

Genetics and population analysis

HypercubeME: two hundred million combinatorially complete datasets from a single experiment

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Abstract

Motivation: Epistasis, the context-dependence of the contribution of an amino acid substitution to fitness, is common in evolution. To detect epistasis, fitness must be measured for at least four genotypes: the reference genotype, two different single mutants and a double mutant with both of the single mutations. For higher-order epistasis of the order n , fitness has to be measured for all 2^n genotypes of an n -dimensional hypercube in genotype space forming a ‘combinatorially complete dataset’. So far, only a handful of such datasets have been produced by manual curation. Concurrently, random mutagenesis experiments have produced measurements of fitness and other phenotypes in a high-throughput manner, potentially containing a number of combinatorially complete datasets.

Results: We present an effective recursive algorithm for finding all hypercube structures in random mutagenesis experimental data. To test the algorithm, we applied it to the data from a recent HIS3 protein dataset and found all 199 847 053 unique combinatorially complete genotype combinations of dimensionality ranging from 2 to 12. The algorithm may be useful for researchers looking for higher-order epistasis in their high-throughput experimental data.

Availability and implementation: <https://github.com/ivankovlab/HypercubeME.git>.

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Supplementary information: [Supplementary data](#) are available at *Bioinformatics* online.

1 Introduction

Epistasis, the dependence of the impact of a mutation on the genetic context, is abundant and important phenomenon in molecular evolution (Breen *et al.*, 2012). Formally, epistasis is characterized by coefficients α having two or more indices in the following representation of fitness f as a function of a genotype g (assuming, for simplicity, that maximum of one mutation is allowed at any position):

$$f(g) = \text{const} + \sum_{i=1}^N \alpha_i \delta_i + \sum_{i=1}^N \sum_{j>i}^N \alpha_{ij} \delta_i \delta_j + \sum_{i=1}^N \sum_{j>i}^N \sum_{k>j}^N \alpha_{ijk} \delta_i \delta_j \delta_k + \dots$$

where sums are taken over N considered positions, $\delta_i = 1$ if i -th

position is mutated in genotype g ; otherwise $\delta_i = 0$ or $\delta_i = -1$, depending on the formalism of epistasis description (Poelwijk *et al.*, 2016). Coefficients α_i correspond to a single effect of the mutation in the i -th position. Coefficients α_{ij} , having two indices, represent the pairwise epistasis between positions i and j , while coefficients α having three or more indices correspond to ‘higher-order epistasis’ (de Araujo and Guimaraes, 2016; Otwinowski *et al.*, 2018; Poelwijk *et al.*, 2016; Sailer and Harms, 2017a, b, c; Tuo, 2018; Weinreich *et al.*, 2013, 2018).

To detect epistatic terms of the order n by means of Walsh transform (Weinreich *et al.*, 2013), one has to measure phenotypes of all 2^n genotypes forming an n -dimensional hypercube in genotype space. Such experimental datasets are called ‘combinatorially complete datasets’ (Weinreich *et al.*, 2013). Up to now, higher-order

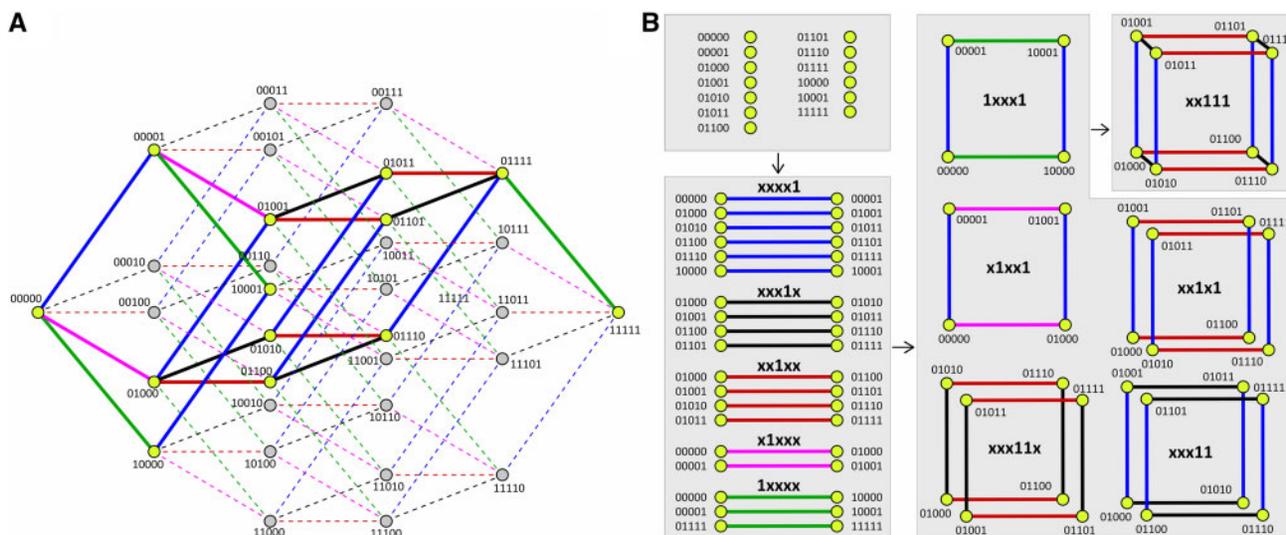


Fig. 1. Example illustrating the work of the algorithm. (A) A graph corresponding to a five-dimensional hypercube is given, where measured genotypes are drawn in yellow-green and non-measured genotypes are drawn in grey. Vertices are given in binary notation, where each digit corresponds to one of five substitution sites. The digit is zero if the corresponding substitution site contains an amino acid of the reference '00000' genotype; otherwise, it is one. The graph is drawn so that: (1) all vertices are visible (that is, no vertex is shaded by another one); (2) all vertices having the same number of zeros belong to the same vertical line; (3) the edges parallel in five-dimensional cube are drawn parallel to each other, and in different colors, for convenience. Blue, black, red, pink and green edges correspond to substitutions in the fifth, fourth, third, second and first sites, respectively. (B) The algorithm is applied to an example of random mutagenesis data from the panel (A). The diagonal for each group of hypercubes is given in bold

epistasis was studied using only a handful of examples carefully designed to have all 2^n combinations (Weinreich *et al.*, 2013). On the other hand, high-throughput experiments using (quasi-)random mutagenesis have produced vast amounts of data: 51 715 measured genotypes for GFP (Sarkisyan *et al.*, 2016), over 65 000 for arginine tRNA (Li *et al.*, 2016), 956 648 for HIS3 protein (Pokusaeva *et al.*, 2019), to name a few. These experiments may contain a number of combinatorially complete datasets as subsets of a general dataset. However, the extraction of such hypercubes from a large dataset is not straightforward, and may not be feasible to do in a brute-force manner.

2 Algorithm

The algorithm uses the fact that any n -dimensional hypercube contains two opposite hyperfacets, which are parallel to each other. Those hyperfacets are hypercubes of dimensionality $(n-1)$, which, in turn, are built from parallel hypercubes of dimensionality $(n-2)$, etc. down to hypercubes of 0-th dimensionality (which are simply points in genotype space, i.e. genotypes).

The algorithm consists of repeating steps. At each step, all possible n -dimensional hypercubes are generated from the set of $(n-1)$ -dimensional hypercubes. Informally, at each step, the algorithm takes all pairs of existing parallel hypercubes and if the distance between the hypercubes in the pair is one, the pair composes the hypercube of a higher dimensionality (Fig. 1). We need to define the diagonal of a combinatorially complete dataset as a list of mutations

```

01 Input: list of genotypes
02 Output: list of found hypercubes
03 FOR each Genotype from Input:
04   HCube.Diagonal <- empty list
05   HCube.First <- Genotype
06   HCube.Last <- Genotype
07   ADD HCube to ListHCubes[0]
08 N <- 0 # current dimensionality
09 REPEAT:
10   FOR each Group from ListHCubes[N] with same
      Diagonal:

```

```

11   FOR each pair of hypercubes HCube1, HCube2 from
      the Group:
12     IF mutation from HCube1.First to HCube2.First is
      single:
13       Forward <- mutation from HCube1.First to
      HCube2.First
14       Reverse <- mutation from HCube2.First to
      HCube1.First
15       IF Forward is alphabetically less than Reverse:
16         NewDiagonal <- list(Diagonal, Forward)
17         HCube <- (NewDiagonal, HCube1.First,
      HCube2.Last)
18       ELSE:
19         NewDiagonal <- list(Diagonal, Reverse)
20         HCube <- (NewDiagonal, HCube2.First,
      HCube1.Last)
21       IF Mutations in NewDiagonal are alphabetic-
      ally ordered:
22         ADD HCube to ListHCubes[N + 1]
23       SORT ListHCubes[N + 1] by Diagonal
24     N <- N + 1
25 UNTIL no new hypercubes are found

```

transforming a genotype of the dataset to the most distant genotype of the same dataset. An n -dimensional combinatorially complete dataset therefore contains 2^{n-1} diagonals, each of which (if not empty) can be written in the forward and reverse direction.

Formally, the algorithm consists of the following steps:

Each step of the algorithm can be easily parallelized. The multi-processor version can be found at <https://github.com/ivankovlab/HypercubeME.git>.

3 Results

We have applied the algorithm to the recently published fitness landscape for HIS3 protein (Pokusaeva *et al.*, 2019), the biggest fitness

landscape published so far. The HIS3 protein was divided into 12 segments, and quasi-random mutagenesis has been done in each segment separately. We had to exclude indels and mutations outside the segment, so the number of considered experimentally measured genotypes ranged from 16 182 for segment S7 to 82 081 for segment S2, overall summing up to 721 791 genotypes (Supplementary Table S1).

We have found all 199 847 053 hypercubes having dimensionality from 2 to 12. The single-processor working time ranged from 2 h for S7 to almost 10 days for S5. Among the found hypercubes, the percentage of squares was 12%, while the remaining 88% had dimensionality 3 and higher and, thus, can be used for exploring higher-order epistasis. The number of found hypercubes throughout segments is given in Supplementary Table S2.

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Conflict of Interest: none declared.

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