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PRODUCTION NETWORKS AND ELASTIC LABOR SUPPLY

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INTRODUCTION

The research paper aims to study the main characteristics of the business cycle: GDP and unemployment and demonstrate how they are affected by the elasticity of substitution between leisure and consumption within models of the network structure of the economy.

The question of how sectoral shocks spread throughout the economy and cause macroeconomic fluctuations has become the core of economic studies in the last century. The idea is as follows: sectoral shocks of particular firms could affect the economy throughout the network structure, which designates the interconnection between the sectors that supply each other with goods and services for the production and marketing of final products. The difference between the linkages in the structure lies in the role that particular sector plays. For example, one sector may be a supplier to a disproportionately large number of other sectors, or, conversely, all sectors can play the same role in supplying each other with materials. Thus, it becomes significantly important to analyze the shocks caused by particular firms considering the fact that they can be the source of macroeconomic fluctuations.

In the scientific literature, the question of the impact of the network structure on the output of the economy was not considered in the formulation proposed in our research paper, however, it was disclosed from different perspectives of view. Thereby, there have been several studies focused on the various sources of aggregate fluctuations: industry shocks, granular origin, network structure, etc. In most research works economists assumed an inelastic supply of labor and for simplicity based the analysis on the Cobb-Douglas utility function. Consequently, there was not developed economic base to continue the investigation of the influence of the substitution elasticity between labor and consumption in the class of network models. Our contribution to this issue is in the implementation and development of the universal network model with great tractability based on the CES-function.

The goal of the current study is to evaluate the effect of the elasticity of substitution between leisure and consumption on the aggregate GDP in the class of network economic models. This paper contains the theoretical reasoning for the subsequent practical application and hypothesis testing based on the historical data of 405 industries of the USA economy. Therefore, in the first part of the paper, we analyze the theoretical base: elaborate a mathematical model and express the formulas of aggregate GDP and unemployment through the input-output matrix of the network structure and the elasticity of substitution. In the second part of our research, we assess the consistency of the theoretical derivations with historical data using shock simulations of particular firms.

The result of the research contributes the literature on the economic networks topic and can be used as a good tool in the the macroeconomic policy development for better understanding of fluctuation sources of GDP and employments. It has a great potential to provide a precise forecast for policy regulations. Furthermore, the model is presumed to facilitate the procedure of the development of preventive measures.

1. LITERATURE REVIEW

To begin with, this research paper is related to two strands of literature. The first covers the literature on the topic of intersectoral linkages, with the emphasis on the role each sector plays in the certain economy. The second strand of the literature relates mainly to empirical studies of elasticity of substitution between leisure and consumption.

Why do aggregate variables fluctuate repeatedly around a trend of the same nature? Before the advent of Keynes' general theory, the issue remained open and attempts to solve the problem were called business cycle theory. In 1977 (Lucas, 1977) it was suggested that due to the law of large numbers, idiosyncratic shocks of individual firms should cancel each other out when considering the economy in the aggregate, and therefore the broader impact should not be substantial on the macroeconomic level. But later this hypothesis was questioned: research showed that GDP fluctuations could be explained by industry shocks in the dynamic sectoral models with mini-aggregate shocks (Long, Plosser, 1983; Jovanovic, 1987 and Durlauf, 1993). Later the hypothesis was confirmed by expanding the model to multisectoral (Horvath, 2000; Conley and Dupor, 2003).

For example, Long J. and Plosser C. (1983) suggested that GDP fluctuations could be explained by industry shocks: their model had a small number of sectors and these shocks can be seen as mini-aggregate shocks. Horvath (Horvath, 2000) and Conley and Dupor (2003) further investigated this hypothesis: they found that sector-specific shocks were an important source of aggregate volatility. Finally, a few economists (Horvath, 1998) and (Dupor, 1999) discussed whether N sectors can have volatility that does not decrease according to $\frac{1}{\sqrt{N}}$. And Gabaix X. (2011) found a different approach to this issue: he builds the argument on these previous contributions and explains that the nature of the "thick tail" of industrial shocks is theoretically important because it determines whether the central limit theorem is applicable or it is not. Thus, the power of the law of large numbers is diminished and shocks compensate each other having a huge effect on the aggregate economy. This assumption was supported empirically by the analysis of the largest hundred firms in the United States: the "granular" hypothesis suggested new directions for macroeconomic research, in particular, that macroeconomic issues can be clarified by studying the behaviour of the biggest firms in the economy. The importance of idiosyncratic shocks in cumulative volatility leads to several effects and directions for future research: focusing on major players allows tracking of industry dynamics. Thus, in 2017 one of the last articles on the subject published has proved that industry shocks are the main sources of aggregate fluctuations on the macroeconomic level (Atalay, 2017), while according to previous studies, industry shocks accounted for less than half of total volatility. Furthermore, two years later Baqaee D. and Farhi E. (Baqaee, Farhi, 2019) demonstrated that the macroeconomic impact of firms' shocks largely depends on how quickly factors can be redistributed between production units.

Later the scientific discourse was developed on various reasons of shock sources of aggregate fluctuations: granular origins (Gabaix, 2011), network origins (Acemoglu, Carvalho, Ozdaglar, Tahbaz-Salehi 2012; Dupor, 1999). Overall, the business cycle models were mainly characterized by non-elastic labor supply for simplicity and were based on the Cobb-Douglas function for simplicity (Horvath M. 1998, Acemoglu, Carvalho, Ozdaglar, Tahbaz-Salehi, 2012). Thus, the issue of elasticity of substitution between consumption and leisure time in models with networks still remained open. However, there were models with elastic labor supply, but without elasticity of substitution between leisure and consumption (Long, Plosser, 1983; Shea, 2002).

Most academic sources consider leisure and consumption as substitutes (Browning, Hansen, Heckman, 1999; Aguiar, Hurst, 2005; Meyer, Mok, 2018). But there are a few articles that suggest taking leisure and consumption as complements, which are still accepted as exceptional cases (Blundel, Pistaferri, Saporta-Eksten, 2016). Our model takes into account both cases: when consumption and leisure are substitutes (elasticity parameter is higher than 1) and complements (elasticity parameter converges to 0).

Important contributions were made by internetworking researchers: Jackson M. (2008) devoted his book "Social and Economic Networks" to the description of models and methods for analysing social and economic networks, where he described the theory in details, introducing the “centrality term”. Moreover, Ballester C., Calco-Armengol A. and Zenou Y. (2006) investigated the importance of networking as an example of a finite number of players.

The use of data analysis and simulation in interconnections has also been addressed in one of the recent studies to demonstrate the usage of the model.

In summary, although there is a large volume of literature on the aggregate shocks in economics, intersectoral connections and elastic labor supply, there is a lack of comprehensive research in the interception of these three topics.

2. MODEL

Our research is based on a multi-sectoral model of real business cycles from the article by Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi "The Network Origins of Aggregate Fluctuations" (Acemoglu et al., 2012) with some significant changes. Firstly, while Acemoglu with co-authors used a model with full employment, we suggested labor supply elastic by adding a new variable f – the amount of leisure time the individual consumer has. This variable could also be considered as the level of unemployment. Secondly, we replaced the consumer's utility Cobb-Douglas function with the CES function to examine the elasticity of substitution between consumption and leisure.

We started with the definition of a competitive equilibrium. The competitive equilibrium in the n-sectors economy $\mathcal{E}$ is composed of prices $(p_1, p_1, ..., p_n)$, nominal wage h, consumption bundle $(c_1, c_2, ..., c_n)$ and quantities $(l_i, x_i, (x_{ij}))$ such that:

(a) consumers maximize their utility,

(b) producers maximize their profit,

(c) market clearing conditions are satisfied, that is,

$$c_i + \sum_{j=1}^{n} x_{ji} = x_i$$

$$\sum_{i=1}^{n} l_i = 1 - f$$

2.1. Consumer's side

The representative household owns one unit of labor. Its preferences are described by CES function over n distinct goods and leisure time:

$$U(C, 1-l) = \frac{C^{1-\gamma}-1}{1-\gamma} + \beta\frac{(1-l)^{1-\gamma}-1}{1-\gamma}$$

where $C = \prod_{i=1}^{n} (c_i)^{1/n}$ - consumption of good, $1-l = f$ - amount of leisure time, $\gamma \in (0, +\infty)$ - elasticity of substitution between leisure and consumption, β - relative value of consumption compared to leisure.

The budget constraint is following:

$$\sum_{i=1}^{n} p_i c_i + hf = h$$

where p_i - the price of good i, h - nominal wage.

After solving the consumer utility maximization problem, described in the section Appendix 1, we got the next functions for leisure f and consumption c_i:

$$f = \frac{1}{1 + \beta^{-\frac{1}{\gamma}} \left(\frac{h}{n \prod\limits_{i=1}^{n} (p_i)^{1/n}} \right)^{\frac{1-\gamma}{\gamma}}}$$

$$c_i = \frac{h\,(1-f)}{np_i} = \frac{h}{np_i} \frac{\left(\frac{h}{n \prod\limits_{i=1}^{n} (p_i)^{1/n}} \right)^{\frac{1-\gamma}{\gamma}}}{1 + \left(\frac{h}{n \prod\limits_{i=1}^{n} (p_i)^{1/n}} \right)^{\frac{1-\gamma}{\gamma}}}$$

Further we denoted the logarithm of the real wage by $\omega = log \left(\frac{h}{\prod\limits_{i=1}^{n} (p_i)^{1/n}} \right)$.

Then $e^{\omega} = \frac{h}{\prod\limits_{i=1}^{n} (p_i)^{1/n}}$.

Functions for f and c_i can be rewritten next way:

$$f = \frac{1}{1 + \beta^{-\frac{1}{\gamma}} \left(\frac{e^{\omega}}{n}\right)^{\frac{1-\gamma}{\gamma}}}$$

$$c_i = \frac{h\left(1 - f\right)}{np_i} = \frac{h\left(1 - \frac{1}{1+\beta^{-\frac{1}{\gamma}} \left(\frac{e^{\omega}}{n}\right)^{\frac{1-\gamma}{\gamma}}}\right)}{np_i} = \frac{h}{np_i} \frac{\beta^{-\frac{1}{\gamma}} \left(\frac{e^{\omega}}{n}\right)^{\frac{1-\gamma}{\gamma}}}{1 + \beta^{-\frac{1}{\gamma}} \left(\frac{e^{\omega}}{n}\right)^{\frac{1-\gamma}{\gamma}}}$$

At this stage of our research we expressed unemployment f through the real wage and elasticity of substitution between leisure and consumption.

2.2. Producer's side

In the economy each good is produced by a particular sector and can either be consumed by households or used by other sectors as a material for production. The production function of the company is a Cobb-Douglas function with constant returns to scale:

$$x_i = z_i^{\alpha} l_i^{\alpha} \prod_{j=1}^{n} \left(x_{ij}\right)^{(1-\alpha) w_{ij}}$$

where l_i - the amount of labor employed in the sector i, $\alpha \in$ (0,1) - the share of labor, x_{ij} - the amount of good j which is used in the production of commodity i, and z_i - the productivity shock of the sector i. Shocks z_i are assumed to be independent across sectors, $log(z_i)$ is further denoted by ε_i. The variable $w_{ij} \geq 0$ stands for the share of commodity j in the input of the sector i. Also, $w_{ij} = 0$ means that sector i does not use good j for production.

We imposed the same assumptions as Acemoglu and his co-authors on w_{ij} and productivity shocks' distributions:

1. The sum of the input shares of all sectors is equal to 1: $\sum_{j=1}^{n} w_{ij} = 1$ for $i = 1, 2, ..., n$.

2. For economies $\{\mathcal{E}_n\}_{n \in N}$ for any sector $i \in I_n$ and all values of $n \in N$,

    - $\mathbb{E}\varepsilon_{in} = 0$.
    - $var(\varepsilon_{in}) = \sigma_{in}^2 \in (\underline{\sigma}^2, \overline{\sigma}^2)$, where $0 < \underline{\sigma} < \overline{\sigma}$ independent of n.

The first assumption is a simple normalization, while the second one puts a considerable restriction: variances of log sectoral shock should stay limited as n $\rightarrow \infty$.

After defining assumptions, we solved the problem of the maximization of the producer's profit, described in the section Appendix 2. As a result, we got the following expressions for l_i and x_{ij}:

$$l_i = \frac{\alpha p_i x_i}{h}$$

$$x_{ij} = \frac{(1-\alpha) p_i w_{ij} x_i}{p_j}$$

We substituted the result into the production function of the firm and made a logarithmic transformation of the expression, described in the section Appendix 3):

$$log(p_i) - (1-\alpha)\sum_{j=1}^{n} w_{ij} log(p_j) = \alpha log(h) - \alpha\varepsilon_i - \alpha log(\alpha)$$
$$-(1-\alpha)log(1-\alpha) - (1-\alpha)\sum_{j=1}^{n} w_{ij} log(w_{ij})$$

Next, we moved the nominal wage h to the left side of the equation:

$$log\left(\frac{p_i}{h}\right) - (1-\alpha)\sum_{j=1}^{n} w_{ij} log\left(\frac{p_j}{h}\right) = -\alpha\varepsilon_i - \alpha log(\alpha)$$
$$- (1-\alpha)log(1-\alpha) - (1-\alpha)\sum_{j=1}^{n} w_{ij} log(w_{ij})$$

We represented this system of equations in matrix form. Firstly, we introduced notations: $log\left(\frac{p}{h}\right)$ - is the column-vector of logarithms of prices divided by nominal wages, W - is the input-output matrix composed of w_{ij}, $\widetilde{W}$ - the matrix consisting of $w_{ij}log(w_{ij})$, ε - column vector of shocks ε_i.

$$log\left(\frac{p}{h}\right) = [I - (1-\alpha)W]^{-1}[[-\alpha log(\alpha) - (1-\alpha)log(1-\alpha)]\mathbf{1}-$$
$$- \alpha\varepsilon - (1-\alpha)\widetilde{W}\mathbf{1}]$$

Then

$$\omega = -\frac{1}{n}\mathbf{1}' log\left(\frac{p}{h}\right) = -\frac{1}{n}\mathbf{1}'[I - (1-\alpha)W]^{-1}[[-\alpha log(\alpha)$$
$$- (1-\alpha)log(1-\alpha)]\mathbf{1} - \alpha\varepsilon - (1-\alpha)\widetilde{W}\mathbf{1}]$$

At this stage, we have obtained an equation that determined the real wage, which depended on the structure of the network (W and $\widetilde{W}$) and shocks ε.

2.3. Aggregate output

The actual aggregate output in the model was defined as the logarithm of the real value added.

$$y = log\left(\frac{\sum_{i=1}^{n}\left(p_i x_i - \sum_{j=1}^{n} p_j x_{ij}\right)}{\prod_{i=1}^{n}(p_i)^{1/n}}\right) = log\left(\frac{\sum_{i=1}^{n} hl_i}{\prod_{i=1}^{n}(p_i)^{1/n}}\right) =$$

$$= log\left(\frac{h(1-f)}{\prod_{i=1}^{n}(p_i)^{1/n}}\right) = log(1-f) - log\left(\prod_{i=1}^{n}\left(\frac{p_i}{h}\right)^{1/n}\right) =$$

$$= log(1-f) - \frac{1}{n}\sum_{i=1}^{n} log\left(\frac{p_i}{h}\right)$$

or

$$y = log\left(\frac{\beta^{-\frac{1}{\gamma}}(\frac{e^{\omega}}{n})^{\frac{1-\gamma}{\gamma}}}{1+\beta^{-\frac{1}{\gamma}}(\frac{e^{\omega}}{n})^{\frac{1-\gamma}{\gamma}}}\right) + \omega$$

Where the real wage:

$$\omega = -\frac{1}{n}\mathbf{1}' log\left(\frac{p}{h}\right) = -\frac{1}{n}\mathbf{1}'[I-(1-\alpha)W]^{-1}[[-\alpha log(\alpha)$$
$$-(1-\alpha)log(1-\alpha)]\mathbf{1} - \alpha\varepsilon - (1-\alpha)\widetilde{W}\mathbf{1}]$$

Thus, we derived equations that described main characteristics of business cycles: aggregate output and unemployment. Both of them depended on the elasticity of substitution between leisure and consumption as well as the network structure of the economy through the real wage.

3. EMPIRICAL ANALYSIS

3.1. Data

The historical data used for conducting the empirical research was assembled by the Bureau of Economic Analysis of the US Department of Commerce. The data demonstrates the results of the comprehensive update of the Industry Economic Accounts accomplished in 2018. Generally, the observations are carried out at intervals of 5 years, which gives the potential for expansive scope and comprehensive analysis of the interaction between industries. For empirical analysis in our research was directly involved data from the 2012 year, which describes the economic performance of 405 commodities in the US economy and the mechanism of their interaction.

The data contains a detailed description of commodity-by-commodity total requirements from the Input-Output matrices. The framework of total requirements shows the production amount necessary from each industry and commodity to deliver 1 dollar of a commodity to final users directly or indirectly. In turn, the Input-Output accounts show the exact mechanism of interconnection between the sectors that supply each other with materials for production and final marketing of goods and services. These accounts provide detailed information on the flows of goods and services in the production process.

3.2. Simulations

To assess the consistency of the model we developed the program in Python based on the derived functions of the GDP and unemployment. The interaction between firms was demonstrated by the shock propagation through the intersectoral linkages. One hundred idiosyncratic industry shocks ε were artificially generated by independent and identically distributed (i.i.d) normal vectors. Consequently, the mean and variance of the GDP and unemployment were estimated over generated shock vectors for different values of the elasticity of substitution (γ) between consumption and leisure time.

This allowed us to examine the main characteristics of the business cycle within the model of changing elasticity of substitution. The results are presented in the graphs below.

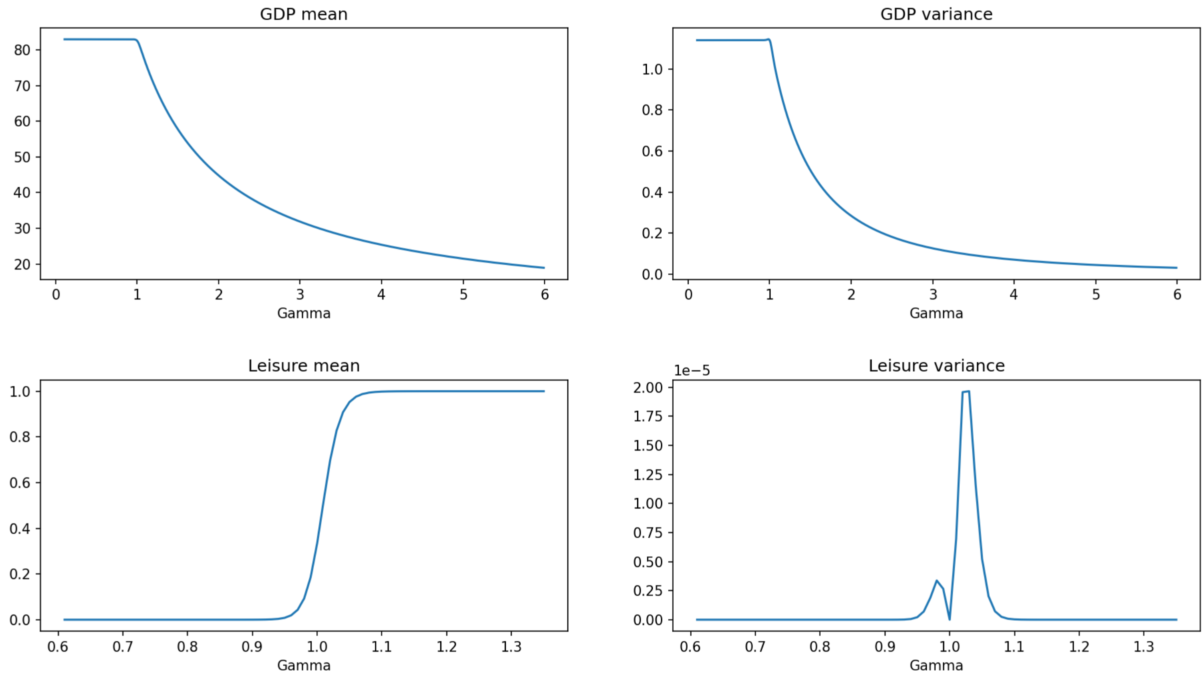

Figure 1: GDP and unemployment mean and variance

If consumption and leisure are complements, both mean and variance of unemployment tend to zero, although the GDP and its variance are constant and take non-zero values. If consumption and leisure are substitutes, the mean and variance of output "drop", while unemployment mean tends to 1. Observed fluctuations in the variance of unemployment around 1 can be explained by the parameter β, which describes the preferences of the consumer regarding consumption and leisure However, these fluctuations are extremely small (y-axis is measured in 10^{-5}) compared to changes in GPD.

Consequently, from the graphs, it could be observed that on the side where consumption and leisure time were complements ($\gamma < 1$) and on the side where they were substitutes ($\gamma > 1$), the behavior of mean and variance of the GDP differs dramatically. Thus, depending on the preferences of the consumer between consumption and leisure, mean and volatility of the GDP function varied greatly. Furthermore, the mean of the GDP, in other words, the trend value, around which fluctuations occurred, dropped sharply when the γ became a little more than 1. In order to explain the behavior of the GDP variance, we have built graphs of functions depending on the real wage.

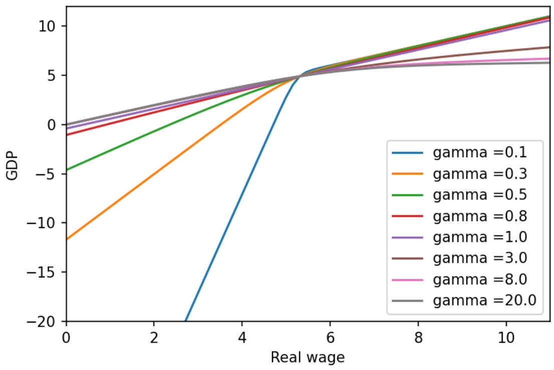

Figure 2: GDP from real wage

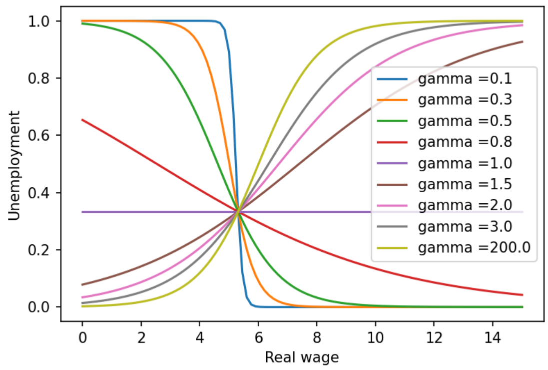

Figure 3: Leisure from real wage

On the other side, when consumption and leisure time are complements, the range of GDP values appears to be wide, whereas when they are substitutes, the graph "shrank". Hence, with substitutes, the range of the function was much smaller. We can explain it intuitively. What does it mean that consumption and leisure are complements? If the price of leisure (i.e. wage) increases, then a person consumes less as he enjoys both consumption and leisure in combination. This means that he is more sensitive to high wages.

As we mentioned earlier, in empirical economic literature leisure and consumption are considered to be substitutes. This fact means that the realistic part of the graphs is shown where values of γ are more than 1. Continuing the reasoning, if consumption and leisure are substitutes, then a person is less sensitive to high wage. If the price of leisure increases, he consumes more, but works less. Thus, a person whose consumption and leisure are substitutes responds less strongly to a wage volatility than one whose consumption and leisure are complements.

To conclude, the elasticity of substitution between leisure and consumption has a significant impact on the characteristics of the business cycle: GDP and unemployment. It influences not only the volatility but also the trend value of the variables.

CONCLUSION

This research work aimed at analyzing how GDP and unemployment are affected by the elasticity of substitution between leisure and consumption within models of the network structure of the economy. The multi-sectoral model of real business cycles from Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) was supplemented by introducing an additional factor - the share of leisure time of a representative agent. The Cobb-Douglas utility function of the consumer was replaced with the CES function which has great tractability.

In the current study, for theoretical analysis, we develop the mathematical model, derive functions for output and unemployment and demonstrate the nontrivial dependence between the network structure and these characteristics of the business cycle. Meanwhile, the original model (Acemoglu, 2012) shows the dependence of the aggregate output only on the real wage, in our model it is decomposed into fluctuations of the real wage and employment. Furthermore, the model shows that both aggregate GDP and unemployment nontrivially depend on the elasticity of substitution between consumption and leisure.

For empirical analysis, we examine the impact of elasticity of substitution between leisure and consumption on GDP and unemployment using historical data and simulations of idiosyncratic shocks of firms. Firstly, we found out that the elasticity of substitution has a significant impact on the characteristics of business cycle and not only on their volatility but also the trend value. Secondly, depending on the consumer's preferences between consumption and leisure time, the mean and volatility of the GDP and unemployment rate changed considerably: the person whose consumption and leisure are substitutes responds less strongly to wage volatility than one whose consumption and leisure are complements.

In conclusion, our model contributes and expands the literature related on the topic of intersectoral input-output linkages. The model serves as a complex instrument for the development and implementation of macroeconomic policy. It has a great perspective for precise forecasting based on the data of a particular economy taking into account all the factors realized and can be used as a base tool for the facilitation of the procedure aimed to solve the unemployment issues.

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APPENDICES

Appendix 1

Consumer utility maximization.

In the given model preferences of a consumer are described by CES-function:

$$U(C, 1-l) = \frac{C^{1-\gamma} - 1}{1-\gamma} + \beta \frac{(1-l)^{1-\gamma} - 1}{1-\gamma}$$

where $C = \prod_{i=1}^{n} (c_i)^{1/n}$ - consumption of good, $1 - l = f$ - amount of leisure time, γ - elasticity of substitution between leisure and consumption.

With a budget constraint:

$$\sum_{i=1}^{n} p_i c_i + hf = h$$

where p_i - the price of good i, h - nominal wage.

The Lagrangian function is:

$$L = \frac{\left(\prod_{i=1}^{n} (c_i)^{1/n}\right)^{1-\gamma} - 1}{1-\gamma} + \alpha \frac{f^{1-\gamma} - 1}{1-\gamma} + \lambda(h - \sum_{i=1}^{n} p_i c_i - hf)$$

First-order condition equations for f и c_i:

$$(1) \quad \frac{\partial L}{\partial f} = \frac{\alpha(1-\gamma)}{1-\gamma} * f^{-\gamma} - \lambda h = 0 \quad \Rightarrow \quad \lambda = \frac{\alpha}{hf^{\gamma}}$$

$$(2) \quad \frac{\partial L}{\partial c_i} = \frac{1-\gamma}{1-\gamma} \left(\prod_{i=1}^{n} (c_i)^{1/n}\right)^{-\gamma} \frac{1}{n} c_i^{-1} \prod_{i=1}^{n} (c_i)^{1/n} - \lambda p_i = 0$$

$$(3) \quad \frac{\partial L}{\partial c_j} = \frac{1-\gamma}{1-\gamma} \left(\prod_{i=1}^{n} (c_i)^{1/n}\right)^{-\gamma} \frac{1}{n} c_j^{-1} \prod_{i=1}^{n} (c_i)^{1/n} - \lambda p_j = 0$$

From (2) and (3) we got:

$$\frac{c_j}{c_i} = \frac{p_i}{p_j} \quad \Rightarrow \quad c_i p_i = c_j p_j$$

According to a budget constraint:

$$\sum_{i=1}^{n} p_i c_i + hf = h \quad \Rightarrow \quad \sum_{i=1}^{n} p_i c_i = h(1-f)$$

Then:

$$n p_i c_i = h(1-f) \quad \Rightarrow \quad c_i = \frac{h(1-f)}{n p_i}$$

From (1) and (2):

$$\frac{\left(\prod_{i=1}^{n} (c_i)^{1/n}\right)^{1-\gamma}}{n c_i p_i} = \frac{\alpha}{f^{\gamma} h}$$

We substituted c_i into upper equation:

$$\frac{\left(\prod_{i=1}^{n} (\frac{h(1-f)}{n p_i})^{1/n}\right)^{1-\gamma}}{n p_i \frac{h(1-f)}{n p_i}} = \frac{\alpha}{f^{\gamma} h} \quad \Rightarrow$$

$$\left(\frac{h(1-f)}{n}\right)^{1-\gamma} \frac{1}{\left(\prod_{i=1}^{n} (p_i)^{1/n}\right)^{1-\gamma}} = \frac{\alpha(1-f)}{f^{\gamma}} \quad \Rightarrow$$

$$\left(\frac{1-f}{f}\right)^{\gamma} = \frac{1}{\alpha} \left(\frac{h}{n \prod_{i=1}^{n} (p_i)^{1/n}}\right)^{1-\gamma} \quad \Rightarrow$$

$$\frac{1-f}{f} = \left(\frac{1}{\alpha}\right)^{\frac{1}{\gamma}} \left(\frac{h}{n\prod\limits_{i=1}^{n}(p_i)^{1/n}}\right)^{\frac{1-\gamma}{\gamma}}$$

Finally, we got:

$$f = \frac{1}{1+\beta^{-\frac{1}{\gamma}}\left(\frac{h}{n\prod\limits_{i=1}^{n}(p_i)^{1/n}}\right)^{\frac{1-\gamma}{\gamma}}}$$

$$c_i = \frac{h\left(1-f\right)}{np_i} = \frac{h}{np_i}\frac{\left(\frac{h}{n\prod\limits_{i=1}^{n}(p_i)^{1/n}}\right)^{\frac{1-\gamma}{\gamma}}}{1+\left(\frac{h}{n\prod\limits_{i=1}^{n}(p_i)^{1/n}}\right)^{\frac{1-\gamma}{\gamma}}}$$

Appendix 2

Producers' profit maximization.

Firm's production function:

$$x_i = z_i^\alpha l_i^\alpha \prod_{j=1}^{n} (x_{ij})^{(1-\alpha)w_{ij}}$$

With a budget constraint:

$$p_i x_i = h l_i + \sum_{j=1}^{n} p_j x_{ij}$$

Marginal product of x_{ij}:

$$\frac{\partial x_i}{\partial x_{ij}} = z_i^\alpha l_i^\alpha (1-\alpha) w_{ij} x_{ij}^{-1} \prod_{j=1}^{n} (x_{ij})^{w_{ij}(1-\alpha)}$$

Marginal product of l_i:

$$\frac{\partial x_i}{\partial l_i} = \alpha z_i^\alpha l_i^{\alpha-1} \prod_{j=1}^{n} (x_{ij})^{w_{ij}(1-\alpha)}$$

First-order condition equations:

$$(1) \quad p_i \frac{\partial x_i}{\partial x_{ij}} = p_j \quad \Rightarrow \quad p_i z_i^\alpha l_i^\alpha (1-\alpha) w_{ij} x_{ij}^{-1} \prod_{j=1}^{n} (x_{ij})^{w_{ij}(1-\alpha)} = p_j \quad \Rightarrow$$

$$p_i x_i (1-\alpha) w_{ij} (x_{ij})^{-1} = p_j \quad \Rightarrow \quad x_{ij} = \frac{(1-\alpha) p_i w_{ij} x_i}{p_j}$$

$$(2) \quad p_i \frac{\partial x_i}{\partial l_i} = h \quad \Rightarrow \quad p_i \alpha z_i^\alpha l_i^{\alpha-1} \prod_{j=1}^{n} (x_{ij})^{w_{ij}(1-\alpha)} = h \quad \Rightarrow$$

$$\frac{p_i \alpha x_i}{l_i} = h \quad \Rightarrow \quad l_i = \frac{\alpha p_i x_i}{h}$$

Optimal amounts of l_i и x_{ij}:

$$l_i = \frac{\alpha p_i x_i}{h}$$

$$x_{ij} = \frac{(1-\alpha) p_i w_{ij} x_i}{p_j}$$

Appendix 3

Derivation of a system of equations that determines equilibrium prices. Taking the logarithm of the production function x_i:

$$x_i = z_i^{\alpha} l_i^{\alpha} \prod_{j=1}^{n} (x_{ij})^{(1-\alpha)w_{ij}}$$

$$log(x_i) = \alpha log(z_i) + \alpha log(l_i) + (1-\alpha) log \prod_{j=1}^{n} {x_{ij}}^{w_{ij}}$$

Substituting the solution of the profit maximization problem l_i и x_{ij}:

$$log(x_i) = \alpha log(z_i) + \alpha log\left(\frac{\alpha p_i x_i}{h}\right) + (1-\alpha) log \prod_{j=1}^{n} \left(\frac{(1-\alpha) p_i w_{ij} x_i}{p_j}\right)^{w_{ij}}$$

$$log(x_i) = \alpha log(z_i) + \alpha log(\alpha) + \alpha log(p_i) + \alpha log(x_i) - \alpha log(h) +$$
$$+ (1-\alpha) \sum_{j=1}^{n} w_{ij} log(1-\alpha) + (1-\alpha) \sum_{j=1}^{n} w_{ij} log(p_i) + (1-\alpha) \sum_{j=1}^{n} w_{ij} log(w_{ij}) -$$
$$- (1-\alpha) \sum_{j=1}^{n} w_{ij} log(p_j) + (1-\alpha) \sum_{j=1}^{n} w_{ij} log(x_i)$$

According to our notations $\varepsilon_i = log(z_i)$. The sum of the cost shares of all sectors used by the i-th sector is equal to 1, i.e. $\sum_{j=1}^{n} w_{ij} = 1$.
After expressing the equilibrium prices, we got the following expression:

$$log(p_i) - (1-\alpha) \sum_{j=1}^{n} w_{ij} log(p_j) = \alpha log(h) - \alpha \varepsilon_i - \alpha log(\alpha)$$
$$- (1-\alpha) log(1-\alpha) - (1-\alpha) \sum_{j=1}^{n} w_{ij} log(w_{ij})$$