# Multiple filtering devices for the estimation of cyclical DSGE models

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**Abstract**

We propose a method to estimate time invariant cyclical DSGE models using the information provided by a variety of filtering approaches. We treat data filtered with alternative procedures as contaminated proxy of the relevant model-based quantities and estimate structural and non-structural parameters jointly using an unobservable component structure. We employ simulated data to illustrate the properties of the procedure and compare our estimates with those obtained when just one filter is used. We revisit the role of money in the transmission of monetary business cycles.

JEL classification: E32, C32.

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# 1 Introduction

DSGE models have become the paradigm for business cycle and policy analyses in academic and policy circles. Relative to earlier structures, current models are of larger scale and feature numerous frictions on the real and nominal side of the economy that help to closely replicate the dynamic responses that structural VARs produce. A few years ago it was standard to informally calibrate these models but today, increased computing power, longer time series and recent developments in system-wide estimation methods allow researchers to routinely employ a variety of full information techniques in structural estimation exercises (see, e.g., Smets and Wouters (2003), Ireland (2004), Rabanal and Rubio Ramirez (2005) among many others).

Despite the increased popularity, structural estimation faces important conceptual and numerical problems. For example, as emphasized by Canova (forthcoming), full information classical estimation makes sense only if the model is assumed to be the data generating process (DGP) of the observables, up to a set of serially uncorrelated measurement errors. Since such an assumption is hard to entrain unless the model is augmented with ad-hoc dynamics, Fukac and Pagan (forthcoming) suggested to complement standard inference with a more robust limited information analysis. It is also well known that there are abundant population identification problems (see Canova and Sala (forthcoming)), that numerical difficulties are widespread, that singularities are often important (there are typically less shocks than endogenous variables in the model) and that errors-in-variables are present (the variables in the model do not often have a direct counterpart in the data). Finally, the vast majority of the models used in the literature are time invariant and intended to explain only the cyclical portion of the data fluctuations while the actual data contains, at a minimum, growth components, cyclical fluctuations and high frequency noise, all of which may be subject to breaks and other forms of slowly moving variations.

When faced with the problem of fitting stationary cyclical DSGE models to the data, applied investigators typically select a sub-sample where time invariance is more likely to hold, filter the raw data with an arbitrary statistical device, and treat the filtered data as the relevant measure of stationary cyclical fluctuations (see e.g. Smets and Wouters (2003), Ireland (2004)). Occasionally, one find authors, see e.g. Kehoe (2007), suggesting that filtering should be applied to both actual data and data simulated by the model but, to the best of our knowledge, such an approach has, so far, no followers in the estimation literature. Alternatively, a unit root in total factor productivity and/or the price of investment is assumed and the data is filtered using a model-driven transformation (see e.g. Fernandez Villaverde and Rubio Ramirez (forthcoming)).

Both statistical and model-based filtering are problematic. While the profession largely agrees that a cyclical model should explain fluctuations with an average periodicity of 8-32 quarters, there is little agreement on how to obtain these fluctuations from the data and only a partial understanding of the consequences that incorrect or suboptimal filtering induce. For example, it is common to use linearly detrended or first differenced data as input in the estimation process, but such transformations do not isolate fluctuations with the required periodicity (see e.g. Canova (1998)). A band pass (BP) filter which, with infinite amount of data, can exactly isolate the fluctuations of interest, it is typically discarded in the estimation literature because its two-sided nature may change the timing of the data information - a similar argument is made also for the Hodrick and Prescott (HP) filter. In addition, with samples of typical length, all filters induce considerable sampling errors in the estimates of the cyclical component which may compound population misspecification problems. Model-driven filtering, on the other hand, fails to extract cycles with the required periodicity (see Canova (2008)) and requires exact knowledge of the number and the nature of the shocks driving the non-cyclical component. Given our general ignorance on the subject, important specification errors are present also with this approach.

Two additional important issues should be mentioned. First, while researchers filter each series separately prior to estimation, a balanced growth path is often used as a working assumption in theoretical models. Hence, should economic theory or pragmatic considerations guide statistical filtering? Second, while real variables typically show long run drifts, nominal variables just display low frequency fluctuations. Should we filter all the data or only real variables? Conversely, should we treat all the fluctuations present in nominal variables as relevant for parameter estimation or not? Since different researchers choose different methods to filter a portion (or all) of the available data prior to estimation, and since measurement error with unknown properties is introduced regardless of the approach one employs, economic inference is likely to be distorted and the magnitude of the distortions may depend on the transformation employed (see Canova (2008)).

This paper proposes a method to estimate the structural parameters of a time invariant cyclical DSGE model using noisy and potentially mismeasured cyclical data. The approach borrows ideas from the recent data-rich environment literature (see Boivin and Giannoni (2005)) to set up an estimated structure where vectors of data filtered with alternative procedures are treated as contaminated estimates of the true cyclical component. We set up a signal extraction framework where the cyclical DSGE is the unobservable factor; vectors of filtered data are contaminated observable proxies, and the parameters of the DSGE model are jointly estimated together with the non-structural parameters. This paper therefore complements those of Canova (2008), who study how to estimate DSGE models when the cyclical component is not solely located at business cycle frequencies and, conversely, the non-cyclical component may play an important role at business cycle frequencies, and of Ferroni (2008), who suggests ways to test trend specifications in DSGE models and compares the properties of one and two step estimators of its structural parameters.

Our approach is advantageous in, at least, three respects. Since we do not have to arbitrarily choose one filtering method prior to the estimation, or select which shock drives the non-cyclical component, we avoid specification errors. Moreover, our method can be used with cyclical data obtained with one-sided and two-sided filters, of both univariate and multivariate nature, as long as the list of filters is sufficiently rich. Finally, if the cyclical components obtained with different filters are relatively similar, their joint use in the estimation permits the elimination of small sample biases in parameter estimates. If, on the other hand, they are relatively different, measurement error may have different time series properties. Since our signal extraction procedure averages the cross-filter information, it may reduce this error and eliminate part of its cyclicality, making estimates of the cyclical components and of the structural parameters better shielded from filtering errors and inference more robust.

We investigate the properties of our approach using experimental data of the typical length employed in macroeconomics. We show that estimating the structural parameters with cyclical data produced by just one filter typically induces large biases. These biases are considerably reduced with our approach. We also show that the unconditional one step ahead mean square error (MSE) produced by our approach is smaller than the MSE obtained with a standard procedure and that the biases of the latter translates in conditional forecasts which are considerably distorted.

To show that the biases are not only statistically but also economically relevant, we revisit the role of money in transmitting monetary business cycles. The recent literature has largely neglected the stock of money when studying monetary business cycles and Ireland (2004) has shown that such an approach is, by and large, appropriate using US data, standard filtering techniques and a maximum likelihood estimation setup. We show that when the information produced by multiple filters is jointly used in the estimation, real balances statistically matter for the transmission of cyclical fluctuations to output and inflation, both directly and indirectly. Furthermore, we show that the propagation of primitive shocks in the estimated economy differs from the one obtained when only one data transformation is used.

We want to be clear for why we insist on working with time invariant cyclical models, rather than considering structures where cyclical and non-cyclical fluctuations are jointly accounted for. On one hand, writing down reasonable models with these features is hard. In theory, in fact, little is known about mechanisms propagating cyclical shocks at longer frequencies (exceptions are Comin and Gertler (2006) or Canova et al. (2007)) or creating important cyclical implications from long run disturbances. On the other, it is convenient to assume that the mechanisms driving cyclical and non-cyclical fluctuations are distinct and orthogonal. Finally, breaks of various sorts make the data largely uninformative about the features of medium/longer term cycles.

The rest of the paper is organized as follows. The next section shows the problems one encounters using a single filtering method to estimate the parameters of DSGE models. Section 3 presents our approach and applies it to experimental data. Section 4 examines the role of money in transmitting monetary business cycles. Section 5 concludes.

## 2 Statistical filtering and structural estimation

To show why statistical filtering induces important measurement errors in the estimated cyclical components and to investigate how these errors affect structural estimates, we simulate data from a textbook New-Keynesian model (see Gali (2008)), where agents face a labor-leisure choice, production is carried out with labor, firms face an exogenous probability of price adjustments and monetary policy is represented with a conventional Taylor rule. The log-linearized equilibrium conditions, in deviation from the steady states, are:

$$\lambda_t = \chi_t - \frac{\sigma_c}{1-h}(y_t - hy_{t-1}) \tag{1}$$
$$y_t = z_t + (1-\alpha)n_t \tag{2}$$
$$w_t = -\lambda_t + \sigma_n n_t \tag{3}$$
$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\rho_\pi \pi_t + \rho_y y_t) + v_t \tag{4}$$
$$\lambda_t = E_t(\lambda_{t+1} + r_t - \pi_{t+1}) \tag{5}$$
$$\pi_t = k_p(w_t + n_t - y_t + \mu_t) + \beta E_t \pi_{t+1} \tag{6}$$

where $k_p = \frac{(1-\beta\zeta_p)(1-\zeta_p)}{\zeta_p}\frac{1-\alpha}{1-\alpha+\epsilon\alpha}$, $\lambda_t$ is the Lagrangian on the consumer budget constraint, $y_t$ is output, $n_t$ is hours, $w_t$ is the real wage and $r_t$ the nominal interest rate; $z_t$ is a technology shock, $\chi_t$ a preference shock, $v_t$ is a monetary policy shock and $\mu_t$ a markup shock. The structural parameters of the model are $\beta$, the discount factor, $\sigma_c$ the risk aversion coefficient, $h$ the coefficient of consumption habit, $1-\alpha$ the share of labor in production, $\sigma_n$ the inverse of Frisch elasticity, $\epsilon$ the elasticity among consumption varieties, $\zeta_p$ the probability of changing prices, while $\rho_\pi, \rho_y, \rho_r$ are parameters of the monetary policy rule. In addition, the parameter vector includes the autoregressive parameters and the standard deviations of the shocks.

For the sake of illustration assume that the preference shock $\chi_t$ has two components (a stationary autoregressive and a unit root), the technology shock is a stationary AR(1), and the monetary policy and the markup shocks are iid. In the simulations we set $\beta = 0.99; \sigma_c = 3.00; h = 0.70; \sigma_n = 0.70; \epsilon = 7.0; \alpha = 0.6; \rho_r = 0.2; \rho_\pi = 1.30; \rho_y = 0.05; \zeta_p = 0.8$, and $\rho_\chi = 0.5; \rho_z = 0.8; \sigma_\chi = 0.0112; \sigma_z = 0.0051; \sigma_v = 0.0010; \sigma_\mu = 0.2060$, while the standard deviation of the shock driving the unit root component in preferences is $\sigma_{\chi,nc} = 0.0061$. None of the conclusions we highlight depend on whether the preference or the technology shock has two components; on whether the non-cyclical component is driven by a unit root or a stochastic linear trend, and on the exact values of the standard deviations of the two components of the preference shocks, as long as $\sigma_{\chi,nc}$ is small relative to $\sigma_\chi$ (see Canova (2008)). We simulate 550 data points for four observable variables $(y_t, w_t, \pi_t, r_t)$, discard 300 initial observations to eliminate the effect of initial conditions, and use the last 100 for forecasting exercises. This means that the sample we use has 150 observations.

Table 1 presents the variability of filtered output and filtered inflation when linear (LT), Hodrick and Prescott (HP), band pass (BP) and first order difference (FOD) filtering are used together with the variability of the true cyclical component of output and inflation. Figure 1 graphs the autocorrelation function of filtered output, of filtered and unfiltered inflation, and of the true cyclical component of the two variables.

| | LT | HP | BP | FOD | True |
|---|---|---|---|---|---|
| Output | 0.36 | 0.08 | 0.18 | 0.13 | 0.21 |
| Inflation | 0.12 | 0.07 | 0.07 | 0.13 | 0.12 |

Table 1: Standard deviation of filtered and true cyclical components; simulated data, scale $10^{-2}$.

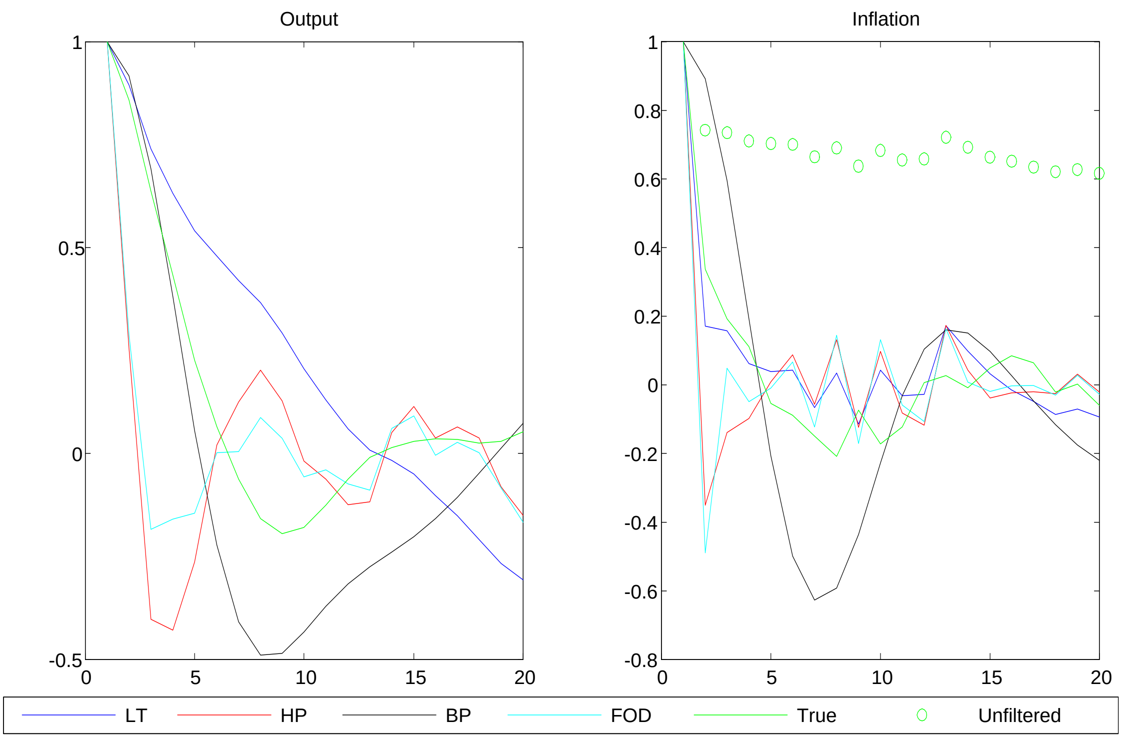


Figure 1: Autocorrelation functions of cyclical output and cyclical inflation

Clearly, the variability of the cyclical component of the two variables is generally mismeasured, and the serial correlation properties of the two variables are similarly distorted. Moreover, the direction and the magnitude of the distortions is filter and variable dependent - the HP and BP filters significantly underestimate both cyclical variabilities, while LT and FOD over or underestimate the cyclical variability of output only. Finally, despite the fact that the model features a unit root shock, unfiltered inflation is clearly stationary and the filtered and unfiltered inflation series have different persistence. Hence, it is generally going to matter for structural parameter estimation whether the model is fitted to filtered or unfiltered inflation.

A compact way to highlight what each of these filtering transformation do to the data is to plot the difference between log spectrum of filtered output and of filtered inflation and the log spectrum of the cyclical component of these two variables. If one filtering transformation exactly recovers the cyclical component, the difference will be zero at all frequencies. Imperfect isolation in certain bands of the spectrum will be evident when the plot is above or below zero at these frequencies. To facilitate the discussion, we separate frequencies into low, medium (business cycle), and high and, in figure 2, we separate the frequencies corresponding to cycles of 8-32 quarters from the others with two vertical bars.

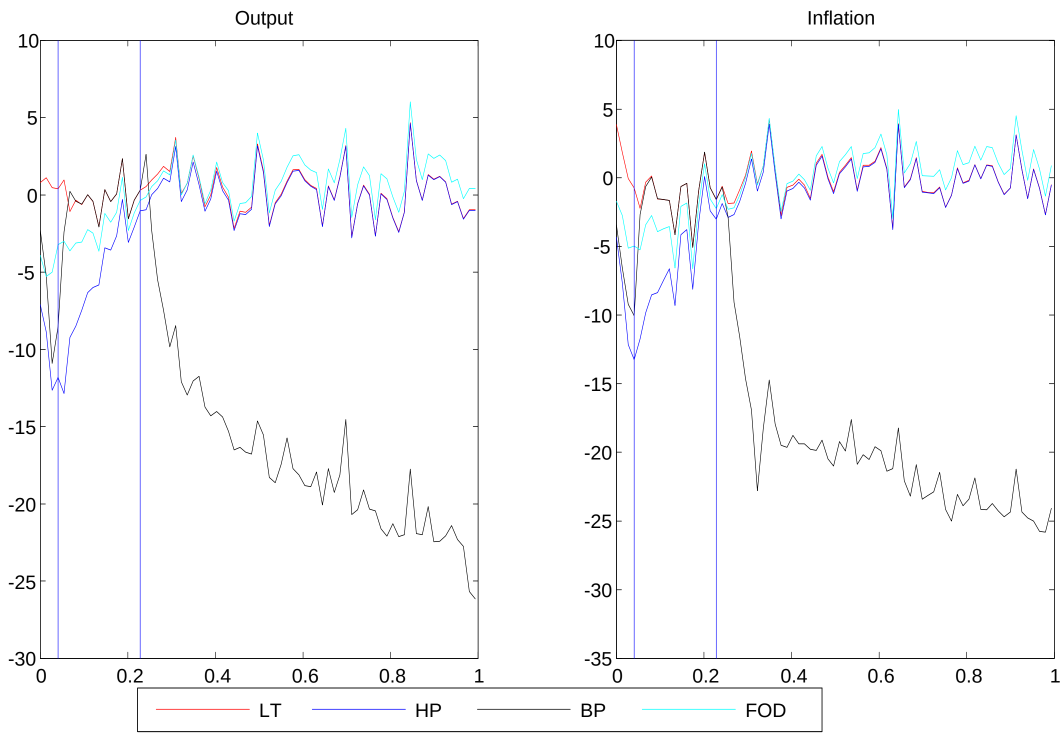


Figure 2: Log spectra of filtering error

Figure 2 shows that all filters imperfectly capture the cyclical component of the data. Moreover, the measurement error is not only located in the high frequencies and its frequency distribution is filter dependent. For example, LT detrended data have much stronger low frequency components than the actual cyclical component while the other three have smaller low frequency components. In addition, all filtered series display significant compression at business cycle frequencies - none of the filter exactly extracts the cyclical component or the business cycle frequencies of the cyclical component. Distortions in the low frequencies of the spectrum are particularly concerning because the perceived income and substitution effects in the economy are different from the true income and substitution effects and this is likely to make estimates of the structural parameters generally distorted.

| Parameter | Distribution | Mean | Standard Deviation |
|---|---|---|---|
| $\sigma_c$ | $\Gamma(20, 0.1)$ | 2.00 | 0.45 |
| $\sigma_n$ | $\Gamma(20, 0.1)$ | 2.00 | 0.45 |
| $h$ | $B(10, 3)$ | 0.76 | 0.11 |
| $\alpha$ | $B(3, 8)$ | 0.27 | 0.13 |
| $\epsilon$ | $N(6, 0.5)$ | 6.00 | 0.50 |
| $\rho_r$ | $B(10, 6)$ | 0.71 | 0.09 |
| $\rho_\pi$ | $N(1.5, 0.2)$ | 1.50 | 0.20 |
| $\rho_y$ | $N(0.4, 0.2)$ | 0.40 | 0.20 |
| $\zeta_p$ | $B(6, 6)$ | 0.50 | 0.14 |
| $\rho_\chi$ | $B(10, 6)$ | 0.71 | 0.09 |
| $\rho_z$ | $B(10, 6)$ | 0.71 | 0.09 |
| $\sigma_\chi$ | $\Gamma^{-1}(10, 20)$ | 0.0056 | 0.0020 |
| $\sigma_z$ | $\Gamma^{-1}(10, 20)$ | 0.0056 | 0.0020 |
| $\sigma_v$ | $\Gamma^{-1}(10, 20)$ | 0.0055 | 0.0020 |
| $\sigma_\mu$ | $\Gamma^{-1}(10, 20)$ | 0.0056 | 0.0020 |

Table 2: Prior distributions for the structural parameters. $\Gamma$ is the gamma distribution, $B$ is the beta distribution, $N$ the normal distribution.

To show that indeed the errors produced by imperfect filtering distorts our ability to understand the features of the true economy and that the amount of distortions depends on whether some or all variables are filtered, we take the experimental data we have constructed and estimate the structural parameters prefiltering the raw data with LT, HP, BP and FOD filters. Estimation is conducted using Bayesian methods. We choose loose priors for all the parameters (see table 2) and, to give the best chance to the routine, start estimation at the true parameter values. Posterior estimates are obtained with a random walk Metropolis algorithm, where the jumping variable has a t-distribution with 5 degrees of freedom and variance is tuned up to have an acceptance rate of about 30 percent for each filtering approach. Half a million draws were made for each filtered/DGP combination; convergence was checked with a standard CUMSUM statistic and achieved after less than 250000 iterations. We keep one out of hundred of the last 100,000 draws to compute statistics of the posterior distribution.

Table 3 reports the median and the standard deviation of the posterior of each structural parameter. The top panel refers to the situation when all variables are independently filtered prior to estimation. The bottom panel to the case when only output and the real wage are independently filtered.

The table shows that there are important estimation biases and the magnitude of these biases can be larger than 100 percent for some important economic parameters. Since measurement error has important low frequency components, the parameters regulating the relative magnitude of income and substitution effects (the risk aversion coefficient $\sigma_c$, the inverse of the Frisch elasticity $\sigma_n$, and persistence of the shocks) are all distorted, implying a much stronger income effect that the true data displays. Furthermore, as expected, the distortions in the persistence parameters are larger with filtering procedures that eliminate "too much" low frequency components from the actual data. Finally, for the DGP we consider, distortions obtained when only real variables are filtered are comparable to those obtained when all variables are filtered. This depends on the fact that the cyclical component of inflation (and the nominal rate) is not very persistent. When cyclical inflation has higher persistence, the combination of filtered and filtered data " unbalances" the likelihood - some equations become more misspecified than others - and this makes estimates obtained filtering only part of the data more distorted. Since likelihood based methods produce parameters estimates which minimize the largest discrepancy between the model and the data, biases tend to be more relevant in this case.

How could one eliminate the distortions induced by imperfect filtering? With a fixed sample size, there is not much one can do other than attempt to design filters which are sufficiently flexible to adapt to the (unknown) features of the cyclical components. If the cyclical component of the data was truly located only at business cycle frequencies - figure 2 shows that this is, in general, not the case - one could try to use filters which are capable to better isolate the frequencies of interest, even in small samples. Filters like these exist in the literature, see e.g. the one suggested by Christiano and Fitzgerald (2003), but their unconventional features (in particular their asymmetry and their time varying weights) may have important and uncontrollable consequences on parameter estimates.

## 3 The idea of the paper

Our suggestion is to use the information contained in the cyclical data obtained with different filters to reduce measurement error induced by imperfect filtering, in particular in the low frequencies of

| Filter | | LT | HP | FOD | BP | $\text{Factor}_u$ | $\text{Factor}_r$ |
|---|---|---|---|---|---|---|---|
| | True | Median (s.e.) | Median (s.e.) | Median (s.e.) | Median(s.e.) | Median (s.e.) | Median(s.e.) |
| All filtered | | | | | | | |
| $\sigma_c$ | 3.00 | 2.00( 0.14) | 1.96( 0.12) | 2.08( 0.12) | 1.95( 0.11) | 2.34( 0.18) | 2.99( 0.33) |
| $\sigma_n$ | 0.70 | 1.24( 0.06) | 1.25( 0.05) | 1.25( 0.05) | 1.33( 0.06) | 0.10( 0.03) | 0.42( 0.03) |
| $h$ | 0.70 | 0.47( 0.04) | 0.45( 0.03) | 0.43( 0.04) | 0.57( 0.03) | 0.73( 0.04) | 0.72( 0.05) |
| $\alpha$ | 0.60 | 0.14( 0.02) | 0.12( 0.02) | 0.14( 0.02) | 0.15( 0.01) | 0.49( 0.02) | 0.44( 0.02) |
| $\epsilon$ | 7.00 | 4.00( 0.13) | 4.70( 0.17) | 3.65( 0.15) | 3.89( 0.13) | 6.13( 0.09) | 6.34( 0.08) |
| $\rho_r$ | 0.20 | 0.20( 0.07) | 0.14( 0.06) | 0.35( 0.09) | 0.23( 0.05) | 0.28( 0.05) | 0.26( 0.12) |
| $\rho_\pi$ | 1.30 | 1.58( 0.06) | 1.66( 0.09) | 1.55( 0.11) | 1.56( 0.06) | 1.51( 0.02) | 1.56( 0.02) |
| $\rho_y$ | 0.05 | 0.81( 0.05) | 0.84( 0.07) | 0.68( 0.04) | 0.71( 0.04) | 0.26( 0.02) | 0.25( 0.05) |
| $\zeta_p$ | 0.80 | 0.93( 0.03) | 0.93( 0.03) | 0.91( 0.03) | 0.95( 0.03) | 0.83( 0.04) | 0.83( 0.03) |
| $\rho_\chi$ | 0.50 | 0.93( 0.03) | 0.94( 0.03) | 0.91( 0.03) | 0.98( 0.03) | 0.72( 0.05) | 0.61( 0.10) |
| $\rho_z$ | 0.80 | 0.87( 0.03) | 0.88( 0.03) | 0.85( 0.04) | 0.95( 0.03) | 0.68( 0.03) | 0.71( 0.05) |
| $\sigma_\chi$ | 1.10 | 0.12( 0.02) | 0.12( 0.02) | 0.12( 0.02) | 0.12( 0.02) | 0.21( 0.04) | 0.29( 0.09) |
| $\sigma_z$ | 0.57 | 0.08( 0.01) | 0.09( 0.01) | 0.08( 0.01) | 0.08( 0.01) | 0.32( 0.08) | 0.37( 0.09) |
| $\sigma_{mp}$ | 0.12 | 0.06( 0.01) | 0.06( 0.01) | 0.06( 0.01) | 0.06( 0.01) | 0.08( 0.01) | 0.08( 0.01) |
| $\sigma_\mu$ | 20.64 | 4.59( 0.59) | 4.30( 0.52) | 3.03( 0.65) | 3.28( 0.52) | 4.78( 0.36) | 5.45( 0.73) |
| Real filtered | | | | | | | |
| $\sigma_c$ | 3.00 | 1.99( 0.14) | 2.02( 0.13) | 1.90( 0.14) | 2.12( 0.12) | 2.40( 0.36) | 2.90( 0.40) |
| $\sigma_n$ | 0.70 | 1.24( 0.05) | 1.25( 0.05) | 1.24( 0.05) | 1.43( 0.08) | 0.12( 0.03) | 0.45( 0.03) |
| $h$ | 0.70 | 0.45( 0.04) | 0.49( 0.03) | 0.36( 0.04) | 0.61( 0.03) | 0.66( 0.04) | 0.62( 0.04) |
| $\alpha$ | 0.60 | 0.13( 0.02) | 0.14( 0.02) | 0.14( 0.02) | 0.15( 0.03) | 0.48( 0.02) | 0.36( 0.04) |
| $\epsilon$ | 7.00 | 3.63( 0.14) | 4.37( 0.15) | 3.77( 0.16) | 4.24( 0.16) | 6.17( 0.09) | 6.37( 0.11) |
| $\rho_r$ | 0.20 | 0.16( 0.05) | 0.23( 0.04) | 0.38( 0.08) | 0.09( 0.04) | 0.36( 0.05) | 0.29( 0.07) |
| $\rho_\pi$ | 1.30 | 1.60( 0.09) | 1.66( 0.08) | 1.63( 0.10) | 1.68( 0.06) | 1.48( 0.02) | 1.49( 0.02) |
| $\rho_y$ | 0.05 | 0.83( 0.05) | 0.79( 0.08) | 0.77( 0.06) | 0.71( 0.08) | 0.33( 0.02) | 0.33( 0.02) |
| $\zeta_p$ | 0.80 | 0.93( 0.03) | 0.90( 0.03) | 0.94( 0.03) | 0.93( 0.03) | 0.78( 0.05) | 0.83( 0.05) |
| $\rho_\chi$ | 0.50 | 0.94( 0.03) | 0.93( 0.03) | 0.94( 0.03) | 0.97( 0.03) | 0.65( 0.04) | 0.68( 0.06) |
| $\rho_z$ | 0.80 | 0.88( 0.03) | 0.86( 0.03) | 0.93( 0.03) | 0.94( 0.03) | 0.66( 0.03) | 0.66( 0.05) |
| $\sigma_\chi$ | 1.10 | 0.12( 0.02) | 0.11( 0.02) | 0.13( 0.02) | 0.12( 0.02) | 0.21( 0.04) | 0.26( 0.08) |
| $\sigma_z$ | 0.57 | 0.08( 0.01) | 0.08( 0.01) | 0.09( 0.01) | 0.08( 0.01) | 0.29( 0.08) | 0.36( 0.09) |
| $\sigma_{mp}$ | 0.12 | 0.06( 0.01) | 0.06( 0.01) | 0.06( 0.01) | 0.06( 0.01) | 0.08( 0.01) | 0.08( 0.01) |
| $\sigma_\mu$ | 20.64 | 6.02( 0.52) | 2.47( 0.69) | 11.72( 0.94) | 7.56( 0.66) | 4.88( 0.36) | 5.60( 0.67) |

Table 3: Parameters estimates obtained using different filters; the DGP has a preference shock with two components, a stationary AR(1) and a unit root.

the spectrum. In other words, rather than arbitrarily selecting one filter and estimating the model with the resulting filtered data, we treat cyclical data extracted with various filtering methods as contaminated estimates of an unobservable cyclical component and use the information provided by different filters jointly in the estimation of the structural parameters. If the measurement error has similar features across filtering methods, the joint use of multiple cyclical data in the estimation reduces small sample biases in parameter estimates - this is the same idea that the literature on stochastic pooling has emphasized - and make inference more robust. If the measurement error is close to be idiosyncratic across filtering methods, less distortions should be present and more precise estimates of the cyclical features of the economy should be obtained. Hence, our approach also uses ideas of Boivin and Giannoni (2005), who suggest that a data rich environment can help to estimate the structural parameters of a DSGE model and more precisely forecast out-of-sample.

Let the log-linearized solution of a cyclical DSGE model be of the form:

$$x_{1t} = RR(\theta)x_{2t-1} + SS(\theta)x_{3t} \quad (7)$$

$$x_{2t} = PP(\theta)x_{2t-1} + QQ(\theta)x_{3t} \quad (8)$$

$$x_{3t+1} = NN(\theta)x_{3t} + \iota_t \quad \iota_t \sim (0, \Sigma(\theta)) \quad (9)$$

where $PP, QQ, RR, SS$ are time invariant matrices which are functions of the vector of structural parameters $\theta = (\theta_1, \ldots, \theta_k)$, $x_{2t} = \tilde{x}_{2t} - \bar{x}_2$ includes predetermined states, $x_t = \tilde{x}_{1t} - \bar{x}_1$ the endogenous variables, $x_{3t}$ the exogenous disturbances and $\bar{x}_i, i = 1, 2$ are the steady states of $\tilde{x}_{1t}$ and $\tilde{x}_{2t}$. We let $x_t^m = S[x_{1t}, x_{2t}, x_{3t+1}]'$, be a $n \times 1$ vector where $S$ is a selection matrix picking those variables which are observable and interesting from the point of view of the analysis. Even though we suppress the dependence of $x_t^m$ on $\theta$, it should be understood that the data produced by the model is in fact conditional on the choice of $\theta$.

Let $x_t^i$ be the vector of filtered observable time series obtained with method $i = 1, 2, ...g$ and let $x_t^d = [x_t^1, x_t^2, \ldots, x_t^g]'$. Assume the following structure:

$$x_t^d = \lambda_0 + \lambda_1 x_t^m + u_t \quad (10)$$

where $\lambda_0$ is a $ng \times 1$ vector of constants, $\lambda_1$ a $ng \times n$ matrix of non-structural parameters and $u_t$ is a $ng \times 1$ vector of possibly serially correlated errors. For estimation purposes, we normalize the $n \times n$ block $\lambda_1^1 = I$ so that the remaining blocks of the matrix $\lambda_1$ can be interpreted as loadings relative to those of the first method. Joint estimation of the structural parameters $\theta$ and the non-structural parameters $\lambda_j$, $j = 0, 1$ is now possible because (7)-(9)and (10) represent a state space system with the latter being a measurement equation and the former state equations. Specifically, these equations can be cast into the state space system

$$s_{t+1} = Fs_t + Ga_{t+1} \quad (11)$$

$$o_t = Hs_t + \eta_t \quad (12)$$

by setting

$$
\begin{aligned}
s_{t+1} &= \begin{pmatrix} x_{1t} & x_{2t} & x_{3t+1} \end{pmatrix}' \\
F &= \begin{pmatrix} 0 & RR & SS \\ 0 & PP & QQ \\ 0 & 0 & NN \end{pmatrix} \\
G &= \begin{pmatrix} 0, & 0, & I \end{pmatrix}' \\
a_{t+1} &= \iota_{t+1} \\
o_t &= \left( x_t^i - \lambda_0^i, i = 1, 2, \ldots, g \right)' \\
H &= \lambda_1 S \\
\lambda_1 &= \text{diag } \left( \lambda_1^i, i = 1, 2, \ldots, g \right)', \ \lambda_1^1 = I \\
\eta_t &= u_t
\end{aligned}
$$

The likelihood of (11)-(12) can be computed with the Kalman filter. In our context the vector of parameters of interest is $\nu$, which includes the structural parameters $\theta$ and the non-structural parameters $(\lambda_0^i, \lambda_1^{i+1}, \sigma_\eta^k, \ i = 1, 2, \ldots, g; \ k = 1, \ldots, ng)$. If Bayesian estimation is preferred, the non-normalized posterior distribution of $\nu$, can be obtained with Monte Carlo Markov Chain simulators. For example, the following algorithm appears to give reasonable results in estimation. Starting from an initial value $\nu^{\ell-1}$, given a $\Sigma$, and a prior $g(\nu)$:

1. Draw a shock vector $\upsilon$ from $t(0, \Sigma, 5)$ and construct a candidate $\nu^* = \nu^{\ell-1} + \upsilon$
2. Solve the model using $\nu^*$; if the solution is indeterminate or no solution is found set $\mathcal{L}(\nu|o) = 0$. Otherwise, evaluate the likelihood of the observables $o_t$ at $\nu^*$ $\mathcal{L}(\nu^*|o)$ with the Kalman filter.
3. Calculate $\breve{g}(\nu^*|o) = g(\nu^*)\mathcal{L}(\nu^*|o)$ and the ratio $MR^* = \frac{\breve{g}(\nu^*|o)}{\breve{g}(\nu^{\ell-1}|o)}$
4. Draw $\varsigma$ from $U[0, 1]$; if $MR^* > \varsigma$ set $\nu^\ell = \nu^*$, otherwise set $\nu^\ell = \nu^{\ell-1}$

Iterated a large number of times, the algorithm ensures that the sample $(\nu^{\bar{L}}, \nu^{\bar{L}+1}, \ldots, )$ for an appropriately chosen $\bar{L}$ is a draw from the target distribution that we need to sample from (for further details see Canova (2007a)).

In (10) different cyclical estimates $x_t^i$ are treated as contaminated proxies of the true cyclical component. They are contaminated in two senses: they introduce fluctuations which are non-cyclical, i.e. their periodicity is outside the 8-32 quarter range; they compress the power of the spectrum of the series at cyclical frequencies. The amount of information they contain for the model relevant concepts of cyclical fluctuations is measured by the vector $\lambda_0$ and the matrix $\lambda_1$. Ideally, $\lambda_0$ is a vector of zeros and $\lambda_1$ a matrix with the identity in each $n \times n$ block, so that each measure is an unbiased and perfectly correlated although noisy signal of the true cyclical component. In general, we expect either $\lambda_0^i \neq 0$ or $\lambda_1^{i+1} \neq I$, or both, for some or all $i$'s. Since we have normalized $\lambda_1^1$, estimates of $\lambda_1^{i+1}$ gives us an idea of the amount of correlation distortions each method displays relative to the first.

This setup is advantageous in, at least, three respects. First, since we do not have to arbitrarily choose one filtering approach prior to the estimation or select which shock drives the non-cyclical component, we avoid specification errors. Second, our approach can use as observables cyclical components obtained with one-sided and two-sided filters, both of univariate and multivariate nature and cyclical components obtained assuming that cyclical and non-cyclical components are correlated or not, as long as the list of filters is sufficiently rich. Third, if the cyclical components obtained with different filters are relatively similar, their joint use in the estimation reduces small sample biases in parameter estimates. If, on the other hand, they are relatively different, measurement error may have different time series properties. Since our signal extraction procedure averages the cross-filter information, it may reduce this error and eliminate part of its cyclicality, making estimates of the cyclical components and of the structural parameters better shielded from filtering errors and inference more robust.

It is important to stress that our analysis is conditional on two important assumptions. First, we assume that the model generating $x_t^m$ is correctly specified; that is, there are no missing variables or shocks. When this is not the case, the interpretation of the $\lambda$'s becomes more difficult and there is no guarantee that the signal extraction approach we describe has better properties than any of the standard approaches. Second, we assume that the cyclical and the non-cyclical components of the data are uncorrelated. While the majority of models used in the literature employs this simplifying assumption, the presence of such a correlation generates additional sources of misspecifications and biases which are neglected in this paper.

The literature is largely silent about the issues we address in this paper. Cogley (2001) and Gorodnichenko and Ng (2007) are concerned with the problem of estimating the structural parameters of a cyclical DSGE when the trend specification is incorrect, but they do not investigate what are the consequences of small sample filtering nor their implications for structural estimates. Giannone et al. (2006) emphasize that if the variables of the model are measured with error, the solution has a natural factor structure and exploit this feature to compare responses obtained from VAR and factor models. Rather than considering a factor structure for the endogenous variables in terms of the states, we construct an estimable structure where vectors of filtered observable data have a factor structure in terms of a subset of the variables of the model. However, as in Giannone et al., we emphasize that important measurement error with low frequency components may exist.

The paper which is closest in spirit to ours is Boivin and Giannoni (2005). Their main point is that the model variables do not have an exact counterpart in the real world and that some external indicators to the model may have important information for interesting variables. The point here is somewhat similar. The cyclical component of the model does not have an exact counterpart in the data because none of the existing filters is able to exactly extract the fluctuations of interest. Moreover, if different cyclical vectors have idiosyncratic error components, this error may be averaged out with our signal extraction approach.

### 3.1 How does the procedure fare with simulated data?

We estimate the structural parameters of the model using the suggested approach and the same experimental data used in section 2. As input in our procedure, we employ the vector of LT, HP and FOD filtered data. Thus, the vector of observable variables $o_t$ is $12 \times 1$ (the model produces implications for four variables and there are three filtering methods). As shown in figure 2, these three set of data show considerable spectral similarities. Thus, the small sample bias reduction effect will probably be more important than the measurement error reduction effect. As mentioned, the latter will dominate if the vector of filtered data has sufficiently idiosyncratic spectral properties - an example of this situation will be considered in section 4. Nevertheless, since it is unlikely that applied investigators will resort to exotic filtering approaches to generate cyclical components with sufficiently idiosyncratic features, the results we present are sufficient to illustrate the advantages of using multiple filtering devices in the estimation of cyclical models and probably more relevant in practice than those obtained under ideal but impractical conditions.

We employ the same Bayesian approach used in section 2, assuming the same priors on the structural parameters shown in table 2 and loose priors on the non-structural parameters entering (10). In particular, we assume that the prior for each element of $\lambda_0$ is normally distributed, centered at zero with variance equal to 0.5; the prior for the free diagonal elements of $\lambda_1$ is normal, centered at 1 with variance 0.5: and the prior for the standard deviation of the $u_t$'s is inverted gamma with mean equal to 0.0056 and variance equal to 0.002.

We report results obtained with two specifications: one where the non-structural parameters are filter and series specific (in this case there are 32 non-structural parameters to be estimated) and another where the constants and the loadings in (10) are common across series for each filter (in this case, there are 17 non-structural parameters). We refer to the first specification as the unrestricted factor model; the second one to the restricted factor model.

The last two columns of table 3 present the posterior median and the posterior standard deviation for the structural parameters obtained with these two specifications, when all variables are filtered (top panel) and when only real variables are filtered (bottom panel). In general, the biases present when only one set of cyclical data is used in the estimation are reduced or eliminated. For example, the risk aversion coefficient and the Frisch elasticity of labor supply are better estimated with reasonably tight standard deviations, and the persistence of the shocks much closer to the true values. The restricted specification appears to be slightly superior when all variables are filtered, but differences are small. Notice also, that the approach we suggest does reasonably well both when all data is filtered and when only real data is filtered. The variability of the structural shocks is still poorly estimated but this outcome is not very surprising since these parameters are weakly identified regardless of which set of cyclical data is used.

To see how these estimates compare with the true ones and with those obtained with standard approaches in terms of economically meaningful statistics, we first compute the unconditional autocorrelation function of the cyclical components of output and inflation, where by this we mean the component generated by the non-unit root shocks, when the posterior median estimates of the parameters obtained when all variables are filtered is used.

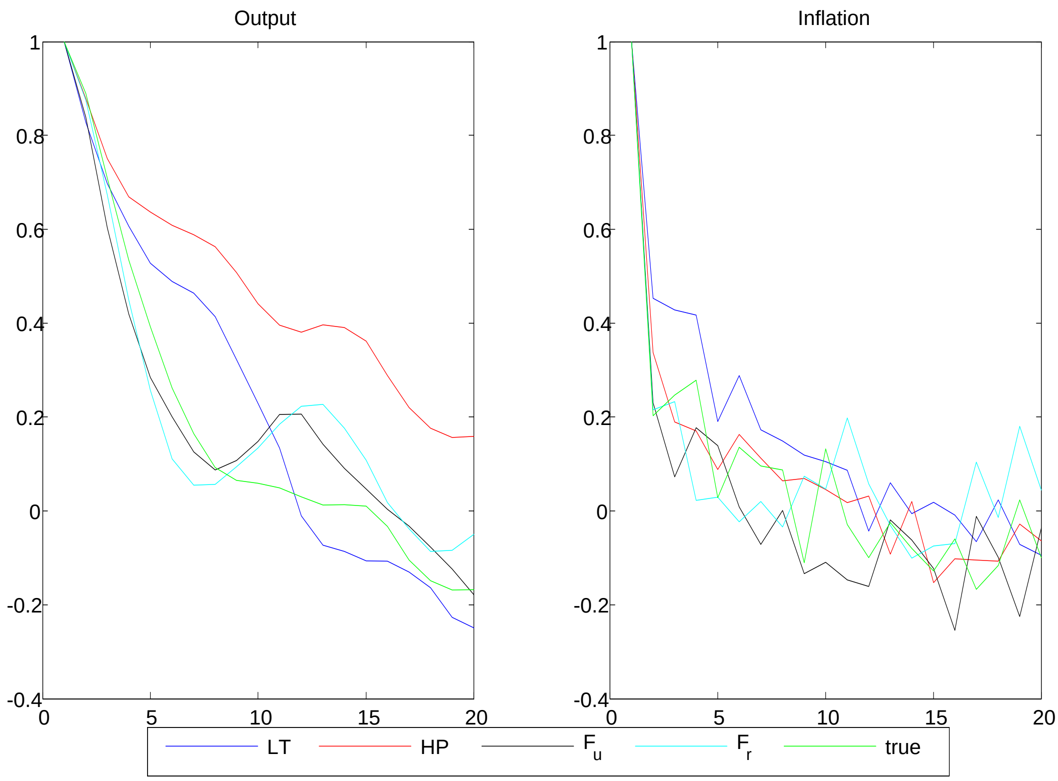


Figure 3: Autocorrelation functions of estimated and true cyclical components

Figure 3 confirms the conclusion that parameter estimates obtained with our approach are superior to those obtained with just one set of cyclical data. For cyclical output, the autocorrelation function obtained with our two specifications is close to the true one: at short horizons they are practically identical; at longer horizons differences are small. One can also see that there is a significant difference with the autocorrelation function of cyclical output obtained with standard approaches. For cyclical inflation the improvement over standard methods is less dramatic and the match with the true autocorrelation not as impressive, primarily because true inflation persistence is low. Nevertheless, even in this case, both the restricted and the unrestricted factor approaches reproduce relatively well the first few terms of the true autocorrelation function of inflation.

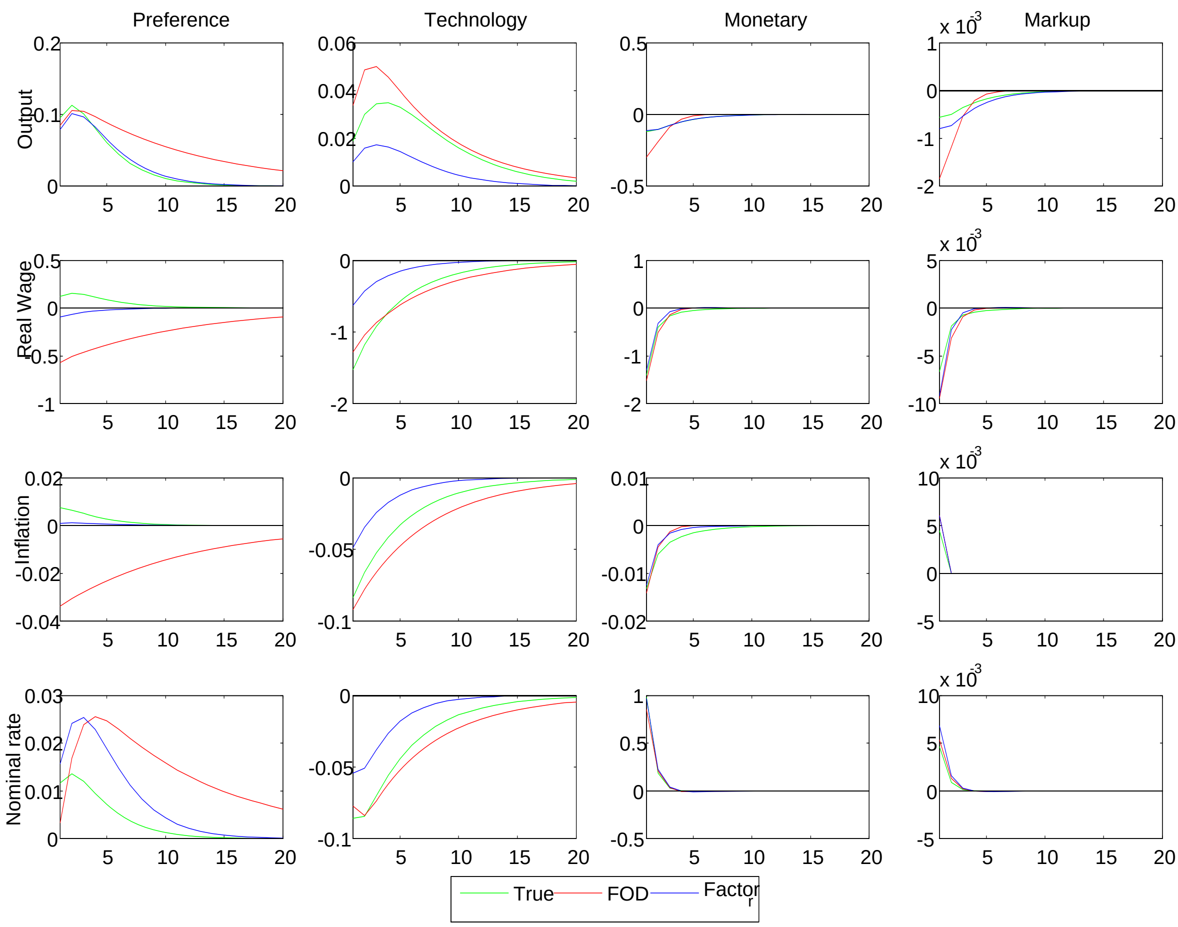


Figure 4: Impulse responses to shocks

The good unconditional performance of our approach is also confirmed when looking at the conditional responses of the endogenous variables to the four structural shocks. Figure 4 presents the responses produced with the true parameters, those generated with the posterior median estimates obtained with the restricted factor model and those obtained with FOD filtered data, which are the most appropriate terms of comparison, given the selected DGP. Few features of figure 4 are worth commenting upon. First, both the shape and the persistence of the conditional responses are well captured by our setup. There is one case where the impact sign is wrong - the response of the real wage to preference shocks - but differences are not large a-posteriori. Second, in comparison, responses obtained with FOD estimates display several distortions. For example, the impact response of output to monetary and markup shocks is too large and the responses of the four variables to preference shocks are poorly captured in terms of sign (see e.g. inflation), magnitude (see e.g. real wages) and persistence (see e.g. nominal rate). Third, it is clear that, in relative terms, our approach trades off a reduced precision in capturing the responses to technology shocks for a better match in reproducing the responses to preference shocks - exactly the opposite of what FOD estimates do.

The statistics we have presented in figures 3 and 4 are in-sample ones. Since DSGE models are nowadays used in policy institutions for out-of-sample unconditional and conditional forecasting exercises, it is worth examining the out-of-sample performance of our setup relative to traditional ones. We conduct two forecasting exercises. In the first case, we compute the sequence of one step ahead forecast errors for output and inflation, when we take as parameter values the posterior median estimates obtained when all data are filtered, setting all the shocks in the forecasting period to zero. The MSE is computed over 100 forecasting periods, when no updating of the parameters in the forecasting sample is performed, and appears in table 4.

| Series | LT | HP | FOD | BP | $\text{Factor}_u$ | $\text{Factor}_r$ |
|---|---|---|---|---|---|---|
| Output | 0.614 | 0.659 | 0.590 | 0.684 | 0.485 | 0.084 |
| Inflation | 0.380 | 0.454 | 0.365 | 0.454 | 0.377 | 0.346 |

Table 4: Mean square error of the unconditional forecasts; simulated data; scale $10^{-4}$.

Figure 5 instead traces out the one-step ahead path of cyclical output and cyclical inflation that would have obtained with posterior median estimates of the parameters when monetary shocks were drawn so as to keep the nominal interest rate fixed over the forecasting path. That is, we allow the nominal interest rate to endogenously react to output and inflation but make sure that the monetary shocks we draw are such that the nominal rate is constant over the forecasting path and equal to the value taken at the date prior to the forecasting period (time 0 in the figure).

Table 4 and Figure 5 indicate that the differences in the estimates shown in table 3 lead to important differences in forecasting performance. Our two specifications are superior to traditional approaches in unconditionally forecasting one-step ahead cyclical output and better in unconditionally forecasting of cyclical inflation. For output the reduction in MSE exceeds 20 percent, for inflation is about 5 percent. Our two specifications are also much better in conditional forecasting. Figure 5 shows that the bias introduced by traditional procedures translates in conditional output forecasts which are consistently different from the true ones while the differences between the forecasts produced by our two specifications and the true ones are statistically insignificant. For inflation differences with standard methods are even more evident since the biases in estimated parameters produce counterintuitive fluctuations, which are absent from the true forecasts.

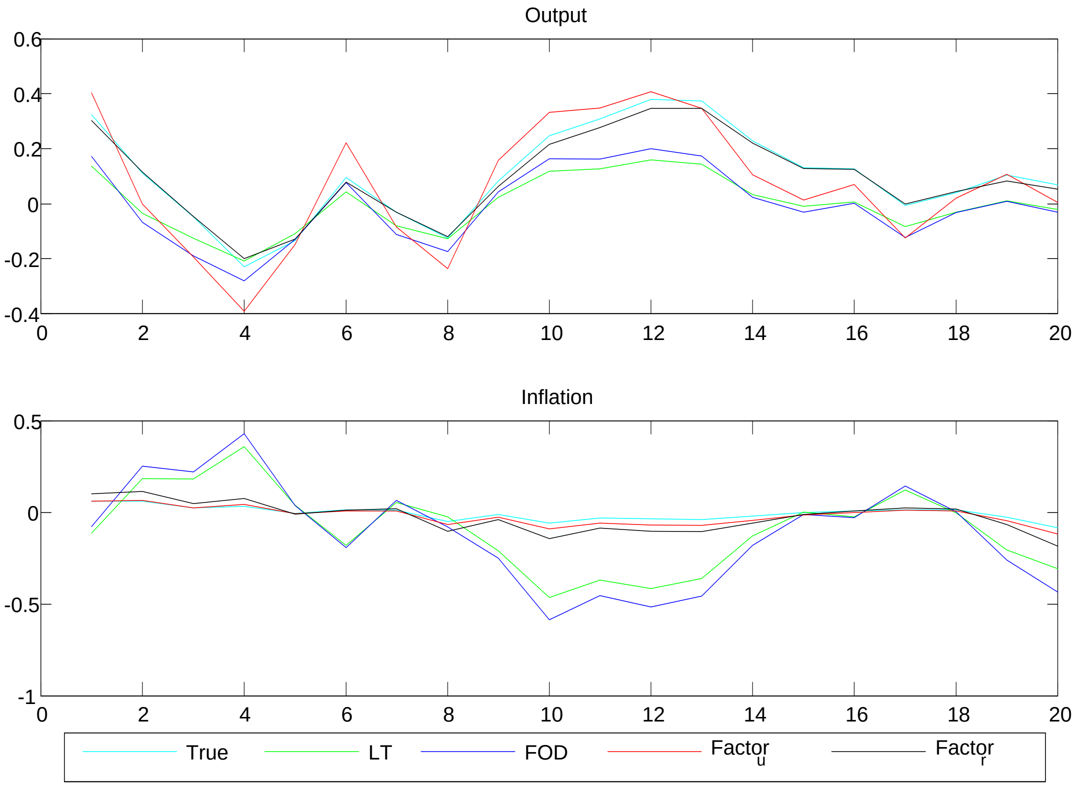


Figure 5: One step ahead forecasts, conditional on a constant interest rate path.

To conclude, the biases that a standard procedure induces in parameter estimates have important consequences for our understanding of both the unconditional autocorrelation properties of the cyclical component of the data and of the conditional responses to shocks. Overall, both statistics appear to be much better reproduced with the specifications we suggest. The out-of-sample forecasts produced by standard approaches inherit and magnify parameter biases providing a distorted picture of the cyclicality of the variables of interest. These problems are, to a large extent, eliminated when multiple sources of cyclical data are used in the estimation.

It is worth mentioning that the results we have presented hold also for alternative specifications of the DGP. In the appendix to this paper we show, for example, that the features of the estimates we have emphasized here hold when the non-cyclical component has power at business cycle frequencies; when the non-cyclical component is driven by a unit root in technology, rather than a unit root in preferences; and when the non-cyclical component is driven by a shock which displays stochastic fluctuations around a linear trend.

# 4 Does money matter in transmitting monetary business cycles?

To show that the additional information our procedure uses may be relevant for understanding important economic phenomena, we reconsider the role of money in transmitting monetary business cycles. The majority of the monetary models nowadays used in the policy and academic literature attributes a minimal role to the stock of money. In most of the cases these models make no reference whatsoever to monetary aggregates, and when they do, they use a specification where a money demand function determines how much money needs to be supplied, given predetermined levels of output, inflation and the nominal rate. As a consequence, changes in the nominal (and real) quantity of money play no direct or indirect role in shaping the dynamics of output and inflation.

Ireland (2004) has constructed a general specification in the class of textbook New Keynesian models where real balances may have a role in influencing the dynamics of output and inflation. He estimated the relevant parameters by likelihood techniques using post 1980 US data and found evidence supporting current theoretical practices. To construct the likelihood of this cyclical model, Ireland first transforms the actual data, taking away a separate linear trend from per-capita GDP and per-capita real balances and demeaning inflation and the nominal interest rate.

In this section, we conduct a similar exercise using post 1959 US data and the cyclical versions of real per-capita output, real per-capita money balances, inflation, and nominal rate series obtained with a number of filtering procedures. As a benchmark, we also estimate the model employing Ireland's preferred transformation over the same sample.

## 4.1 The model economy

The model is close to the one employed by Ireland (2004), except that it also permits real balances to play an indirect role, via its effects on the interest rate. Relative to the model we have considered in section 2, we allow the real stock of money to potentially matter for the determination of the output and inflation; consider frictions in the form of adjustment costs to changing prices rather than with a Calvo staggered-price device; and set the habit parameter to zero.

Since the economy is quite standard, we only briefly describe its features. There is a representative household, a representative final good producing firm, a continuum of intermediate goods-producing firms supplying the differentiated commodity $i \in [0,1]$ and a monetary authority. At each $t$ the representative household maximizes

$$E_t \sum_t \beta^t \chi_t [U(c_t, \frac{M_t}{p_t e_t}) - \eta n_t] \tag{13}$$

where $0 < \beta < 1,\ \eta > 0$, subject to the sequence of budget constraints

$$M_{t-1} + T_t + B_{t-1} + W_t h_t + D_t = P_t c_t + \frac{B_t}{R_t} + M_t \tag{14}$$

where $c_t$ is consumption, $n_t$ are hours worked, $p_t$ is the price level, $M_t$ are nominal balances, $W_t$ is the nominal wage and $B_t$ are one period nominal bonds with gross nominal interest rate $R_t$; $T_t$ are lump sum nominal transfers made by the monetary authority at the beginning of each $t$, and $D_t$ nominal dividends distributed by the intermediate firms. $\chi_t$ and $e_t$ are disturbances to preferences and the money demand whose properties will be described below. Let $m_t \equiv \frac{M_t}{p_t}$ denote real balances and $\pi_t \equiv \frac{p_t}{p_{t-1}}$ the period $t$ gross inflation rate.

The representative final good producing firm uses $y_t^i$ units of intermediate good $i$, purchased at the price $p_t^i$ to manufacture $y_t$ units of final goods according to the constant return to scale technology $y_t = [\int_0^1 (y_t^i)^{(\epsilon-1)/\epsilon} di]^{\epsilon/(1-\epsilon)}$ with $\epsilon > 1$, where $\epsilon$ is the constant price elasticity of demand for each intermediate good. Profit maximization produces demand functions

$$y_t^i = (\frac{p_t^i}{p_t})^{-\epsilon} y_t \tag{15}$$

Competition within the sector implies that $p_t = (\int_0^1 (p_t^1)^{1-\epsilon} di)^{1/(1-\epsilon)}$

The intermediate good producing firm i, hires $n_t^i$ units of labor from the representative household to produce $y_t^i$ units of intermediate good $i$ using the production function $y_t^i = z_t n_t^i$, where $z_t$ is an aggregate productivity shock. Intermediate goods substitute imperfectly for one another in producing finished goods. Hence, intermediate firms can set the price of their good but must satisfy (15) at the chosen price. We assume a quadratic cost in adjusting prices, measured in finished goods, given by

$$\frac{\phi}{2}(\frac{p_t^i}{\pi^s p_{t-1}^i} - 1)^2 y_t \tag{16}$$

where $\phi > 0$ and $\pi^s$ measures steady state inflation. Optimal prices are chosen to maximize

$$E \sum_t \beta^t \chi_t [U_1(c_t, \frac{M_t}{p_t e_t})](\frac{D_t^i}{p_t}) \tag{17}$$

subject to (15), where $\beta^t \chi_t U_1(c_t, \frac{M_t}{p_t e_t})$ measures the marginal utility value to the household of an additional unit of profits $t$ and real dividends are

$$\frac{D_t^i}{p_t} = (\frac{p_t^i}{p_t})^{1-\epsilon} y_t - (\frac{p_t^i}{p_t})^{-\epsilon} (\frac{w_t y_t}{z_t}) - \frac{\phi}{2} (\frac{p_t^i}{\pi p_{t-1}^i} - 1)^2 y_t \tag{18}$$

The monetary authority sets the nominal interest rate according to

$$R_t = R_{t-1}^{\rho_r} y_{t-1}^{(1-\rho_r)\rho_y} \pi_{t-1}^{(1-\rho_r)\rho_\pi} \Delta M_t^{(1-\rho_r)\rho_m} v_t \tag{19}$$

where $\rho_r, \rho_y, \rho_\pi, \rho_m \geq 0$ are parameters and $v_t$ is a monetary policy shock.

The law of motion of the disturbances of the model $d_t = (\chi_t, e_t, z_t, v_t)$ is $\log d_t = \bar{d} + N \log d_{t-1} + \iota_t$, where $N$ is a diagonal matrix with entries $\rho_\chi, \rho_e, \rho_z, 0$, respectively. The covariance matrix of the structural shocks $\Sigma$ is diagonal with entries $\sigma_\chi^2, \sigma_e^2, \sigma_z^2, \sigma_v^2$. In a symmetric equilibrium all firms make identical choices so $y_t^i = y_t, n_t^i = n_t, p_t^i = p_t, D_t^i = D_t$.

Log-linearizing the model around the steady state produces the following equilibrium conditions

$$\begin{aligned}
\hat{y}_t &= E_t\hat{y}_{t+1} - \omega_1((\hat{R}_t - E_t\hat{\pi}_{t+1}) - (\hat{\chi}_t - E_t\hat{\chi}_{t+1})) + \omega_2((\hat{m}_t - \hat{e}_t) - (E_t\hat{m}_{t+1} - E_t\hat{e}_{t+1})) && (20)\\
\hat{m}_t &= \gamma_1\hat{y}_y - \gamma_2\hat{R}_t + (1 - (R^s - 1)\gamma_2)\hat{e}_t && (21)\\
\hat{\pi}_t &= \beta E_t\hat{\pi}_{t+1} + \psi(\frac{1}{\omega_1}\hat{y}_t - \frac{\omega_2}{\omega_1}(\hat{m}_t - \hat{e}_t) - \hat{z}_t) && (22)\\
\hat{R}_t &= \rho_r\hat{R}_{t-1} + (1-\rho_r)\rho_y\hat{y}_{t-1} + (1-\rho_r)\rho_\pi\hat{\pi}_{t-1} + (1-\rho_r)\rho_m(\Delta\hat{m}_t + \hat{\pi}) + \hat{v}_t && (23)
\end{aligned}$$

where

$$\begin{aligned}
\omega_1 &= -\frac{U_1(c_t, \frac{m_t}{e_t})}{y^s U_{11}(c_t, \frac{m_t}{p_t e_t})} && (24)\\
\omega_2 &= -\frac{m^s}{e^s}\frac{U_{12}(c_t, \frac{m_t}{e_t})}{y^s U_{11}(c_t, \frac{m_t}{e_t})} && (25)\\
\gamma_1 &= (R^s - 1 + \frac{y^s r^s \omega_2}{m^s})(\frac{\gamma_2}{\omega_1}) && (26)\\
\gamma_2 &= \frac{R^s}{(R^s-1)(m^s/e^s)}(\frac{U_2(c_t, \frac{m_t}{e_t})}{(R^s-1)e^s U_{12}(c_t, \frac{m_t}{e_t}) - R^s U_{22}(c_t, \frac{m_t}{e_t})}) && (27)\\
\psi &= \frac{\epsilon - 1}{\phi} && (28)
\end{aligned}$$

the superscript $s$ denotes steady state values of the variables, $U_j$ is the first derivative of $U$ with respect to argument $j = 1, 2$ and $U_{ij}$ is the second order derivative of the utility function, $i, j = 1, 2$.

The log-linearized Euler condition (equation (20)) includes terms involving real money balances and the money demand shocks. They drop out from the expression if and only if utility is separable in consumption and real balances, i.e. $U_{12} = 0$ (see equation (25)). Similarly, real balances play a role in the forward looking Phillips curve (equation (22)), as long as $\omega_2 \neq 0$, which in turn again implies that $U_{12} \neq 0$ is necessary for real balances to matter. Thus, real balances play a direct role in determining output and inflation if and only if real balances and consumption enter non-separably in the utility function. On the other hand, the posited policy rule implies that the growth rate of nominal balances may be an important determinant of output and inflation indirectly, via interest rate determination. When $\omega_2$ and $\rho_m$ are both zero real balances have no direct or indirect role in propagating cyclical fluctuations.

## 4.2 Estimation

We estimate the model with quarterly US data spanning the period 1959:1-2008:2. All the data comes from the FRED data bank at the Federal Reserve Bank of Saint Louis and it is seasonally adjusted. For real GDP we take the GDPC96 series, which is a chain weighted real value of domestic production, convert it in per-capita terms dividing it by the civilian non-istitutional population, age 16 and over (CNP16OV) and log it. For real balances, we use the stock of M2 (M2SL), divide it by the GDP deflator (GDPDEF), convert it into per-capita terms scaling it by the civilian non-istitutional population, age 16 and over and log it. Inflation is calculated annualizing the quarterly growth rate of the GDP deflator and a three months T-bill (TB3M) is our measure of interest rates.

We employ 8 procedures to extract the cyclical component of the data. The first transformation (POLY) fits a second order deterministic polynomial to each series separately, allowing for a change in all the parameters at 1980:3. The cyclical component is the residual in the regression. The second transformation takes the first difference of all the series (FOD) as an estimate of the cyclical component. The third and the forth transformations are obtained with a HP filter with $\lambda = 1600$ and with a BP filter extracting cycles with 8 to 32 quarters periodicity. In this latter case we use the particular implementation of Baxter and King (1994). The fifth transformation is a univariate Beveridge and Nelson decomposition (BN) which fits an ARIMA(1,1,1) model to each series separately and takes as estimate of the cyclical component the difference between the original series and its model-based long run forecast. The sixth transformation is a multivariate version of this procedure (MBN) which fits a VAR with 6 lags to the four variables and takes as an estimate of the cyclical component, the difference between the level of the variables and their long run path implied by the model. The seventh transformation is a classical decomposition (CD) which assumes an additive representation of the components, fits a linear trend to the data and takes the residuals as the cyclical component. Finally, the last transformation employs an unobservable component (UC) decomposition which assumes that the non-cyclical component is a random walk and that the cyclical component has a trigonometric representation (see Canova (2007)). This implies that each of the series has an ARIMA(2,1,0) representation. The cyclical component is then estimated with the projected values of an AR(2) regression of the growth rate of each variable. Note that we have selected these procedures to introduce as much idiosyncratic features in the vectors of observables as possible. In fact, among the procedures we consider there are some where the non-cyclical component is deterministic, some where it is stochastic, and some where it is smooth; some use univariate and other multivariate information; some imply that cyclical and non-cyclical components are independent and some that they are correlated. Finally, some filtering procedures are two-sided and some one-sided.

We estimate the parameters of the model using Bayesian methods. The vector of observables is $32 \times 1$ (four series, 8 filtering methods) and the vector of states is $4 \times 1$. Since we set $\beta = 0.99$ and steady state inflation to 2 percent, there are 9 structural parameters $(\omega_1, \omega_2, \psi, \gamma_1, \gamma_2, \rho_r, \rho_p, \rho_y, \rho_m)$ - $\epsilon$ and $\phi$ are not separately identifiable - and seven auxiliary parameters $(\rho_\chi, \rho_e, \rho_z, \sigma_\chi, \sigma_e, \sigma_z, \sigma_v)$ to be estimated. We parameterize the link between the model and the cyclical data, allowing one intercept and one slope per filter, independent of the series, but allow the idiosyncratic term to be series and filter dependent. This implies that the intercept measures the average (across series and time) bias of each procedure in constructing the cyclical component and the slope measures the correlation between the data produced by each method and the model based quantities (again, on average across series). Since we normalize the slope of the first procedure to the identity, we have a total of 47 non-structural parameters to be estimated (8 intercepts, 7 slopes and 32 variances). We have also experimented with specifications which restrict the variances of the idiosyncratic component to be either series specific (independent of the filtering method) or filter specific (independent of the series) but discarded them because the model fit is relatively poor.

We draw 500,000 elements of the MCMC chain using the algorithm described in section 3. Convergence was achieved in less than 100,000 draws for each model specification we present. Posterior statistics are computed using one every 100 of the last 200,000 draws.

To compare our results, we estimate the parameters of interest using as vector of observables linearly detrending per-capita output and per-capita real balances and demeaned inflation and demeaned nominal rate, allowing for measurement error in each of the four equations. This is the right specification to employ for comparison purposes since our approach has idiosyncratic error built in (10).

## 4.3 The results

Before presenting estimates of the relevant parameters, we briefly comment on the estimates of the non-structural parameters we have obtained. First, the vector of $\lambda_0$ is estimated to be zero with very small standard errors. Therefore, all filtered data do not display level biases relative to the cyclical components produced by the model. Second, the loadings parameters are estimated to be between 0.60 (with UC filtered data) and 0.86 (with CD filtered data). Therefore, there appears to be sufficient idiosyncratic information in the cyclical data obtained with the procedures we employ. Since posterior standard errors are tight, bilateral differences in the loadings are generally a-posteriori relevant. Third, the error $u_t$ appears to have a highly idiosyncratic variance, both across series and across filtering methods. This reflects the fact that the variability of each individual series depends on the filtering approach and that vectors of filtered series contain different amount of cyclical information. This is the reason for why, for example, a restricted version of the setup we use, where only one parameter characterizes the variability across series or across filtering methods, produces a poor fit.

Table 5 presents the marginal likelihood of the unrestricted specification, where both direct and indirect effects of money are allowed, and for three restricted specifications, where either the direct effect is eliminated ($\omega_2 = 0$), the indirect effect is eliminated $\rho_m = 0$, or both are eliminated and the estimates of $\omega_2$ and $\rho_m$ obtained in the various cases.

| Specification | Acceptance rate | Marginal log Likelihood | $\omega_2$ | $\rho_m$ |
|---|---|---|---|---|
| Unrestricted | 33.86 | 16274 | 0.44 (0.02) | 0.48 (0.02) |
| $\omega_2 = 0$ | 33.64 | 16237 | 0 | 0.96(0.01) |
| $\rho_m = 0$ | 38.15 | 16212 | 0.43(0.02) | 0 |
| $\omega_2 = 0$, $\rho_m = 0$ | 33.77 | 16220 | 0 | 0 |
| Standard | | | 0.03 (0.02) | 0.04 (0.03) |

Table 5: Marginal likelihood and posterior estimates.

A specification where both effects are present is preferable to the other specifications in terms of in-sample fit. Furthermore, restricting both $\rho_m = 0$ and $\omega_2 = 0$ is preferable to restricting only $\rho_m = 0$. Overall, at least in terms of in-sample fit, both the direct and indirect effects of money are important. This conclusion is confirmed when looking at location measures of the posterior of the two relevant parameters. Statistically, both parameters are estimated tightly and both are a-posteriori different from zero. Economically, our estimates imply that money has a moderate influence on output and inflation fluctuations. On the contrary, the standard specification implies that both the direct and the indirect effect of money are statistically quite small and economically unimportant.

How different are the implications of the model estimated with our procedure relative to the one estimated with a standard approach? Figure 6 presents responses to unitary impulses in the four shocks in the two specifications. Responses look qualitatively similar in the two cases, but there are important differences in the magnitude and the persistence of the responses to shocks. In particular, when our approach is used, the persistence of the responses to monetary shocks is reduced, the persistence of the responses to technology shocks is increased and the responses to money demand shocks have both different magnitude and different persistence.

In sum, with our estimation approach money plays a role in transmitting fluctuations to output and inflation while this is not the case when a standard procedure is used. Given the experimental evidence we have collected in the previous section, it is likely that our estimates display less biases than those obtained with a standard approach, as far as persistence of the shocks and measurement of the substitution and income effects are concerned, reinforcing the conclusion that leaving money out of the model induces important specification and measurement errors.

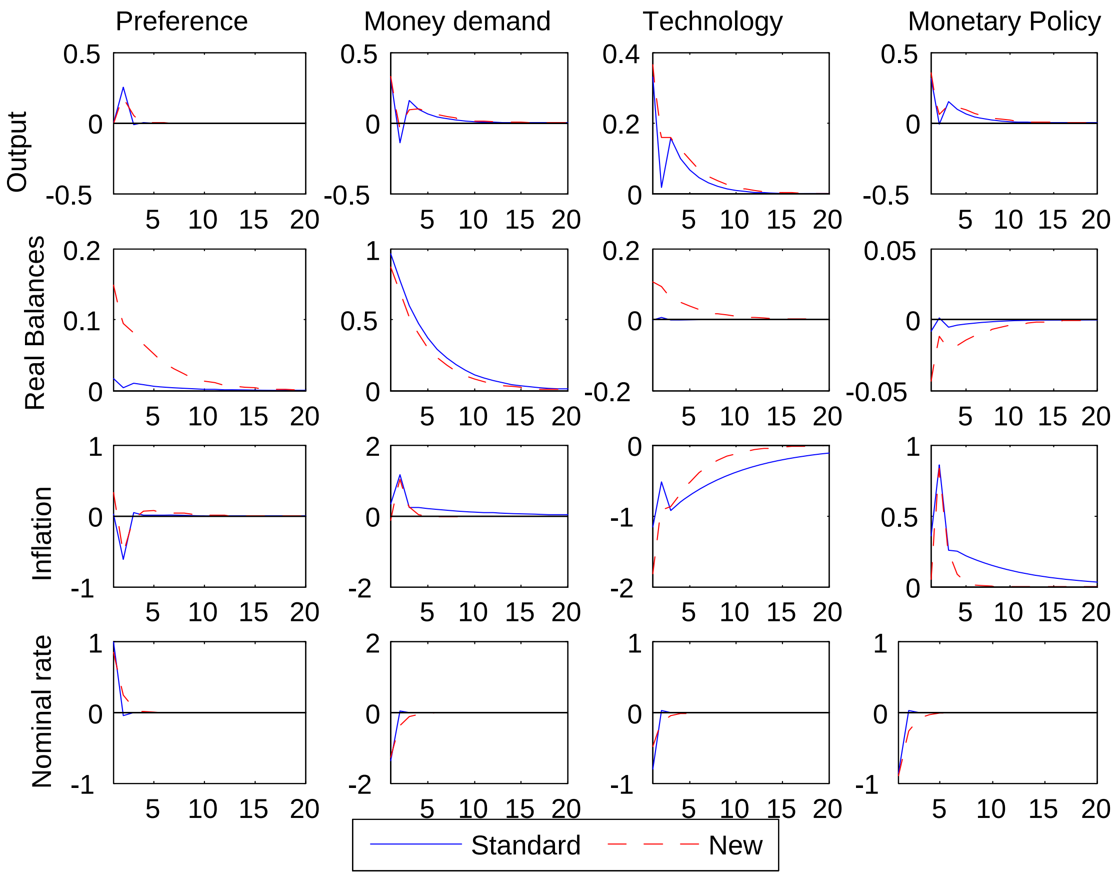


Figure 6: Impulse responses, standard and new approaches

# 5 Conclusions

This paper proposes a method to estimate the structural parameters of a time invariant cyclical DSGE model using multiple sources of cyclical information. The approach borrows ideas from the recent literature employing data-rich environments to estimate DSGE models (see Boivin and Giannoni (2005)), and uses vectors of filtered data obtained with alternative filtering procedures as potentially biased indicators of the true cyclical component. We set up an estimation framework where the cyclical DSGE model is the unobservable factor; vectors of filtered data are contaminated observable proxies; and the parameters of the DSGE model are jointly estimated together with non-structural parameters linking the DSGE model and the observables using signal extraction techniques.

Our approach is advantageous in, at least, two respects. First, since we do not have to arbitrarily choose one filtering approach prior to the estimation, we avoid specification errors. In fact, our approach can use as observables cyclical components obtained with one-sided and two-sided filters, both of univariate and multivariate nature, and cyclical components obtained assuming that cyclical and non-cyclical components are correlated or not, as long as the list of filters is sufficiently rich. The only constraint to the number of the vectors of filtered data used in the estimation is the RAM capacity of the computer and the ability of the investigator to limit the exponential proliferation of non-structural parameters. Second, if the cyclical components obtained with different filters are relatively similar, their joint use in the estimation permits the elimination of small sample biases in parameter estimates. On the other hand, if different filters have sufficiently different features, measurement error may have different time series properties. Since our signal extraction procedure averages the cross sectional information, it may reduce measurement error and eliminate part of its cyclicality, making estimates of the cyclical components more reliable, estimates of the structural parameter better shielded from filtering errors and inference more robust.

Using experimental data, we show that standard approaches employing just one arbitrary filter to estimate structural parameters typically induces large biases in the estimates and that these biases are considerably reduced with our approach. We also show that the estimates obtained with our procedure have superior properties in selected out-of-sample forecasting exercises.

To demonstrate that the biases induced by standard estimation approaches may have relevant economic implications, we revisit the role of money in the monetary business cycle. The literature has largely neglected the stock of money when studying monetary business cycles and Ireland (2004) has shown that such an approach is, by and large, appropriate using US data and a standard estimation setup. We show that when the cyclical information produced by alternative filters is jointly used in estimation, both the direct and the indirect channels through which money propagate fluctuations to output and inflation, are statistically important and economically significant and that the propagation of primitive shocks in the estimated economy is different from the one obtain if only one data transformation is used.

One may wonder why the literature uses time invariant cyclical models in the first place and does not, instead, employ (time varying) models which can jointly explain the cyclical and the non-cyclical properties of the data. We think there are three reasons for why such an approach is currently unfeasible. First, jointly modeling cyclical and non-cyclical fluctuations is an ambitious task since there are few theoretical mechanisms which are able to propagate temporary shocks for a long period of time (we need, for example, R&D, as in Comin and Gertler (2006) or Schumpeterian creative destruction, as in Canova, et al. (2007)) or create important cyclical implications from long run disturbances. Second, it is convenient to assume that the mechanism driving growth and cyclical fluctuations are distinct and orthogonal. Third, time varying structures are difficult to deal with in theory and hard to handle computationally (see e.g. Fernandez Villaverde and Rubio Ramirez (2007)).

Given these problems, this paper provides a simple setup where specification and measurement error biases in the estimates of the parameters of a cyclical DSGE model could be reduced, making inference less prone to arbitrary choices that researchers may make. In this sense, this work complements those of Canova (2008) and Ferroni (2008), who provided new methodologies to reduce specification and small sample errors in the estimation of cyclical DSGE models. Future work in the area will include studying the properties of the procedure using more complex experimental designs and the reconsideration of known puzzles in the macroeconomic literature using the approach this paper proposes.

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# Appendix to
# Multiple filtering devices for the estimation of cyclical DSGE models

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This appendix reports parameter estimates obtained with standard approaches and the approach suggested in the paper for alternative specifications of the DGP of the non-cyclical component of the model. Table A.1 reports estimates obtained when the non-cyclical component is still driven by a unit root in preference but the non-cyclical component has significant power at business cycle frequencies. Table A.2 reports estimates obtained when the non-cyclical component is driven by a unit root in technology. Table A.3 presents estimates obtained when the non-cyclical component is driven by a shock to technology which randomly fluctuates around a linear trend.

| Filter | | LT | HP | FOD | BP | $\text{Factor}_u$ | $\text{Factor}_r$ |
|---|---|---|---|---|---|---|---|
| | true | Median (s.e.) | Median (s.e.) | Median (s.e.) | Median(s.e.) | Median (s.e.) | Median(s.e.) |
| All filtered | | | | | | | |
| $\sigma_c$ | 3.00 | 1.63 (0.09) | 1.70 (0.14) | 1.98 (0.08) | 1.87 (0.10) | 1.58 (0.11) | 2.08 (0.20) |
| $\sigma_n$ | 0.70 | 1.56 (0.07) | 1.63 (0.10) | 1.64 (0.07) | 1.49 (0.06) | 0.67 (0.16) | 0.49 (0.09) |
| $h$ | 0.70 | 0.64 (0.02) | 0.65 (0.02) | 0.49 (0.02) | 0.64 (0.02) | 0.60 (0.04) | 0.77 (0.10) |
| $\alpha$ | 0.60 | 0.32 (0.02) | 0.20 (0.04) | 0.52 (0.02) | 0.15 (0.02) | 0.41 (0.03) | 0.48 (0.02) |
| $\epsilon$ | 7.00 | 3.99 (0.13) | 4.09 (0.14) | 4.07 (0.13) | 3.85 (0.13) | 6.21 (0.12) | 6.38 (0.12) |
| $\rho_r$ | 0.20 | 0.44 (0.05) | 0.32 (0.03) | 0.53 (0.02) | 0.44 (0.04) | 0.31 (0.07) | 0.26 (0.05) |
| $\rho_\pi$ | 1.30 | 2.01 (0.07) | 2.05 (0.08) | 1.68 (0.05) | 2.03 (0.07) | 1.50 (0.02) | 1.50 (0.05) |
| $\rho_y$ | 0.05 | 0.11 (0.02) | 0.15 (0.02) | 0.11 (0.00) | 0.18 (0.02) | 0.43 (0.03) | 0.21 (0.02) |
| $\zeta_p$ | 0.80 | 0.92 (0.03) | 0.93 (0.03) | 0.87 (0.03) | 0.94 (0.03) | 0.81 (0.02) | 0.79 (0.02) |
| $\rho_\chi$ | 0.50 | 0.98 (0.03) | 0.98 (0.03) | 1.00 (0.03) | 0.99 (0.03) | 0.93 (0.01) | 0.85 (0.01) |
| $\rho_z$ | 0.80 | 0.92 (0.03) | 0.94 (0.03) | 0.92 (0.03) | 0.95 (0.03) | 0.62 (0.02) | 0.59 (0.04) |
| $\sigma_\chi$ | 1.10 | 0.19 (0.03) | 0.25 (0.04) | 1.34 (0.45) | 0.20 (0.04) | 0.86 (0.16) | 2.36 (0.48) |
| $\sigma_z$ | 0.57 | 0.64 (0.08) | 0.59 (0.08) | 3.67 (0.28) | 0.19 (0.03) | 0.66 (0.23) | 0.47 (0.12) |
| $\sigma_v$ | 0.12 | 0.05 (0.01) | 0.05 (0.01) | 0.06 (0.01) | 0.05 (0.01) | 0.08 (0.01) | 0.09 (0.01) |
| $\sigma_\mu$ | 20.64 | 6.42 (0.36) | 11.25 (0.82) | 6.35 (0.23) | 3.77(0.23) | 5.17 (0.64) | 6.38 (1.00) |
| $\sigma_\chi^{nc}$ | 4.00 | | | | | | |
| Real variables filtered | | | | | | | |
| $\sigma_c$ | 3.00 | 1.92 (0.07) | 1.89 (0.07) | 1.93 (0.07) | 1.96 (0.09) | 1.98 (0.22) | 1.73 (0.26) |
| $\sigma_n$ | 0.70 | 2.10 (0.08) | 2.11 (0.08) | 2.04 (0.08) | 2.03 (0.09) | 0.68 (0.17) | 0.69 (0.09) |
| $h$ | 0.70 | 0.58 (0.02) | 0.58 (0.02) | 0.52 (0.02) | 0.62 (0.02) | 0.68 (0.02) | 0.76 (0.03) |
| $\alpha$ | 0.60 | 0.48 (0.02) | 0.47 (0.02) | 0.52 (0.02) | 0.43 (0.02) | 0.66 (0.04) | 0.57 (0.04) |
| $\epsilon$ | 7.00 | 3.71 (0.13) | 4.21 (0.15) | 4.00 (0.13) | 3.73 (0.15) | 6.27 (0.10) | 6.37 (0.12) |
| $\rho_r$ | 0.20 | 0.53 (0.04) | 0.55 (0.05) | 0.42 (0.01) | 0.19 (0.03) | 0.44 (0.03) | 0.33 (0.09) |
| $\rho_\pi$ | 1.30 | 1.26 (0.05) | 1.32 (0.05) | 1.01 (0.03) | 1.22 (0.04) | 1.53 (0.04) | 1.53 (0.05) |
| $\rho_y$ | 0.05 | -0.17 (0.01) | -0.04 (0.02) | -0.00 (0.00) | -0.14 (0.02) | 0.30 (0.06) | 0.17 (0.05) |
| $\zeta_p$ | 0.80 | 0.76 (0.02) | 0.76 (0.03) | 0.68 (0.02) | 0.72 (0.02) | 0.82 (0.05) | 0.84 (0.04) |
| $\rho_\chi$ | 0.50 | 1.00 (0.03) | 1.00 (0.03) | 1.00 (0.03) | 1.00 (0.03) | 0.96 (0.01) | 0.96 (0.01) |
| $\rho_z$ | 0.80 | 0.84 (0.03) | 0.90 (0.03) | 0.87 (0.03) | 0.85 (0.03) | 0.95 (0.02) | 0.95 (0.01) |
| $\sigma_\chi$ | 1.10 | 0.09 (0.01) | 0.21 (0.05) | 3.07 (0.14) | 0.16 (0.02) | 1.07 (0.20) | 1.66 (0.21) |
| $\sigma_z$ | 0.57 | 0.32 (0.04) | 0.25 (0.03) | 6.55 (0.24) | 0.12 (0.01) | 0.33 (0.07) | 0.49 (0.21) |
| $\sigma_{mp}$ | 0.12 | 0.07 (0.01) | 0.06 (0.01) | 0.05 (0.01) | 0.06 (0.01) | 0.08 (0.01) | 0.09 (0.01) |
| $\sigma_\mu$ | 20.64 | 13.29 (0.64) | 16.11 (1.03) | 8.03 (0.26) | 12.70 (0.63) | 5.36 (0.88) | 6.94 (1.02) |
| $\sigma_\chi^{nc}$ | 4.00 | | | | | | |

Table A-1: Parameters estimates using different filters; The non-cyclical component is driven by a unit root shock to preferences.

| Filter | | LT | HP | FOD | BP | $\text{Factor}_u$ | $\text{Factor}_r$ |
|---|---|---|---|---|---|---|---|
| | true | Median (s.e.) | Median (s.e.) | Median (s.e.) | Median(s.e.) | Median (s.e.) | Median(s.e.) |
| | | | | All filtered | | | |
| $\sigma_c$ | 3.00 | 1.88 (0.09) | 1.84 (0.13) | 1.84 (0.08) | 1.95 (0.10) | 2.34 (0.32) | 2.89 (0.64) |
| $\sigma_n$ | 0.70 | 1.57 (0.07) | 1.61 (0.08) | 1.70 (0.07) | 1.51 (0.06) | 0.19 (0.04) | 0.53 (0.28) |
| $h$ | 0.70 | 0.67 (0.02) | 0.66 (0.02) | 0.48 (0.02) | 0.65 (0.02) | 0.53 (0.03) | 0.77 (0.11) |
| $\alpha$ | 0.60 | 0.19 (0.02) | 0.18 (0.02) | 0.53 (0.02) | 0.16 (0.01) | 0.42 (0.02) | 0.37 (0.07) |
| $\epsilon$ | 7.00 | 4.10 (0.13) | 4.08 (0.13) | 3.98 (0.13) | 3.98 (0.13) | 6.27 (0.06) | 6.46 (0.11) |
| $\rho_r$ | 0.20 | 0.50 (0.03) | 0.42 (0.04) | 0.50 (0.02) | 0.47 (0.04) | 0.42 (0.04) | 0.28 (0.15) |
| $\rho_\pi$ | 1.30 | 1.87 (0.06) | 1.97 (0.10) | 1.62 (0.05) | 2.11 (0.07) | 1.51 (0.03) | 1.56 (0.03) |
| $\rho_y$ | 0.05 | 0.27 (0.01) | 0.31 (0.04) | 0.10 (0.00) | 0.20 (0.02) | 0.32 (0.03) | -0.04 (0.12) |
| $\zeta_p$ | 0.80 | 0.93 (0.03) | 0.93 (0.03) | 0.86 (0.03) | 0.95 (0.03) | 0.86 (0.03) | 0.96 (0.05) |
| $\rho_\chi$ | 0.50 | 0.98 (0.03) | 0.98 (0.03) | 1.00 (0.03) | 0.99 (0.03) | 0.73 (0.05) | 0.78 (0.03) |
| $\rho_z$ | 0.80 | 0.94 (0.03) | 0.94 (0.03) | 0.92 (0.03) | 0.95 (0.03) | 0.69 (0.03) | 0.91 (0.06) |
| $\sigma_\chi$ | 1.10 | 2.07 (0.22) | 2.16 (0.32) | 1.33 (0.29) | 0.24 (0.05) | 0.20 (0.04) | 1.52 (0.12) |
| $\sigma_z$ | 0.57 | 0.14 (0.02) | 0.13 (0.02) | 3.93 (0.21) | 0.19 (0.03) | 0.28 (0.06) | 0.36 (0.12) |
| $\sigma_{mp}$ | 0.12 | 0.06 (0.01) | 0.05 (0.01) | 0.06 (0.01) | 0.05 (0.01) | 0.07 (0.01) | 0.07 (0.01) |
| $\sigma_\mu$ | 20.64 | 8.48 (0.29) | 8.85 (0.48) | 4.97 (0.23) | 5.19 (0.35) | 4.66 (0.80) | 5.61 (0.53) |
| $\sigma_z^{nc}$ | 3.01 | | | | | | |
| | | | | Real filtered | | | |
| $\sigma_c$ | 3.00 | 1.73 (0.26) | 2.64 (0.11) | 2.58 (0.09) | 2.54 (0.11) | 2.51 (0.32) | 2.66 (0.82) |
| $\sigma_n$ | 0.70 | 2.16 (0.15) | 1.72 (0.09) | 2.05 (0.08) | 1.63 (0.08) | 0.56 (0.15) | 0.49 (0.21) |
| $h$ | 0.70 | 0.38 (0.02) | 0.57 (0.03) | 0.50 (0.02) | 0.59 (0.02) | 0.80 (0.09) | 0.84 (0.14) |
| $\alpha$ | 0.60 | 0.07 (0.02) | 0.19 (0.02) | 0.40 (0.02) | 0.17 (0.02) | 0.33 (0.03) | 0.47 (0.02) |
| $\epsilon$ | 7.00 | 4.03 (0.14) | 3.24 (0.11) | 4.02 (0.13) | 3.87 (0.14) | 6.03 (0.08) | 6.54 (0.13) |
| $\rho_r$ | 0.20 | 0.66 (0.04) | 0.80 (0.03) | 0.59 (0.02) | 0.46 (0.06) | 0.36 (0.07) | 0.32 (0.17) |
| $\rho_\pi$ | 1.30 | 1.63 (0.07) | 1.39 (0.05) | 1.12 (0.04) | 1.11 (0.05) | 1.66 (0.06) | 1.54 (0.05) |
| $\rho_y$ | 0.05 | 0.38 (0.04) | 0.03 (0.01) | -0.10 (0.01) | -0.07 (0.02) | -0.01 (0.05) | -0.08 (0.10) |
| $\zeta_p$ | 0.80 | 0.84 (0.03) | 0.86 (0.03) | 0.52 (0.04) | 0.80 (0.03) | 0.88 (0.04) | 0.87 (0.07) |
| $\rho_\chi$ | 0.50 | 0.97 (0.03) | 0.85 (0.03) | 0.96 (0.03) | 0.84 (0.03) | 0.73 (0.06) | 0.68 (0.04) |
| $\rho_z$ | 0.80 | 0.81 (0.03) | 0.98 (0.03) | 0.98 (0.03) | 0.98 (0.03) | 0.91 (0.04) | 0.95 (0.06) |
| $\sigma_\chi$ | 1.10 | 0.35 (0.06) | 0.17 (0.02) | 6.96 (0.23) | 0.16 (0.02) | 1.17 (0.33) | 0.64 (0.23) |
| $\sigma_z$ | 0.57 | 0.12 (0.02) | 0.09 (0.01) | 2.48 (0.26) | 0.08 (0.01) | 0.13 (0.02) | 0.37 (0.12) |
| $\sigma_{mp}$ | 0.12 | 0.06 (0.01) | 0.05 (0.01) | 2.46 (0.25) | 0.05 (0.01) | 0.08 (0.01) | 0.09 (0.01) |
| $\sigma_\mu$ | 20.67 | 14.57 (1.04) | 8.66 (0.38) | 11.90 (0.38) | 5.53 (0.30) | 7.74 (0.56) | 6.24 (0.80) |
| $\sigma_z^{nc}$ | 3.01 | | | | | | |

Table A-2: Parameters Estimates using different filters. The non-cyclical component is driven by a unit root shock to technology.

| Filter | | LT | HP | FOD | BP | $\text{Factor}_u$ | $\text{Factor}_r$ |
|---|---|---|---|---|---|---|---|
| | true | Median (s.e.) | Median (s.e.) | Median (s.e.) | Median(s.e.) | Median (s.e.) | Median(s.e.) |
| | | | | All filtered | | | |
| $\sigma_c$ | 3.00 | 1.23 (0.12) | 1.54 (0.15) | 1.12 (0.10) | 1.52 (0.13) | 2.56 (0.24) | 2.78 (0.45) |
| $\sigma_n$ | 0.70 | 1.67 (0.13) | 1.51 (0.10) | 1.87 (0.11) | 1.51 (0.09) | 0.27 (0.04) | 0.56 (0.21) |
| $h$ | 0.70 | 0.59 (0.04) | 0.56 (0.03) | 0.41 (0.04) | 0.55 (0.02) | 0.55 (0.03) | 0.78 (0.13) |
| $\alpha$ | 0.60 | 0.23 (0.03) | 0.19 (0.02) | 0.40 (0.02) | 0.17 (0.01) | 0.47 (0.02) | 0.48 (0.07) |
| $\epsilon$ | 7.00 | 4.04 (0.18) | 3.99 (0.15) | 3.54 (0.17) | 4.01 (0.14) | 6.11 (0.08) | 6.55 (0.13) |
| $\rho_r$ | 0.20 | 0.46 (0.04) | 0.40 (0.05) | 0.51 (0.05) | 0.40 (0.04) | 0.39 (0.05) | 0.30 (0.15) |
| $\rho_\pi$ | 1.30 | 1.99 (0.07) | 1.80 (0.12) | 1.75 (0.08) | 1.71 (0.09) | 1.44 (0.03) | 1.41 (0.05) |
| $\rho_y$ | 0.05 | 0.33 (0.02) | 0.30 (0.03) | 0.17 (0.02) | 0.28 (0.02) | 0.21 (0.03) | -0.06 (0.17) |
| $\zeta_p$ | 0.80 | 0.94 (0.03) | 0.90 (0.06) | 0.89 (0.03) | 0.90 (0.03) | 0.87 (0.03) | 0.85 (0.04) |
| $\rho_\chi$ | 0.80 | 0.88 (0.05) | 0.84 (0.02) | 0.87 (0.03) | 0.85 (0.03) | 0.84 (0.05) | 0.79 (0.03) |
| $\rho_z$ | 0.50 | 0.97 (0.05) | 0.95 (0.02) | 0.94 (0.04) | 0.95 (0.04) | 0.66 (0.04) | 0.62 (0.07) |
| $\sigma_\chi$ | 1.10 | 1.99 (0.26) | 2.10 (0.32) | 1.33 (0.37) | 0.94 (0.16) | 0.27 (0.05) | 1.37 (0.17) |
| $\sigma_z$ | 0.57 | 0.11 (0.03) | 0.17 (0.03) | 5.12 (0.29) | 0.21 (0.03) | 0.42 (0.07) | 0.30 (0.18) |
| $\sigma_{mp}$ | 0.12 | 0.04 (0.05) | 0.06 (0.02) | 0.09 (0.02) | 0.06 (0.01) | 0.09 (0.01) | 0.08 (0.01) |
| $\sigma_\mu$ | 20.64 | 9.41 (0.39) | 8.53 (0.58) | 3.99 (0.33) | 7.19 (0.52) | 5.61 (0.70) | 7.66 (0.49) |
| $\sigma_z^{nc}$ | 0.05 | | | | | | |

Table A-3: Parameters Estimates using different filters. The non-cyclical component is driven by a shock to technology which randomly fluctuates around a linear trend.