Asymmetric Effects of Monetary Policy in the US:
Positive vs. Negative or Big vs. Small?

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Abstract

This paper reconsiders the empirical evidence on the asymmetric output effects of monetary policy. Asymmetric effects is a common feature of many theoretical models, and there are many different versions of such asymmetries. We concentrate on the distinctions between positive and negative money-supply changes, big and small changes in money-supply, and possible combinations of the two asymmetries. Earlier research has found empirical evidence in favor of the former of these in US data. Using M1 as the monetary variable we find evidence in favor of neutrality of big shocks and non-neutrality of small shocks. The results may, however, be affected by structural instability of M1 demand. Thus, we substitute M1 with the federal funds rate. In these data we find that only small negative shocks affect real aggregate activity. The results are interpreted in terms of menu-cost models.

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1 Introduction

In this paper we will test whether there are asymmetric effects of money-supply on real aggregate activity using US postwar quarterly data. Asymmetric effects of monetary policy can arise in a number of different ways and can hypothetically have strong implications for the conduct of economic policy. We focus on the possible asymmetries in the way that different types of monetary policy shocks affect real activity. Other types of asymmetric effects have been explored by Garcia and Schaller (1996) who examine whether monetary policy affects output differently in different phases of the business cycle and Ravn and Sola (1997) who look at the effects of monetary policy on the transitional dynamics. Hooker and Knetter (1996) analyze whether military procurement spending has asymmetric effects on employment and they find that “large” negative procurement shocks have proportionally larger effects on employment growth than large positive shocks or small shocks to procurement. Hooker (1996) examines whether there are asymmetries in the relationship between oil-price shocks and US macroeconomic variables. He finds that the asymmetric effects in this relationship are fragile. We shall, however, focus on the relationship between money-supply shocks and aggregate activity.

So far, the literature have focused on a particular asymmetry that we will denote “the traditional Keynesian asymmetry”, which states that positive money-supply shocks are neutral while negative money-supply shocks have real effects. This asymmetry can be derived under the assumption of either downwards (upwards) sticky (flexible) nominal wages or sticky prices together with rationing of demand. Cover (1992) and DeLong and Summers (1988) have tested for this asymmetry in US data. Cover (1992) finds firm support for the hypothesis in quarterly postwar data and he shows that the results are robust to the specification of money-supply and to the inclusion of lagged money-supply shocks in the output equation. DeLong and Summers (1988) find that negative money-supply shocks have a greater output effect than positive money-supply shocks. Karras (1996) have analyzed data for a number of European countries and finds strong evidence in favor of the traditional Keynesian asymmetry hypothesis. Macklem, Paquet and Phaneuf (1996) find results for Canada and the US in line with those quoted above when including evidence from the yield curve (the spread) and when controlling for foreign factors. Sensier (1996) finds less firm support for the asymmetry hypothesis in UK data.

We will take as a starting point the type of asymmetric effects that arise in models with menu-costs and we will test these asymmetries against the “traditional Keynesian” asymmetry. In static (non-stochastic) settings, a standard menu-cost model implies that “big” money-supply shocks (relative to the degree of real rigidity and the size of the menu-cost) are neutral because firms would be subject to first-order costs by following a fixed price strategy. In contrast, “small” money-supply costs would have real effects because a strategy of keeping nominal prices fixed would be associated with only a second-order cost. Hence, “small” shocks to nominal demand have real effects while “big” shocks are neutral. Note that this asymmetry is different from “the traditional Keynesian asymmetry”
described above.

These results are slightly different if one considers a stochastic setting. Consider e.g. a model in which firms have to choose between indexing their prices (at the cost of paying the menu-cost) or non-indexation of prices before the money-supply shock is observed. As shown by Ball and Romer (1990), firms will choose indexation (non-indexation) if the variance of money-supply is high (low) and high-variance (low-variance) shocks would therefore be neutral (have real effects). One slightly disadvantageous feature of this is that in this case one should either always have neutrality or always have non-neutrality of monetary policy. However, we extend the analysis by assuming that the money-supply process can change between having a “high” variance or a “low” variance. In this setting, the relevant distinction is between low-variance shocks (which have real effects) and high variance shocks (which are neutral). This set-up may produce periods of neutrality and periods of non-neutrality according to the predictability of monetary policy. This is the second version of asymmetric effects that we shall test for using US quarterly data.

A final case that we look at is an intermediate case in which only “small negative” shocks to nominal demand affect real aggregate activity. This case can be derived from the dynamic menu-cost model in which there is positive steady-state inflation and has been analyzed by Ball and Mankiw (1994). They look at a model where firms can change prices costlessly every second period but, if firms want to change prices in between the two periods, they must pay the menu-cost. This gives rise to an asymmetric pricing rule in which “inaction” (not changing prices in between periods in response to an unanticipated money-supply shock) is optimal for a wider range of negative shocks than positive shocks. We call this case for the “hybrid” asymmetry because it has similarities both to the traditional Keynesian asymmetry and to the standard menu-cost asymmetry.

The procedure that we propose consists of estimating a money-supply process in which one allows for changes-in-regime. We shall be using the regime-switching model of Hamilton (1988) appropriately modified to allow for lagged exogenous variables that are not allowed to switch between regimes. We model the money-supply as a regime-switching process that can have changes in the mean and in the variance of the innovations to the process. Because of the regime-switching technique, we can distinguish between four different shocks to money supply: “big positive shocks”, “big negative shocks”, “small positive shocks” and “small negative shocks”. The distinction between “big” and “small” is here made by reference to the variance of the innovations in the two states (since this is the prediction of the menu-cost type models in stochastic settings).

This technique allows us to test simultaneously for the existence of the three versions of asymmetric effects that are our primary interest, i.e. whether there is evidence of the traditional Keynesian asymmetry, the asymmetry of “big” and “small” shocks, and the

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1Garcia and Schaller (1995) also apply the Hamilton (1989) Markov-Switching methodology to investigate the question of asymmetric effects of monetary policy. They focus, however, on whether the effects of monetary policy depends on the state of the economy, i.e. whether there is a relationship between the phase of output and the output effects of monetary policy.
"hybrid" asymmetry. We estimate a simultaneous system consisting of a money-supply equation and an output equation. The current shocks from the "money-supply" relationship (i.e. unanticipated money-supply shocks) feeds into the output equation. We then test for asymmetries by introducing various parameter restrictions on the four different types of unanticipated money-supply shocks in the output equation and by applying likelihood ratio tests.

We look at two different sets of quarterly data for the US for the postwar period. We first look at the data considered previously by Cover (1992). This data set covers the period 1947-1987. In this application we measure money-supply as M1 and real activity as GNP in constant prices (as Cover, 1992). We investigate these data because Cover (1992) has previously documented evidence in favor of the traditional Keynesian asymmetry using these data and we wish to check whether the results are robust to the inclusion of alternative theories of asymmetric effects. The second set of data we look at are data for the period 1960-1995 which has previously been investigated by, among others, Christiano, Eichenbaum and Evans (1996). In this application we measure money-supply as (minus) the federal funds rate\(^2\). The motivation for using the federal funds rate rather than M1 (or other measures such as M2 or M3) is that the federal funds rate is likely to be a less endogenous and more stable measure of monetary policy than M1. It is well-documented for the US that M1 money-demands are unstable and, hence, it might be difficult to identify M1-shocks with changes in money-supply.

In the analysis of the data previously analyzed in detail by Cover (1992) we find that there have indeed been regime-changes in the money-supply relationship. Our results imply that the money-supply is divided into a low-growth, low-variance regime that spans the period from the beginning of the sample up to around 1967 and a high-mean, high-variance regime that takes over for the majority of the period after 1968. Furthermore, when we test for the presence of asymmetric effects, we find strong evidence in favor of neutrality of money-supply shocks when the variance is high and non-neutrality when the variance is low. This result is in line with the predictions of the standard menu-cost model in a stochastic environment. However, as discussed above, because there is evidence other places that the shocks that we identify as money-supply shocks may indeed be money-demand shocks, one may be somewhat sceptical about these results.

For this reason we also look at the alternative data set using the (minus) federal funds rate as the measure of monetary policy. Again, the money-supply process is divided into a regime with a low mean and a low variance and another regime with a high mean and a high variance of money-supply. The timing of the two regimes is, however, very different from the results using M1. We find that the low-mean, low-variance, regime dominates the money-supply for most of the period. The periods where the other regime dominates are identified as a short period in the mid-seventies and the Volcker period. When we test for asymmetric effects using these alternative data we find that there is very strong

\(^2\)We use minus the federal funds rate because a positive (negative) money-supply shock then corresponds to a loosening (tightening) of monetary policy.
evidence in favor of the “hybrid asymmetry”, i.e. that only small negative money-supply shocks have real effects. This finding is in line with the menu-cost model analyzed by Ball and Mankiw (1994) for an economy with positive steady-state inflation. Furthermore, the results once more imply that unless one allows for alternative asymmetries, one would accept the traditional Keynesian asymmetry.

The remainder of the paper is organized as follows. In Section 2 we look into the implications for asymmetric effects of standard menu-cost models. Section 3 is devoted to a description of the empirical method that we will apply. In Section 4 we examine the two alternative sets of US data and test for the presence of asymmetries. Section 5 summarizes and draws some conclusions.

2 Theoretical Considerations

Asymmetries in the relationship between nominal demand and aggregate output is a feature that can arise in many theoretical models and in many different versions. Here we will consider some of these theoretical possibilities in order to motivate the empirical analysis. This will also help clarifying the different versions of asymmetric effects.

In the traditional Keynesian literature building on sticky wages or sticky prices, the natural candidate for asymmetric effects is related to different real effects of positive and negative changes in nominal demand. Consider a textbook Keynesian model with downwards (upwards) sticky (flexible) nominal wages. Assume also that the labor market initially clears at the nominal wage that corresponds to the price level (and expected price level) consistent with the current level nominal demand and assume that the long run supply curve is vertical. In this model the supply curve will be vertical at the expected price level but positively sloped for price levels below the expected price level. Hence, an unanticipated increase in nominal demand will be neutral since nominal wages are flexible upwards but an unanticipated decrease in nominal demand will be associated with lower output and employment since nominal wages are inflexible downwards.

This is the type of asymmetry that has been tested empirically for the US by DeLong and Summers (1988) and by Cover (1992). Both of these studies find empirical support for this phenomenon. This result is potentially important for the conduct of monetary policy and imply that average activity can be increased by lowering the variance of money-supply shocks. The problem with the analysis above is the lack of clear microfoundations. Economic agents may adjust to the economic environment and such adjustments may have implications for the result on asymmetric effects. Hence, it is important to consider models in which decision rules are explicitly derived. We will consider if such asymmetries can arise in menu-cost type models and derive the specific types of non-linearities in the relationship between activity and nominal demand.

Akerlof and Yellen (1985), Mankiw (1985) and Blanchard and Kiyotaki (1987) have analyzed how menu-cost models or near-rationality may affect the pricing decisions of
firms and how this affects the real effects of changes in nominal demand. Here we follow the presentation in Ball and Romer (1990) and Ball and Mankiw (1994). Consider an economy with a large number of price setting agents each of whom acts as a producer/consumer. Each agent produces a single differentiated good which is sold at the nominal price $P_i$. It is assumed that there is a small cost, the menu-cost, of changing nominal prices denoted by $s > 0$. Let the utility of agent $i$ be given as:

$$U_i = G\left(Y, \frac{P_i}{P}\right) - sD_i$$

(1)

where $Y$ denotes aggregate real spending, $P$ denotes the aggregate price level, and $D_i$ is a dummy variable which equals one if prices are changed and zero otherwise. The function $G$ can be thought as incorporating the effect of aggregate income on the demand for each agent’s good (through $Y$) and the point on the demand curve at which the agent is producing (through $P_i/P$). Alternatively, one can think of (1) in terms of a monopolistic competition model in which firms are owned by the agents. Following Ball and Romer (1990) let us assume that velocity is equal to unity, i.e. $Y = M/P$, where $M$ denotes the nominal money stock. (1) can then written as:

$$U_i = G\left(M, \frac{P_i}{P}\right) - sD_i$$

(2)

In the absence of menu-costs (i.e. $s = 0$), the first-order condition for each agent is that $G_2\left(Y, \frac{P_i}{P}\right) = 0$, where $G_2$ denotes the derivative of $G$ with respect to the second argument. In this case changes in $M$ are neutral (in a symmetric equilibrium) since the first-order condition is homogenous of degree zero. It is assume that there is such a symmetric equilibrium and that it corresponds to $M = P = P_i = 1^3$. Consider now an experiment where prices of all producers are set based on an expected money-supply equal to 1 but after this a level of the money supply, $M \neq 1$, is realized. Each producer now has to decide whether to change nominal prices (setting them equal to $P_i^*$) and pay the menu-cost or whether to maintain prices ($P_i$). If all producers change prices, the unanticipated change in nominal demand will be neutral. In contrast, if no producer changes the price, the nominal demand shock will affect aggregate activity.

Assume first that every price setter except $i$ expect every other price setter not to change prices. The utility of not changing the price for agent $i$ is then given as $U^{NA} = G(M, 1)$. If the agent decides to change the price of good $i$, utility is given by $U^{CP} = G(M, P_i^*/P) - s$. Hence, inaction is an equilibrium if:

$$U^{NA} - U^{CP} > 0 \Rightarrow G\left(M, \frac{P_i^*}{P}\right) - G(M, 1) < s$$

(3)

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3 Strictly speaking, one also needs to assume that the second-order condition is fulfilled and that the equilibrium is stable (i.e. $G_{22}(1, 1) < 0$ and $G_{12}(1, 1) > 0$).
This implies that there is a range of money-supplies for which inaction is a possible equilibrium\(^4\). Making a second-order Taylor approximation around \(M = 1\) gives us that this range is given by:

\[
M \in (1 - M^*; 1 + M^*), \quad M^* = \sqrt{-\frac{2G_{22}s}{G_{12}^2}}
\]

(4)

Similarly, there is a range of money-supply shocks for which the producer finds it optimal to change prices even in expectation of all other producers not changing prices. This range is given by:

\[
M \in (-\infty; M^{**}) \cup (M^{**}; \infty), \quad M^{**} = \sqrt{-\frac{2s}{G_{22}}}
\]

(5)

The insight from this is that there is an asymmetry in how money-supply affects aggregate output. If \(M \in (1 - M^*; 1 + M^*)\), money-supply changes may have real effects, i.e. “small” changes in nominal demand should be non-neutral. In contrast, “big” changes \((M \in (-\infty; M^{**}) \cup (M^{**}; \infty))\) will be neutral since all agents will find it optimal to change prices. Hence, the basic menu-cost model implies a different asymmetric effect than the traditional Keynesian model. With menu-costs and no other features, it is the size of the change in nominal demand that matters (not the sign of the change).

Above, the changes in money-supply are to be regarded as zero-probability events. Price setters do not perceive the possibility of a change in money-supply and act only after observing the realization of money-supply. Assume now alternatively that private agents know that money-supply is distributed with a mean \(M\) and a variance \(\sigma^2\) and, as Ball and Romer (1990), that agents must decide whether to pay the menu-cost before they observe the current money-supply shock. Thus, by construction, agents either choose indexation (pay the menu-cost) or non-indexation. Ball and Romer (1989, 1990) show that in this model, non-indexation is an equilibrium for:

\[
EG\left(\frac{M}{P_0}, \frac{P^*_t}{P_0}\right) - EG\left(\frac{M}{P_0}, 1\right) \simeq -\frac{G_{12}^2}{2G_{22}} \sigma^2 < s
\]

(6)

where \(1/P_0 \simeq 1 - \sigma^2 G_{211}^2/(2G_{22})\). The difference between this case and the analysis above is that the decision of whether or not to index prices, i.e. to pay the menu-cost, is determined by the variance of the money-supply shock. If the variance is high, firms index nominal prices because there is a high probability of a “big” shock, and they choose inaction for a low variance. In the former case, monetary disturbances are neutral, while they have short-run effects in the latter case. This result resembles the previous analysis where it predicts that monetary policy is either always neutral or always

\(^4\)It is possible that this range overlaps with a range of money-supplies for which it is also optimal for all agents to change prices. There is no straightforward criteria available for choosing among these equilibria if the two ranges overlap.
non-neutral. In terms of empirical tests, this results is rather unsatisfactory because it simply implies that both neutrality and non-neutrality could be consistent with the theory and it does as such not have testable implications except if one was able to estimate the various parameters that enter into (6).

This rather unsatisfactory implication can be overturned by a slight modification of the analysis. Assume that money-supply can be in either of two states of nature. In one state the variance of money-supply is low and equal to \( \sigma^2 \) and in the other state the variance of money-supply is high and equal to \( \sigma^2 > \sigma^2 \). Let us also assume that the state variable that dictates the variance of money-supply follows a first-order Markov-process. Let \( \pi_{ij} \) be the probability that, given that the observed state today is \( i \), the realized state tomorrow is \( j \). Then the probability transition matrix is given by:

\[
\Pi = \begin{bmatrix} \pi_00 & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix}
\]

where each row sums to 1. Assume also that agents observe the current state when setting the initial price and when deciding whether to pay the menu-cost or not. Then using the same reasoning as above shows that inaction is an equilibrium when:

\[
-\frac{G^2_{12}}{2G_{22}} \left( \pi_{00}\sigma_0^2 + \pi_{01}\sigma_1^2 \right) < s \quad \text{when the current state is 0}
\]

\[
-\frac{G^2_{12}}{2G_{22}} \left( \pi_{10}\sigma_0^2 + \pi_{11}\sigma_1^2 \right) < s \quad \text{when the current state is 1}
\]

There are essentially three possible outcomes here. If (i) the difference between \( \sigma_0^2 \) and \( \sigma_1^2 \) is small, or (ii) either \( \pi_{01} \) or \( \pi_{10} \) is close to 1, then there will either be indexation or in non-indexation in both states. These two possibilities are similar to the analysis above of the one-state model of Ball and Romer (1990). The more interesting possibility is where neither of these two conditions are fulfilled. If there is a non-trivial difference between the two variances and the states are relatively persistent, then it is possible that there will be indexation if today’s state is 1 and non-indexation if today’s state is 0. Hence, as in the standard menu-cost model, firms’ actions depend upon the monetary policy that they observe and their expectations on tomorrow’s monetary policy. If there is persistence in monetary policy regimes and they are sufficiently different, firms will choose to index prices when they observe highly unpredictable monetary disturbances but will revert to non-indexation when monetary policy becomes more stable\(^5\).

The implication of this is that there are asymmetric effects of monetary policy but in contrast to the menu-cost model with zero-probability events, the distinction is between situations with “big” variance of unanticipated shocks to money-supply and situations with “low” variance of the unanticipated money-supply shocks. Hence, once again the

\(^5\)These results can be extended to the case where also the mean changes between regimes and to a case where agent firm solves a dynamic problem.
menu-cost model does not produce the traditional Keynesian asymmetry but rather a distinction between “big” and “small”. Notice also that the above result implies that firms may change their indexation rules over time.

Ball and Mankiw (1994) look at a menu-cost model in which firms face a two-period problem and in which there is positive steady-state inflation (equal to $\hat{p}$). In this set-up they derive the possibility for an outcome that resembles the traditional Keynesian distinction between positive and negative innovations to money-supply. In their analysis, each firm initially sets a price. This price can be changed next period but, in this period, a change in the nominal price is subject to a menu-cost. They also assume the loss functions are quadratic such that, for a big enough menu-cost, firms will choose a price that equals half the steady-state inflation rate in both periods\footnote{If we let $\dot{p}$ denote the steady-state inflation rate, then with a quadratic loss function, it is optimal to set prices at $\dot{p}/2$ in both periods given that $s > \dot{p}^2/2$.}. If an unanticipated shock arrives in period 1 it might be optimal for firms to pay the menu-cost and change prices. Since the optimal price in period 1 ($\dot{p}$) is already above the price set at period 0 ($\dot{p}/2$), it is clear that positive disturbances will lead to a greater incentive to change prices than negative disturbances. This implies that there will be an asymmetric price adjustment rule. In particular, they show that in the quadratic set-up, the range of non-action (i.e. the set of money-supplies that are non-neutral) is given as $M \in \left(-\sqrt{s-\dot{p}/2}; \sqrt{s-\dot{p}/2}\right)$, which is a symmetric interval around $-\dot{p}/2$ but asymmetric around 0.

This implies that positive money-supply shocks are more likely to be neutral than negative money-supply shocks. The reason is that in the face of the positive steady-state inflation, “moderate” negative shocks bring the actual price closer to the optimal price and oppositely so for positive shocks. The model therefore implies an asymmetry which is similar to both the basic menu-cost results discussed above and to the traditional Keynesian asymmetry. We will call this for a “hybrid” asymmetry.

All of the cases discussed above relate to how different monetary policy shocks affect output. This is the kind of asymmetry that we shall investigate empirically, but this may not be the only sort of asymmetric effects in the relationship between nominal demand and real activity. An alternative asymmetry is that monetary policy may affect aggregate output and employment differently in booms and recessions. In booms credit and liquidity may be readily available and it is thus likely that monetary shocks are neutral (or close to being neutral). In recessions, however, firms and consumers may find it harder to obtain funds for investment, production or consumption and monetary policy might have real effects through the credit and liquidity channels. This is the mechanism looked into in the research on financial market imperfections, see e.g. Bernanke and Gertler (1989), Gertler (1992), Greenwald and Stiglitz (1993) and Shleifer and Vishny (1992) among many others. Albeit of great interest, we shall not be concerned with this possible asymmetry but concentrate on the above versions of asymmetric effects.

Finally, it is worth mentioning that the above discussion concentrated on sticky prices
rather than sticky wages. Imperfections in the labor market such as the existence of efficiency wage considerations or insider-outsider phenomena can be coupled with the menu-cost models and has been investigated by Akerlof and Yellen (1985) and Ball and Romer (1990). The importance of such considerations for our interests, is that real rigidities increase the importance of nominal rigidities. Ball and Romer (1990) show that nominal rigidities by themselves may not lead to major real effects of nominal shocks but that real rigidities (such as imperfections in the labor market) can increase these effects.

3 Empirical Methodology

In this section we describe in some detail the empirical methodology that will be applied to the tests for asymmetric effects. DeLong and Summers (1988) and Cover (1992) have previously examined the US evidence on the asymmetric effects of money-supply shocks. Both of these papers apply a method that can be seen as an extension of the procedure used for testing the New Classical theories of imperfect information-based non-neutralities (developed by Lucas, 1972, 1975) in Barro (1977,1978), Barro and Herchowitz (1980), Boschen and Grossman (1982), and, in particular, Mishkin (1982). Two relationships are estimated simultaneously. The first of these is meant to capture money-supply. From this relationship one obtains estimates of the anticipated and the unanticipated money-supply shocks. These shocks feed into the next equation meant to capture aggregate output. Both DeLong and Summers (1988) and Cover (1992) tested for whether positive and negative unanticipated money-supply shocks have different effects on real activity. Both of these studies found strong support for the traditional Keynesian asymmetry in US data.

Their methodology can be summarized as follows. First, the following two relationships are estimated (simultaneously):

$$\Delta m_t = \Phi (L) \Delta m_{t-1} + \Theta x_{t-1} + \varepsilon_t$$  

$$\Delta y_t = \psi z_t + \beta^+ \varepsilon_t^+ + \beta^- \varepsilon_t^- + \xi_t$$  

where $\Delta$ is the first-difference operator, $m_t$ is the measure of the monetary policy, $\Phi (L)$ is a lag-polynomial, $\Theta$ is a vector of parameters, $x_{t-1}$ is a vector of exogenous regressors, $y_t$ is the measure of real aggregate activity, $\psi$ is a parameter-vector, and $z_t$ is a vector of regressors. The instruments $z_t$ can include lagged values of dependent and independent variables. $\varepsilon_t^+$ and $\varepsilon_t^-$ are the positive and negative parts of $\varepsilon_t$ from equation (9) and are defined as:

$$\varepsilon_t^+ \equiv \max (0, \varepsilon_t), \varepsilon_t^- \equiv \min (0, \varepsilon_t)$$

\footnote{For example in terms of the relationship in (4), real rigidities can interpreted in as affecting the range of inaction through $-G_{12}/G_{22}$, see Ball and Romer (1990).}
Equation (9) is to be thought of as a money-supply process and equation (10) as an aggregate output equation. The asymmetry hypothesis is a test on whether $\beta^+$ equals $\beta^-$; rejection of this restriction together with $\beta^+$ being insignificantly different from zero and $\beta^-$ significantly different from zero, supports the hypothesis.

We extend the above analysis. In order to do so we will distinguish not only positive and negative money supply shocks, but also between “big” and “small” shocks. Furthermore, as made clear above, in a stochastic menu-cost model the relevant distinction between big and small is based on the variance of the unanticipated money-supply shock. Hence, we shall estimate a money-supply relationship that allows for this distinction between “big” and “small” shocks. In order to do so we apply the discrete-state regime-switching technique of Hamilton (1989) to the modelling of money-supply.

Such a technique has been used widely to characterize movements that arise when the moments of the variables under scrutiny change behavior over time, see e.g. Hamilton (1988, 1989, 1991), Phillips (1991), Sola and Drifill (1994), and Ravn and Sola (1995). The basic elements of the method are described extensively in Hamilton (1994). Garcia and Schaller (1995) also apply a switching-regime method to investigate the issue of asymmetric effects of monetary policy. The question they address, however, is different from the one that we will look at. They model output as a regime switching process and test whether monetary policy has different effects depending on the current state of the economy.

According to the regime-switching methodology, a time series is modelled as having discrete changes in its unconditional mean and/or variance. The variable that dictates the changes-in-regime is modelled as an unobservable discrete-valued state variable, $s_t$, that takes on the values 0 or 1. We also add to the switching regression a set of conditioning variables which are not subject to changes in regime. With this modification, we estimate a money-supply equation that allows for changes in mean and variance. This leads us to the following specification of money-supply:

$$\Delta m_t - \mu(s_t) = \Phi(L) (\Delta m_{t-1} - \mu(s_{t-1})) + \Theta x'_{t-1} + \sigma(s_t) \eta_t$$

(12)

where $\Phi(L)$ is a lag-polynomial, $\Theta$ is a vector of parameters, $x'_{t-1}$ is a vector of de-meaned exogenous variables\(^8\) defined as $x - \mu_x$, $\mu(s_t)$ is a state-dependent mean, $s_t$ is the discrete-valued state variable that can take on the values 0 and 1, and $\eta_t$ is an i.i.d $N(0,1)$ error term which is independent of $s_t$.

The money-supply process can now have two different means, $\mu_0$ and $\mu_1$ with associated variances $\sigma_0^2$ and $\sigma_1^2$. In the practical application these are estimated as $\mu_0 + \Delta \mu s_t$ and $\sigma_0 + \Delta \sigma s_t$. It is assumed that the (unobserved) states are generated by a two-state Markov-process. Let $\pi_{ij}$ be defined as $\pi_{ij} = P(s_t=i \mid s_{t-1}=j)$, $i,j = 0,1$, then the

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\(^8\) We de-mean the non-switching exogenous variables so that $\mu(s_t)$ can be interpreted as the unconditional mean of money growth.
probability transition matrix is given as:

$$\Pi = \begin{pmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{pmatrix}$$

where each of the transition probabilities are restricted to be non-negative and belong to the unit interval.

In this approach we have not allowed for regime-switching in the exogenous variables. To allow these variables to have changes-in-regime will require either to impose that they all switch simultaneously with money-supply (see e.g. Sola and Driffill, 1994) or that each variable is allowed to switch independently (see e.g. Ravn and Sola, 1995). The first approach is applicable when the variables are closely related as would be the case for example for interest rates of bonds of different maturities\(^9\), but does not seem applicable in the present analysis. The second approach has the disadvantage that the increase in the number of states quickly makes it intractable.

The division into “big” and “small” shocks is done in the following fashion. Consider first the expected money-growth in period \(t\) given the realized state at time \(t-1\). Expected money-growth is given as:

\[
\begin{align*}
E_{t-1} \Delta m_t &= (\mu_0 + \Delta \mu \pi_{11} + \Phi (\Delta m_{t-1} - \mu_0) + \Theta x_{t-1}) \text{ if } s_{t-1} = 0 \\
E_{t-1} \Delta m_t &= (\mu_0 + \Delta \mu \pi_{22} + \Phi (\Delta m_{t-1} - (\mu_0 + \Delta \mu)) + \Theta x_{t-1}) \text{ if } s_{t-1} = 1
\end{align*}
\]

The residuals (i.e. unexpected money-supply shocks) in these two cases can then be defined as:

\[
\begin{align*}
\varepsilon_{t|0} &= \Delta m_t - (\mu_0 + \Delta \mu \pi_{11} + \Phi (\Delta m_{t-1} - \mu_0) + \Theta x_{t-1}) \sim N\left(0, \sigma_0^2\right) \\
\varepsilon_{t|1} &= \Delta m_t - (\mu_0 + \Delta \mu \pi_{22} + \Phi (\Delta m_{t-1} - (\mu_0 + \Delta \mu)) + \Theta x_{t-1}) \sim N\left(0, \sigma_1^2\right)
\end{align*}
\]

Recall, however, that the states are unobservable for the econometrician and, thus, the above distinction requires one to draw inference on the regimes. We do this by using the estimates of the probabilities of being in each of the two regimes. Let \(P(s_{t-1} = i)\) be the (estimated) probability conditional on information available at time \(t-1\) that the state is equal to “\(i\)” at time \(t-1\) using the (modified) Hamilton filter. Assume also that state 0 is the state where the variance of unanticipated money-supply shocks is low. We can then define the two shocks in the following manner:

\[
\begin{align*}
\varepsilon^S_t &= P(s_{t-1} = 0) \times (\Delta m_t - \left[\mu_0 + \Delta \mu \pi_{11} + \Phi (\Delta m_{t-1} - \mu_0) + \Theta x_{t-1}\right]) \quad (13) \\
\varepsilon^N_t &= P(s_{t-1} = 1) \times (\Delta m_t - \left[\mu_0 + \Delta \mu \pi_{22} + \Phi (\Delta m_{t-1} - (\mu_0 + \Delta \mu)) + \Theta x_{t-1}\right]) (14)
\end{align*}
\]

\(^9\)In the case of the term structure there are, of course, very strong theoretical reasons why the state variables that dictate the changes-in-regime should be closely related.
Next, each of these two shocks can be divided into their positive and negative parts, which we denote by + (positive) and - (negative), using the same technique as in the previous section. Accordingly, we end up with four money supply shocks, \(\{e_t^{B+}, e_t^{B-}, e_t^{S+}, e_t^{S-}\}\). This construction allows us to test for the presence of asymmetric effects using the following procedure\(^{10}\).

We estimate jointly the money-supply equation (12) and the following version of the output equation:

\[
\Delta y_t = \psi z_t + \beta^{B+} e_t^{B+} + \beta^{B-} e_t^{B-} + \beta^{S+} e_t^{S+} + \beta^{S-} e_t^{S-} + \xi_t
\]  

(15)

The initial regression estimates these relationships, (12) and (15), imposing no parameter restrictions. We will call this for Case 1. At this point one can look at the significance of each of the shocks as a check on signs of asymmetric effects. The real tests, however, are carried out in a sequential manner using LR-tests. This means that we impose parameter restrictions on the coefficients on the money-supply shocks, \(\{e_t^{B+}, e_t^{B-}, e_t^{S+}, e_t^{S-}\}\). The first set of restrictions we impose is that:

**Case 2:** \(H_0 : \beta^{B+} = \beta^{B-} = \beta^{S+} = \beta^{S-}\)

Testing Case 2 against Case 1 is a test of the absence of any asymmetry and can be performed as an LR-test which is \(\chi^2\)-distributed with 3 degrees of freedom under the null. If these restrictions are rejected, the tests for the two versions of asymmetric effects are carried out by imposing a number of different parameter restrictions.

Consider first the case where one wishes to test for the asymmetry that positive and negative money-supply shocks have different effects. This hypothesis can be tested for in two steps. First, according to this hypothesis, it should not matter whether a given money-supply shock is “big” or “small”. Hence, we estimate the system imposing the following restrictions:

**Case 3:** \(H_0 : \beta^{B+} = \beta^{S+} \text{ and } \beta^{B-} = \beta^{S-}\)

Testing Case 3 against Case 1 (an LR-test which is \(\chi^2\)-distributed with 2 degrees of freedom) constitutes the first part of this hypothesis. The second part of the hypothesis is that positive shocks are neutral. Hence, we then impose that:

**Case 4:** \(H_0 : \beta^{B+} = \beta^{S+} = 0 \text{ and } \beta^{B-} = \beta^{S-}\)

Testing Case 4 against Case 3 is then a test of whether positive shocks are neutral. If these tests are passed, and the coefficient on the negative shocks is significantly positive, then the data supports the “traditional Keynesian asymmetry” hypothesis.

\(^{10}\)Alternatively, one can use a method of simulated moments method to obtain estimates of the unexpected money growth.
The other asymmetry hypothesis can be tested in a similar manner. The hypothesis is that only small money-supply shocks have real effects and, a priori, there is no reason that positive and negative shocks should have different effects. Hence, we first estimate the following case:

**Case 5:** $H_0 : \beta^{B+} = \beta^{B-}$ and $\beta^{S+} = \beta^{S-}$

Testing Case 5 against Case 1 (an LR-test which is $\chi^2$-distributed 2 degrees of freedom under the null), constitutes the first part of the hypothesis. The second part imposes that “big” shocks are neutral:

**Case 6:** $H_0 : \beta^{B+} = \beta^{B-} = 0$ and $\beta^{S+} = \beta^{S-}$

Again, we test this specification against Case 5, and if the test is passed (and the coefficient on the “small” shocks is positive and significant) then the hypothesis is backed by the data.

A last case to consider is that the relevant hypothesis is a hybrid version in which only “small negative” shocks have real effects. We can test the hybrid version in any of the two sequences outlined above. This can be performed by imposing:

**Case 7:** $H_0 : \beta^{B+} = \beta^{B-} = \beta^{S+} = 0$

The validity of this restriction can be tested against either Case 4 if this case is passed or against Case 6 if this case is passed. Case 7 can, of course, also be tested directly against Case 1 if only the “small negative” shock is significant.

Before proceeding to discuss the results, it is worthwhile to relate our procedure to alternative methods used in the literature. The main difference between our approach and other applications is the definition of “big” and “small” shocks. Demery (1993) makes the distinction between “small” and “big” shocks by defining the former as those that are in a two standard-error interval around zero and the latter as those not belonging to this interval\(^{11}\). This definition corresponds to viewing “big” shocks as outliers and the procedure is not robust to structural changes in the mean and the variance. This definition is not appropriate in the light of the analysis in section 2 and may produce estimates of the money-supply shocks that are wrongly identified. As an example, consider the case where the variance (but not the mean) changes over time. In this case the above procedure will identify “big” shocks as a mixture of the low and high variance shocks. In contrast, our procedure should identify the two different regimes if they have been observed in the sample.

\(^{11}\)Caballero and Engel (1993) apply a similar strategy when testing for asymmetries in the price-adjustments of firms in the face of nominal rigidities and changes in demand.
4 Empirical Tests for the US

In this section we will empirically test for the different varieties of asymmetric effects of nominal demand on real activity discussed in the previous section. We will test this on two alternative sets of quarterly data for the US\textsuperscript{12}. The first data set consists of data for the period 1948-1987 and has previously been examined by Cover (1992). The second set of data covers the period 1960-1995 and has been looked at previously by Christiano, Eichenbaum and Evans (1996).

In both applications we will use the empirical method described above but the two applications differ in the measure of monetary policy that is used to derive the money-supply shocks. In the application on Cover’s (1992) data we use the logarithm of M1 as the measure of money-supply while we use the (minus of the ) federal funds rate when looking at the more recent data.

Another issue that has to be addressed is the specification of the relationships for money-supply and output. Cover (1992) experiments with a number of different specifications of the money-supply relationship and with two different version of the output equation. The money-supply process is specified either as in Barro and Rush (1980), as in Mishkin (1982) (modified slightly), or as an “Optimal” money-supply\textsuperscript{13}. The two different specifications of the output equation have in common that $z_t$ includes a constant, lagged change in real output, $\varepsilon^+_t$, and $\varepsilon^-_t$ but differ in whether the change in the t-bill rate is included or not.

In all of the above money-supply processes there are signs of misspecification related to the existence of an outlier (at the first quarter of 1983 when the Volcker regime ended) in the money-supply residuals and of heteroscedasticity of the money-supply residuals\textsuperscript{14}. For these reasons, we estimate (using a general-to-specific approach) an alternative money-supply process from the data used by Cover (1992)\textsuperscript{15,16}. The process that we estimate includes the first lag of M1 growth, the 4th to the 6th quarter lag of the (log of) federal

\textsuperscript{12}The data is described in more detail in the appendix.

\textsuperscript{13}In the “Barro-Rush” specification, the vector of regressors $x_{t-1}$ includes a constant, the unemployment rate, and the contemporaneous real federal expenditure to normal expenditure. In the “Modified Mishkin” specification, $x_{t-1}$ contains a constant, lagged changes in money-supply, lagged changes in the treasury-bill rate, and lagged values of the federal government’s budget surplus. The “Optimal” specification includes various elements of the above variables as well as lagged values of the changes in the monetary base.

\textsuperscript{14}Results are available upon request (these results are given in the previous version of this paper, Ravn and Sola, 1996).

\textsuperscript{15}It also turns out that, once one corrects for the presence of the outlier, the results on asymmetric effects are no longer valid. Specifically, one can in this case no longer reject a hypothesis that the positive and negative shocks have the same effect on output and that they are neutral. Details on results are given in Ravn and Sola, 1996, table 3.

\textsuperscript{16}Belongia (1996) documents another problem with the asymmetry result for the US data. Belongia (1996) shows that if one uses a divisor index for the money-stock, then one cannot reject the hypothesis that positive and negative money-supply shocks have symmetric effects.
government’s budget surplus, the first, fifth and sixth lag of the log difference of monetary base, the two-quarter lag of the unemployment rate, the second and the sixth lag of output growth, and the first, third and fifth lag of the first difference of the t-bill rate. This relationship was identified by testing downwards from a relationship that initially included six lags of all the variables.

In the second application on the data previously analyzed by Christiano, Eichenbaum and Evans (1996) we use minus the federal funds rate as the measure of monetary policy. It is well-documented that the federal funds rate is a more stable indicator monetary policy than M1 and one should probably have greater faith in the results using this variable rather than those above using M1. The problem with using M1 is that there is evidence of structural instability of M1 demand for the period after the first oil-price shock. Hence, shocks identified as money-supply shocks may well be money-demand shocks. We use here the minus of the federal funds rate such that a positive shock to the money-supply process can be interpreted as a loosening of monetary policy. In this application we specify the vector of regressors in the money-supply relationship as in Christiano, Eichenbaum and Evans (1996). This vector includes four lags of the (minus of the) federal funds rate, four lags of the log difference of GDP, four lags of the log difference of non-borrowed reserves, four lags of the log difference of total reserves, and four lags of the log difference of the implicit GDP deflator\(^\text{17}\).

For both sets of data we specified the output equation (15) as in Cover (1992) such that it includes one lag of output growth, the first difference of the t-bill rate and the lag of the first-differenced t-bill rate. The results are robust to changes in this relationship (including modelling the output equation as an autoregressive process).

### 4.1 Results for M1

In Cover’s (1992) study he found strong evidence in favor of the traditional Keynesian asymmetry and we are will check whether the same result appears when testing also for other versions of asymmetric effects.

#### 4.1.1 Single Equation Estimates of Money-Supply

First we turn to the results of single equation estimates of the money-supply process with changes in regime. Figure 1 illustrates the first difference of the log of M1. From this picture we see that the mean and variance of M1-growth appears to be higher in the post-1970 period than in the pre-1970 period. This may indicate that there has been a change in the properties of money-supply around 1970, but it is, of course, also possible

\(^{17}\)We also experimented with including the unemployment rate but this variable did not affect the results. It should also be noted that we have not included the index of sensitive commodity prices that Christiano, Eichenbaum and Evans (1996) introduce in order to address the “price puzzle”. This issue is not important for our analysis.
that this change is explained by the exogenous variables included in the money-supply relationship.

Table 1 reports the results for the estimation of the money-supply process with changes in regime. The results are encouraging since both the change in the mean of the process and the change in the variance of the process are significant. The estimates suggest that there is a low-mean low-variance regime where the mean is around 0.7 percent per quarter and the standard deviation around 0.4 percent, and a high-mean high-variance regime where the mean is around 1.65 percent per quarter and the standard deviation around 0.8. This implies that the mean and standard deviation of the innovation in the “high” state (state 1) of money-supply are estimated to be roughly twice the corresponding numbers in the “low” state (state 0). Note also that both regimes are quite persistent since the diagonal elements of the transition matrix are both in excess of 0.98\textsuperscript{18}.

Figure 2 illustrates the estimated probabilities of being in regime 1, the regime where the mean and the variance are both high. The filter divides the sample very clearly into the two regimes and the estimates imply that money growth and the variance of money growth were low from the start of the sample until 1967. From 1967 to 1987.4 the probability of being in the regime with high mean and high variance is practically equal to one, with the exception of the last three quarters of 1976 and the final three observations. It should also be noted that there are no signs of specification errors in the regression residuals.

4.1.2 Tests for Asymmetric Effects Using M1

The first step is to estimate jointly the two relationships, (12) and (15), imposing no parameter restrictions in (15). The results of this are reported in table 2, column 1. There are noticeable differences in the level of significance of the various money-supply shocks. The only single component that is significant at conventional levels of confidence is the positive shock in the regime with low variance. “Big” positive shocks are at the other extreme, entering with zero coefficients. Negative shocks, these being either big or small, are less significant than the small positive shocks. Hence, one may suspect that there is some version of asymmetric effects of money-supply in the data. To investigate this, we first estimate a version of the system imposing the parameter restriction of Case 2, i.e. imposing that all components of the money-supply shocks enter with the same parameter. The results are given in the second column of table 2. The estimates imply that the unanticipated money-supply shock is significant in the output equation. However, the LRT-test indicates that the parameter restrictions are rejected. Thus, we proceed to look into whether the data supports any of the asymmetry hypotheses discussed in section 2.

First, we impose the restrictions of Case 3. This restriction forms the first step in

\textsuperscript{18}The expected time in each regime (when the process is in either of these) is given by the inverse of one minus each of the diagonal elements.
testing for the presence of the traditional Keynesian asymmetry and imply that “big” and “small” shocks enter with the same coefficients, i.e. that the only important distinction is between positive and negative money-supply shocks. The results of this are given in column 3 of table 2. The parameter estimates now imply that negative shocks enter with a coefficient of around 0.92 and that these are highly significant. Positive shocks enter with a coefficient close to zero and are insignificant. Hence, the t-ratios support the hypothesis. Nevertheless, the probability of observing the value of the LR-test under the null is only 4.4%. One further piece of evidence that points towards rejection of the hypothesis is that the “small positive” money-supply shock, which in the unrestricted specification (Case 1) was the single most significant component, is now insignificant.\footnote{Given that Case 3 is rejected we do not wish to interpret the LR-test of Case 4 against Case 3 since one cannot be sure that the test statistic is $\chi^2$-distributed.}

The other alternative to be tested is whether “big” and “small” money-supply shocks have asymmetric effects on output. The first step in testing this is to impose the restrictions under Case 5, i.e. that it is irrelevant whether the shocks are positive or negative. The results are given in column 5 of table 2. Under this specification only “small” shocks are significant and they enter with a coefficient of 1.04. “Big” shocks are insignificant at any level of confidence and enter with a coefficient very close to zero. Furthermore, the restrictions imposed on the output equation cannot be rejected using the LR-test which has a probability value as high as 27.4%. Notice also that any test would lead one to prefer Case 5 to Case 3 as the preferred simplification of the unrestricted model: not only is the likelihood much higher for Case 5, but an encompassing test (see e.g. Mizon and Richard (1986)), a technique that can be used for testing non-nested hypotheses, would also accept Case 5 and reject Case 3 as appropriate simplifications of Case 1.

It still needs to be tested whether high-variance shocks are neutral. Hence, we impose this extra condition and estimate the model under the restrictions of Case 6. The results, reported in column 6 of table 2, imply that this restriction cannot be rejected (the probability value for the LR-test is as high as 61.3%).

The last case to be looked at is the “hybrid” model in which only small negative money-supply shocks affect real activity. We test this against Case 6 since this is the preferred specification so far. The results are given in column 7 of table 2. The results are very clear and imply that the Case 7 is strongly rejected (the probability value for the LR-test is 0.09%).

The evidence is therefore that the data supports the hypothesis that there are asymmetric effects of money-supply shocks on output in the US. However, contrary to previous results, we do not find evidence in favor of the hypothesis that negative shocks have real effects and that positive shocks are neutral. The hypothesis that we find to be supported by the data is that “big” shocks (high-variance) are neutral but “small” (low-variance) shocks have real effects. This asymmetry would be implied by models in which firms face a menu-cost of changing their nominal prices in a stochastic setting. It may be of interest to inspect the results of our estimates of the money-supply equation for the joint
estimation with the restricted output equation. The parameter estimates are summarized in table 3 and are very similar to the single equation results. The estimates imply that there is low-mean, low-variance regime and a high-mean, high-variance regime. Figure 3 plots the estimated probability of being in the regime where the innovation variance and the mean money-growth are “high”. We estimate the state with low mean and low variance to have dominated from the start of the sample up to around 1967. The state with high mean growth and high innovation variance then takes over in 1968 and dominates most of the period up to around 1975 and again from the second quarter of 1978 to 1987. The state with low mean and low innovation variance dominates most of the intermediary period of 1975-1978. In interpreting these estimates it should be kept in mind that we include additional conditioning variables and the results do therefore not relate to standard univariate estimates of the money-supply process.

### 4.2 Results for the Federal Funds Rate

As discussed previously, there are reasons to expect that the results above might be hampered by structural instability of M1 demand. It has been shown in previous research that M1 demand has been relatively unstable in the 1980’s and the 1990’s. This implies that the shocks identified above as “money-supply” shock may well indeed be a mixture of money-demand and money-supply shocks (see e.g. Baba, Hendry and Starr, 1992, or Stock and Watson, 1993, for a discussion). It has also been claimed that the federal funds rate may be a better indicator of monetary policy. For these reasons we now turn to an analysis of the question of asymmetric effects using the federal funds rate rather than M1. In order to facilitate an easy comparison with the analysis for M1, we will transform the federal funds rate and measure it by the minus of the federal funds rate such that positive shocks indicate a loosening of monetary policy. The federal funds rate is illustrated graphically in Figure 4. One notices from this plot that there seems to have been periods of low and stable federal funds rates (in the early sixties, the last part of the seventies, and the last part of the eighties) and periods with higher and more volatile federal funds rates (the late sixties, a period around the first oil-price shock, and the early eighties).

#### 4.2.1 Single Equation Estimates

As above, we start the analysis by looking at the results of single equation estimates of the federal funds rate process using the regime-switching technique. We specify the federal funds rate process as Christiano, Eichenbaum and Evans (1996) and it includes four lags of the following five variables: (i) the federal funds rate, (ii) the log-difference of GDP, (iii) the log-difference of the implicit GDP deflator, (iv) the log-difference of non-borrowed

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20 Hamilton, 1996, provides an excellent discussion and analysis of the Federal Funds daily market.
reserves, and (v) the log-difference of total reserves\(^{21}\).

Table 4 reports the single equation results of the estimation of the process (12) for the federal funds rate. As for M1, we find that there are clear signs of changes in regime. The results imply that there is a low-mean, low-variance, regime and a high-mean, high-variance, regime. In the “low” regime the mean of the federal funds rate is estimated to be around 6.4 percent and the standard deviation to be 0.42 percent. In the “high” regime, the mean is estimated to be around 8.4 percent and the standard deviation to be 2.2 percent\(^{22}\). Evidently, it is the change in the variance that dominates the change in regime in this process. Furthermore, from the estimates of the Markov transition probabilities, it is seen that the “low” regime is much more persistent than the “high” regime. Conditional upon being in either of these regime, the probabilities imply that the expected duration of the “low” regime is close to 15 years while the expected duration of the low regime is exactly equal to 2 years.

Figure 5 illustrates the estimated probabilities of each of the two regimes. The regime with low funds rates and a low-variance of the innovations is estimated to dominate most of the sample period. There are two periods in which the regime with high funds rates and high volatility takes over. The first period is the period immediately after the first oil-price shock, 1973.3-1975.4. The second period is, non-surprisingly, the Volcker period 1979.3-1982.3 (one might also include 1982.4 in this regime but our estimates imply that the probability of the “high” regime is 13.4 percent for this observation). These results seem much more sensible than the dating of regimes in the application using M1.

### 4.2.2 Tests for Asymmetric Effects Using the Federal Funds Rate

We now turn to the tests for asymmetric effects of monetary policy. Once again, we estimate simultaneously the two equations (12) and (15). In this application the measure that we use of real activity is GDP rather than GNP used for the analysis with the M1 data. The results of this exercise are reported in table 5.

Column 1 of table 5 reports the results of estimating (15) with each of the four money-supply shocks entering unrestricted. The results imply that the only component of the money-supply shocks that affects GDP significantly is the “small” negative shock. The point estimates of all other components of the money-supply shocks are extremely small and very insignificant. Given this observation we proceed directly to test Case 7 against the unrestricted case since the other specifications under these circumstances do not necessarily nest Case 7.

\(^{21}\)The results are robust to changes in the federal funds rate process. We experimented with the inclusion of the unemployment rate, with using the CPI rather than the GDP deflator, and with using industrial production rather than GDP. We also experimented with alternative lag lengths and got the same results as those reported here.

\(^{22}\)By “high” we mean that the variance is high. This is the more contractionary regime since the mean interest rate is higher but it is also more volatile since the variance is high and it is the latter aspect that we are interested in.
Column 7 of table 5 lists the results for the estimation of the system introducing the restriction of Case 7, i.e. that only “small negative” money-supply shocks have real effects. This test is performed as an LR-test which has 3 degrees of freedom. The value of the LR-test statistic is as low as 0.011 and the probability is as high as 99.968. This implies that the restrictions of Case 7 cannot be rejected at any conventional significance level. Furthermore, the coefficient of the “small negative” money-supply shock is significant (the t-ratio is 2.29) and this component, thus, has real effects. In conclusion, the results indicate very strong empirical evidence in favor of the “hybrid” asymmetry analyzed in Ball and Mankiw (1994).

Finally, we report the estimates of the federal funds rate process and the estimated probabilities of the two regimes in table 6 and figure 6. The results are basically identical to those obtained for the single equation estimations and still imply that the “high” regime is associated with 1973.3-1975.4 and 1979.3-1982.3.

The results therefore indicate that using the federal funds data as opposed to using M1 as the monetary policy measure leads to a different conclusion on the asymmetric effects of monetary policy. When using the federal funds rate, which we would expect being a better indicator of monetary policy, we find that only small negative money-supply shocks have real effects.

5 Summary and Conclusion

Asymmetries in the relationship between real aggregate activity and monetary policy is a phenomenon that can arise under a variety of different assumptions about the economy. The specific version of the asymmetry differs between competing theories and it is often difficult to test the underlying assumptions directly on macroeconomic data. Furthermore, it is not clear that tests of the assumptions at the household or firm level necessarily carry over to the aggregate level. Since, such asymmetric effects in principle can have strong implications not only for the way we think about the macroeconomy, but also for the conduct of economic policy, it thus seems important to empirically examine the evidence on these asymmetries using aggregate data.

In this paper we have focused on the possible asymmetries in the way that different monetary policy shocks affect real aggregate activity. Our principal aim of this investigation has been to test indirectly for the asymmetries that may arise in macroeconomic models with menu-costs but the analysis may be thought of more broadly in terms of models with imperfections in goods and labor markets. We highlighted the possible distinctions between different monetary policy shocks that may arise in such models and we compared these to the traditional Keynesian asymmetry that have been investigated empirically in a number of papers.

In principle the menu-costs models imply a different type of asymmetric effects than the distinction between positive and negative shocks tested for previous papers. The most
important distinction in basic menu-costs models is between “big” and “small” shocks as
distinguished either by their size (in a non-stochastic environment) or by their variance
(in a stochastic environment). However, with steady-state inflation (as analyzed by Ball
and Mankiw, 1994) there may also be a distinction between positive and negative shocks
but the implied asymmetry is different from the traditional Keynesian asymmetry since
the latter does not distinguish shocks by their size.

We developed an empirical framework within which we were able to distinguish be-
tween these competing theories and to test for each of them and we applied this to US
postwar data. Our results indicated that when using M1, there is evidence in favor of the
basic menu-cost model in a stochastic framework (based on the same stochastic properties
as our application), i.e. a distinction between high and low variance shocks. As implied
by theory, the former of these shocks were found to be neutral and the latter to have real
effects. These results differed from previous results using the same data. These results,
however, may be hampered by the instability of M1 demand and we thus considered the
same analysis using the federal funds rate as the monetary policy measure rather than
M1. In these data, which we have more faith in, we found very strong evidence in favor
of only “small negative” shocks having real effects. Thus, the US data seems to indicate
evidence in favor of the asymmetry implied by menu-cost models in environments with
positive steady-state inflation, as analyzed by Ball and Mankiw (1994).

It would be interesting to extend this analysis along two lines. First, one might wish to
look into other versions of asymmetric effects. One particular possibility is to look into how
economic policy affects output in different phases of the business cycle. Another possibility
is to look into the effects of nominal demand shocks and their possible asymmetric effects
in stochastic dynamic general equilibrium models. We plan to investigate these matters
in future research.

6 References


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7 Appendices

7.1 Data Description

All variables studied in this paper are sampled at the quarterly frequency and were de-seasonalized from the source. The first set of data were kindly supplied by James Peery Cover and are described in detail in Cover (1992). The sample period covers 1948.1-1987.4. The variables used here are defined as:

\[ \begin{align*}
    m1 &= \text{The logarithm of M1} \\
    y &= \text{The logarithm of GNP in constant prices} \\
    b &= \text{The logarithm of the monetary base} \\
    u &= \text{The unemployment rate} \\
    fs &= \text{The logarithm of the federal government’s budget surplus} \\
    tbr &= \text{The t-bill rate}
\end{align*} \]

The second set of data corresponds to the data set studied in Christiano, Eichenbaum and Evans (1996). These data were obtained from the Datastream Database. The sample period starts in 1959.3 and ends in 1995.3. The variables used here are defined as:

\[ \begin{align*}
    -ff &= \text{The minus of the federal funds rate} \\
    y &= \text{The logarithm of GDP in constant prices} \\
    nbr &= \text{The logarithm of the sum of non-borrowed reserves and extended credit} \\
    py &= \text{The logarithm of the implicit GDP deflator} \\
    trs &= \text{The logarithm of total reserves} \\
    tbr &= \text{The t-bill rate}
\end{align*} \]

7.2 The Filter

It is assumed that one of the variables included in the filter is governed by a scalar state variable. The other variable(s) is(are) not allowed to switch and is(are) de-meaned. The filter involves the following five steps.

\textbf{Step 1.} Let \( y \) and \( x \) be the variables that are observed and let \( s \) be the unobserved state variable. Calculate the density of the \( m \) past states and the current state conditional on the information included in \( y_{t-1}, x_{t-1} \) and all past values of \( y \) and \( x \):

\[ p \left( s_t, s_{t-1}, \ldots, s_{t-m} \mid y_{t-1}, y_{t-2}, \ldots, y_0, x_{t-1}, x_{t-2}, \ldots, x_0 \right) \]

\[ = p \left( s_t \mid s_{t-1} \right) p \left( s_{t-1}, s_{t-2}, \ldots, s_{t-m} \mid y_{t-1}, y_{t-2}, \ldots, y_0, x_{t-1}, x_{t-2}, \ldots, x_0 \right) \] (A.1)

where \( p(s_t \mid s_{t-1}) \) is the transition probability matrix of the states which are assumed to follow a Markov process. As in all subsequent steps, the second term on the right-hand-side is known from the preceding step of the filter. In the present case the probability on
the left-hand side of 16) is known from the input to the filter which in turn represents the result of the iteration at date \( t - 1 \) (from step 5 described below).

Initial values for the parameters and the initial conditions for the Markov process are required to start the filter. The unconditional distribution, \( p(s_m, s_{m-1}, s_0) \) has been chosen for the first observation.

**Step 2.** Calculate the joint conditional density of \( y_t \) and \( (s_t, s_{t-1}, s_{t-m}) \),

\[
p(s_t, s_{t-1}, s_{t-m} \mid y_{t-1}, y_{t-2}, y_0, x_{t-1}, x_{t-2}, x_0)
\]

(A.2)

where we assume that

\[
p(s_t, s_{t-1}, s_{t-m} \mid y_{t-1}, y_{t-2}, y_0, x_{t-1}, x_{t-2}, x_0)
\]

\[
= \frac{1}{(2\pi)^{0.5} (\sigma_0 + \Delta \sigma s_t)} \exp \left( -\left( \frac{(y_t - (\mu_0 + \Delta \mu s_t))^2}{(2(\sigma_0 + \Delta \sigma s_t))^2} \right) \right)
\]

where

\[
u_{st} \equiv y_t - (\mu_0 + \Delta \mu s_t) - \Phi (L) (y_{t-1} - (\mu_0 + \Delta \mu s_{t-1})) - \Theta (L) x_t
\]

It should be noted that \( p(y_t \mid s_t, s_{t-m}, y_{t-1}, y_0, x_{t-1}, x_0) \) involves \((s_t, s_{t-m})\) which is a vector that can take on \(2^{m+1}\) values.

**Step 3.** Marginalize the previous joint densities with respect to the states which gives the conditional density from which the (conditional) likelihood function is calculated:

\[
p(y_t \mid y_{t-1}, y_0, x_{t-1}, x_0) = \sum_{s_t=0}^{1} \sum_{s_{t-1}=0}^{1} \cdots \sum_{s_{t-m}=0}^{1} p(y_t, s_t, s_{t-m} \mid y_{t-1}, y_0, x_{t-1}, x_0)
\]

(A.3)

**Step 4.** Combining the results from steps 2 and 3, calculate the joint density of the state conditional on the observed current and past realizations of \( y \),

\[
p(s_t, s_{t-1}, s_{t-m} \mid y_t, y_0, x_t, x_0) = \frac{p(y_t, s_t, s_{t-m} \mid y_{t-1}, y_0, x_{t-1}, x_0)}{p(y_t \mid y_{t-1}, y_0, x_{t-1}, x_0)}
\]

(A.4)

**Step 5.** The desired output is then obtained from:

\[
p(s_t, s_{t-1}, s_{t-m+1} \mid y_t, y_0, x_t, x_0) = \sum_{s_{t-m}=0}^{1} p(s_t, s_{t-1}, s_{t-m} \mid y_t, y_0, x_t, x_0)
\]

(A.5)

The output of step 5 is used as an input to the filter in the next iteration. Estimates of the parameters are calculated by maximizing the sample likelihood, which can be calculated from step 3.
8 Tables and Figures

Table 1. Money-Supply with Changes in Regime:
M1-process Single Equation Results

<table>
<thead>
<tr>
<th>variable</th>
<th>estimate</th>
<th>variable</th>
<th>estimate</th>
<th>Test-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta m_{t-1})</td>
<td>0.257 (0.072)</td>
<td>(\mu_0)</td>
<td>0.726 (0.131)</td>
<td>(Q(1) = 0.295 ] [0.587]</td>
</tr>
<tr>
<td>(u_{t-2})</td>
<td>0.403 (0.189)</td>
<td>(\Delta \mu)</td>
<td>0.934 (0.235)</td>
<td>(Q(10) = 9.521 ] [0.709]</td>
</tr>
<tr>
<td>(f_{s_{t-4}})</td>
<td>-1.160 (0.608)</td>
<td>(\sigma_0)</td>
<td>0.417 (0.041)</td>
<td>(QQ(1) = 2.264 ] [0.132]</td>
</tr>
<tr>
<td>(f_{s_{t-5}})</td>
<td>2.254 (0.843)</td>
<td>(\Delta \sigma)</td>
<td>0.368 (0.077)</td>
<td>(QQ(10) = 16.120 ] [0.096]</td>
</tr>
<tr>
<td>(f_{s_{t-6}})</td>
<td>-1.234 (0.528)</td>
<td>(\pi_{00})</td>
<td>0.985 (0.015)</td>
<td></td>
</tr>
<tr>
<td>(\Delta b_{t-1})</td>
<td>0.078 (0.092)</td>
<td>(\pi_{11})</td>
<td>0.989 (0.013)</td>
<td></td>
</tr>
<tr>
<td>(\Delta b_{t-5})</td>
<td>0.141 (0.079)</td>
<td> </td>
<td> </td>
<td></td>
</tr>
<tr>
<td>(\Delta b_{t-6})</td>
<td>-0.180 (0.089)</td>
<td> </td>
<td> </td>
<td></td>
</tr>
<tr>
<td>(\Delta y_{t-2})</td>
<td>0.144 (0.042)</td>
<td> </td>
<td> </td>
<td></td>
</tr>
<tr>
<td>(\Delta y_{t-6})</td>
<td>0.115 (0.048)</td>
<td> </td>
<td> </td>
<td></td>
</tr>
<tr>
<td>(\Delta tbr_{t-1})</td>
<td>-0.405 (0.054)</td>
<td> </td>
<td> </td>
<td></td>
</tr>
<tr>
<td>(\Delta tbr_{t-3})</td>
<td>-0.199 (0.058)</td>
<td> </td>
<td> </td>
<td></td>
</tr>
<tr>
<td>(\Delta tbr_{t-5})</td>
<td>-0.144 (0.056)</td>
<td> </td>
<td> </td>
<td></td>
</tr>
</tbody>
</table>

Notes: Sample period: 1948.1-1987.4. \(\Delta m1\) is the log difference of M1, \(\Delta b\) is the log difference of the monetary base, \(u\) is the unemployment rate, \(\Delta y\) is the log difference of GNP, \(f_{s}\) is the log of the federal government’s budget surplus, \(\Delta tbr\) is the difference of the t-bill rate. \(Q(x) \) (\(QQ(x)\)) is the Box-Pierce test for autocorrelation in the standardized residuals (squared standardized residuals) of order \(x\). Numbers in parentheses are standard errors. Numbers in brackets are probabilities.
Table 2. Output Equation ML Estimates: M1 Measure

<table>
<thead>
<tr>
<th>Variable</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.552(0.175)</td>
<td>0.528(0.097)</td>
<td>0.791(0.163)</td>
<td>0.805(0.118)</td>
<td>0.475(0.100)</td>
<td>0.469(0.100)</td>
<td>0.611(0.103)</td>
</tr>
<tr>
<td>$\Delta y_{t-1}$</td>
<td>0.318(0.073)</td>
<td>0.333(0.073)</td>
<td>0.307(0.072)</td>
<td>0.304(0.072)</td>
<td>0.324(0.072)</td>
<td>0.324(0.073)</td>
<td>0.328(0.074)</td>
</tr>
<tr>
<td>$\Delta tbr_{t}$</td>
<td>0.285(0.064)</td>
<td>0.291(0.066)</td>
<td>0.297(0.065)</td>
<td>0.295(0.066)</td>
<td>0.276(0.064)</td>
<td>0.275(0.064)</td>
<td>0.268(0.067)</td>
</tr>
<tr>
<td>$\Delta tbr_{t-1}$</td>
<td>0.088(0.071)</td>
<td>0.084(0.072)</td>
<td>0.067(0.071)</td>
<td>0.066(0.070)</td>
<td>0.085(0.068)</td>
<td>0.084(0.068)</td>
<td>0.077(0.070)</td>
</tr>
<tr>
<td>$e_{t}^{S+}$</td>
<td>0.868(0.366)</td>
<td>0.381(0.122)</td>
<td>0.021(0.212)</td>
<td>-</td>
<td>1.042(0.275)</td>
<td>1.077(0.269)</td>
<td>-</td>
</tr>
<tr>
<td>$e_{t}^{S-}$</td>
<td>0.752(0.472)</td>
<td>0.381(0.122)</td>
<td>0.919(0.289)</td>
<td>0.945(0.246)</td>
<td>1.042(0.275)</td>
<td>1.077(0.269)</td>
<td>1.040(0.513)</td>
</tr>
<tr>
<td>$e_{t}^{B+}$</td>
<td>0.000(0.249)</td>
<td>0.381(0.122)</td>
<td>0.021(0.212)</td>
<td>-</td>
<td>0.080(0.158)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$e_{t}^{B-}$</td>
<td>0.495(0.336)</td>
<td>0.381(0.122)</td>
<td>0.919(0.289)</td>
<td>0.945(0.246)</td>
<td>0.080(0.158)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{y}$</td>
<td>0.901(0.535)</td>
<td>0.938(0.680)</td>
<td>0.914(0.554)</td>
<td>0.915(0.561)</td>
<td>0.915(0.361)</td>
<td>0.915(0.324)</td>
<td>0.955(0.558)</td>
</tr>
<tr>
<td>$Q(1)$</td>
<td>0.386(0.535)</td>
<td>0.170(0.680)</td>
<td>0.350(0.554)</td>
<td>0.338(0.561)</td>
<td>0.834(0.361)</td>
<td>0.971(0.324)</td>
<td>0.343(0.558)</td>
</tr>
<tr>
<td>$Q(10)$</td>
<td>8.484(0.582)</td>
<td>9.476(0.488)</td>
<td>9.125(0.520)</td>
<td>9.003(0.532)</td>
<td>7.509(0.677)</td>
<td>7.359(0.691)</td>
<td>11.999(0.385)</td>
</tr>
<tr>
<td>$QQ(1)$</td>
<td>1.226(0.268)</td>
<td>1.832(0.176)</td>
<td>3.070(0.080)</td>
<td>3.261(0.071)</td>
<td>2.229(0.135)</td>
<td>2.405(0.121)</td>
<td>2.929(0.087)</td>
</tr>
<tr>
<td>$QQ(10)$</td>
<td>8.609(0.570)</td>
<td>8.738(0.557)</td>
<td>11.445(0.324)</td>
<td>11.076(0.352)</td>
<td>9.670(0.470)</td>
<td>9.577(0.478)</td>
<td>8.915(0.540)</td>
</tr>
<tr>
<td>Log.lik.</td>
<td>-349.59</td>
<td>-354.56</td>
<td>-352.71</td>
<td>-352.84</td>
<td>-350.88</td>
<td>-351.09</td>
<td>-356.53</td>
</tr>
<tr>
<td>LR-test</td>
<td>-</td>
<td>9.951$^{(1)}$</td>
<td>6.254$^{(2)}$</td>
<td>0.241$^{(3)}$</td>
<td>2.592$^{(4)}$</td>
<td>0.255$^{(5)}$</td>
<td>11.046$^{(6)}$</td>
</tr>
</tbody>
</table>

Notes: See notes to Table 1. The monetary shocks refer to unanticipated shocks.

1) LR-test of Case 2 vs. Case 1 2) LR-test of Case 3 vs. Case 1 3) LR-test of Case 4 vs. Case 3 4) LR-test of Case 5 vs. Case 1 5) LR-test of Case 6 vs. Case 5 6) LR-test of Case 7 vs. Case 6
Table 3. Money-Supply with Changes in Regime:
M1-process Jointly Estimated with Output under Case 6

<table>
<thead>
<tr>
<th>variable</th>
<th>estimate</th>
<th>variable</th>
<th>estimate</th>
<th>Test-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m_{t-1}$</td>
<td>0.218 (0.069)</td>
<td>$\mu_0$</td>
<td>0.727 (0.085)</td>
<td>$Q(1) = 0.191$ [0.662]</td>
</tr>
<tr>
<td>$w_{t-2}$</td>
<td>0.212 (0.165)</td>
<td>$\Delta \mu$</td>
<td>0.914 (0.172)</td>
<td>$Q(10) = 8.848$ [0.547]</td>
</tr>
<tr>
<td>$f_{s_{t-4}}$</td>
<td>−1.337 (0.560)</td>
<td>$\sigma_0$</td>
<td>0.415 (0.038)</td>
<td>$QQ(1) = 3.138$ [0.077]</td>
</tr>
<tr>
<td>$f_{s_{t-5}}$</td>
<td>2.607 (0.760)</td>
<td>$\Delta \sigma$</td>
<td>0.380 (0.077)</td>
<td>$QQ(10) = 14.746$ [0.142]</td>
</tr>
<tr>
<td>$f_{s_{t-6}}$</td>
<td>−1.340 (0.468)</td>
<td>$\pi_{00}$</td>
<td>0.979 (0.014)</td>
<td></td>
</tr>
<tr>
<td>$\Delta b_{t-1}$</td>
<td>0.136 (0.083)</td>
<td>$\pi_{11}$</td>
<td>0.986 (0.015)</td>
<td></td>
</tr>
<tr>
<td>$\Delta b_{t-2}$</td>
<td>0.122 (0.068)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta b_{t-6}$</td>
<td>−0.142 (0.082)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-2}$</td>
<td>0.131 (0.040)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-6}$</td>
<td>0.063 (0.047)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta thr_{t-1}$</td>
<td>−0.408 (0.053)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta thr_{t-3}$</td>
<td>−0.209 (0.059)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta thr_{t-5}$</td>
<td>−0.143 (0.055)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: See notes to table 1.
Table 4. Money-Supply with Changes in Regime: 
Federal funds Rate Process Single Equation Results

<table>
<thead>
<tr>
<th>variable</th>
<th>estimate</th>
<th>variable</th>
<th>estimate</th>
<th>Test-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-ff_{t-1}$</td>
<td>1.226</td>
<td>$\Delta tr_{t-1}$</td>
<td>-0.178 (0.044)</td>
<td>$Q(1) = 0.002$ [0.968]</td>
</tr>
<tr>
<td>$-ff_{t-2}$</td>
<td>-0.283 (0.180)</td>
<td>$\Delta tr_{t-2}$</td>
<td>0.147 (0.053)</td>
<td>$Q(10) = 7.612$ [0.667]</td>
</tr>
<tr>
<td>$-ff_{t-3}$</td>
<td>0.049 (0.163)</td>
<td>$\Delta tr_{t-3}$</td>
<td>-0.011 (0.059)</td>
<td>$QQ(1) = 0.305$ [0.581]</td>
</tr>
<tr>
<td>$-ff_{t-4}$</td>
<td>-0.052 (0.093)</td>
<td>$\Delta tr_{t-4}$</td>
<td>-0.024 (0.046)</td>
<td>$QQ(10) = 7.378$ [0.689]</td>
</tr>
<tr>
<td>$\Delta nbr_{t-1}$</td>
<td>0.145 (0.038)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta nbr_{t-2}$</td>
<td>-0.113 (0.043)</td>
<td>$\mu_0$</td>
<td>-6.362 (0.675)</td>
<td></td>
</tr>
<tr>
<td>$\Delta nbr_{t-3}$</td>
<td>-0.041 (0.045)</td>
<td>$\Delta \mu$</td>
<td>-2.031 (0.486)</td>
<td></td>
</tr>
<tr>
<td>$\Delta nbr_{t-4}$</td>
<td>0.047 (0.037)</td>
<td>$\sigma_0$</td>
<td>0.418 (0.027)</td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-1}$</td>
<td>-0.173 (0.057)</td>
<td>$\Delta \sigma$</td>
<td>1.760 (0.390)</td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-2}$</td>
<td>-0.053 (0.055)</td>
<td>$\pi_{00}$</td>
<td>0.875 (0.080)</td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-3}$</td>
<td>-0.069 (0.054)</td>
<td>$\pi_{11}$</td>
<td>0.984 (0.011)</td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-4}$</td>
<td>0.021 (0.051)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta p_{t-1}$</td>
<td>-0.377 (0.172)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta p_{t-2}$</td>
<td>-0.285 (0.162)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta p_{t-3}$</td>
<td>0.217 (0.168)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta p_{t-4}$</td>
<td>0.207 (0.168)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Sample period: 1959.3-1995.3. $-ff$ is the minus of the federal funds rate, $\Delta nbr$ is the log difference of non-borrowed reserves, $\Delta tr$ is the log difference of total reserves, $\Delta y$ is the log difference of GDP, $\Delta py$ is the log difference of the implicit GDP deflator.
Table 5. Output Equation ML Estimates: Federal Funds Rate Measure

<table>
<thead>
<tr>
<th>Variable</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.733</td>
<td>0.609</td>
<td>0.698</td>
<td>0.698</td>
<td>0.615</td>
<td>0.615</td>
<td>0.733</td>
</tr>
<tr>
<td></td>
<td>(0.151)</td>
<td>(0.100)</td>
<td>(0.151)</td>
<td>(0.110)</td>
<td>(0.101)</td>
<td>(0.101)</td>
<td>(0.116)</td>
</tr>
<tr>
<td>$\Delta y_{t-1}$</td>
<td>0.208</td>
<td>0.243</td>
<td>0.204</td>
<td>0.203</td>
<td>0.235</td>
<td>0.235</td>
<td>0.209</td>
</tr>
<tr>
<td></td>
<td>(0.085)</td>
<td>(0.082)</td>
<td>(0.084)</td>
<td>(0.084)</td>
<td>(0.084)</td>
<td>(0.083)</td>
<td>(0.084)</td>
</tr>
<tr>
<td>$\Delta tbr_t$</td>
<td>0.33</td>
<td>0.26</td>
<td>0.32</td>
<td>0.32</td>
<td>0.28</td>
<td>0.28</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.07)</td>
<td>(0.10)</td>
<td>(0.08)</td>
<td>(0.01)</td>
<td>(0.08)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$\Delta tbr_{t-1}$</td>
<td>0.18</td>
<td>0.12</td>
<td>0.17</td>
<td>0.17</td>
<td>0.14</td>
<td>0.14</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.08)</td>
<td>(0.08)</td>
<td>(0.08)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$e_{t+}^{S_+}$</td>
<td>0.000</td>
<td>0.065</td>
<td>0.000</td>
<td>-</td>
<td>0.170</td>
<td>0.170</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.441)</td>
<td>(0.080)</td>
<td>(0.184)</td>
<td>-</td>
<td>(0.184)</td>
<td>(0.184)</td>
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</tr>
<tr>
<td>$e_{t-}^{S_-}$</td>
<td>0.473</td>
<td>0.065</td>
<td>0.243</td>
<td>0.238</td>
<td>0.170</td>
<td>0.170</td>
<td>0.474</td>
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<td>(0.210)</td>
<td>(0.080)</td>
<td>(0.180)</td>
<td>(0.124)</td>
<td>(0.184)</td>
<td>(0.184)</td>
<td>(0.207)</td>
</tr>
<tr>
<td>$e_{t+}^{B+}$</td>
<td>0.000</td>
<td>0.065</td>
<td>0.000</td>
<td>-</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.395)</td>
<td>(0.080)</td>
<td>(0.184)</td>
<td>-</td>
<td>(0.180)</td>
<td>-</td>
<td>-</td>
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<tr>
<td>$e_{t-}^{B-}$</td>
<td>0.005</td>
<td>0.065</td>
<td>0.243</td>
<td>0.238</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
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<tr>
<td></td>
<td>(0.399)</td>
<td>(0.080)</td>
<td>(0.180)</td>
<td>(0.124)</td>
<td>(0.180)</td>
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</tr>
<tr>
<td>$\sigma_Y$</td>
<td>0.833</td>
<td>0.857</td>
<td>0.838</td>
<td>0.839</td>
<td>0.847</td>
<td>0.847</td>
<td>0.833</td>
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<tr>
<td>$Q(1)$</td>
<td>0.009</td>
<td>0.013</td>
<td>0.040</td>
<td>0.051</td>
<td>0.018</td>
<td>0.018</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>(0.924)</td>
<td>(0.909)</td>
<td>(0.841)</td>
<td>(0.821)</td>
<td>(0.893)</td>
<td>(0.893)</td>
<td>(0.916)</td>
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<td></td>
<td>(0.944)</td>
<td>(0.432)</td>
<td>(0.940)</td>
<td>(0.935)</td>
<td>(0.736)</td>
<td>(0.736)</td>
<td>(0.942)</td>
</tr>
<tr>
<td>$QQ(1)$</td>
<td>0.470</td>
<td>0.193</td>
<td>0.299</td>
<td>0.299</td>
<td>0.590</td>
<td>0.591</td>
<td>0.473</td>
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<tr>
<td></td>
<td>(0.493)</td>
<td>(0.661)</td>
<td>(0.585)</td>
<td>(0.585)</td>
<td>(0.442)</td>
<td>(0.442)</td>
<td>(0.492)</td>
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<tr>
<td></td>
<td>(0.453)</td>
<td>(0.555)</td>
<td>(0.365)</td>
<td>(0.365)</td>
<td>(0.377)</td>
<td>(0.377)</td>
<td>(0.451)</td>
</tr>
<tr>
<td>LR-test</td>
<td>-</td>
<td>30.340(4)</td>
<td>0.840(2)</td>
<td>0.019(9)</td>
<td>3.797(4)</td>
<td>0.000(4)</td>
<td>0.011(9)</td>
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<tr>
<td></td>
<td></td>
<td>(0.000)</td>
<td>(0.656)</td>
<td>(0.892)</td>
<td>(0.150)</td>
<td>(0.984)</td>
<td>(0.999)</td>
</tr>
</tbody>
</table>

Notes: See notes to table 4. $\Delta tbr$ is the difference of the t-bill rate. The monetary shocks refer to unanticipated shocks.

1) LR-test of Case 2 vs. Case 1 2) LR-test of Case 3 vs. Case 1 3) LR-test of Case 4 vs. Case 3 4) LR-test of Case 5 vs. Case 1 5) LR-test of Case 6 vs. Case 5 6) LR-test of Case 7 vs. Case 1
Table 6. Money-Supply with Changes in Regime:
Federal Funds Rate Process Case 7 Results

<table>
<thead>
<tr>
<th>variable</th>
<th>estimate</th>
<th>variable</th>
<th>estimate</th>
<th>Test-statistic</th>
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</thead>
<tbody>
<tr>
<td>$-f_{t-1}$</td>
<td>1.231</td>
<td>$\Delta tr_{t-1}$</td>
<td>-0.173</td>
<td>$Q(1) = 0.031$</td>
</tr>
<tr>
<td>$-f_{t-2}$</td>
<td>-0.314</td>
<td>$\Delta tr_{t-2}$</td>
<td>0.159</td>
<td>$Q(10) = 6.881$</td>
</tr>
<tr>
<td>$-f_{t-3}$</td>
<td>0.063</td>
<td>$\Delta tr_{t-3}$</td>
<td>-0.018</td>
<td>$QQ(1) = 0.276$</td>
</tr>
<tr>
<td>$-f_{t-4}$</td>
<td>-0.043</td>
<td>$\Delta tr_{t-4}$</td>
<td>-0.014</td>
<td>$QQ(10) = 8.390$</td>
</tr>
<tr>
<td>$\Delta nbr_{t-1}$</td>
<td>0.138</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta nbr_{t-2}$</td>
<td>-0.117</td>
<td>$\mu_0$</td>
<td>-6.535</td>
<td>(0.624)</td>
</tr>
<tr>
<td>$\Delta nbr_{t-3}$</td>
<td>-0.037</td>
<td>$\Delta \mu$</td>
<td>-2.167</td>
<td>(0.366)</td>
</tr>
<tr>
<td>$\Delta nbr_{t-4}$</td>
<td>0.034</td>
<td>$\sigma_0$</td>
<td>0.417</td>
<td>(0.027)</td>
</tr>
<tr>
<td>$\Delta y_{t-1}$</td>
<td>-0.165</td>
<td>$\Delta \sigma$</td>
<td>1.690</td>
<td>(0.373)</td>
</tr>
<tr>
<td>$\Delta y_{t-2}$</td>
<td>-0.054</td>
<td>$\pi_{00}$</td>
<td>0.819</td>
<td>(0.080)</td>
</tr>
<tr>
<td>$\Delta y_{t-3}$</td>
<td>-0.056</td>
<td>$\pi_{11}$</td>
<td>0.984</td>
<td>(0.012)</td>
</tr>
<tr>
<td>$\Delta y_{t-4}$</td>
<td>0.001</td>
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<tr>
<td>$\Delta p_{t-1}$</td>
<td>-0.402</td>
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<tr>
<td>$\Delta p_{t-2}$</td>
<td>-0.307</td>
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<td></td>
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<tr>
<td>$\Delta p_{t-3}$</td>
<td>0.259</td>
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<tr>
<td>$\Delta p_{t-4}$</td>
<td>0.208</td>
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</tr>
</tbody>
</table>

Notes: See notes to table 4.
Figure 1: The Growth Rate of M1

Figure 2: Estimated Probabilities: Single Equation Results for M1
Figure 3: Estimated Probabilities: Results for Case 6 for M1

Figure 4: Federal Funds Rate
Figure 5: Estimated Probabilities: Single Equation Results for Minus Federal Funds Rate

Figure 6: Estimated Probabilities: Results for Case 7 for Minus Federal Funds Rate