A fraction \( \rho \) of agents is poor \((p)\), and the corresponding share \( 1 - \rho \) is rich \((r)\). There is a range \( J \subset \mathbb{R}_+ \) of final product sectors, each producing one corresponding good \( j \in J \). In addition, a range \( Q \subset \mathbb{R}_+ \) of sectors produces intermediate goods after industrialization. Let \( \Pi_j \) denote total profits of industry sector \( j \), and correspondingly \( \pi_j = \Pi_j/N \) sector \( j \)'s profits per person. In this setup, giving \( \pi_j \) to each agent would mean completely equal distribution of sector \( j \)'s profits. To introduce inequality, we define \( \tau_{p,j} = \pi_{p,j}/\pi_j \), where \( \pi_{p,j} \) are profits from sector \( j \) given to each poor person. Consequently, the inequality measure \( \tau_{p,j} \) indicates the profits of sector \( j \) given to a poor person relative to the profits an average individual receives from sector \( j \). Similarly, \( \tau_{r,j} = \pi_{r,j}/\pi_j \) is the inequality measure for the rich, and it is straightforward to derive \( \tau_{r,j} = (1 - \rho \cdot \delta_{p,j}) / (1 - \rho) \). Note that inequality in the distribution of sector \( j \)'s profits is increasing in \( \rho \), provided \( \tau_{r,j} > 1 \). Total income of an agents in period \( t \) is then given by \( I_{i,t} = w_t + \tau_{i,L}(r_L L/N) + \int_{j \in J} \tau_{i,j} \pi_{j,t} + \int_{q \in Q} \tau_{i,m} \pi_{m,t} \), \( i \in \{r, p\} \), where \( w_t \) is the wage, and the second term reflects the distribution of total land rents \( r_L L \). This equation reflects the underlying assumptions that all agents provide one unit of labor of the same quality, and that the source of inequality between rich and poor is the distribution of profits.

3.1 Consumers

Consumers have hierarchic preferences over a continuum \([a, \infty)\) of consumption goods, where \( a \) is a positive number close to zero. Goods are indivisible and consumption is a take-it-or-leave-it decision. When consuming good \( j \in [a, \infty) \), a consumer derives marginal utility \( 1/j \). Thus, the index \( j \) captures the hierarchic nature of preferences by assigning a high marginal utility to low-\( j \) (basic needs) goods and a low marginal utility to high-\( j \) (luxury) goods.\(^5\) Let \( c_p \ (c_r) \) denote the last good consumed by a poor (rich) agent. Then instantaneous utility takes the form \( u(c_{i,t}) = \ln(c_{i,t}) - \ln(a) \) for \( i \in \{r, p\} \).\(^6\) Consequently, the index \( c_i \) is a welfare measure for agent \( i \). Regarding the range of consumption, let \( J_A = [a, 1] \) denote agriculture and food products (in the following referred to as agriculture), and \( J_M = (1, \infty) \) all other, more luxurious goods (for example, manufactured products such as cotton, clothing, or ceramics). In the following we will refer to the range \( J_M \) as manufacturing products. This definition reflects the implicit assumption that consumers first satisfy their need for food before moving on to industrial products along the hierarchy of goods. Given disposable income \( I_{i,t} \), consumers maximize utility in each period, which yields

\(^5\)This approach is similar to Murphy et.al. (1989) and Zweimüller (2004). Deviating from the former, we use the interval \([a, \infty)\) rather than \((0, \infty)\) because this approach enables us to have hierarchic preferences also for goods \( q < 1 \).

\(^6\)In this step we need the consumption range \([a, \infty)\) rather than \((0, \infty)\), since only the former allows us to find a finite value for the integral \( \int_a^\infty \frac{1}{y} dy \).
\( c_{i,t} = \begin{cases} 
  \frac{I_{i,t}}{p_A} & \text{if } I_{i,t} \leq p_A \\
  \frac{(I_{i,t} - p_A)}{p_M + 1} & \text{if } I_{i,t} > p_A 
\end{cases} 
\)  

(1)

where \( p_A \) and \( p_M \) are the price of agricultural and manufacturing goods, respectively.\(^7\) To keep matters as simple as possible we treat the saving rate as an exogenously given maximum share of total income that is fully or partially utilized whenever up-front costs need to be paid. This point will be explained in more detail below. The setup of the consumption side of the model implies that a final good \( j \in [a, c_p) \) is demanded by every agent in the population, that is, it is demanded by the rich and the poor - from now on indexed by the superscript \( rp \). A good \( j \in (c_p, c_r) \) is purchased by the rich only - from now on indexed by the superscript \( r \), and finally, goods \( j \in (c_r, \infty) \) are not demanded or produced. This implies the following demand function for final goods

\[
\begin{align*}
  y_{d,j,t}^d &\equiv \begin{cases} 
  y_{d,rp,j,t}^d & \text{if } j \leq c_p \\
  (1-\rho)N & \text{if } c_p < j \leq c_r \\
  0 & \text{if } j > c_r 
\end{cases} 
\end{align*}
\]

(2)

### 3.2 Producers

The economy’s production side consists of two ranges of final sectors: agriculture and industry, and a range of intermediate product sectors. Following Murphy et al. (1989), each final product \( j \in [a, \infty) \) is produced in its own sector, and each sector contains two types of firms: First, a competitive fringe of firms with freely accessible pre-industrialization technology and second, a single industrialized firm, provided that this firm paid up-front costs \( C_j \) to gain access to a higher-productivity industrialized technology. While prices are determined in the competitive fringe, an industrialized firm has the exclusive access to the advanced technology in its sector and can thus make profits due to its lower variable costs (taking prices and demand as given). In order for industrialization to be worthwhile in a given sector, expected discounted life-time profits after industrialization must exceed the up-front costs.

The competitive fringe in agriculture uses land and labor as inputs in the production function:

\[
f^{A}_{c}(A_{A,t}, l, n) = A_{A,t} l^\gamma^{pre} n^{1-\gamma^{pre}}
\]

(3)

where \( l \) is land, \( n_A \) is labor and \( 0 < \gamma^{pre} < 1 \). TFP in pre-industrialized agriculture in period \( t \), \( A_{A,t} \), is calculated as \( A_{A,t} = (1 + \kappa_t) A_A \) where \( A_A \) is average pre-industrialized TFP in agriculture and \( \kappa_t \) is a shock to agricultural TFP following the AR(1) process \( \kappa_{t+1} = \theta \kappa_t + \epsilon_t \). Therein, \( \theta \) is the autocorrelation and \( \epsilon_t \sim N(0, \sigma^2) \). We choose this representation of the shock since \( \kappa_t \) can be interpreted as percentage deviation from average productivity. The shock should be interpreted as caused by weather

\(^7\)Strictly speaking, this result is an approximation that is valid if \( a \) is sufficiently close to zero and if \( p_A = p_A \) for all agricultural goods and \( p_M = p_M \) for all manufacturing goods. These conditions will be validated later on. Note also that utility maximization of an agent who consumes only agricultural products (i.e., \( c_{i,t} \leq 1 \)) implies that the inequality \( 1/c_{i,t} \geq p_A/p_M \) must hold in order to maintain the hierarchic consumption pattern. Otherwise, the agent would have an incentive to consume at least one manufacturing product (i.e., the first one in the manufacturing range) before finishing the consumption of all food products.
conditions rather than changes in technology. The autocorrelation then results from the abundance or shortage of grain in periods following positive or negative shocks. After paying up-front cost $F_j, j \in [a, 1]$, an agricultural firm gains access to the technology:

$$f^n_A \left( l, n_A, \{x_q, A\}_{q \in Q_A} \right) = \overline{A}_A l^{\gamma_{post}} n_A^{(1-\alpha_A)(1-\gamma_{post})} \left( \int_0^{Q_A} x_{q, A} dq \right)^{2A(1-\gamma_{post})}$$

(4)

where $\overline{A}_A$ is TFP in industrialized agriculture, $0 < \alpha_A < 1$ and $x_{q, A}$ is intermediate input $q$ taken from a range $[0, Q_A]$ of available intermediate inputs. There is no shock to industrialized agriculture’s TFP. The parameter $\beta \in (0, 1)$ denotes decreasing returns with respect to each individual intermediate input. Note that both production functions exhibit constant returns to scale with respect to all inputs. However, production in post-industrialized agriculture is subject to increasing returns with respect to intermediate input variety $Q_A$.9

In manufacturing, the competitive fringe uses the technology $f^n_M (n) = A_M n_M$, where $A_M$ is pre-industrialized TFP. After paying $F_j, j \in (1, c_v]$, a monopolistic firm has access to industrialized production:

$$f^n_M \left( n_M, \{x_{q, M}\}_{q \in Q_M} \right) = \overline{A}_M n_M^{(1-\alpha_M)} \left( \int_0^{Q_M} x_{q, M} dq \right)^{2M}$$

(5)

where $\overline{A}_M$ is TFP in industrialized manufacturing and $0 < \alpha_M < 1$. The returns-to-scale properties in this production function are the same as those in (3).

Finally, a pre-industrial competitive fringe produces intermediate products with the technology $f^n_I (n_I) = A_I n_I$, where $A_I$ is pre-industrial TFP. In the pre-industrial stage the range of intermediate products that can be produced is restricted to $Q < Q$, which reflects the fact that the more advanced production of intermediate products requires industrialized technology. A classical example is the use of steam engines for pumping water in coal mines. The payment of $F_I$ provides access to the industrialized production $f^n_I (n_I) = \overline{A}_I n_I$, where $\overline{A}_I$ reflects the increased productivity. Under the new technology the range of intermediate inputs $Q$ is unrestricted. Note that before industrialization no intermediate products are needed in final goods production and thus $Q_{pre} = 0$. As in agriculture and manufacturing, the price of intermediate inputs is restricted by the price at which the competitive fringe can produce.

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8This fact alone is not sufficient to explain the relatively high autocorrelation of wages ($\theta = 0.62$) observed in the data between 1600 and 1780 in England. As we will argue later, $\theta$ picks up the overall stickiness related a shock to agriculture.

9This is a somewhat extreme representation of the fact that industrialized agriculture can react better to weather events such as droughts. However, none of our results (except for the post-industrialized variation of output) depend on this assumption.

To see this, note that if prices are the same for each intermediate input, $x_i = \overline{x}$, $\forall i$, and thus $3$ becomes $f^n_A (\cdot) = \overline{A}_A l^{\gamma_{post}(1-\alpha_A)(1-\gamma_{post})} \left( Q_A \overline{A}^{\alpha_A(1-\gamma_{post})} Q_A^{1/\beta} \right)^{2A(1-\gamma_{post})}$. For a given total amount of intermediate inputs, $Q_A \overline{x}$, and given land and labor, we then have increasing returns to the variety $Q_A$, which is due to the decreasing returns to each individual intermediate input $x_i$. 

9This is a somewhat extreme representation of the fact that industrialized agriculture can react better to weather events such as droughts. However, none of our results (except for the post-industrialized variation of output) depend on this assumption.
In our model, industrialization means the wide-spread transition from a largely labor-input based production towards a higher-productivity production that uses a variety of intermediate inputs. Historically, the up-front costs necessary to access the new technology are related to the purchase of engines (e.g., steam or spinning) and the erection of the necessary infrastructure, e.g., buildings (Feinstein and Pollard 1988, v.Tunzelmann 1978, Koehn 1995). Intermediate inputs (e.g., coal, cotton or fertilizer) were then used together with labor (and land) to fabricate final products. Note that there is no capital in the production functions. In the pre-industrial period, this assumption is realistic, as manpower was by far the most important factor of production. In industrialized production, we can think of the up-front costs as being the capital stock. In this interpretation, firms must erect all production facilities before starting production. Moreover, each firm size requires a corresponding up-front investment. Returns to scale, broadly defined (including capital), thus depend on the relationship between up-front costs and firm size. Similarly, returns to capital depend on the form of productivity advance as a function of up-front costs. If this is a convex (concave) function, we have decreasing (increasing) returns to capital.

From now on, we refer to a division $J$ as a collection of similar sectors, i.e., the collection of all agricultural sectors $j = A, \forall j \in J_A$, all manufacturing sectors $j = M, \forall j \in J_M$, and all intermediate sectors $q = I, \forall q \in J_I \equiv Q$. We assume that within each division, the available technology is identical across sectors $j$. Within divisions, sectors then differ with respect to two properties: First, the status of industrialization determines whether there is a competitive fringe or one monopolistic firm. This will be represented in the following by $J_{ind}$ and $J_{nonind}$, respectively. Second, the number of people demanding the sector’s product determines whether it produces for all people or for the rich only. This demand-related property will be represented by $\omega = \{rp, r\}$, denoting rich-poor and rich-only demand, respectively.

### 3.3 Allocation of Labor and Factor Payments

The assumption that all firms in a division have access to the same technology simplifies our analysis. We can then reduce the continuum of producers in each division $J$ to four types of representative firms: There are competitive non-industrialized firms producing for rich and poor (supplying $y^{s,rp}_{j,nonind}$) and producing for rich only ($y^{s,r}_{j,nonind}$). In addition, we (potentially) have monopolistic industrialized firms, producing for all consumers ($y^{s,rp}_{j,ind}$) and others supplying the rich only ($y^{s,r}_{j,ind}$). Figure 1 illustrates the divisions in the economy.\(^1\)

We normalize the price of agricultural goods to unity. Factor prices for labor and land are then determined in non-industrialized agriculture for the following reasons: First, provided that the condition $1/c_{l,t} \geq p_A/p_M$ holds, agents consume all agricultural products along the hierarchy of preferences before going on to the first manufacturing good. Perfect competition in non-industrialized production then implies that agricultural firms pay their workers the marginal product of labor and manufacturing

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\(^{11}\)Figure 1 is showing a simplified case where the line of industrialized sectors is not interrupted by pre-industrial sectors. This however, can be the case since industrialization in our model does not necessarily proceed along the hierarchy of preferences.
firms cannot overbid this wage since $1/c_{i,t} \geq p_A/p_M$. Underbidding is not feasible, either, since then manufacturing workers would shift to agriculture. Second, industrialized firms have no incentive to pay wages higher then those in the competitive fringe. This holds even if the whole economy is industrialized, which becomes clear if we think of the competitive fringe as the backup-home-production sector that is used in case that agents find no other employment. Then the wage paid in non-industrialized agriculture is also the reservation wage of agents. Due to equivalent factor prices for rich-poor and rich-only production and constant returns to scale (provided that all final firms use the full amount of available intermediate inputs, $Q$), goods prices are independent of the production scale, i.e., $p^\omega_j = p_j, \omega = \{rp, r\}$.

There are three types of inputs in this economy: labor, intermediate inputs and land (used in agriculture only). Labor is used in all sectors, but within the divisions $J_A$ or $J_M$ there may be different labor allocations across sectors, $n^a_j$ or $n_j^r$, and $n^{a,ind}_j$ or $n^{r,ind}_j$, depending on whether the sector is producing for all consumers or for rich people only, and depending on its state of industrialization. In the intermediate sector labor demand is $n_{J_I}$ or $n^{I,ind}_{J_I}$. Labor is also needed to build infrastructure and machines as sectors industrialize. This type of labor is paid for with the up-front costs, and we have $n_F = F_t/w_t$, where $F_t$ is the total up-front cost paid in period $t$. Finally, the capital stock (i.e., the up-front-costs paid) of industrialized sectors depreciates at $\delta$. Repairing the capital stock requires labor $n_\delta = \delta K_t/w_t$, where $K_t$ is the total capital stock in the economy, which corresponds to the sum of all up-front payments up to $t$ in sectors that are still industrialized in $t$. All labor $N$ is used, which yields the resource constraint

$$\sum_{J=(J_A, J_M, J_A, J_M, J_A, J_M)} \sum_{\omega=(rp, r)} n^{a_j}_j m^\omega_j + \sum_{J=(J_I, J_I, ind)} n_j m_j + n_F + n_\delta = N \quad (6)$$

where $m_J$ is the mass of sectors in $J$. As discussed above, firms have increasing returns with respect to intermediate product variety. This, together with the assumption

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**Figure 1:** An example for the model economy where $c_p > 1$. Some agriculture and some manufacturing are industrialized.
that all intermediate firms (and thus prices of intermediate inputs) are identical, implies that industrialized final producers (i) use the whole variety of available intermediate products, that is, \( Q_A = Q_M = Q \) and (ii) use identical amounts of each intermediate input, i.e., \( x^{rp}_{q,J} = x^{rp}_{J} \) and \( x^{r}_{q,J,t} = x^{r}_{J,t}, \forall q \in Q \) for \( J = \{J_{A,ind}, J_{M,ind}\} \). The fact that all intermediate inputs must be used yields the constraint

\[
\sum_{J=\{J_{A,ind}, J_{M,ind}\}} \sum_{\omega=\{rp,r\}} x^{\omega}_{J} m^{\omega}_{J} = y_{q}
\]

where \( y_{q} \) is the output of intermediate sector \( q \), and the condition must hold for all \( q \in Q \). If \( c_p < 1 \), some agricultural sectors produce for the rich only. In this case the land rental rates in rich-poor and rich-only agriculture must equalized, which determines the size of land used in the respective sectors, \( l^{rp} \) and \( l^{r} \). If an agricultural sector industrializes, it uses all the land available to the competitive fringe in the sector, that is, if a rich-poor sector decides to industrialize it has \( l^{rp} > l^{r} \) at its disposal. Industrializing firms thus take land as exogenously given. All available fertile land \( L \) is used in agriculture, which requires

\[
\sum_{J=\{J_{A,ind}, J_{M,ind}\}} \sum_{\omega=\{rp,r\}} l^{\omega}_{J} m^{\omega}_{J} = L
\]

### 3.4 Timing

To keep matters simple, we assume that there is no financial intermediation for poor people. Poor agents consume all their income \( I_{p,t} \) in each period as in equation (1). Rich agents provide a maximum share \( s_r \) of their income \( I_{r,t} \) to finance up-front investments whenever an entrepreneur promises them an expected return \( r_{K,j} \geq r_K \) for project \( j \), i.e., for the industrialization of a firm in sector \( j \). Consequently, the maximum available funds in each period are \( s_r I_{r,t} \). If no industrialization project promises a return that exceeds \( r_K \), the rich do not invest and consume all their income. Similarly, if only a few projects are regarded worthwhile investing by the rich, they invest a share smaller than \( s_r \) of their income and consume the remainder. The fact that industrialization in Europe was initially funded mainly by rich individuals is described, among others, in Feinstein and Pollard (1988) and DaRin and Hellmann (2002). Moreover, Crouzet (1965) argues that

"the capital which made possible the creation of large scale 'factory' industries came . . . mainly from industry itself . . . the simple answer to this question how industrial expansion was financed is the overwhelming predominance of self-finance"

In each final sector \( j \), and in each intermediate sector \( q \in Q \) there is initially a competitive fringe producing, and within each fringe there is a potential entrepreneur. In each period potential entrepreneurs observe the demand for their sector’s product, following the shock to agricultural productivity. However, before undertaking a project,
the duration $\theta$ of shocks is only known with uncertainty. Optimists believe that a positive shock lasts longer while a negative shock will be over sooner. There is therefore heterogeneity of beliefs at any one point in time amongst agents about the duration of shocks.\textsuperscript{13} This attitude means that they expect higher returns to industrialization projects in their sector. The contrary applies to pessimists. That is, heterogeneous entrepreneurs expect a range of returns $r_{subj}^{K,j} = r_{obj}^{K,j} \pm 4r_{K,j}$, where the objectively expected return $r_{obj}^{K,j}$ is calculated using the true value of $\theta$. An entrepreneur in a sector needs to borrow money from the rich to finance the up-front costs. If the subjectively expected return to an industrialization project exceeds the rich’s reservation value, i.e., $r_{subj}^{K,j} \geq r_{K}$, the entrepreneur contacts the rich for lending. The rich have no a-priori information about projects; they have to believe in the entrepreneur’s promised return $r_{subj}^{K,j}$. In return for financing and letting him manage the project, the entrepreneur gives the rich all property rights of the monopolistic firm that he creates. Once a project is realized, the entrepreneur learns the actual duration $\theta$ of shocks and can thus calculate the objectively expected return $r_{obj}^{K,j}$ of his project.\textsuperscript{14} In all subsequent periods he reports $r_{obj}^{K,j}$ to the lenders, and if the objectively expected return falls short of $r_{K}$ for a consecutive number of periods $t > T_l$ (denoting lenders’ leniency), his efforts are considered as failed and the lenders stop the project, sending the sector back to its pre-industrial state.\textsuperscript{15} For sectors that fall back to pre-IR, the game with the potential entrepreneurs starts again. We use the variable $B_{j,t}$ to track the state of industrialization of all final and intermediate sectors, where $B_{j,t} = 1$ if sector $j$ is industrialized and zero otherwise. Following the above discussion, the law of motion for $B_{j,t}$ is

$$B_{j,t+1} = B_{j,t} + dB_{j,t}, \text{ where}$$

$$dB_{j,t} = \begin{cases} 
0 & \text{if } B_{j,t} = 0 \text{ and } r_{K,j,t} < \Sigma K \\
1 & \text{if } B_{j,t} = 0 \text{ and } r_{K,j,t} \geq \Sigma K \\
-1 & \text{if } B_{j,t} = 1 \text{ and } r_{K,j,t} < \Sigma K, \forall \tilde{t} \in [t - T_l, t] \\
0 & \text{if } B_{j,t} = 1 \text{ and } r_{K,j,t} \geq \Sigma K \text{ for some } \tilde{t} \in [t - T_l, t] 
\end{cases}$$

3.5 Equilibrium

Let $y_{j,t}^\omega$ denote the output of sector $j$ producing for $\omega \in \{rp, r\}$. We have now completed the setup of the model and can define the equilibrium.

\textbf{Definition 1} A static equilibrium in period $t$ is a collection of allocations, prices, profits, and up-front costs of industrialization project \((c_{i,t}, l_{i,t}, n_{j,t}^F, n_{j,t}^A, n_{j,t}, \lambda_{j,t}^F, \lambda_{j,t}^A, y_{j,t}^\omega, p_{j,t}, x_{j,t}^\omega, P_{j,t}, T_{j,t}, \Pi_{j,t})\) for $i = \{r, p\}, \omega \in \{rp, r\}, j \in \{J_A,t, J_A,ind,t, J_M,t, J_M,ind,t\}$ and $q \in Q_t$ such that, given the state variables $A_{A,t}, Q_t$ and $B_{j,t}$,

\textsuperscript{13}In this regard, our model is similar in spirit to Mankiw and Reis (2003).
\textsuperscript{14}The assumption of rapid entrepreneurial learning is not crucial for our results, and solely serves to simplify the model.
\textsuperscript{15}This assumption is not only realistic but also necessary in order to avoid one-time extremely negative shocks to end industrialization of a large proportion of the economy.
(i) the choice variable $c_{i,t}$ fulfills equation (1), (ii) final goods markets clear, i.e., $y_{d,ω}^{t} = y_{s,ω}^{t} = y_{ω}^{t}$, ∀$j,ω$, (iii) agents are indifferent between working in agriculture, manufacturing or in the intermediate sector, i.e., $w_{j,t} = w_{j}$, ∀$j$, (iv) interest rates in rich-poor and rich-only production must equalize, that is, $r^{r}_{r} = r^{r}$, (v) final producers use the full variety of intermediate inputs, i.e., $Q_{ω}^{t} = Q^{t}$, ∀$j,ω$, (vi) intermediate sectors are symmetric, that is, $p_{q,t} = p_{I,t}$ and $x_{ω}^{q,j,t} = x_{ω}^{I,j,t}$ ∀$q \in Q^{t}$ and (vii) the resource constraints (6-8) are satisfied.

Definition 2 A dynamic equilibrium is a sequence of static equilibria for $t=0,1,2,...$, together with sequences for $\{ A_{A,t}, Q_{t}, B_{j,t} \}_{t=0}^{∞}$, such that, given an exogenous sequence of shocks to non-industrialized agriculture $\{ ε_{t} \}_{t=0}^{∞}$ and given the initial conditions $A_{A,0}$ and $B_{j,0}$, the evolution of the economy satisfies the law of motion in equation (9) and the constraints $A_{A,t} ≥ 0$ and $Q_{t} ≥ 0$ in all periods $t$.

We solve the model by simulation, as described in Appendix A.1. In the following we outline the calibration of the model for the industrial revolution in England from 1780 to 1850.

3.6 Calibration

We normalize population, $N = 1$, and choose land $L = 5$ such that its rental rate is 6%. The share of poor is set to $ρ = 85%$. This definition follows from the revised figures from Massie, as described in Lindert and Williamson (1982), where we include amongst the "poor" everyone with an income of 40 pounds or less – giving 86% of all families in 1759, with an average income of sterling 22.7. To reflect inequality in profit distribution we set $τ_{p} = 0.2$ for the distribution of land rents and profits from intermediate producers and manufacturing. These sectors were mainly in the hand of people falling into the rich category, according to our definition. The corresponding coefficient for the rich is then $τ_{r} = 5.53$. These numbers mean that, as compared to an average agent, a rich person, receives about five times more profits while a poor agent receives about one fifth. We choose $τ_{p} > 0$ to reflect the redistribution from rich to poor in England’s Speenhamland system. The magnitude is reasonable, considering that with this calibration our model gives the income of an average rich person as about 2.5 times the income of an average poor person in 1780. Considering the distribution of profits from industrialized agriculture, we choose a more equal distribution of profits, that is, $τ_{p}^{A} = τ_{r}^{A} = 1$. The reason for this choice of parameters is that agriculture was run mainly by poor people who profited when agriculture became more productive. In the setup of our model, however, wages are determined by the non-industrialized competitive fringe in agriculture, so that it does not rise in the progress of industrialization. Therefore, we use the distribution of profits in industrialized agriculture to represent the rise in poor people’s income during the industrial revolution. For example, Feinstein’s (1998) estimates suggest that wages increased by 35% between 1780 and 1850.

16As we also show later, the resulting inequality in consumption produced by our model matches the data well.

17For the period 1760 to 1850, the increase was only 24% (Voth 2003).
Pre-industrial TFP in agriculture has a direct effect on income and, due to the inequality between rich and poor, on the share of total manufacturing output sold to these two classes of people. This mechanism works as follows: if pre-industrial agricultural productivity rises, wages rise, which benefits the poor relatively more than the rich, as their income is mainly composed of wage. If the poor have already finished all food consumption (i.e., $c_p > 1$), a further rise of income makes the large number of poor consume more manufacturing output. Thus, the share of manufacturing products sold to the poor rises. Crafts (1985) gives the percentage of industrial purchases to workers as about 40% in 1760 and 26% in 1780. We calibrate pre-industrial TFP in agriculture such that about 30% of industrial purchases are made by the poor. The 30% consumption share of the poor imply $c_p \approx 1.1$ in 1780. We then derive pre-industrial TFP in manufacturing such that for $c_p = 1$: $p_M = (w/A_M) = p_A$. This ensures that the condition $1/c_{i,t} \geq p_A/p_M$ always holds. We choose initial TFP in pre-industrial intermediate sectors such that it equals TFP in manufacturing.

According to Crafts (2004b, table 1), British labor productivity grew at an average rate of 1.11% per year during the period 1780-1850. Crafts also shows that this growth occurred almost solely in agriculture and modern sectors (cottons, woollens, iron, canals, railways and ships). The share of these sectors in domestic value added was about 60% in 1841 (calculated from the input-output table in Horrell et.al. 1994). Assuming that all other sectors (services, housing, public, and defense) did not contribute to labor productivity growth, we find that output per worker in agriculture and modern sectors must have grown at an average annual rate of 1.32%, or a factor of about 2.5 between 1780 and 1850.18 We then use this factor to calculate TFP growth in agriculture and modern sectors as described in Appendix A.2.

To derive the magnitude and persistence of shocks in the agricultural sector, we use real wage data for England 1600-1780 from Wrigley and Schofield (1997). The underlying assumption when working with wages rather than output is a strong positive correlation of wages with agricultural productivity before 1780. With fixed labor supply and agriculture the dominant sector, productivity shocks will have an immediate knock-on effect on real wages in the economy. This is especially true since wages were largely fixed in nominal terms, and most of the variation in the Phelps-Brown/Hopkins wage series arises from changes in agricultural prices (Wrigley and Schofield 1997). Figure 2 shows the real wage index and its trend.19 From this time series we derive the HP-filtered series of percentage shocks to real wages, i.e., the shocks in the form $\kappa_{t+1} = \theta \kappa_t + \epsilon_t$. We estimate the parameters of this AR(1) process to be $\theta = 0.62$ ($t=10.31$) and $\sigma_\epsilon = 0.073$. Note that the autocorrelation of shocks is certainly not solely related to weather events, as this could not explain a coefficient of over 0.5. Rather, we think of the autocorrelation as originating from frictions within the economy that make shocks keep their influence over several periods.

18There is no agreement in the literature as to whether productivity growth in agriculture was faster, slower or equal to productivity growth in modern sectors. For example, Crafts (1985: 70-89) shows that productivity growth in agriculture was rapid, and in some periods surpassed manufacturing productivity growth. We therefore assume that the growth of labor productivity was broadly speaking the same in modern sectors and agriculture.

19The standard deviation of real wages is very similar to the standard deviation of agricultural output in later years.
However, since pre-industrial agricultural productivity is the only source of fluctuation in our model, we use the above-found autocorrelation to introduce an economy-wide shock.

To calibrate the labor and intermediate input shares of the production functions, we perform some calculations based on the input-output table for 1841 by Horrell et.al. (1994). We explain our methodology in Appendix A.3.

The adoption of steam engine technology is intrinsically related to the industrial revolution. When calibrating up-front costs in our model we thus refer to costs of steam power adoption. Feinstein and Pollard (1982, table 7.6) provide cost figures for steam engines and boilers. Von Tunzelmann (1978, Table 4.2 and Table 4.10) presents figures on additional costs related to engine house, steam pipes, erection and framework for several engine sizes and years between 1795 and 1830. Analyzing these numbers, we find that the ratio of total non-engine cost over the cost of engine and boiler is relatively stable over engine sizes and time at 0.45. We use this number, together with Feinstein and Pollard’s data on the patent premium charged by Watt and Boulton, to calculate total up-front costs of steam engine usage depending on engine sizes. Figure 3 shows our results.

Size and total costs of steam engines are highly correlated, but there is a fixed component in these costs - the intercept is at 337 Pounds. Therefore, average cost per horsepower falls with engine size. To work with these figures in our model we need two further components: First, we derive the average size of steam engines used in England during the industrial revolution from Feinstein and Pollard (1982, table 7.5), who provide estimates of the number of steam engines and total horsepower installed.

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20 This approach is also applied by Stokey (2001).
in various industrial towns, 1800-1850. Engine sizes there vary between 12 and 29hp with a weighted average of about 20hp. We consequently assume that the typical industrialized firm in England used engines of 20hp. Our a-priori belief is that only firms producing for a broad share of population had incentives to industrialize, whereas the rich-only production was in the hands of artisans. Therefore, we assume that an engine of 20hp is needed to shift to industrial production. A sector $j$ that receives demand $N$ can thus be interpreted as producing a product that receives enough regional or interregional demand to make a kick-off worthwhile. Note that under this view goods $j$ can be interpreted to be differentiated by kind and/or region, and $N$ can be seen as a parameter reflecting population density rather than total population. For example, a country like China with a huge population but also a large surface certainly had several regional markets for each product.

Second, we need to fit the Pounds or Pounds per hp into the units of our model. Since up-front costs are not included in the production function (e.g., in terms of capital), we cannot use the numbers from Figure 3 directly. What is represented in our model, however, are material costs of industrialized production, i.e., the cost of intermediate inputs. Appendix A.4 shows the calculation of total material cost in sectors producing for rich and poor, i.e., receiving demand $N$. Having found these figures, we can relate them to fixed costs using the relation of up-front costs to annual material costs in industrial production. Von Tunzelmann (1978, Table 4.11) provides data on up-front costs and annual material cost for 10hp and 30hp steam engines between 1795 and 1835. We use these figures to compute fixed cost and material cost for a hypothetical 20hp engine around 1800 and find the ratio up-front costs over annual material cost.
\[ \phi = 8. \] Stokey’s (2001) figures suggest a similar ratio of about 8 for 30hp engines. Crafts (2004b) uses the rate of growth of horsepower to calculate the growth rate of the capital stock. While such an approach works well in a growth accounting exercise, it cannot be applied in our approach, since we explicitly need cost figures. Having found \( \phi \), we can calculate up-front costs for a 20hp engine in the three divisions as

\[ F_J(20\text{hp}) = \phi \cdot TMC_J(20\text{hp}) \]  

where \( TMC_J \) denotes total material costs in division \( J \). If a sector producing for the rich only decided to industrialize, it would not want to use engines as large as those used in sectors producing for rich and poor, since its demand is lower. On the other hand, a sector in another country, supplying more people than the rich-poor English sectors, would optimally buy larger engines when industrializing. In order to calculate up-front costs for differently sized engines we run a regression of total cost on engine size, \( \text{Cost}(\text{hp}) = a_0 + a_1 \cdot \text{hp} \), using the data presented in Figure 3. We find \( a_0 = 337 \) and \( a_1 = 68.7 \) with \( R^2=0.99 \). Using these estimates and the known value \( F_J(20\text{hp}) \), we can calculate \( F_J \) for different engine sizes.

There are at least two potential biases in our method of calibrating up-front costs: (1) The annual cost of intermediate inputs may overestimate the annual material cost for steam engines as defined in v. Tunzelmann (1978). For example, intermediate inputs may include products that are not used to run engines but merely to be processed in the engines. This upward bias would overestimate up-front costs. (2) The cost of purchasing and installing steam engines may underestimate real up-front costs of industrializing. For example, industrialization may require the complete change of production logistics and infrastructure, leading to large costs in addition to purely technological costs. Analyzing whether the former or the latter effect is dominating goes far beyond the scope of this paper and we leave it open for future research, taking our approach as the current best guess for finding up-front costs of industrialization.

Due to its lower usage of intermediate inputs, up-front costs in agriculture are lower than up-front costs in industry. On the other hand, because of the unfavorable weight-value ratio of most agricultural commodities, few products from the primary sector could be marketed throughout the country. We assume that effective market size for agriculture is smaller by a scale factor of \( v_A \). There are at least two reasons to believe that \( v_A > 1 \) is reasonable: First, agricultural products are generally less diversified than manufacturing products, so that for the same number of people demanding a product, one expects to have relatively more firms in agriculture supplying this product. Second, the weight-value ratio is higher for agricultural product, making them relatively more expensive to transport. Thus, one would expect agricultural markets to be more regionally bounded, and thus smaller, than markets for manufacturing products. Our results are not sensitive to the precise value for \( v_A \) chosen; we run the calibration with \( v_A = 3 \), indicating that the market for a manufacturing firm is about three times larger than for an agricultural firm.

Our calibration of up-front costs yields the following results: \( F_A = 0.63 \), \( F_M = 2.01 \), and \( F_I = 0.21 \). We have \( F_M \approx v_A F_A \), due to the different market sizes.\footnote{This relation holds only approximately since the intermediate input share are different in agriculture and manufacturing and because up-front costs per hp are a non-linear function of size (see Figure 3).}
The up-front costs for the intermediate division are relatively low due to the smaller scale of production in each intermediate sector and the small (hypothetical) share of intermediate inputs in intermediate goods production. This also has the implication that agriculture never industrializes without manufacturing having made the transition.

In order to reflect entrepreneurial heterogeneity with respect to the rate of return that they expect for industrialization projects, we use the following figures: $\Delta r_{K,M} = \pm 5\% \text{ and } \Delta r_{K,A} = \Delta r_{K,I} \pm 2.5\%$. We draw the subjectively expected return of an entrepreneur in division $J$, $r_{K,J}^{subj}$, using a uniform distribution over the interval $[r_{K,J}^{obj} - \Delta r_{K,J}, r_{K,J}^{obj} + \Delta r_{K,J}]$. Recall from the discussion above that the heterogeneity of entrepreneurs originates from different expectation regarding the duration of shocks. The influence of such shocks is larger in manufacturing than in agriculture or intermediate sectors. The reason for this is that a positive shock raises poor people’s income, such that the poor demand a wider range of manufacturing products. But once the positive shock is over, demand drops back immediately. Now imagine being an entrepreneur in one of the sectors that had only demand from the rich before the shock and receive demand $N$ during the positive shock. As long as demand is high, industrialized production is highly profitable, but it is less so once the shock is over. Thus, the expectation of shock duration is crucial in manufacturing. Plugging $\Delta r_{K,M} = \pm 5\%$ into our model, we find that it is roughly equivalent to expecting an autocorrelation of 0.8 for positive shocks and 0.4 for negative shocks (where 0.62 is the correct value). This 25% deviation from the objective duration of shocks seems reasonable.

In agriculture, on the other hand, demand is basically always $N$, since the simulation starts out at $c_P > 1$ (see the above discussion on the calibration of pre-industrial TFP in agriculture). Therefore, the duration of the shock is less decisive than in manufacturing, and we choose a smaller heterogeneity for agricultural entrepreneurs. We still allow for some heterogeneity to reflect other influences like different access to, or special tastes for new technology (driving the subjective return up or down). The same arguments apply to the intermediate sector, where a shock also has a relatively smaller influence as compared to manufacturing. The demand for intermediate products is determined by the state of industrialization in, and the scale of demand for, agriculture and manufacturing products. Since the state of industrialization does not react immediately to shocks (recall the discussion regarding leniency of lenders), the expected duration is not as crucial as in manufacturing.

In order to reflect lenders’ leniency, we choose a maximum of $T_l = 3$ consecutive periods during which the entrepreneur’s expected return can be below the minimum required return $E_K$. Finally, we choose the range of intermediate products that can be produced with pre-industrial technology as $Q = 0.1$. This is a guess with some underlying intuition: as explained above, in 1780 our model is calibrated such that $c_p \approx 1.1$. As a consequence of receiving rich and poor demand, it is already profitable for the range $(1, 1.1]$ of manufacturing sectors to industrialize, although intermediate inputs are not yet produced industrially. We could think of these firms as already being organized in larger units (e.g., manufactories) but not yet using steam power to run their engines (e.g., using water mills instead). If a range of size 0.1 is producing final products with a pretty much industrial technology, a reasonable thing to assume is that a similar range of intermediate firms is able to produce intermediate inputs for the final
production. We therefore choose $Q = 0.1$. In order to find the maximum share of income that rich people are willing to invest, $s_r$, we use the aggregate saving rate $s$. According to Crafts (1985, p. 95), the average saving rate 1700-1850 was 0.2. The corresponding investments, however, went into old and new technology, and what we need in our model are investments in the latter, only. Feinstein (1978) showed that net capital formation increased from 7 to 14 percent of GDP. It is reasonable to assume that the increase in capital formation reflect the investment in modern technology. Using a conservative estimate, we assume that roughly 5% of GDP was used in capital formation of industrializing sectors by the end of the period, which gives an industrialization-specific investment rate of $s = 5\%$. Recall that only rich people invest in our model. Depending on the share and relative income of poor and rich people, the overall saving rate of 5% translates into a rate $s_r \approx 15\%$. All other parameters used in the model are fairly standard, and are given in Table 3 together with the calibration we discussed above.

**TABLE 3: Parameter Values**

<table>
<thead>
<tr>
<th>Population and income distribution</th>
<th>$N = 1$</th>
<th>$L = 5$</th>
<th>$\rho = 0.85$</th>
<th>$\tau_p = 0.2$</th>
<th>$\tau_A = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>$\Delta_A = 0.82$</td>
<td>$\Delta_M = 0.94$</td>
<td>$\Delta_f = 0.94$</td>
<td>$\bar{A}_A = 2.25$</td>
<td>$\bar{A}_M = 4.36$</td>
</tr>
<tr>
<td>Shock to $\Delta_A$</td>
<td>$\theta = 0.62$</td>
<td>$\sigma_z = 0.073$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor Shares</td>
<td>$\gamma^{pre} = 0.3$</td>
<td>$\gamma^{post} = 0.3$</td>
<td>$\alpha_A = 0.36$</td>
<td>$\alpha_M = 0.44$</td>
<td>$\alpha_I = 0.27$</td>
</tr>
<tr>
<td>Up-front costs</td>
<td>$F_A = 0.63$</td>
<td>$F_M = 2.01$</td>
<td>$F_A = 0.21$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrepreneurial Heterogeneity</td>
<td>$\Delta r_{K,A} \pm 2.5%$</td>
<td>$\Delta r_{K,M} = \pm 5%$</td>
<td>$\Delta r_{K,I} = \pm 2.5%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other parameters</td>
<td>$\delta = 0.05$</td>
<td>$s = 0.05$</td>
<td>$\beta = 0.8$</td>
<td>$Q = 0.1$</td>
<td>$\nu_A = 3$</td>
</tr>
</tbody>
</table>

**4 Results**

Industrialization in our model is interpreted as the widespread adoption of high-productivity technology that depends on intermediate inputs. This process follows various steps. Initially the economy is in its pre-industrial state with agriculture and intermediate input production occurring in the low-productivity competitive fringe. For England, our calibration yields $c_p \approx 1.1$ in 1780. Thus, some manufacturing firms receive demand from rich and poor people, which makes it worthwhile for them to produce on a larger scale.\textsuperscript{22} In the language of our model, this means that they sink the up-front costs to become monopolists in their sectors and gain access to the technology from equation (5). However, at this stage intermediate inputs are not yet produced industrially, so

\textsuperscript{22}Our calibration of up-front costs yields that for these manufacturing firms the expected return of industrialization is large enough to kick off.
that only a small range $Q$ is available, and intermediate inputs are expensive. Historically, these early firms producing at a larger scale can be seen as manufactories run with water mills and slowly substituting steam power for water. Figures for coal prices in v.Tunzelmann (1978) support the fact that the price of intermediate inputs dropped significantly between 1780 and 1850.

If enough manufacturing firms demand intermediate products, it becomes worthwhile for the intermediate sector to industrialize. As shown in Appendix A.1, the price of intermediate inputs then drops by about 50%. Moreover, it becomes worthwhile for more intermediate firms to enter the market such that $Q$ grows beyond $Q_*$. These changes in the supply of intermediate inputs make industrialization worthwhile in agriculture, as well as for further manufacturing firms. This, in turn, creates higher demand for intermediate inputs, which results in a further extension of $Q$. This process goes on throughout the transition from the pre- to the post-industrial economy. In the following we provide a glance at some features of the pre-industrial economy, i.e., at how the world in our model would look like without a kick-off; only being driven by shocks to agriculture. Thereafter, we analyze incentives for industrialization in a pre-industrial world. Finally, we turn to industrialization itself and examine results of our model regarding the transition period.

4.1 The pre-industrial economy

Suppose that modern industrial technology had not been available in 1780, then only the shock to agricultural TFP would have driven income and consumption, without ever initiating a kick-off. In the following we analyze this hypothetical economy. The upper left panel of Figure 4 shows income $I_\omega$ and consumption $c_\omega$ of rich and poor people as a result of shocks to agricultural TFP. A remarkable feature is that although the rich’s income increases as a result of positive shocks, their consumption scale (i.e., the last good that they consume along the hierarchy) decreases. The intuition behind this finding is that as agricultural TFP increases, wages (that are determined in agriculture) increase and thus the cost of producing manufacturing goods rises. Since the largest share of rich people’s consumption are manufacturing goods, rising prices of these goods offset the higher wages. The poor, having the largest part of consumption in agriculture, are less affected by rising prices of manufacturing products and thus profit from positive shocks.

The upper right part of Figure 4 shows the composition of GDP. Note that for $c_p > 1$, agricultural output stagnates since all agricultural demand is satisfied. The bottom left figure shows the composition of the labor force, which compares well with the data: Crafts (1985, p. 62) suggests that – abstracting from the service sector – 31% of the English male labor force were employed in manufacturing in 1760 (with

$23$ Note that to the right of the $c_p = 1$ line, $c_p$ grows up to the point where the shock is zero. The reason is that some low-$j$ manufacturing sectors (i.e., close to 1) are already performing the monopolistic large-scale production, paying profits to the rich. Profits grow with final demand that in turn depends on poor people’s income, being driven by the shock. The additional income of the rich from profits then is enough to offset the rise in manufacturing prices. Due to our restriction of analyzing the non-kick-off economy, no further manufacturing firms industrialize in the range of positive shocks, so that the price-effect is the determining factor.
linear interpolation, the figure for 1780 is 37%). Finally, the consumption share of manufacturing goods are very sensitive to shocks. Variations in the range of ±20%, as seen in Figure 2, let the consumption shares fluctuate between 0 and 50%. This variation is probably too high but not unreasonable, considering the variating shares in Table 2 and the fact that an extremely negative shock to the poor’s income may well constrain them to food consumption in that period.

Using the numbers from Table 2, we can perform a plausibility-check of our calibration with respect to inequality in 1780, i.e., the consumption of the rich relative to the poor. When using $\rho = 0.85$ for the share of the poor, as used in our calibration, the manufacturing consumption of a representative rich person relative to a poor person was about 16, according to the data. In our model, we have $c^p_{1780} = 1.1$ and $c^r_{1780} = 2.58$. Since manufacturing goods have index $j > 1$, the relative manufacturing consumption of rich to poor is $1.58/0.1 = 15.8$. This close similarity to the data is certainly a coincidence; but since it is not a direct result of the calibration, it provides a check for inequality in the model.
4.2 Incentives for Industrialization

We saw above that following a positive shock the range of products consumed by rich and poor ($c_p$) increases. But is this sufficient to induce the kick-off of firms? We analyze this question in the following, still sticking to the pre-industrial economy. The left panel of Figure 5 visualizes the expected returns of industrialization projects under full information about the duration of shocks. Strictly speaking, we analyze in each division the (objectively) expected returns of the sector that is awaited to industrialize next. These are the lowest-$j$ agricultural sector, an arbitrary intermediate sector, and the lowest-$j$ manufacturing sector that still has the competitive fringe. We see that for manufacturing expected returns are negative for negative shocks and rise sharply as shocks become positive. The reason for this observation is that industrialization of manufacturing is profitable only if rich and poor people demand the product. But for the sector we look at, this is only the case for positive shocks. The more positive the shock, the longer the time-span over which the objective entrepreneur expects a high demand, and the higher is his expected return. The demand-profit relation can also be seen in the right panel of Figure 5. The distance between price and marginal cost of manufacturing production is not changing much with the shock. Thus, the decisive point in order to recoup up-front costs must be the amount of output sold.

For agriculture, things look different. The expected return declines with a more positive shock. The reason for this becomes obvious when looking at prices and variable costs of agricultural goods: While $p_A$ does not change (numeraire), the variable costs of industrialized agriculture increase. Intuitively, this follows because wages are determined in non-industrial agriculture so that they rise with a positive shock, thus affecting the labor cost of industrialized agriculture. An alternative explanation is that the shock affects only pre-industrial agriculture, while leaving unaffected industrialized production (recall the intuition we provided above for industrialized agriculture being more capable to cope with weather events). Thus, the relative advantage of industrialized over non-industrialized agriculture diminishes with more positive shocks. The decisive event pushing expected returns of agriculture upwards is not visible in Figure 5 - the drop in intermediate input prices following the industrialization of the intermediate sector. This feature will be analyzed in the next section.

Finally, an intermediate sector charges the mark-up $\frac{1}{\beta}$ over its marginal cost.24 Thus, as was the case for manufacturing, demand is the driving force for expected returns. However, expected returns in the intermediate sector are less volatile than those in manufacturing. The explanation is that intermediate sectors’ demand does not depend directly on the volatile range of poor people’s consumption, but on the usage of intermediate products in final production, and thus on the extent of industrialization in final sectors. This fact becomes obvious in the left panel of Figure 5, where intermediate profits have a hump just to the left of a zero-shock. This is the range where a negative shock takes away poor people’s final demand from manufacturing, such that the monopolistically producing range ($1 < j \lesssim 1.1$) supplies the rich only, therefore demanding less intermediate inputs. If the shock is negative enough (around -15%), the poor stop all manufacturing consumption, and intermediate entrepreneurs expect

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24In Figure 5 we list the price that a monopolistic intermediate producer would charge, not the (higher) price of the competitive fringe.
them to return to manufacturing demand only in the far future. An even more negative shock then matters less for expected profits of intermediate producers.

Figure 5: Incentives for industrialization

Looking again at the left panel of Figure 5, we see that a 30% shock pushes the (objectively) expected return of industrialization in manufacturing above the lenders’ constraint $r_K = 10\%$. However, due to the entrepreneurial heterogeneity, it is not compulsory that the expected return passes this level. In fact, extremely optimistic entrepreneurs in manufacturing may already decide to industrialize their sector at objective returns of 5%. Figure 5 shows that this threshold is already passed for shocks smaller than 10%. On the other hand, if in a sector that is expected to kick off next, the entrepreneur is pessimistic, he may require objective returns of up to 15%. This level is never reached within a reasonable range of shocks. Whether or not an industrialization occurs thus depends on several factors: (i) a sufficiently positive shock to raise expected returns in manufacturing sectors that are awaited to kick off (i.e., the lowest-$j$ sectors that are not yet industrialized); (ii) a sufficiently optimistic attitude of entrepreneurs in these sectors; (iii) a sufficiently long duration of the positive shock such that enough manufacturing sectors industrialize, creating the demand for the intermediate sector; (iv) an optimistic attitude of entrepreneurs in the intermediate sector at the point in time when demand for intermediate products is high.

4.3 Industrialization

If the previously mentioned four conditions are fulfilled, the economy goes through a period of industrialization. Our simulations suggest that there may be failed kick-offs if a positive shock is followed by a long period of negative shocks. If however, industrialization reaches the agricultural sector, the process continues even during a series of negative shocks. The reason is that, as explained above, the returns of industrialization in agriculture increase with negative shocks to pre-industrial agriculture. In the following we analyze a typical kick-off by imposing a large enough shock over some periods and then letting the shock be zero for the remaining time. Figure 6 shows some features of this ”deterministic” industrialization. In the upper left panel, a positive shock
This process continues until the post-industrialization equilibrium is reached after approximately half a century. This time-span may seem short compared to the time-span 1780-1850 usually quoted for England’s industrialization. It should be noted, however, that there are no failed kick-offs in this ‘deterministic’ simulation, so that all projects are successful, leading to a fast progress. Moreover, our model only considers the first adoption of modern technology and not its improvement, and there are no constraints in the model with respect to technology availability. Introducing such frictions would enlarge the time-span needed for full industrialization.

The upper right panel of Figure 6 shows that industrialization spreads relatively smoothly in agriculture and manufacturing, whereas it is a more uneven process for intermediate sectors. Since the size of the industrialized intermediate division is endogenous, we derive its optimal extent of industrialization as follows: Whenever it is profitable for the intermediate division to increase its extent of industrialization, the model calculates the new range such that the expected returns of each intermediate firm are 10%. Thus, whenever the intermediate division decides to industrialize, the extent of this step depends on how much more demand for intermediate products has been created by agriculture and manufacturing compared to the last time when the intermediate sector adjusted. This leads to the more uneven industrialization process in the intermediate division.

Looking at the lower left part of Figure 6 reveals that the inequality between rich and poor increases throughout the industrial revolution. Recent evidence presented by Feinstein (1998) suggests that the gains from industrialization primarily accrued to capital owners. The lower right panel shows the actual return of industrialized firms producing for rich and poor. Note that the returns are very smooth, which is due to the fact that there are no shocks in this analysis. Therefore, the presented returns are a best-case scenario, without demand ever dropping to rich-only for industrialized firms. Manufacturing profits are higher than the others, around 25% as compared to 10%. But this is justified because manufacturing firms are much more affected by shocks - so in the best-case scenario they require higher returns. In fact, when we introduce shocks, the average profits of manufacturing firms are the same as those of the other sectors. The returns of agriculture being around 10% is a result of our calibration of the market size parameter $v_A$, while the returns of the intermediate sector follow from the ongoing adjustment of the range of intermediate production.

The left panel of Figure 7 shows GDP and up-front costs paid during the industrialization. GDP grows by about 50%, which matches the data well: According to Crafts and Harley 1992, and Crafts 1985, per capita output growth is 1760-1800 is 0.2% p.a., 1800-1830 0.5%, 1830-60, 1.1% . This means an increase of 57% for 1760-1850. In the right panel we present the targeted and the actual saving rates of rich investors. The

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25 The more intermediate firms there are, the smaller is the demand for each firm’s output.
26 This was originally argued by Williamson (1985), but his evidence was questioned decisively by Feinstein (1988).
Figure 6: A ‘deterministic’ industrialization

former is the rate at which investors plan their investment at the end of the previous period, the latter is the actual investment rate when up-front costs are paid. The actual rate may exceed the target if investment is planned in a period of a positive shock but the shock in the following period is more negative or due to the approximation procedure in our simulation, as explained in Appendix A.1.\textsuperscript{27} We see that during industrialization, the availability of funds is a binding constraint in each period, at least in the absence of shocks.

4.4 Probabilities of industrialization in various countries

Why did the industrial revolution occur in England first? Could it have been France, Belgium or China? In our model, industrialization occurs stochastically, with the probability of a kick-off depending crucially on the initial income and consumption of the poor. Figure 8 shows the probability of a kick-off occurring in England after 1780,\textsuperscript{26}

\textsuperscript{27}We approximate the continuum of sectors by small intervals of width 0.01. Then, for example, if targeted investment would be sufficient to let a range of 0.0258 sectors industrialize, we round this number, which yields 0.03. Actual investment that must be paid for 0.03 then exceeds the targeted investment. As Figure 7 shows, however, this approximation does not create large deviations.
Given that consumption of rich and poor remains constant at its 1780 level,\(^{28}\) we see that a kick-off occurring within 50 years after 1780 is very likely.

In order to compare the kick-off probability in 1780 England with that in other countries, we would ideally need a cross-section of income, population size, transport cost and income distribution figures. While those available are not of as a high a quality as one might like, we can derive some basic probabilities based on rough estimates. Income data are presented in Table 4.4. We present data on population density, since in the 18th century transport costs were high so that the population that could be reached with reasonable transport effort presumably determined marked size, not merely total

\(^{28}\)If we impose an exogenous increase in per-capita consumption, the marginal probabilities of a kick-off in a certain period grow over time.
population. At a horizon of 100 years, France in 1780 had an industrialization probability of 20%.\textsuperscript{29} This is largely the result of a very steep trade-off between population size and per capita income – the vast majority of the population in France was so poor that access to manufactured goods was out of the question. Lower population density did not help. In the case of China, population size is essentially irrelevant because the average income is so low that industrialization will almost never happen (the probability is less than 1/1000). Despite the very large number of inhabitants, the number of rich is never sufficient to enable the move to advanced manufacturing.

The Poor Laws are not important for England’s higher industrialization probability. With redistribution of 2.5% of GDP, “mass incomes” were bolstered at the expense of the more privileged groups in society. However, without this redistribution, industrialization probabilities at a horizon of 100 years are still essentially 100%, even if the probabilities at shorter horizons are somewhat lower.

<table>
<thead>
<tr>
<th>TABLE 4: Income and population in other countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>England</td>
</tr>
<tr>
<td>France</td>
</tr>
<tr>
<td>China</td>
</tr>
</tbody>
</table>

Sources: Maddison (2001) and CIA (2005) for land area

5 Conclusion

We argue in this paper that combining hierarchical preferences with a big-push model can help us understand why England (and Europe) industrialized first. We show that in an economy with minimum fixed capital requirements, the distribution of income, population size as well as the average per capita income will influence the probability of making the transition from “Malthus to Solow”. If starting conditions are sufficiently favorable, a series of positive shock to agricultural productivity has the potential to start the industrialization process. It may be interrupted – as growth in early modern Europe often was – but the right kind of persistent, benign fluctuations will eventually lead to wide-spread adoption of new manufacturing techniques. This transition was markedly easier in the case of England (and Europe) since starting conditions were unusually favorable. The low-pressure European marriage pattern was crucial in maintaining living standards of the wider population at a level that was sufficiently high for them to start consuming manufactured goods as soon as favorable income shocks occurred. In the English case, the relatively generous Poor Law system also reinforced this effect. We calibrate our model with data for England prior to the classic Industrial Revolution, and show how we can match many of the crucial features isolated by economic historians in recent decades. We also present quantitative results for the ex ante industrialization probabilities of other countries – in this case, France and China – and show that England’s better chances are largely the result of higher per capita income before the

\textsuperscript{29}The corresponding methodology is presented in Appendix A.5.
start of industrialization, which we argue largely reflects more a favorable demographic regime.
APPENDIX

A.1. Simulating the Model

We derive the FOC from the production functions for the optimal input of land, labor, and intermediate inputs in final production. Together with the conditions mentioned in the definition of the equilibria we simulate the model, finding the static equilibrium for each period and using the laws of motion to go the next period. 

A.2. Calibration of Total Factor Productivity Growth

In the following, productivity means output per worker, while total factor productivity (TFP) is simply the $A$ in a production function. As discussed in the section Calibration, labor productivity in agriculture and modern sectors grew at a factor of $\Delta \text{prd} \simeq 2.5$ between 1780 and 1850. In order to find TFP growth we must solve the equation

$$\frac{y_{1850}}{y_{1780}} = \Delta \text{prd}\frac{y_{1850}}{y_{1780}}$$

(A-1)

for agriculture and manufacturing sectors, using the pre-industrial figures for 1780 and the industrialized ones for 1850. In this equation, $y_{1850}$ is value added, which is different from total output if intermediate inputs are used, i.e., in industrial production. Equation (A-1) thus becomes

$$\frac{p_j^{1780} y_j^{1850} - \int_{0}^{Q_{1850}} y_{1850} p_{q,j}^{1850} dq}{n_j^{1850}} = \Delta \text{prd}\frac{y_{1850}}{y_{1780}}$$

(A-2)

where $y_j$ denotes total output of sector $j$. Solving this equation for agriculture yields:

$$\overline{A}_A = \left\{ \left[ A_A \Delta \text{prd} \left( \frac{\gamma_{\text{pre}}}{1 - \gamma_{\text{pre}}} \frac{p_{1850}}{w_{1850}} \right)^{\gamma_{\text{pre}}} \left( \frac{1 - \alpha_A p_{1850}^{1850}}{\gamma_{\text{post}}} \frac{1}{\alpha_A} \right) \right] \left( \frac{1 - \beta}{Q_{1850}} \right)^{(1 - \beta) \frac{\alpha_A}{\mu}} \left( \frac{1}{\gamma_{\text{post}}} \right)^{(1 - \gamma_{\text{post}})} \left( \frac{1}{w_{1850}} \right) \right\}$$

(A-3)

where we face the problem that while 1780 TFP and thus factor prices are known, 1850 variables are endogenously depending on $\overline{A}_A$. We solve this issue as follows: First, since $w$ is fixed by non-industrialized agriculture, its expected value is the same in 1780 and 1850. Second, the price of intermediate inputs $p_{1850}^{1850}$ can be derived from the 1850 technology of the intermediate sector, which is known independently of $\overline{A}_A$ (see...
equation (A-5)). Third, $Q_{1850}$ can be approximated by its initial value $Q$. Although this parameter grows larger than $Q$ during industrialization, this approximation is valid since $\overline{A}_A$ reacts inelastically to changes in $Q$. Finally, to approximate output and land in 1850 we assume that $c_{1850}^p > 1$ such that all agricultural sectors produce for rich and poor, which gives output $y_{A,1850}^p$. Land $l_{A,1850}^p$ is then given by the land use in the competitive fringe. Solving equation (A-2) for manufacturing implies:

$$\overline{A}_M = \left( \frac{1}{Q_{1850}} \right)^{(1-\beta)\alpha_M} \left[ \overline{A}_M \Delta prd \left( \frac{1 - \alpha_M p_{1850}}{\alpha_M w_{1850}} \right)^{\alpha_M} + \frac{p_{1780}}{\overline{p}_M} \right]$$

where we use the same arguments as above to approximate 1850 variables. Finally, the TFP change in intermediate production can be calculated easily since equation (A-1) simplifies to

$$\overline{A}_I = \Delta prd \overline{A}_I$$

which follows the fact that labor is the only input in the linear production function of intermediate sectors such that TFP is equivalent to output per worker.

### A.3. Calibration of Parameters in the Production Function

In this section we use the input-output table for 1841 by Horrell et.al. (1994). We choose agriculture and food, drink and tobacco to represent the division $J_A$ in our model. We then define sectors as intermediate if the ratio of intermediate demand over total demand exceeds 50%. This is the case for mining and quarrying (77%), metal manufacturing (70%), and bricks, pottery, and glass (64%). Finally, manufacturing sectors in our model correspond to soap and dyes, textiles and clothing, metal goods, and other manufacturing. In order to derive $\alpha_A$, $\alpha_M$, and $\alpha_I$, we must consider that total cost $C_J$ in our model (as derived in Appendix A.1.) do not include the payment of profits to capital, they only result from payments to land, labor, and intermediate inputs. In order to find $C_J$ we must thus subtract the cost of capital from total cost figures provided in the input-output table, i.e., $C_J = TC_J - r_K K_J$, where $TC_J$ denotes total cost including payments to capital. Horrell et. al. (1994, p.564) give the capital rental rate as 8.8%, and they also provide data on employment and capital stock per sector in their input output table.

For agriculture, our calibration follows from the equation $Q_{pr} x^A_{I,A} = \alpha_A [C (y_A^p) - r_K l^p]$ which is derived from equations (???) and (???) above. It follows the intermediate input share

$$\alpha_A = \frac{Q_{pr} x^A_{I,A}}{TC (y_A^p) - r_K K_A - r_L l^p}$$

where we use $\omega = r_p$ in the calibration since in our model industrialization in agriculture occurs only in the rich-poor sectors. Moreover, we can think of 1841 as being
close enough to 1850, where the industrialization is finished in our model, such that the capital stock $K_A$ refers to rich-poor agriculture only. One more correction is needed in agriculture: just by the nature of the agricultural production process, intermediate inputs (e.g., seeds stemming from the previous year’s harvest) have a high share in total output. But not all of these intermediate inputs can be considered as being used only after the industrialization of agriculture. Since our model requires this interpretation, we would over-estimate the importance of intermediate inputs in industrialized agriculture when taking the figures straight from the input-output table. To correct this potential bias, we calculate the ratio of agricultural inputs to all other inputs used in manufacturing ($\approx 30\%$) and suppose that this number reflects the typical industrial usage of agricultural inputs that is used in agriculture, as well. Table A.1 summarizes our calculations.

The calibration for the manufacturing sector is even more simple than the one for agriculture, since we neither face the above bias problem nor need to consider land. The basic equation is $Q_{pI}x_{I,M}^y = \alpha_M C(y_M^z)$, which is derived from (???). The intermediate input share is then determined by

$$\alpha_M = \frac{Q_{pI}x_{I,M}^y}{TC(y_M^z) - r_K K_M^z}$$

(13)

where again we use $\omega = r_p$. Finally, recall that the cost in intermediate production (without payments to capital) is $C(y_I) = n_I w / A_I$, that is, in the model the intermediate sector itself does not use intermediate inputs. In Appendix A.4, however, it will become necessary to have some measure for the ‘hypothetical’ intermediate input share $\alpha_I$, fulfilling the equation $[\text{Cost of interm. inputs}] = \alpha_I C(y_I)$. We calculate this number using the input-output data in the same manner as above:

$$\alpha_I = \frac{[\text{Cost of interm. inputs}]}{TC(y_I) - r_K K_I}$$

(14)

Table A.1 shows the calibration procedure and the results. In order to check the plausibility of our approach, we also calculate the implicit wage from the number of workers in each sector and payments to labor (which is the residual of value added after subtracting capital and land rents). We implicitly find quite similar wages for the three divisions agriculture, manufacturing and intermediate products.
TABLE A.1: Calibration of intermediate input shares

<table>
<thead>
<tr>
<th>(in pounds m)</th>
<th>Agriculture and Food</th>
<th>Manufacturing</th>
<th>Intermediate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital rental rate</td>
<td>8.8%</td>
<td>8.8%</td>
<td>8.8%</td>
</tr>
<tr>
<td>Share of land in value added</td>
<td>0.25</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Total capital</td>
<td>210</td>
<td>73</td>
<td>25</td>
</tr>
<tr>
<td>Total labor</td>
<td>1846</td>
<td>1882</td>
<td>488</td>
</tr>
<tr>
<td>Total intermed. inputs</td>
<td>125.7</td>
<td>67.6</td>
<td>9.0</td>
</tr>
<tr>
<td>Intermed. inputs w/o agric.</td>
<td>48.0</td>
<td>51.9</td>
<td>8.0</td>
</tr>
<tr>
<td>Ratio agr inputs/all other inp.</td>
<td>30.3% *</td>
<td>30.3%</td>
<td>12.5%</td>
</tr>
<tr>
<td>Industrial agric. input</td>
<td>14.5</td>
<td>15.7</td>
<td>1.0</td>
</tr>
<tr>
<td>Industrial interm. inp. ((Qp_Ix_{I,J}))</td>
<td>62.5</td>
<td>67.7</td>
<td>9.0</td>
</tr>
<tr>
<td>Value added</td>
<td>173.5</td>
<td>91.0</td>
<td>26.6</td>
</tr>
<tr>
<td>Capital ((r_KK_J))</td>
<td>18.5</td>
<td>6.4</td>
<td>2.2</td>
</tr>
<tr>
<td>(implicit capital share)</td>
<td>10.7%</td>
<td>7.1%</td>
<td>8.3%</td>
</tr>
<tr>
<td>Land ((r_Ll))</td>
<td>43.4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Labor (residual)</td>
<td>111.6</td>
<td>84.6</td>
<td>24.4</td>
</tr>
<tr>
<td>(implicit wage)</td>
<td>0.060</td>
<td>0.045</td>
<td>0.050</td>
</tr>
<tr>
<td>Total cost ((TC_J))</td>
<td>236.0</td>
<td>158.6</td>
<td>35.6</td>
</tr>
<tr>
<td>(\alpha_J)</td>
<td>0.36</td>
<td>0.44</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Source: Data from Horrell et al. (1994)

*Imposed ratio, see discussion in the text

A.4 Calibration of the Cost of Intermediate Inputs

The usage of intermediate inputs results from the firms’ cost-minimization problem, as shown in Appendix A.1. Using the above equations for \(x_{I,J}\), we calculate total material cost, \(TMC_J\), as the cost of intermediate inputs used in rich-poor production. For agriculture,

\[
TMC_A = Qp_Ix_{I,A} = \alpha_A \left( \frac{1}{1} \right) \left( \frac{1}{y_{A}} \right)^{1-\alpha_A} \left( \frac{w}{1-\alpha_A} \right)^{1} \left( \frac{1}{Q} \right)^{(1-\beta)\alpha_A} \left( \frac{p_I}{\alpha_A} \right)^{\alpha_A} \left( \frac{1}{1-\alpha_A} \right)^{1} \left( y_{A}^{rp} \right)^{1-\alpha_A} \left( (1-\beta)\alpha_A \right) \left( \frac{w}{1-\alpha_A} \right)^{1} \left( y_{A}^{rp} \right)^{1-\alpha_A} \left( (1-\beta)\alpha_A \right)
\]

(15)

where, as discussed above, land \(I^{rp}\) is determined in the competitive fringe and is taken as given by the industrialized sector. For manufacturing we have

\[
TMC_M = Qp_Ix_{I,M} = \alpha_M \left( \frac{1}{1} \right) \left( \frac{1}{y_{M}^{rp}} \right)^{(1-\beta)\alpha_M} \left( \frac{p_I}{\alpha_M} \right)^{\alpha_M} \left( \frac{w}{1-\alpha_M} \right)^{1-\alpha_M} \left( y_{M}^{rp} \right)^{1-\alpha_M} \left( (1-\beta)\alpha_A \right) \left( \frac{w}{1-\alpha_A} \right)^{1} \left( y_{A}^{rp} \right)^{1-\alpha_A} \left( (1-\beta)\alpha_A \right) \left( \frac{w}{1-\alpha_A} \right)^{1} \left( y_{A}^{rp} \right)^{1-\alpha_A} \left( (1-\beta)\alpha_A \right)
\]

(16)

Since the intermediate sector in our model is only using labor inputs, we approximate for its intermediate input usage with the ‘hypothetical’ intermediate input share \(\alpha_I\), as calculated in equation (14). We use \(\alpha_I\) in the following procedure: First, we
calculate the total demand for each intermediate product, \( y_I \), at the point in time where the industrial revolution starts, i.e., in 1780.\(^{30}\) Second, we calculate the (hypothetical) total material cost in the intermediate sector as the share \( \alpha_I \) of total cost, that is,

\[
TMC_I = \alpha_I \frac{w}{A_I} y_I \tag{17}
\]

A.5. Relative Probabilities of a kick-off

In order to find relative probabilities, we use the income figures from Maddison in our model. That is, we update the pre-industrial productivity in each year such that income growth reflects the actual data. With this adjusted productivity we calculate the probabilities of a kick-off over a range of 100 years for each country, starting in 1760. In order to reflect the hypothetical situation "What if England had not industrialized in 1780", we down-scale the Maddison income figures for England after 1780. [The writing of this section is in progress]

\(^{30}\)Note that since \( c_p > 1 \) at this point in time (as we explained in the calibration of pre-industrial agricultural productivity), there are already some industrialized manufacturing firms demanding intermediate products.
References


[34] Mankiw, Greg, Reis, Ricardo, and Justin Wolfers (2003), 'Disagreement about Inflation Expectations’, NBER Macro Annual 18.


