Asymptotic Error Exponents in Energy-Detector and Estimator-Correlator Signal Detection

Josep Font-Segura, Gregori Vazquez, and Jaume Riba
Signal Theory and Communications Department, Technical University of Catalonia (UPC)
D5-{214, 204, 116}, Campus Nord UPC, Jordi Girona 1-3, 08034 Barcelona, Spain
Email: {josep.font-segura, gregori.vazquez, jaume.riba}@upc.edu

***Abstract*—The performance in signal detection is evaluated by the error (false-alarm and missed-detection) probabilities. However, calculating these probabilities is a difficult task in practice. This paper studies the asymptotic behavior of the energy-detector and the estimator-correlator by means of the Stein's lemma. The Stein's lemma is an information-theory result that provides the best achievable error exponent in the error probabilities when the number of observations goes to infinity. The derived closed-form expressions explain how detection performance is driven by the detector parameters and the second-order statistics of the problem. More specifically, it is shown that the error exponents depend on the signal-to-noise ratio (SNR) and the observation size. The prime focus is to establish a link between the required observation size for a fixed error probability as a function of the SNR. Numerical results show the tightness of the lemma.**

***Index Terms*—Stein's lemma, Kullback-Leibler divergence, likelihood ratio test, energy-detector, estimator-correlator, spectrum sensing, cognitive radio.**

I. Introduction

Signal detection, as well as many other engineering problems, can be cast as deciding between two alternative explanations of the observations. The error probabilities associated to the detection are fundamental for understanding the performance of detection problems. In most practical situations these expressions are not available in a closed-form solution, because the probability distribution of the sufficient detection statistics in the test is difficult to obtain. Nonetheless, the asymptotic properties of the statistics are sometimes useful to characterize the behavior and to obtain performance bounds. The Stein's lemma [1] is a fundamental result that provides the asymptotic behavior of the error probabilities associated to detection problems when the number of observations grows to infinity. More specifically, it shows that the error probabilities decay exponentially with the number of observations [2], hence playing the role of error exponent as in coding theory.

The potentials of the Stein's lemma and the Kullback-Leibler divergence (KLD) as a measure of distance have been recently explored in a wide variety of information-theory and communication problems, from MIMO radar [3] to sensor networks [4]. In the field of cognitive radio, the Stein's lemma has been employed in asymptotic performance analysis in collaborative spectrum sensing [5] and quadratic likelihood detection [6].

This work has been partially funded by the Spanish Government under TEC2010-21245-C02-01 (DYNACS), CONSOLIDER INGENIO CSD2008-00010 (COMONSENS), CENIT CEN-20101019 (THOFU), and the Catalan Government (DURSI) under Grant 2009SGR1236 and Fellowship FI-2010.

In this paper, the error exponents for the energy-detector and the estimator-correlator are derived. Both the energy-detector and the estimator-correlator are optimal likelihood ratio tests under the Neyman-Pearson criterion and have a central role in spectrum sensing for cognitive radio [7]. It is shown that the error exponents of the false-alarm and missed-detection probabilities in detection depend on the observation size and the second-order statistics of the problem, i.e., the signal-to-noise ratio (SNR). The main focus of the paper is to establish an asymptotic relation between the observation size and the SNR for a fixed error probability. In general, it is shown that the observation size scales as the inverse of a monotonically increasing function of the SNR, i.e.,

$$N \propto \frac{1}{f(\text{SNR})}. \tag{1}$$

The main contribution of the paper is the derivation of simple, closed-form expressions of $f(\text{SNR})$ that permit the evaluation of the main factors yielded in the signal detection task.

The rest of the paper is organized as follows. Sec. II briefly introduces the Stein's lemma together with the system model. Sec. III derives the error exponents of the energy-detector and the estimator-correlator. Sec. IV provides numerical results to assess the tightness of the lemma, and Sec. V concludes the paper.

II. The Stein's Lemma

In a general framework of binary hypothesis testing problems based on block signal processing, we are given a set of i.i.d. vector observations $\mathbf{X} \doteq (\mathbf{x}_1, \ldots, \mathbf{x}_M)$, each one distributed according to the probability density function $p(\mathbf{x})$. Each vector $\mathbf{x}_m$ is composed by N consecutive samples, being N the observation size large enough to arise cross-correlation between samples and guarantee independence between samples within two separate vectors. Despite the total available amount of samples is MN, in this paper we assume that N is a design parameter, while M can be arbitrarily large. The probability density function $p(\mathbf{x})$ takes one of the following forms under the two hypotheses

$$\mathcal{H}_0 \quad : \quad p(\mathbf{x}) = p_0(\mathbf{x}) \tag{2a}$$

$$\mathcal{H}_1 \quad : \quad p(\mathbf{x}) = p_1(\mathbf{x}). \tag{2b}$$

The detection problem consists of designing a decision function or test $T(\mathbf{x})$ whose output implies accepting either $\mathcal{H}_0$ or $\mathcal{H}_1$, as a function of the observations and the statistics of the problem. The test is specified by the detection sets $\mathcal{T}_0$ and its complementary $\mathcal{T}_1$ over which $\mathcal{H}_0$ or $\mathcal{H}_1$ is decided, respectively. Given the boundary which defines the aforementioned sets, the error probabilities associated to the test $T(\mathbf{x})$ are defined as

$$\alpha = \mathbb{P}\left[T(\mathbf{X}) \in \mathcal{T}_1 \mid \mathcal{H}_0\right] \quad (3a)$$

$$\beta = \mathbb{P}\left[T(\mathbf{X}) \in \mathcal{T}_0 \mid \mathcal{H}_1\right]. \quad (3b)$$

In communications problems, the hypothesis $\mathcal{H}_0$ denotes the only noise situation (i.e., $\mathbf{x} = \mathbf{w}$), whereas $\mathcal{H}_1$ denotes the signal plus noise situation (i.e., $\mathbf{x} = \mathbf{s} + \mathbf{w}$). Therefore, α is called the false-alarm probability,and β the missed-detection probability. In cognitive radio, α is important to guarantee opportunistic communication for the secondary users, while β protects the legacy systems. According to the Neyman-Pearson criterion, the likelihood ratio test (LRT) $T(\mathbf{X})$ defined as

$$T(\mathbf{X}) = \frac{p_1(\mathbf{X})}{p_0(\mathbf{X})} \quad (4)$$

provides optimal error probabilities pair (α, β) in the sense that for one fixed error probability (e.g., the false-alarm probability), any other test will provide a higher probability pair (e.g., higher missed-detection probability). The LRT (4) makes decisions by comparing to a threshold γ, which defines the decision sets as $\mathcal{T}_0 = \{T(\mathbf{X}) < \gamma\}$, and $\mathcal{T}_1 = \{T(\mathbf{X}) \geq \gamma\}$.

The Stein's lemma may be expressed in two versions, which are summarized in the following lemmas.

Lemma 1 (False-alarm probability Stein's lemma)**.** *Consider the binary hypothesis testing problem* (2) *and the likelihood ratio test* (4)*. For a fixed missed-detection probability $\beta \leq \beta_0$, the false-alarm probability asymptotically behaves as*

$$\lim_{M\to\infty} \frac{1}{M} \log \alpha = -\mathcal{D}(\mathcal{H}_1 \| \mathcal{H}_0) \quad (5)$$

where $\mathcal{D}(\mathcal{H}_1 \| \mathcal{H}_0)$ evaluates the Kullback-Leibler divergence (KLD) given by

$$\mathcal{D}(\mathcal{H}_1 \| \mathcal{H}_0) = \int p_1(\mathbf{x}) \log \frac{p_1(\mathbf{x})}{p_0(\mathbf{x})} d\mathbf{x}. \quad (6)$$

Lemma 2 (Missed-detection probability Stein's lemma)**.** *Consider the binary hypothesis testing problem* (2) *and the likelihood ratio test* (4)*. For a fixed false-alarm probability $\alpha \leq \alpha_0$, the missed-detection probability asymptotically behaves as*

$$\lim_{M\to\infty} \frac{1}{M} \log \beta = -\mathcal{D}(\mathcal{H}_0 \| \mathcal{H}_1). \quad (7)$$

A direct consequence of Lemmas 1 and 2 is that both error probabilities decay, as M grows to infinity, exponentially with respect to each associated KLDs, i.e.,

$$\alpha \approx u(M) e^{-M\mathcal{D}(\mathcal{H}_1 \| \mathcal{H}_0)} \quad (8a)$$

$$\beta \approx v(M) e^{-M\mathcal{D}(\mathcal{H}_0 \| \mathcal{H}_1)}, \quad (8b)$$

where $u(M)$ and $v(M)$ are slow-varying functions compared to the exponential, such that $\lim_{M\to\infty} \frac{1}{M} \log u(M) = \lim_{M\to\infty} \frac{1}{M} \log v(M) = 0$. Therefore, given a number of observations, the detection performance exclusively depends on the KLD between hypotheses, which in the sequel we show that it is related to the observation size N and the second-order statistics of the problem.

III. Application to Spectrum Sensing

In the sequel, the expressions of the KLDs for the energy-detector and estimator-correlator LRTs are analyzed.

A. Energy-Detector

Though its simplicity, the energy-detector is a low-complexity and well-studied test that has been adopted in recent standards [8] as a fast-sensing algorithm. The IEEE 802.22 standard defines the sensing requirements for detecting TV white spaces for wireless regional area network (WRAN) devices opportunities. The spectrum sensing defined in the standard is based on two stages: fast and fine sensing. The energy-detector is employed in the fast sensing stage as a coarse detector, whereas a more sophisticated detector is used in the fine sensing stage when the fast sensing stage detects the presence of the signal. Hence, the energy-detector is still an important statistical test for practical engineering problems as it allows simple formulations to obtain insights on the required observation size and SNR.

The energy-detector is the optimal test in the Neyman-Pearson criterion when the primary signal and noise have Gaussian white statistics, and is given by [9]

$$T_{\mathrm{ED}}(\mathbf{x}) = \mathrm{tr}(\mathbf{X}\mathbf{X}^H). \quad (9)$$

Under this assumption, the statistics of the observations are

$$p_0(\mathbf{x}) = \mathcal{C}_N(\mathbf{0}, \sigma_w^2 \mathbf{I}) \quad (10a)$$

$$p_1(\mathbf{x}) = \mathcal{C}_N(\mathbf{0}, \sigma_s^2 \mathbf{I} + \sigma_w^2 \mathbf{I}), \quad (10b)$$

where σ_s^2 and σ_w^2 are the signal and noise powers, respectively. As a consequence, the KLD that define the error exponents in α and β for the energy-detector are[1]

$$\mathcal{D}(\mathcal{H}_1 \| \mathcal{H}_0) = N\left[\mathrm{SNR} - \log(1 + \mathrm{SNR})\right] \quad (12a)$$

$$\mathcal{D}(\mathcal{H}_0 \| \mathcal{H}_1) = N\left[\log(1 + \mathrm{SNR}) - \frac{\mathrm{SNR}}{1 + \mathrm{SNR}}\right], \quad (12b)$$

respectively, where $\mathrm{SNR} \doteq \sigma_s^2/\sigma_w^2$ is the signal-to-noise ratio (SNR) of the problem.

Due to the white statistics under both hypotheses, the KLD (6) equals N times the KLD of each individual observation. Hence, the error exponents of the energy-detector (9) grow linearly with the observation size N in the same way it linearly grows with the number of observations M, in the sense that

[1]Let $p_0(\mathbf{x}) = \mathcal{C}_N(\mathbf{0}, \mathbf{R}_0)$ and $p_1(\mathbf{x}) = \mathcal{C}_N(\mathbf{0}, \mathbf{R}_1)$. The KLD (6) is then

$$\mathcal{D}(\mathcal{H}_1 \| \mathcal{H}_0) = \mathrm{tr}(\mathbf{R}_0^{-1}\mathbf{R}_1) - \log\left[\det{}^{-1}(\mathbf{R}_0)\det(\mathbf{R}_1)\right] - N. \quad (11)$$

It follows that the KLD between two Gaussian processes depends on three terms: the ratio between the second-order statistics, the log-radio of determinants of the covariance matrices, and a penalty term of dimensionality.

the equivalent total number of available samples is MN. In other words, no correlation needs to be exploited.

The slope of the error probabilities is given by functions of the SNR, which from (12), are given by $\text{SNR} - \log(1+\text{SNR})$ and $\log(1+\text{SNR}) - \text{SNR}(1+\text{SNR})^{-1}$. The nonnegativity of the error exponents is guaranteed provided that the former functions are nonnegative for all $\text{SNR} \geq 0$. Additionally, they are monotonically increasing functions in SNR, which ensures that in the limit of the high-SNR regime the detection is error-free. Another interpretation of (12) is how SNR and the observation size scale to preserve the error probabilities in the asymptotic cases of low-SNR (i.e., $\text{SNR} \to 0$) and high-SNR (i.e., $\text{SNR} \to \infty$) regimes.

1) False-alarm probability: The error exponent associated to the false-alarm probability admits the following approximations. In the low-SNR regime, it can be approximated by the second degree polynomial $\log(1+\text{SNR}) \approx \text{SNR} - \frac{1}{2}\text{SNR}^2$. Therefore, (12a) approximates by

$$\mathcal{D}(\mathcal{H}_1 \| \mathcal{H}_0) \approx \frac{1}{2} N \cdot \text{SNR}^2. \tag{13}$$

This means that the required observation size is inversely proportional to the squared value of the SNR, i.e.,

$$N \propto \frac{1}{\text{SNR}^2} \tag{14}$$

to preserve a target false-alarm probability α_0. Contrarily, the approximation $\text{SNR} - \log(1+\text{SNR}) \approx \text{SNR}$ is valid in the high-SNR regime. In that case, the error exponent may be approximated by

$$\mathcal{D}(\mathcal{H}_1 \| \mathcal{H}_0) \approx N \cdot \text{SNR}, \tag{15}$$

for which the required observation size is related to the SNR by

$$N \propto \frac{1}{\text{SNR}}. \tag{16}$$

This concludes that the energy-detector is more sensitive to a change in channel conditions (i.e., SNR) when operating in the low-SNR regime rather than in the high-SNR regime.

2) Missed-detection probability: Regarding the error exponent associated to the missed-detection probability, the following two approximations $\log(1+\text{SNR}) \approx \text{SNR} - \frac{1}{2}\text{SNR}^2$ and $\text{SNR}(1+\text{SNR})^{-1} \approx \text{SNR} - \text{SNR}^2$ apply in the low-SNR regime. Hence, (14) holds for β, as $\mathcal{D}(\mathcal{H}_0 \| \mathcal{H}_1) \approx \frac{1}{2} N \text{SNR}^2$ as well. In the high-SNR regime, the behavior of the error exponent is more conservative with the SNR, because by the approximation $\log(1+\text{SNR}) - \text{SNR}(1+\text{SNR})^{-1} \approx \log(\text{SNR})$ as $\text{SNR} \to \infty$ it follows that

$$\mathcal{D}(\mathcal{H}_0 \| \mathcal{H}_1) \approx N \cdot \log(\text{SNR}). \tag{17}$$

In other words, the missed-detection probability is more restrictive in the observation size, in the sense that is inversely proportional to the logarithm of the SNR,

$$N \propto \frac{1}{\log(\text{SNR})}. \tag{18}$$

As a common factor in, the observation size is always inversely proportional to a monotonically increasing function of the SNR, as claimed by (1), which has the closed-form expressions in (12)–(18).

B. Estimator-Correlator

The detection of a signal in Gaussian noise describes many real engineering situations, including the energy-detector discussed above. In the problem in hand, it is also a valid assumption that the signal to be detected has Gaussian distribution. While it facilitates the analysis, it is reasonable in signal detection problems in low-SNR regimes as the Gaussian distribution provides optimum second-order treatment [10], and acts as a worst-case distribution.

Claimed by the KLD, the performance of the detection in terms of error probabilities depends on the distinctness between the two hypotheses (2). For zero-mean Gaussian signals, this distinctness is reflected by the cross-correlation between hypotheses. The correlation can be found in the time, frequency, or space domains. In what follows, the temporal correlation of the signal to be detected is exploited when both noise and signal correlation matrices are known.

In this setting, the observations are distributed according to

$$\begin{aligned} p_0(\mathbf{x}) &= \mathcal{C}_N(\mathbf{0}, \mathbf{R}_w) && (19a) \\ p_1(\mathbf{x}) &= \mathcal{C}_N(\mathbf{0}, \mathbf{R}_s + \mathbf{R}_w), && (19b) \end{aligned}$$

under $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively; where $\mathbf{R}_s$ and $\mathbf{R}_w$ are the correlation matrices of the signal and noise, respectively, defined as $\mathbf{R}_s \doteq \mathbb{E}[\mathbf{s}\mathbf{s}^H]$ and $\mathbf{R}_w \doteq \mathbb{E}[\mathbf{w}\mathbf{w}^H]$. The optimal test in the Neyman-Pearson criterion is the estimator-correlator, given by [9]

$$T_{\text{EC}}(\mathbf{x}) = \text{tr}\left[\mathbf{R}_w^{-1}\mathbf{R}_s\left(\mathbf{R}_s + \mathbf{R}_w\right)^{-1}\mathbf{X}\mathbf{X}^H\right]. \tag{20}$$

From (11), it follows that the KLDs that define the error exponents in α and β for the estimator-correlator are

$$\begin{aligned} \mathcal{D}(\mathcal{H}_1 \| \mathcal{H}_0) &= \text{tr}(\mathbf{S}) - \log\det(\mathbf{I} + \mathbf{S}) && (21a) \\ \mathcal{D}(\mathcal{H}_0 \| \mathcal{H}_1) &= \log\det(\mathbf{I} + \mathbf{S}) - \text{tr}\left[\mathbf{S}\left(\mathbf{I} + \mathbf{S}\right)^{-1}\right], && (21b) \end{aligned}$$

respectively. The SNR matrix has been defined as $\mathbf{S} \doteq \mathbf{R}_w^{-1}\mathbf{R}_s$. As it can be appreciated, the KLDs (12) and (21) share the property that they exclusively depend on the ratio of signal and noise second-order statistics, as well as the observation size N. For the estimator-correlator, however, the effect of temporal correlation and the effect of the observation size are both inherent in the structure of the SNR matrix $\mathbf{S}$.

For clarity and comparison purposes, we also provide the expressions of the error exponents for asymptotically large observation size, i.e., $N \to \infty$. It is a well-known result in statistical signal processing that any autocorrelation matrix asymptotically follows the eigenvalue decomposition $\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, where the eigenvectors in $\mathbf{U}$ are related to the Fourier matrix and independent of the process, and the eigenvalues in $\mathbf{\Lambda}$ are equal to the power spectral density at the $f_n \doteq n/N$ frequency [9]. Therefore, $\mathbf{S}$ is asymptotically equivalent to

$$\mathbf{S} \to \mathbf{U}\mathbf{\Lambda}_w^{-1}\mathbf{\Lambda}_s\mathbf{U}^H, \tag{22}$$

in a weak norm sense. That is, its eigenvalues reflect the SNR at the f_n frequency, denoted by $[\mathbf{\Lambda}_w^{-1}\mathbf{\Lambda}_s]_{nn} = \text{SNR}_n$. The expressions in (21) and the result (22) allow the following interpretations.

1) False-alarm probability: For (21a), by applying the following Taylor series approximation $\log\det(\mathbf{I}+\mathbf{S}) \approx \text{tr}(\mathbf{S}) - \frac{1}{2}\text{tr}(\mathbf{S}^2)$ when $\text{SNR} \to 0$, it follows that in the low-SNR regime,

$$\mathcal{D}(\mathcal{H}_1\|\mathcal{H}_0) \approx \frac{1}{2}\text{tr}(\mathbf{S}^2) \to \frac{1}{2}N \cdot \text{SNR}_q^2, \tag{23}$$

where the quadratic mean of the SNR has been defined as $\text{SNR}_q \doteq \sqrt{\frac{1}{N}\sum_n \text{SNR}_n^2}$. That is, the error exponent associated to α is proportional to the squared value of the quadratic mean of the SNR. Furthermore, the observation size scales as

$$N \propto \frac{1}{\text{SNR}_q^2}, \tag{24}$$

which shows consistency with (14) for white power spectral densities. Contrarily, at the high-SNR regime, we can approximate the KLD by

$$\mathcal{D}(\mathcal{H}_1\|\mathcal{H}_0) \approx \text{tr}(\mathbf{S}) \to N \cdot \text{SNR}_a, \tag{25}$$

i.e., N times the arithmetc mean of the SNR defined as $\text{SNR}_a \doteq \frac{1}{N}\sum_n \text{SNR}_n$. Therefore, at the high-SNR regime, the α error exponent of the estimator-correlator scales linearly with the observation size and arithmetic mean of SNR

$$N \propto \frac{1}{\text{SNR}_a}. \tag{26}$$

2) Missed-detection probability: Regarding (21b), on the one hand, the same low-SNR approximation aforementioned can be applied to $\mathcal{D}(\mathcal{H}_0\|\mathcal{H}_1)$, together with the Taylor series approximation of the second term $\text{tr}[\mathbf{S}(\mathbf{I}+\mathbf{S})^{-1}] \approx \text{tr}(\mathbf{S}) - \text{tr}(\mathbf{S}^2)$ as $\text{SNR} \to 0$. As a consequence, this leads to the same approximation (23), i.e., $\mathcal{D}(\mathcal{H}_0\|\mathcal{H}_1) \approx \frac{1}{2}\text{tr}(\mathbf{S}^2)$ and (24). This shows that both error probabilities, α and β, show the same behavior in the low-SNR regime in proportion to the squared of the second-order statistics. On the other hand, as $\text{SNR} \to \infty$, the term $\text{tr}\left[\mathbf{S}\left(\mathbf{I}+\mathbf{S}\right)^{-1}\right] \approx \text{tr}(\mathbf{I})$ can be neglected in front of the first term $\log\det(\mathbf{I}+\mathbf{S}) \approx \log\det(\mathbf{S})$. Therefore,

$$\mathcal{D}(\mathcal{H}_0\|\mathcal{H}_1) \approx \log\det(\mathbf{S}) \to\doteq N \cdot \log(\text{SNR}_g), \tag{27}$$

which coincides with the asymptotic behavior of the channel capacity formula. We have further defined the geometric mean of the SNR as $\text{SNR}_g \doteq \sqrt[n]{\prod_n \text{SNR}_n}$. This concludes that the error exponent of the missed-detection probability scales with the logarithm of the geometric mean of the SNR, whereas the required observation size becomes proportional to

$$N \propto \frac{1}{\log(\text{SNR}_g)}. \tag{28}$$

We note that (28) reduces to (18) for white statistics.

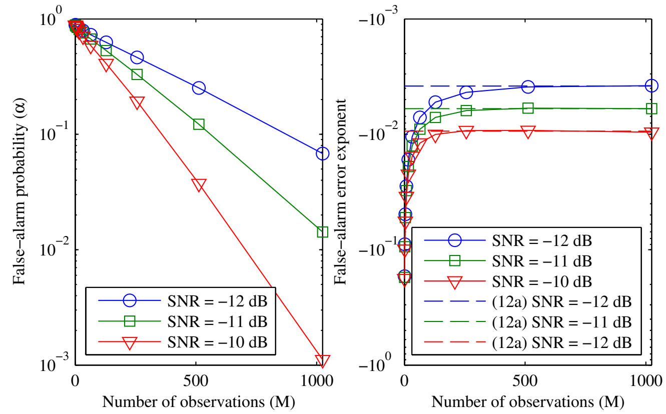

Fig. 1. False-alarm probability versus observation size (left) and simulated false-alarm error exponent versus the number of observations compared to the theoretical error exponents (12a) for the energy-detector (9).

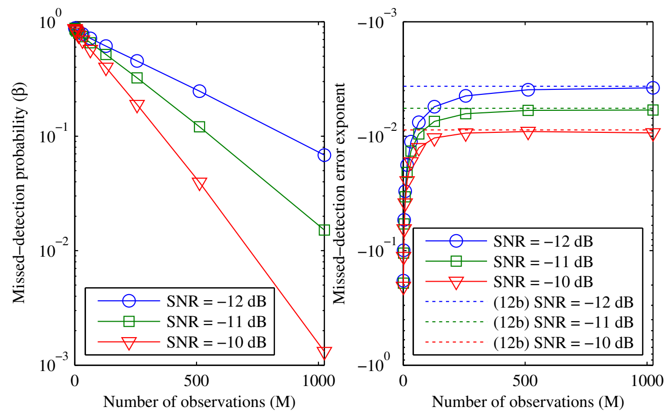

Fig. 2. Missed-detection probability versus observation size (left) and simulated missed-detection error exponent versus the number of observations compared to the theoretical error exponents (12b) for the energy-detector (9).

In conclusion, we have shown that signal detection based on the estimator-correlator asymptotically (as M and N go to infinity) depends on the quadratic mean of the SNR profile in the low-SNR regime (i.e., wideband signals), and on the arithmetic and geometric means in the high-SNR regime (i.e., narrowband signals). In general, we have shown that the required observation size scales as inversely proportional to a function of the SNR (1), whose closed-form expressions have been derived.

IV. Numerical Results

Simulation results are provided to assess the behavior of the error probabilities of the energy-detector (9) and the estimator-correlator (20) in low-SNR conditions. More specifically, the linear behavior of the logarithm of the error probabilities as well as the tightness of the Stein's lemma are evaluated. In the sequel, the number of observations M vary from 1 to 1024 and are equally spaced in a base-2 logarithmic scale. The observation size N is set, as an example, to $N = 2$, as in the estimator-correlator case the detection of a signal that presents correlation only in consecutive samples is considered.

A. Energy-Detector

The Stein's lemmas on the false-alarm probability and missed-detection probability for the energy-detector (9) are evaluated in Figs. 1 and 2, respectively.

As it can be appreciated in both figures, the error probabilities in signal detection obey a linear scaling with the number of observations as $M \to \infty$. Furthermore, the rate at which the error probabilities diminish with M, i.e., the slope associated to the error exponents, is asymptotically given by the KLDs (12) which depend on the SNR of the problem, as well as the observation size N. As the scenario conditions are in the low-SNR regime, the asymptotic error exponents depend on the squared value of the SNR. Therefore, the three error exponents in the right hand of the Figs. are equally spaced, as the SNR points are equally spaced as well in the logarithmic scale. Also, because of the low-SNR regime, both false-alarm and missed-detection probabilities show very similar performances in terms of achievable error exponents. This corroborates the fact that the KLDs (21) have the same approximation in the low-SNR regime.

B. Estimator-Correlator

Finally, the asymptotic performance of the estimator-correlator (20) is evaluated in terms of Monte Carlo trials on the error probabilities in several low-SNR conditions. In this example, the following simple correlation matrix $\mathbf{R}_s = \text{SNR}[1\ -0.5; -0.5\ 1]$ has been employed, and $\mathbf{R}_w = \mathbf{I}$.

The false-alarm probability and its associated error exponent is depicted in Fig. 3, whereas the missed-detection probability and its associated error exponent is depicted in Fig. 4. Under the same conditions of SNR, observation size, and number of samples, the estimator-correlator performs slightly better than the energy-detector as it is able to exploit the correlation of the signal in $\mathbf{R}_s$. This is seen in the asymptotic error exponents (21) which, for white Gaussian noise, reduce to $\mathbf{S} = \frac{1}{\sigma_w^2}\mathbf{R}_s$. Finally, as expected, for arbitrarily large M, e.g., for $M \geq 100$, the linear behavior of the logarithm of the error probabilities with M is observed, as well as the tightness of the Stein's lemma for $M \geq 500$.

V. Conclusions

In this paper, the asymptotic behavior of the error probabilities in signal detection has been addressed by means of the Stein's lemma. It has been shown that the error exponents in the energy-detector and the estimator-correlator depend on the observation size and the SNR profile of the problem. Closed-form expressions of the asymptotic relation between observation size and SNR have been obtained. Simulation results have been reported to assess the linear behavior of the logarithm of the error probabilities, as well as to evaluate the tightness of the lemma.

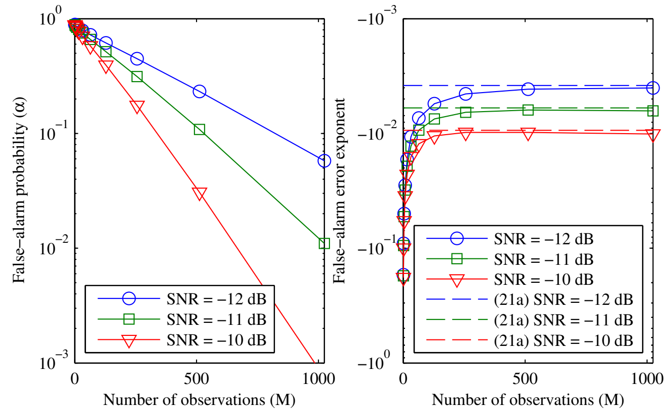

Fig. 3. False-alarm probability versus number of observations (left) and simulated false-alarm error exponents versus number of observations compared to the theoretical error exponents (21a) for the estimator-correlator (20).

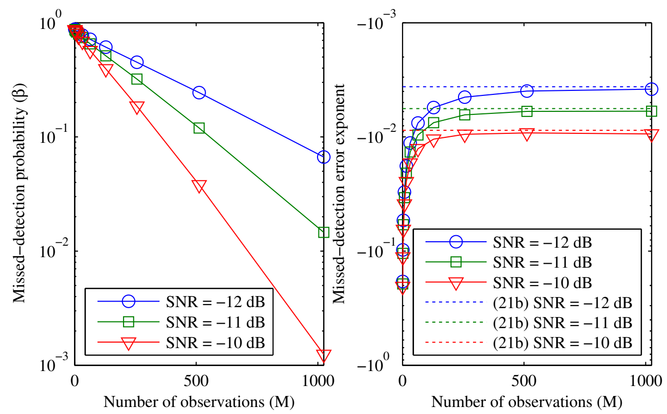

Fig. 4. Missed-detection probability versus observation size (left) and simulated missed-detection error exponent versus the number of observations compared to the theoretical error exponents (21b) for the estimator-correlator (20).

References

[1] T. Cover and J. A. Thomas, *Elements of Information Theory*. John Wiley and Sons, Inc., 1991.

[2] E. Tuncel, "On error exponents in hypothesis testing," *IEEE Trans. Inf. Theory*, vol. 51, no. 8, pp. 2945–2950, Aug. 2005.

[3] J. Tang, N. Li, Y. Wu, and Y. Peng, "On detection performance of MIMO radar: A relative entropy-based study," *IEEE Signal Process. Lett.*, vol. 16, no. 3, pp. 184–187, Mar. 2009.

[4] S. Misra and L. Tong, "Error exponents for the detection of Gauss-Markov signals using randomly spaced sensors," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 3385–3396, Aug. 2008.

[5] A. Rawat, P. Anand, H. Chen, and P. Varshney, "Collaborative spectrum sensing in the presence of Byzantine attacks in cognitive radio networks," *IEEE Trans. Signal Process.*, vol. 59, no. 2, pp. 774–786, Feb. 2011.

[6] W. Zhang, H. Poor, and Z. Quan, "Frequency-domain correlation: An asymptotically optimum approximation of quadratic likelihood ratio detectors," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 969–979, Mar. 2010.

[7] A. Ghasemi and E. Sousa, "Spectrum sensing in cognitive radio networks: requirements, challenges and design trade-offs," *IEEE Commun. Mag.*, vol. 46, no. 4, pp. 32–39, Apr. 2008.

[8] C. Cordeiro, K. Challapali, D. Birru, and N. Sai Shankar, "IEEE 802.22: the first worldwide wireless standard based on cognitive radios," in *Proc. IEEE Int. Symp. New Frontiers in Dynamic Spectr. Access Networks (DySPAN)*, Nov. 2005, pp. 328–337.

[9] S. M. Kay, *Fundamentals of Statistical Signal Processing*. Upper Saddle River, NJ: Prentice Hall, 1998, vol. 2 (Detection Theory).

[10] J. Villares and G. Vazquez, "The Gaussian assumption in second-order estimation problems in digital communications," *IEEE Trans. Signal Process.*, vol. 55, no. 10, pp. 4994–5002, Oct. 2007.