Liquidity Sentiments†

By Vladimir Asriyan, William Fuchs, and Brett Green*

We develop a rational theory of liquidity sentiments in which the market outcome in any given period depends on agents’ expectations about market conditions in future periods. Our theory is based on the interaction between adverse selection and resale considerations giving rise to an intertemporal coordination problem that yields multiple self-fulfilling equilibria. We construct “sentiment” equilibria in which sunspots generate fluctuations in prices, volume, and welfare, all of which are positively correlated. The intertemporal nature of the coordination problem disciplines the set of possible sentiment dynamics. In particular, sentiments must be sufficiently persistent and transitions must be stochastic. We consider an extension with production in which asset quality is endogenously determined and provide conditions under which sentiments are a necessary feature of any equilibrium. A testable implication is that assets produced in good times are of lower average quality than those produced in bad times. (JEL D84, D82, E32, E44, G12)

In a frictionless market, all gains from trade are realized and durable assets are held by parties that value them the most. As a result, competitive prices reflect not only the gains from trade today, but also all expected future gains from trade. In the presence of frictions, some gains from trade may remain unrealized, which depresses prices. In such an environment, there is a close connection between liquidity—the ease with which assets are reallocated to their most productive use—and asset prices. In this paper, we explore the extent to which rational expectations

*Asriyan: CREI, Ramon Trias Fargas, 25-27, Merce Rodoreda Bldg., Barcelona, 08005, Spain, UPF, Barcelona GSE, and CEPR (email: vasriyan@crei.cat); Fuchs: McCombs School of Business, University of Texas, 2110 Speedway, Austin, TX 78705, and Universidad Carlos III de Madrid (email: wfuchs@gmail.com); Green: Olin Business School, Washington University, 1 Brookings Drive, Campus Box 1133, St. Louis MO, 63130 (email: b.green@wustl.edu). Mikhail Golosov was the coeditor for this article. Asriyan acknowledges financial support from the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563). Fuchs gratefully acknowledges support from the ERC grant 681575. We thank Manuel Amador, Fernando Broner, Jason Donaldson, Hugo Hopenhayn, Sergei Kovbasyuk, Arvind Krishnamurthy, Pablo Kurlat, Ricardo Lagos, Alberto Martin, Vincent Maurin, Guillermo Ordoñez, Giorgia Piacentino, Giacomo Ponzetto, Edouard Schaal, Anton Tsy, Jaume Ventura, Victoria Vanasco, Venky Venkateswaran, Pierre-Olivier Weill, and seminar participants at BYU, CEMFI, CREI-UPF, Frankfurt School Finance and Management, HEC Paris, London Business School, Sciences Po, Universidad Carlos III de Madrid, UT Austin, OTC Markets and Securities Workshop at LAEF in Santa Barbara, Transpyrenean Macro Workshop CREI-TSE, the Workshop on Information Asymmetries in Macro-Finance at the Barcelona GSE Summer Forum, 8th Summer Workshop in Macro-Finance in Paris, 6th Rome Workshop on Macroeconomics, 13th CSEF-IGIER Symposium on Economics and Institutions in Capri, SED (Edinburgh), FIRS (Barcelona), FTG (Minnesota), and MFS (Boston) for useful suggestions and feedback. We thank Janko Heineken, Ilja Kantorovich, and Joon Sup Park for excellent research assistance. A prior version of this paper was circulated under the title “Sentiment, Liquidity, and Asset Prices.”

†Go to https://doi.org/10.1257/aer.20180998 to visit the article page for additional materials and author disclosure statements.
about future liquidity or *sentiments* can fluctuate over time and influence prices and quantities.

We analyze a dynamic economy in which asset owners are privately informed about the common value component or quality of their asset, which is either high or low. Gains from trade arise over time because owners experience idiosyncratic shocks to their private value of ownership or “productivity.” Potential buyers compete for assets, but they face a lemons problem, as in Akerlof (1970), since they do not observe the quality of owners’ assets. A buyer who purchases an asset in any given period becomes an owner in the next period. The important feature of our environment is that buyers must worry not only about the quality of the assets for which they bid, but also about market liquidity in the future if they want to resell. In order to highlight our main results, it is useful to mention that when only one of the two considerations (i.e., either adverse selection or resale considerations) are present, then the equilibrium is unique and the economy features no aggregate volatility.

Our main result is that the interaction between adverse selection and resale concerns generates an intertemporal coordination problem, which gives rise to multiple self-fulfilling equilibria and generates endogenous volatility. The reason is that when buyers anticipate the need to sell assets in the future, their willingness to pay for them today depends on their beliefs about future market conditions. If buyers today expect that buyers in the future will offer high (pooling) prices, then their perceived difference between high and low quality assets is smaller rendering the adverse selection problem less of a concern. Hence, they are willing to make a high offer today, which attracts a better pool of sellers. That is, the expectation of future market liquidity generates liquidity in the market today. Conversely, if buyers today expect that future buyers will only offer low prices, then buying a lemon today becomes more of a concern, which renders the adverse selection problem more severe making it more difficult to consummate a trade today.

To illustrate these ideas, we first consider candidate equilibria that we term *non-sentiment equilibria*. A defining property of these equilibria is that market liquidity is constant over time and, thus, so too are the agents’ expectations about future liquidity. This class includes an efficient equilibrium, in which all owners with low productivity trade their assets immediately and, as a result, prices and welfare are permanently high. It also includes an inefficient equilibrium, in which only low quality asset owners trade and, as a result, prices and welfare are permanently low. We show that there exists (i) a lower bound, \( \pi_1 \), on the proportion of high quality assets, \( \pi \), such that the efficient equilibrium exists when \( \pi \geq \pi_1 \), and (ii) an upper bound, \( \bar{\pi} \), such that the inefficient equilibrium exists when \( \pi \leq \bar{\pi} \). Importantly, \( \pi_1 < \bar{\pi} \) and, therefore, the two equilibria coexist for intermediate \( \pi \).

We then consider *sentiment equilibria*, in which agents’ (rational) expectations about future market liquidity depend nontrivially on a publicly observable sunspot process that is extrinsic to the economy (i.e., unrelated to fundamentals). Sentiment equilibria are characterized by sets of “good” and “bad” states as well as a transition matrix for how the sentiment process evolves. In good states, agents have a positive outlook on future market conditions leading to high prices today and, as a result, all gains from trade being realized. In bad states, agents correctly anticipate that the market is likely to be illiquid in the future and, as a result, the market is illiquid today. We demonstrate that the coexistence of multiple non-sentiment equilibria is
necessary and sufficient for the existence of sentiment equilibria. Moreover, the set of equilibrium sentiment dynamics is disciplined by the primitives of the model. That is, unlike repeated static coordination problems, sentiment equilibria cannot be driven by an arbitrary sunspot process, but rather must exhibit certain properties. Most notably, the sentiment process must be sufficiently persistent and transitions must be stochastic.

We extend our model to incorporate endogenous asset production. This extension allows us to (i) determine the distribution of asset quality endogenously and (ii) provide conditions under which sentiments are a necessary feature of any equilibrium. In each period, a mass of producers supply assets. Each producer exerts unobservable costly effort, which affects the quality of the asset produced. After production, producers can trade their assets in the market. We first show that, in any sentiment equilibrium, the quality of assets produced is countercyclical, lower in good times and higher in bad times. The intuition for this result is that when all assets trade quickly and at the same price, there is less incentive to create high quality assets. Second, we show that when production costs are intermediate, any equilibrium must involve sentiments. That is, prices and liquidity must be endogenously volatile. Intuitively, if agents expect liquid markets and high prices to persist indefinitely, the quality of produced assets would be too low, and future buyers would not be willing to offer high prices, which renders the market illiquid in the future and contradicts expectations. Conversely, if agents expect illiquidity and low prices to persist indefinitely, the quality of assets produced would be too high, and future buyers would make aggressive offers thereby inducing a liquid market, which again contradicts expectations. Thus, non-sentiment equilibria are unsustainable.

Though our model abstracts from institutional details of specific markets, we discuss several interpretations of the model and explore the predictions of sentiment equilibria within the context of these applications. The first application is the (re)allocation of capital among firms. Within this context, the model’s predictions match the stylized (and still somewhat puzzling) facts in Eisfeldt and Rampini (2006). In particular, that reallocation of capital is pro-cyclical, but the cross-sectional dispersion of productivity is countercyclical. In the model, only high productivity firms operate capital in good times whereas, in bad times, some capital is operated by low productivity firms. In addition, aggregate TFP in our economy is endogenous and fluctuates with market sentiments. Thus, sentiments can be an important source of macroeconomic volatility.

Second, we consider an application in which, due to financial frictions, entrepreneurs must sell their existing projects in order to undertake new ones. In sentiment equilibria, good states involve high growth fueled by liquid secondary markets enabling all new investment opportunities to be pursued. In bad states, growth is lower and some new investments are not pursued because entrepreneurs forego them in favor of managing their existing project. This application of our model is related to work by Eisfeldt (2004), Kurlat (2013), and Bigio (2015). One important difference is that heterogeneity in project quality is short-lived in their models, whereas it is long-lived in ours. Thus, while the existing literature has shown that (short-lived) adverse selection can serve to amplify aggregate shocks, we demonstrate that (long-lived) adverse selection can, in fact, be the source of the aggregate shocks.
The model’s predictions also match stylized facts in housing markets which exhibit strong positive correlation between prices and transaction volume and negative correlation between prices and time on the market (Mayer 2011). Large movements in housing prices are difficult to explain based on fundamentals and are thus often interpreted as “bubbles.” The sentiment equilibria in our model exhibit similar time-series behavior: prices and volume rise and fall despite no obvious changes in fundamentals. More generally, our model suggests that sentiments, liquidity, and prices are intrinsically connected even when agents are fully rational and prices are competitive. Thus, sentiments cannot be separated from fundamentals; both are essential for determining asset valuations.

**Related Literature**

Our paper naturally relates to the recent and growing literature that embeds adverse selection into dynamic economies. This literature highlights that adverse selection can serve as a channel for amplification and that novel dynamics can emerge because the joint distribution of assets for sale and gains from trade changes over time. In a competitive framework, Janssen and Karamychev (2002) and Janssen and Roy (2002) show that when the gains from trade are persistent, past liquidity has a negative effect on current liquidity. This can lead to deterministic liquidity cycles as also shown by Maurin (2016).

Daley and Green (2016) and Fuchs, Green, and Papanikolaou (2016) explicitly model re-trade considerations and construct equilibria with time-varying trading volume. Guerrieri and Shimer (2014) also model resale considerations within a search framework where sellers with higher quality assets post higher prices but take longer to sell. All of these papers have a signaling component where delay or posted price can serve as a signal of quality. In contrast, we intentionally focus on a setting without any scope for signaling and where agents’ beliefs about asset quality are constant across different sellers and over time. In our model, the novel dynamics emerge as a result of an intertemporal coordination problem.

Chiu and Koeppl (2016) model the interaction between adverse selection and search frictions. They focus on steady state equilibria and policies designed to alleviate the adverse selection problem. That multiple steady state equilibria can exist in their model is closely related to our finding in Theorem 1. However, they do not analyze the possibility for sentiment equilibria.

There are a number of papers that study the potential for coordination problems and equilibrium multiplicity in economies with adverse selection (e.g., Plantin 2009; Malherbe 2014; Asriyan, Fuchs, and Green 2017; Benhabib, Dong, and Wang 2018). It is important to highlight that equilibrium multiplicity in these papers arises due to intratemporal strategic complementaries, whereas in our paper it arises due to an intertemporal coordination. This feature puts discipline on the set of equilibrium sentiment dynamics.

---

1 See, for example, Eisfeldt (2004); Martin (2005); Kurlat (2013); Chari, Shourideh, and Zetlin-Jones (2014); Gorton and Ordoñez (2014, forthcoming); Golosov, Lorenzoni, and Tsyvinski (2014); Bigio (2015), Mäkinen and Palazzo (2017).

2 The importance of re-trade considerations in asset markets goes back to Harrison and Kreps (1978). See also Lagos and Zhang (2015, forthcoming) for recent related work within search-theoretic environment.
It is also important to contrast our findings with the potential for multiplicity in a static environment with adverse selection as first noted by Wilson (1980). We intentionally avoid this type of multiplicity by assuming the buyers are identical and strategic. Hence, the static version of our model features a unique equilibrium. Our contribution is to demonstrate that multiplicity and the potential for sentiments emerge once dynamics are present.

Sentiment equilibria exhibit some features that are similar to those in the broad literature on rational bubbles. Yet, there are important differences. In our model, the assets generate real output and the price is always pinned down by fundamentals. That is, sentiment equilibria do not require a violation of the transversality condition. Fiat assets can also have positive prices when used as a medium of exchange as in the search-theoretic monetary models in the spirit of Kiyotaki and Wright (1989). In contrast to this literature, assets in our economy do not serve as a medium of exchange. Recently, Trejos and Wright (2016) developed a framework that integrates the role of assets as payment instruments with assets as dividend-generating instruments. Relatedly, there is work that explores how money can help alleviate the adverse selection problem associated with other media of exchange (e.g., Williamson and Wright 1994).

Dating back to Shell (1977) and Cass and Shell (1983), there is a rich literature on sunspots in macroeconomics, where an extrinsic random variable can affect economic outcomes. In our model, the nature of the coordination problem combined with strategic trade puts discipline on how the sunspot is mapped into sentiments. In particular, the sentiment process needs to be both stochastic and sufficiently persistent in order for agents to be willing to coordinate on it; how persistent, depends on model parameters.

There are other classes of models which discipline sunspot dynamics due to general equilibrium considerations rather than strategic ones. For example, in the literature on rational bubbles, crashes must occur with sufficiently small probability; otherwise, bubbles grow too fast and eventually violate feasibility (Weil 1987). In a different context, Schneider and Tornell (2004) show that self-fulfilling financial crises must endogenously be rare events. There is also an earlier literature that puts structure on the set of admissible sunspots within neoclassical competitive general equilibrium economies (see Chiappori and Guesnerie 1991 for a survey).

A natural question is what drives sentiments? In the core of the paper we consider them being driven by a stochastic process extrinsic to the economy. In Section IVB, we discuss how sentiments could be driven by real economic variables, in which case one can interpret the sentiments in our model as an amplification mechanism (Manuelli and Peck 1992).

Recently, there has been a renewed interest among macroeconomists in understanding how sentiments—in the form of correlated shocks to agents’ information sets—can be drivers of aggregate fluctuations, as in the work of Angeletos and La’O

---

3 See, for example, the early papers by Samuelson (1958); Tirole (1985); Weil (1987); Santos and Woodford (1997); and the more recent work by Martin and Ventura (2012, 2018); Asriyan, Fuchs, and Green (2016); and Miao, Shen, and Wang (2019). Barlevy (2015) provides a thorough overview of various theories of asset bubbles.

4 See Shell (2008) for a survey of the literature on sunspots and Benhabib and Farmer (1999) for a survey of the literature on indeterminacy.
In this literature, dispersion of information among agents about aggregate states is an essential ingredient. We contribute to this literature by showing that, in the presence of adverse selection, sentiments which coordinate agents’ expectations about future market conditions can generate aggregate fluctuations even when information about aggregates is common to all economic agents. Moreover, with endogenous production, these sentiments (i.e., endogenous volatility) must be part of any equilibrium when production costs are not too extreme. A related finding obtains in Golosov and Menzio (2017), who derive endogenous aggregate fluctuations due to a moral hazard problem between firms and workers.

Finally, the link between market liquidity and incentives to produce high quality assets has been explored in Chemla and Hennessy (2014), Vanasco (2017), Fukui (2018), Daley, Green, and Vanasco (forthcoming), as well as in contemporaneous work by Caramp (2017) and Neuhann (2017). It is, by now, well understood that higher liquidity in asset markets reduces the incentive to produce high quality assets. Our contribution is to provide a fully dynamic model in which both liquidity and production quality vary over time endogenously and in the absence of aggregate shocks.

The rest of the paper is organized as follows. In Section I, we present the model. In Sections II and III, we conduct our main analysis. In Section IV, we discuss applications and extensions of the model. We conclude in Section V. All proofs are in the Appendix.

I. The Model

The economy takes place in discrete time with an infinite horizon. Time is indexed by \( t \in \{0,1,\ldots\} \). There is a unit mass of indivisible assets, indexed by \( k \in [0,1] \), which are identical in every respect except for their common value or “quality,” which we denote by \( \theta_k \in \{L,H\} \). The fraction of high-quality assets in the economy is denoted by \( \pi \in (0,1) \). Assets are long lived and qualities are fixed over time.\(^6\)

There is a mass \( M \) of ex ante identical agents, indexed by \( m \in [0,M] \). Each agent can own at most one asset. We refer to agents who own assets as owners and to the rest as potential buyers. At each date \( t \), an owner \( m \), has a private value from asset ownership or “productivity,” which is either low or high and is denoted by \( \omega_{mt} \in \{l,h\} \). The flow value that agent \( m \) derives from owning asset \( k \) at date \( t \) depends on both her private value (\( \omega_{mt} \)) and the quality of the asset (\( \theta_k \)); we denote it by \( u(\theta_k,\omega_{mt}) \). All agents are risk neutral and have a discount factor \( \delta \in (0,1) \).

High-quality assets deliver a higher flow payoff \( u(H,\omega) > u(L,\omega) \), and agents with a high private value of ownership derive a higher flow payoff, \( v_0 \equiv u(\theta,h) > c_0 \equiv u(\theta,l) \). For this reason, when \( \omega_{mt} = l \), we say that owner \( m \) is shocked since there are gains from transferring the asset to an unshocked agent. For simplicity, we assume each owner’s status is i.i.d., each period an owner is shocked with probability \( \Pr(\omega_{mt} = l) = \lambda \in (0,1) \), which is also the fraction of

\(^5\) Other recent work includes papers by Lorenzoni (2009); Hassan and Mertens (2011); Benhabib, Wang, and Wen (2015).

\(^6\) Our results are robust provided there is at least some persistence in asset quality (see Section IVC).
shocked owners in the market in each period. In addition, we assume that $c_H > v_L$, which implies that the common value component is sufficiently important that strategic considerations due to adverse selection remain relevant when the owner is shocked.

The market for assets is competitive; in each period, at least two unshocked buyers are randomly matched with an owner, and they compete for the owner’s asset, à la Bertrand. To ensure that there are a sufficient number of unshocked buyers per seller, we require that $M \geq 1 + (2/(1 - \lambda))$. When an owner receives offers, she decides which (if any) offer to accept. If the owner rejects all offers, she continues to be an owner in the next period and is rematched with a new set of buyers. If the owner accepts an offer, then she sells her asset and enters the pool of potential buyers. A buyer whose offer is accepted, acquires the asset and becomes an owner in the next period, whereas a buyer whose offer is rejected remains a buyer in the next period. We will assume throughout that the agents have “deep pockets,” so their budget constraints do not bind when bidding for assets.

Trade in our economy may be hindered by the presence of asymmetric information. In particular, both asset quality and ownership status are privately known by the asset owner and not observable to buyers. In dynamic environments with asymmetrically informed agents, the history of previous trading behavior can signal information about asset quality. For both parsimony and tractability, we will intentionally abstract from this possibility by assuming that the past trading history of individual assets is not observed by buyers. Therefore, when making offers at date $t$, each asset looks identical to each buyer. Furthermore, because the productivity shocks are i.i.d., the joint distribution of $(\theta, \omega)$ among asset owners at the beginning of any trading period is independent of the history of play and constant over time.

The price of an asset at date $t$ is the maximal bid of the buyers for that asset at date $t$. Since all assets appear identical to buyers, we restrict attention to equilibria in which the price is also the same across all assets at any date $t$. We denote this (common) price by $P_t$.

We refer to an owner of a type-$\theta$ asset whose productivity is $\omega$ as a $(\theta, \omega)$-owner. Given an owner’s private information, the current price, and her expectation about future prices, the problem facing an owner is when, if ever, to accept an offer. Let $V_t(\theta, \omega)$ denote the equilibrium payoff to a type $(\theta, \omega)$-owner at date $t$, which solves

$$V_t(\theta, \omega) = \max_{T \geq t} \left\{ \sum_{s=t}^{T-1} \delta^{s-t} u(\theta, \omega_s) + \delta^{T-t} P_T \right\},$$

That private value shocks are independent over time facilitates tractability, but is not essential for our main results (see Section IVC).

Perfect competition among buyers is not essential for our results (see Section IVC), but it motivates our equilibrium conditions. Further, since shocked buyers have no gains from trading with a seller, it is irrelevant whether or how many are matched with a seller in any period. For simplicity and without loss, we assume that shocked buyers are not matched to a seller.

For example, in each period, each agent receives a sufficiently large endowment of the numeraire good, and agents’ preferences over the numeraire good are linear.

Asymmetric information about the private value is not essential for our main insights, see Section IVC.

That is, $\theta$ and $\omega$ are independently distributed with $\Pr(\theta = H) = \pi$ and $\Pr(\omega = h) = 1 - \lambda$. 

1. That private value shocks are independent over time facilitates tractability, but is not essential for our main results (see Section IVC).
2. Perfect competition among buyers is not essential for our results (see Section IVC), but it motivates our equilibrium conditions. Further, since shocked buyers have no gains from trading with a seller, it is irrelevant whether or how many are matched with a seller in any period. For simplicity and without loss, we assume that shocked buyers are not matched to a seller.
3. For example, in each period, each agent receives a sufficiently large endowment of the numeraire good, and agents’ preferences over the numeraire good are linear.
4. Asymmetric information about the private value is not essential for our main insights, see Section IVC.
5. That is, $\theta$ and $\omega$ are independently distributed with $\Pr(\theta = H) = \pi$ and $\Pr(\omega = h) = 1 - \lambda$. 

1. That private value shocks are independent over time facilitates tractability, but is not essential for our main results (see Section IVC).
2. Perfect competition among buyers is not essential for our results (see Section IVC), but it motivates our equilibrium conditions. Further, since shocked buyers have no gains from trading with a seller, it is irrelevant whether or how many are matched with a seller in any period. For simplicity and without loss, we assume that shocked buyers are not matched to a seller.
3. For example, in each period, each agent receives a sufficiently large endowment of the numeraire good, and agents’ preferences over the numeraire good are linear.
4. Asymmetric information about the private value is not essential for our main insights, see Section IVC.
5. That is, $\theta$ and $\omega$ are independently distributed with $\Pr(\theta = H) = \pi$ and $\Pr(\omega = h) = 1 - \lambda$. 

where $T$ is a stopping time and where $E_t$ denotes the expectation operator conditional on the public information available at date $t$, which includes the current sentiment (see below). The Bellman equation is

\begin{equation}
V_t(\theta, \omega) = \max \left\{ P_t, u(\theta, \omega) + \delta E_t \left\{ V_{t+1}(\theta, \omega') | \theta, \omega \right\} \right\},
\end{equation}

where the next period’s realization of a random variable is denoted with a prime. Clearly, it is optimal for a $(\theta, \omega)$-owner to accept a maximal offer of $p$ at date $t$ if

\begin{equation}
(\theta, \omega) \in \Gamma_t(p) \equiv \left\{ (\theta, \omega) : u(\theta, \omega) + \delta E_t \left\{ V_{t+1}(\theta, \omega') | \theta, \omega \right\} \leq p \right\}.
\end{equation}

Thus, $\Gamma_t$ characterizes owners’ strategy in period $t$. For notational convenience, we adopt the convention that a $(\theta, \omega)$-owner accepts an offer of $p$ in period $t$ with probability 1 if she is indifferent. In equilibrium, only an $(L, h)$-owner may be indifferent and whether she trades in this scenario is irrelevant for asset prices and payoffs.\(^{12}\)

Our equilibrium notion requires that the buyers’ offers must be optimal. Formally, we will impose two conditions. First, because buyers are identical, symmetrically informed, and compete in Bertrand fashion, they must earn zero expected profit conditional on their offer being accepted. That is, if $\Gamma_t(p) \neq \emptyset$ then

\begin{equation}
P_t = E_t \left\{ v_\theta + \delta V_{t+1}(\theta, \omega') | (\theta, \omega) \in \Gamma_t(p) \right\}.
\end{equation}

Second, we require that a buyer cannot profitably deviate from the equilibrium price by making a higher or lower price offer. Any deviant offer below the maximal bid of other buyers (i.e., any $p < P_t$) will be rejected with probability 1; the seller would do strictly better to accept $P_t$. Therefore, a profitable deviation does not exist provided that for all $p \geq P_t$,

\begin{equation}
p \geq E_t \left\{ v_\theta + \delta V_{t+1}(\theta, \omega') | (\theta, \omega) \in \Gamma_t(p) \right\}.
\end{equation}

Agents’ expectations about the future affect the seller’s willingness to trade at a given price today through (2) and (3), and the buyer’s willingness to offer a given price today through (4) and (5). Of course, in equilibrium, these expectations must be rationalized by future behavior. A primary goal of this paper will be to characterize the extent to which these expectations can (rationally) vary over time and then study the implications for aggregate dynamics. In order to economize on notation and technical detail, we will restrict attention to equilibria in which these expectations are stationary with respect to a publicly observable sentiment process that follows a homogenous Markov chain with a finite state space $Z = \{ z_1, z_2, \ldots, z_n \}$.

**DEFINITION 1:** A stationary rational expectations equilibrium (REE) is a value function $V$, a probability distribution characterizing agents’ beliefs, a price function

\(^{12}\) If an $(H, l)$ or $(L, l)$-owner were indifferent but accepted the offer of $p$ with probability less than one, it would be profitable for a buyer to offer $\varepsilon$ more and trade with probability 1.
$P$, and a Markov chain $Z_t$ with state space $\mathcal{Z}$ and transition matrix $Q = [q_{ij}]$, such that for each $Z_t \in \mathcal{Z}$

(i) $V_t(\theta, \omega) = V(\theta, \omega, Z_t)$ solves (1),

(ii) $\Gamma_t(p) = \Gamma(p, Z_t)$ satisfies (3),

(iii) if $\Gamma(P_t, Z_t) \neq \emptyset$ then $P_t = P(Z_t)$ satisfies (4),

(iv) for all $p \geq P_t$, (5) holds, and

(v) agents’ beliefs, which consist of a joint probability distribution over asset qualities, productivity shocks, and sentiments, are consistent with the true underlying distribution.$^{13}$

It is worth emphasizing several points about our equilibrium definition. First, the sentiment process is purely an extrinsic coordination device that is unrelated to the economic payoffs. Second, while we have incorporated the sentiment process into our equilibrium definition, an equilibrium need not involve sentiments in any economically meaningful way (e.g., $\mathcal{Z}$ can be a singleton, see Section IIA). Finally, condition (iv) rules out the possibility of multiple equilibria that may arise in the static Akerlof (1970) model by eliminating any candidate price if there exists a higher one that could be made without generating an expected loss for the buyer.

Henceforth, we will restrict attention to equilibria in which the Markov chain is irreducible, meaning that is possible to get to any state starting from any state (though doing so may involve many transitions)$^{14}$ This restriction is not necessary for most of our results. It is also not without loss with respect to the set of possible dynamics. For example, it rules out equilibria with absorbing states, which can exist. However, imposing irreducibility simplifies exposition and formal statements of results without compromising the main economic insights.

Frictionless Benchmark

Before characterizing equilibria, we briefly remark on a benchmark economy in which asset quality is publicly observable.$^{15}$ Observability of asset quality suffices to ensure that allocations are efficient.

PROPOSITION 1 (Observable Quality): If asset qualities are publicly observable, then the equilibrium is unique. In it, all assets are efficiently allocated and the price of a $\theta$-quality assets is $p_{t} = (1 - \delta)^{-1}v_{\theta}$ for all $t$.

$^{13}$In particular, given agents’ information at time $t$, $(\theta, \omega, Z_{t+1})$ are independently distributed with $\Pr(\theta = H) = \pi$, $\Pr(\omega = h) = 1 - \lambda$, and $\Pr(Z_{t+1} = z|Z_t = z_t) = q_{ij}$.

$^{14}$Formally, irreducibility requires that for any two states $z_i, z_j \in \mathcal{Z}$, there exists an integer $n < \infty$ such that $\Pr(Z_n = z_j|Z_0 = z_i) > 0$.

$^{15}$Formally, our notion of equilibrium can be modified in two ways to accommodate the benchmark. First, prices are indexed by $\theta$. Second, the condition of the expectation in both (4) and (5) (i.e., $(\theta, \omega) \in \Gamma_t(p)$) is replaced by $\theta$. 
This result illustrates that adverse selection is a necessary ingredient for our main results. Without it, buyers can condition offers on quality, which is sufficient to ensure that all gains from trade are realized. The intuition is that if a shocked owner retained a type-$\theta$ asset in any period, then a buyer could profitably deviate by making an offer between the seller’s reservation value and her own value for the asset, which is strictly higher. Finally, because it is always efficiently allocated and markets are competitive, a type-$\theta$ asset is priced at the present discounted value of $v_\theta$.

II. Equilibrium

In this section, we characterize equilibria of our model. We start by providing a partial characterization of any equilibrium, which narrows the set of possible allocations.

**PROPOSITION 2:** In any equilibrium and for all $Z_t \in \mathcal{Z}$,

$$V(L, l, Z_t) = V(L, h, Z_t) = P(Z_t) \leq V(H, l, Z_t) < V(H, h, Z_t).$$

Intuitively, because the flow payoff is higher to an unshocked owner or an owner with a higher quality asset, the values can be ranked according to $V(\theta, l, Z_t) \leq V(\theta, h, Z_t)$ and $V(L, \omega, Z_t) \leq V(H, \omega, Z_t)$. Clearly, an owner’s value is weakly larger than the current price, otherwise she would do better to accept it. However, if $V(L, h, Z_t) > p(Z_t)$ then buyers make positive profits on any transaction with an $(L, l)$-owner. Finally, because there are no gains from trading with them, buyers will never be able to attract an $(H, h)$-owner without making losses in expectation.

An immediate implication is that, in every period and regardless of $Z_t$, $(L, l)$-owners sell their assets, whereas $(H, h)$-owners do not. Further, it is without loss with respect to the set of equilibrium payoffs and prices to restrict attention to equilibria in which $(L, h)$-owners also trade in every period. Thus, with regard to which assets are traded, the only question is whether $(H, l)$-owners trade. If so, then all assets are allocated efficiently in that period and we refer to the market as being *liquid*. If not, then some assets are inefficiently retained by low productivity owners and we refer to the market as being *illiquid*.

A. Non-Sentiment Equilibria

We first consider a simple class of equilibria in which sentiments do not play a role: the allocations are the same in every period and prices are constant. These equilibria help illustrate the link between prices and liquidity as well as how an intertemporal coordination problem can lead to multiplicity.

**DEFINITION 2:** An equilibrium is a non-sentiment equilibrium if market liquidity is the same in every period with probability one.

From Proposition 2, it follows that there can be two types of non-sentiment equilibria, depending on whether $(H, l)$-owners trade. We adopt the following definition in order to distinguish among them.
DEFINITION 3: A non-sentiment equilibrium features **efficient trade** if \((H, l)\)-owners trade in every period. Otherwise, if only low quality assets trade, it features **inefficient trade**.

In the efficient trade equilibrium, all shocked owners trade and the assets are efficiently reallocated each period. Instead, in the inefficient trade equilibrium, the allocation is inefficient because the unproductive owners with high quality assets retain ownership. Given a candidate type of non-sentiment equilibrium, the equilibrium price and value functions are uniquely pinned down. Whether such a candidate is in fact an equilibrium then rests on whether conditions (i) and (iv) of Definition 1 hold (i.e., whether owners or buyers have a profitable deviation).

The following theorem shows that a non-sentiment equilibrium always exists and provides necessary and sufficient conditions for each type of equilibrium.

**THEOREM 1** (Non-Sentiment Equilibrium): There exist thresholds \(\bar{\pi} < \bar{\pi} \in (0, 1)\) such that

(i) the efficient trade equilibrium exists if and only if \(\pi \geq \bar{\pi}\),

(ii) the inefficient trade equilibrium exists if and only if \(\pi \leq \bar{\pi}\).

Notably, the two equilibria coexist when \(\pi \in [\bar{\pi}, \bar{\pi}]\). When they coexist, both the price and welfare are higher in the efficient trade equilibrium (welfare is higher in a Pareto sense).

The unexpected part of the theorem is that the two equilibria coexist for a generic set of parameters. The intuition is that liquidity can be self-fulfilling due to a coordination problem, albeit an intertemporal one. In particular, if buyers today expect that buyers in the future will offer high (pooling) prices, then their perceived difference between high and low quality assets is smaller rendering the adverse selection problem less of a concern. Hence, they are willing to make a high offer today. At this high price, an \((H, l)\)-owner is willing to sell today. That is, the expectation of future market liquidity generates liquidity in the market today. Conversely, if buyers today expect that future buyers will only offer low prices, then buying a lemon today becomes more of a concern, which renders the adverse selection problem more severe, making it more difficult to consummate a trade today.

From the discussion above, it should be clear that dynamic considerations are essential for this coordination problem to arise. The next proposition shows that the scope for multiplicity grows when agents care more about the future, but vanishes as they become arbitrarily impatient (\(\delta \to 0\)).

**PROPOSITION 3:** The wedge \(\bar{\pi} - \bar{\pi}\) is increasing in \(\delta\), and goes to zero as \(\delta \to 0\). Thus, the equilibrium becomes generically unique as resale considerations vanish.

---

16 In a static model (or our model with \(\delta = 0\)), there exists a single (i.e., non-generic) value of \(\pi\) such that both equilibria exist.
Figure 1 illustrates this result graphically by plotting the thresholds $\bar{\pi}$ and $\bar{\pi}$ against the discount factor $\delta$. That the region of multiplicity disappears as $\delta$ goes zero is consistent with Wilson (1980), who shows that strategic buyers eliminate the possibility of multiple equilibria in a static Akerlof (1970) model. Thus, the possibility of multiple equilibria in our setting hinges on the intertemporal coordination problem that arises because, when trading today, the agents care about the future market conditions.

In what follows, we show explicitly how to construct the non-sentiment equilibria, and then formalize the intuition above for why multiple equilibria arise in our setting. We begin with the construction of the efficient trade equilibrium.

Efficient Trade Equilibrium.—Let $p^{ET}$ and $V^{ET}$ denote the candidate equilibrium price and value function in an efficient trade equilibrium. Recall that in an efficient trade equilibrium, all but the $(H,h)$-owners trade every period. Therefore, owners values are given by

$$V^{ET}(L,l) = V^{ET}(L,h) = V^{ET}(H,l) = p^{ET}, \tag{6}$$

$$V^{ET}(H,h) = v_H + \delta E\{V^{ET}(H,\omega')\}. \tag{7}$$

Since buyers are unshocked, the zero-profit condition requires that

$$p^{ET} = \hat{\pi} V^{ET}(H,h) + (1 - \hat{\pi}) (v_L + \delta E\{V^{ET}(L,\omega')\}), \tag{8}$$
where $\hat{\pi} \equiv \lambda \pi / (\lambda \pi + 1 - \pi)$ is the probability that the asset is of high quality, conditional on being sold. A buyer has the same value as an $(H, h)$-owner if the asset turns out to be of high quality (w.p. $\hat{\pi}$), but not the same value as an $(L, h)$-owner if the asset turns out to be of low quality (w.p. $1 - \hat{\pi}$). That is because $(L, h)$-owners sell their asset immediately whereas the buyer must consume the flow payoff for one period before then reselling it. Notably, the buyer understands that, conditional on his offer of $p^E$ being accepted, the probability the asset is of high quality is strictly smaller than $\pi$.

Combining (6) through (8), we arrive at the following analytical expression for the candidate price in an efficient trade equilibrium:

$$p^E = (1 - \delta)^{-1} \left( \hat{\pi} v_H + (1 - \hat{\pi}) v_L + \delta(1 - \hat{\pi})(1 - \lambda) \right) \frac{\hat{\pi}(v_H - v_L)}{1 - \delta(1 - \hat{\pi})(1 - \lambda)}.$$

To verify that such an equilibrium exists, we must rule out profitable deviations. It is clear that there are no deviations for the buyers, since any such deviation would need to attract the $(H, h)$-owner, which is impossible without the buyers making losses in expectation. For owners, it is sufficient to check that an $(H, l)$-owner does not benefit from a one-period deviation (i.e., rejecting $p^E$ for one period). That is,

$$V^E(H, l) = p^E \geq c_H + \delta E\{V^E(H, \omega')\}.$$

Letting $\kappa(\hat{\pi}) \equiv \hat{\pi} v_H + (1 - \hat{\pi}) v_L - c_H$ and using equations (6) through (8), this condition can be written as

$$\kappa(\hat{\pi}) \geq \delta(1 - \hat{\pi}) E\{V^E(H, \omega') - V^E(L, \omega')\}.$$

Thus, the efficient trade equilibrium exists when the static gain from selling in this period, captured by $\kappa(\hat{\pi})$, is greater than the future loss the owner suffers from selling her $H$-asset at a price that pools both types of assets, captured by $\delta(1 - \hat{\pi}) \Delta^E$; we provide closed-form expressions for $\Delta^E$ in the Appendix. The threshold $\pi$ in Theorem 1 is the value of $\pi$ at which condition (11) holds with equality, which can be shown to be interior and unique.

Inefficient Trade Equilibrium.—Let $p^I$ and $V^I$ denote the candidate equilibrium price and value function in an inefficient trade equilibrium. In the inefficient trade equilibrium, only owners of low quality assets trade. Therefore, $(L, \omega)$-owner values are given by

$$V^I(L, l) = V^I(L, h) = p^I,$$

whereas $(H, \omega)$-owners consume the output from their asset today and in the future,

$$V^I(H, \omega) = u(H, \omega) + \delta E\{V^I(H, \omega')\}.$$
Buyers understand that only low quality assets trade today and in future periods. The zero-profit condition requires

\[ p^{IT} = \frac{v_L}{1 - \delta}. \]

For existence of such an equilibrium, we must again rule out profitable deviations for the owners and the buyers. It is straightforward to see that there are no deviations for the owners, since \( H \)-owners strictly prefer to keep (recall that \( c_H > v_L \)), whereas \( L \)-owners prefer to trade. To rule out deviations for the buyers, it suffices to check that the buyers’ profits are non-positive if they make an offer that attracts an \((H, l)\)-owner, i.e., that

\[ V^{IT}(H, l) \geq \hat{\pi} V^{IT}(H, h) + (1 - \hat{\pi}) \left( v_L + \delta E\{ V^{IT}(L, \omega') \} \right). \]

Using (12) through (14), this condition becomes

\[ \kappa(\hat{\pi}) \leq \delta (1 - \hat{\pi}) \underbrace{E\{ V^{IT}(H, \omega') - V^{IT}(L, \omega') \}}_{\Delta^{IT}}. \]

Thus, in contrast the efficient trade equilibrium, the inefficient trade equilibrium exists when the static gain to the \((H, l)\)-owner from selling in this period, captured by \( \kappa(\hat{\pi}) \), is lower than the future loss she suffers by selling her \( H \)-asset at a price that pools both types of assets, captured by \( \delta (1 - \hat{\pi}) \Delta^{IT} \); we provide a closed-form expression for \( \Delta^{IT} \) in the Appendix. The threshold \( \hat{\pi} \) in Theorem 1 is the value of \( \pi \) at which condition (16) holds with equality, which can also be shown to be interior and unique.

**What Is the Source of the Multiplicity?**—The conditions for the existence of each type of equilibrium, i.e., (11) and (16), look remarkably similar except that the inequality is reversed. For the efficient trade equilibrium to exist, the expected difference between the value of a high and low quality asset in the next period must be sufficiently low, but it must be sufficiently high for existence of an inefficient trade equilibrium. Naively, it then seems that they cannot simultaneously hold except for non-generic cases. Yet, Theorem 1 clearly states that there is a positive (Lebesgue) measure of \( \pi \) such that both equilibria exist. The crucial observation is that the difference between the expected value of owning a high versus low quality asset depends on the structure of the equilibrium. In the efficient trade equilibrium, the difference is relatively small since assets are regularly pooled at a common price. Whereas in the inefficient trade equilibrium, \( H \) and \( L \) assets are never pooled which magnifies the difference in their expected value. In short, \( \Delta^{ET} < \Delta^{IT} \), thus multiple non-sentiment equilibria exist whenever

\[ \frac{\kappa(\hat{\pi})}{\delta (1 - \hat{\pi})} \in (\Delta^{ET}, \Delta^{IT}). \]
We have already seen that dynamic considerations are necessary for multiplicity (see Figure 1). Figure 2 illustrates how the region of multiplicity changes with other model parameters. Absent a reason to trade in the future (i.e., if $\lambda = 0$ in panel A of Figure 2 or $1 - \chi = (v_H - c_H)/v_H = 0$ in panel B of Figure 2), buyers do not worry about future liquidity and there is no scope for multiplicity. Further, if the shocks are severe enough, then strategic considerations vanish and shocked sellers always trade. Finally, as illustrated in panel C of Figure 2, in order to activate the potential for multiple equilibria, the difference between high and low quality assets must be sufficiently large.

**B. Sentiment Equilibria**

Thus far, we have considered non-sentiment equilibria, in which the agents’ expectations about the future do not vary over time. But can there also exist equilibria in which expectations, prices, and allocations change over time? By Proposition 2, we can partition $Z$ into two disjointed sets, which we will denote as $Z_1$ and $Z_0$, which correspond to the set of states in which the market is liquid and illiquid respectively. We will sometimes refer to $Z_1$ as “good” states and $Z_0$ as “bad” states.

**DEFINITION 4:** An equilibrium is a sentiment equilibrium if both $Z_0$ and $Z_1$ are non-empty.

Our first result shows that the economy cannot feature deterministic variation in liquidity.

**PROPOSITION 4:** A sentiment equilibrium with deterministic transitions between good and bad states does not exist. That is, $z_i \in Z_1 (Z_0)$ if and only if there exists $z_j$ with $q_{ij} > 0$ such that $z_j \in Z_1 (Z_0)$.

The intuition for this result is as follows. Suppose that the market is liquid at date $t$ but will be illiquid at date $t + 1$ with probability 1. Then, regardless of play beyond $t + 1$, the expected future market conditions are worse starting from
date \( t \) then they are starting from date \( t + 1 \). Therefore, the expected difference in the future value between high and low quality assets is weakly larger at date \( t \) (i.e., \( \Delta_i \geq \Delta_{i+1} \)). In other words, the adverse selection problem is more severe at date \( t \). But, if a buyer and an \((H, l)\)-owner cannot agree to trade at date \( t + 1 \) (our supposition), then they certainly will not be able to agree to trade at date \( t \), a contradiction. By a similar reasoning, we can rule out equilibria in which the market is illiquid at date \( t \) and liquid at date \( t + 1 \) with probability 1.

Although deterministic transitions cannot exist as part of a sentiment equilibrium, as we will show next, our economy can feature sentiment equilibria with stochastic transitions.

**THEOREM 2:** A sentiment equilibrium exists if and only if \( \pi \in (\pi_-, \pi_+) \).

The theorem shows that the conditions for multiplicity of non-sentiment equilibria are exactly the same as the conditions for existence of sentiment equilibria. That \( \pi \in (\pi_-, \pi_+) \) is sufficient for existence of sentiment equilibria is perhaps not surprising. Necessity is less obvious, but we provide an intuition for it after stating our next result.

Theorem 2 does not shed light on the characteristics of sentiment equilibria, which we now address. Given any candidate sentiment process, which is fully characterized by \((\mathcal{Z}_0, \mathcal{Z}_1, Q)\), it is straightforward to construct the associated candidate value functions and prices. Next, define

\[
\Delta(z) \equiv E\{V(H, \omega', Z_{t+1}) - V(L, \omega', Z_{t+1}) | Z_t = z\},
\]

which is analogous to \( \Delta^E_T \) and \( \Delta^I_T \) (see Section IIA), except that the expected difference in asset values can now depend on the current sentiment.

**PROPOSITION 5:** A candidate is a sentiment equilibrium if and only if

\[
\frac{\kappa(\hat{\pi})}{\delta(1 - \hat{\pi})} \in \left[ \max_{z \in \mathcal{Z}_1} \Delta(z), \min_{z \in \mathcal{Z}_0} \Delta(z) \right].
\]

As alluded to in Proposition 4, a crucial feature of any sentiment equilibrium is that the sentiments be sufficiently persistent. That is, in order for the market to be liquid today given some \( z \in \mathcal{Z}_1 \), agents must expect that the market is sufficiently likely to be liquid in the future, meaning \( \Delta(z) \) is relatively small. Conversely, in order for the market to be illiquid today given some \( z \in \mathcal{Z}_0 \), agents must expect that the market is sufficiently likely to be illiquid in the future, meaning \( \Delta(z) \) is relatively large. And of course, future market conditions must rationalize these expectations.

Proposition 5 facilitates an intuition for why \( \pi \in (\pi_, \pi_+) \) is necessary for the existence of sentiment equilibria (as stated in Theorem 2), which is as follows. First, non-sentiment equilibria feature the most extreme expectations regarding future

\[\text{17 See the proof of Proposition 5 for this construction.}\]
liquidity. Hence, in any candidate sentiment equilibrium, \( \Delta(z) \in (\Delta^{ET}, \Delta^{IT}) \) for all \( z \). Next, the proof of Theorem 1 shows that \( \pi < \bar{\pi} \iff \kappa(\hat{\pi})/(\delta(1 - \hat{\pi})) < \Delta^{ET} \). Therefore, if the fraction of high quality assets is below \( \pi \), a liquid market today cannot be sustained for any rational expectation about the future—an \((H, l)\)-owner can profitably deviate by rejecting. Similarly, \( \pi > \bar{\pi} \iff \kappa(\hat{\pi})/(\delta(1 - \hat{\pi})) > \Delta^{IT} \). Therefore, if the fraction of high quality assets is above \( \bar{\pi} \), an illiquid market today cannot be sustained for any rational expectation about the future. A buyer can profitably deviate by making an offer that attracts an \((H, l)\)-owner.

To illustrate the implications of Proposition 5, let us begin by considering a simple class of candidate sentiment equilibria, where \( \mathcal{Z} = \{b, g\} \) and \( Z_t \) follows a symmetric first-order Markov process with persistence parameter \( \rho = \Pr(Z_{t+1} = z|Z_t = z) \in (0, 1) \). In the \( g \) state, agents coordinate on a liquid market (i.e., \( \mathcal{Z}_1 = \{g\} \)), whereas in the \( b \) state they coordinate on an illiquid market (i.e., \( Z_0 = \{b\} \)). We refer to this class of processes as a binary-symmetric sentiment process with persistence \( \rho \).

**COROLLARY 1:** A sentiment equilibrium with a binary-symmetric sentiment process with persistence \( \rho \) exists if and only if \( \pi \in (\pi, \bar{\pi}) \) and \( \rho \geq \rho \), where \( \rho \in [1/2, 1) \) depends on the primitives.

This result emphasizes the role of intertemporal coordination for the existence of multiple equilibria in our setting. The realization of the sentiment must not only signal to the agents what to play today, but it must also be informative about how the equilibrium play will proceed in the future. These two objectives are accomplished precisely by a sentiment process that is sufficiently persistent. To understand why the persistence is needed, note that the more persistent is the process the larger is the expected difference in asset values in the illiquid state, \( \Delta(b) \), and the smaller is the expected difference in asset values in the liquid state, \( \Delta(g) \). As \( \rho \to 1 \), \( \Delta(b) \to \Delta^{IT} \) and \( \Delta(g) \to \Delta^{ET} \). Thus, for \( \rho \) large enough and \( \pi \in (\pi, \bar{\pi}) \),

\[
\frac{\kappa(\hat{\pi})}{\delta(1 - \hat{\pi})} \in [\Delta(g), \Delta(b)],
\]

which, by Proposition 5, completes the argument.

The amount of persistence that a sentiment process needs depends on model parameters, as illustrated in Figure 3. Here, the shaded region depicts the combination of the parameters \( \pi \) and \( \rho \) for which a binary-symmetric sentiment equilibrium exists. The lower boundary of the region depicts the combinations of \( \pi \) and \( \rho \) for which the \((H, l)\)-owner is indifferent between trading and retaining her asset in the good state. It is downward sloping because the pooling bid is higher both when the pool quality is higher and when the good state is expected to last longer. On the other hand, the upper boundary depicts the combinations for which the buyers make exactly zero profits by deviating and attracting the \((H, l)\)-owner in the bad state. It is upward sloping because the buyers’ willingness to pay for the asset pool is higher when the pool quality is higher but lower when the bad state is expected to last longer. In the interior, neither the owners nor the buyers want to profitably deviate from equilibrium play. Figure 3 emphasizes that, in
contrast to intratemporal coordination problems, a sentiment equilibrium cannot be driven by an arbitrary stochastic process, but rather is disciplined by model parameters.

The binary-symmetric sentiment example is perhaps the simplest illustration of how sentiments can drive equilibrium behavior. Yet, the dynamics can be much richer. We illustrate the dynamics of an economy with a richer sentiment process in Figure 4. In this example, $Z = \{1, \ldots, N\}$, the transition matrix has the form

\[
Q = \begin{pmatrix}
\rho & 1 - \rho & 0 & \ldots & 0 \\
1 - \frac{\rho}{2} & \rho & \frac{1 - \rho}{2} & \ldots & 0 \\
0 & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \ldots & 1 - \frac{\rho}{2} & \rho \\
\end{pmatrix},
\]

with $Z_0 = \{1, 2, \ldots, n^* - 1\}$ and $Z_1 = \{n^*, n^* + 1, \ldots, N\}$. Thus, the market is liquid when $Z_t \geq n^*$ and it is illiquid otherwise. A feature of this example that gives rise to richer dynamics is that even if the market remains in a liquid state, agents' expectations about future liquidity can change. For instance, the market is liquid when $Z_t = N$ and when $z_t = n^*$, but when $Z_t = N$, traders expect that the market will remain liquid for at least the next $N/2$ periods, whereas when $Z_t = n^*$, there

\[
\begin{array}{c}
\text{Proportion of high quality assets (}\pi\text{)}
\\
\text{Sunspot persistence (}\rho\text{)}
\end{array}
\]

**Figure 3. Sentiment Equilibrium Existence Set**

*Note:* The figure illustrates all the combination of the parameters $\pi$ and $\rho$ for which the binary-symmetric sentiment equilibrium exists.
is risk of illiquidity in the next period. As in the binary-symmetric example, such a sentiment equilibrium exists provided the parameter $\rho$ is sufficiently high, a property that is reflected in the cyclical dynamics in panels B and C of Figure 4.

### III. Asset Production

In this section, we explore the role of sentiments in determining the distribution of asset quality in the economy. In order to do so, suppose that in each period there is a mass $\mu \in (0,1)$ of “producers” each of whom can create an asset. Each producer chooses an investment level $q$ at cost $c(q)$, with $c'(q) > 0$, $c''(q) \geq 0$. A producer who chooses an investment level $q$ produces an $H$-quality asset with probability $q$ and an $L$-quality asset with probability $1-q$. Thus, more investment corresponds to a higher likelihood of creating an $H$-quality asset but also a higher cost. To keep the environment stationary, we assume that each period a fraction $\mu$ of assets mature before paying off and their owners exit the market. As a result, the factor with which each agent discounts asset payoffs becomes $\delta^t = \delta(1 - \mu)$.

Each asset takes one period to produce: a producer in period $t$ becomes the owner of the asset in period $t+1$ and faces the same i.i.d. process of productivity shocks as other owners in the economy. We assume that the vintage of the asset is observable, which seems plausible and facilitates a tractable analysis. In other words, in each period there will be a different market for each vintage of asset.

Given a candidate equilibrium and the current sentiment $Z_t$, the date-$t$ producer chooses $q_t$ to solve

$$\max_{q \in [0,1]} \left\{ \delta \left( q E\{ V(H, \omega', Z_{t+1} | Z_t) \} + (1-q) E\{ V(L, \omega', Z_{t+1} | Z_t) \} \right) \right\} - c(q) \right\}.$$  

Thus, the first order condition for investment at date $t$ is

$$c'(q_t) = \frac{\delta \left( E\{ V(H, \omega', Z_{t+1}) - V(L, \omega', Z_{t+1} | Z_t) \} \right)}{\Delta(Z_t)}.$$  

18 If vintage is not observable, then the distribution of quality among all assets in the economy can vary over time, which introduces additional nontrivial dynamic considerations.
Combining the first order condition with Proposition 5 gives us the following immediate implication.

**Proposition 6:** If sentiments are part of an equilibrium with endogenous production, then the quality of assets created in good states is lower than the quality of assets created in bad states.

Intuitively, if markets are more likely to be liquid next period, then producers have less incentive to create high quality assets in the current period. This finding has testable implications that we discuss in more detail in Section IV A. However, it does not address the question of whether sentiment equilibria exist when asset production is endogenous. We turn to this question next.

**Proposition 7:** When asset production is endogenous

\[
(i) \text{ efficient trade is an equilibrium } \iff c'(\pi) \leq \xi \equiv \delta \cdot \Delta_{\pi=\bar{\pi}}^{ET},
\]

\[
(ii) \text{ inefficient trade is an equilibrium } \iff c'(\bar{\pi}) \geq \bar{\xi} \equiv \delta \cdot \Delta_{\pi=\bar{\pi}}^{IT},
\]

\[
(iii) \text{ if } c'(\pi) > \xi \text{ and } c'(\bar{\pi}) < \bar{\xi}, \text{ then any equilibrium is a sentiment equilibrium (and a sentiment equilibrium exists).}
\]

The first two statements are perhaps not very surprising. Because the incentive to invest is lower in the efficient trade equilibrium, it can only be sustained as an equilibrium when the marginal cost of production is sufficiently low. Conversely, because the incentive to invest is highest in the inefficient trade equilibrium, it can only be sustained as part of an equilibrium when the marginal cost of production is sufficiently high. The third statement is more interesting: when the marginal costs are intermediate, non-sentiment equilibria cannot be sustained; endogenous production requires that sentiments be part of any equilibrium. \(^{19}\) Figure 5 illustrates this finding. If the marginal cost of production lies entirely in the shaded area (i.e., for all \( \pi \in (\bar{\pi}, \bar{\pi}) \)), then any equilibrium must feature sentiments, whereas if the marginal cost curve lies above the upper line or below the lower line for some \( \pi \in (\bar{\pi}, \bar{\pi}) \), then non-sentiment equilibria can be sustained.\(^{20}\)

We have shown quality is countercyclical in sentiment equilibria, which must be part of any equilibrium provided that marginal production costs are not too extreme.\(^{21}\) However, there is an important distinction between the cyclicality of the quality and the quantity of assets produced. In our model, we have exogenously fixed the quantity of assets produced. If we were to incorporate heterogeneous entry

\(^{19}\)The conditions in part (iii) of Proposition 7, are not necessary for a sentiment equilibrium to exist because sentiment equilibria can coexist with non-sentiment equilibria. A necessary and sufficient condition for the existence of a sentiment equilibrium is that there exists \( \bar{\pi} \in (\pi, \bar{\pi}) \) such that \( c'(\bar{\pi}) \in (\Delta_{\pi=\bar{\pi}}^{ET}, \Delta_{\pi=\bar{\pi}}^{ET}) \).

\(^{20}\)To be more precise, if the marginal cost function lies below the lower line (denoted \( \Delta_{\pi=\bar{\pi}}^{ET} \)) for some (all) \( \pi \in (\bar{\pi}, \bar{\pi}) \), then efficient trade is an (the unique) equilibrium. If the marginal cost function lies above the upper line (denoted \( \Delta_{\pi=\bar{\pi}}^{IT} \)) for some (all) \( \pi \in (\bar{\pi}, \bar{\pi}) \), then inefficient trade is an (the unique) equilibrium.

\(^{21}\)If we relax the restriction that the Markov chain be irreducible, then with observable vintages, one can construct equilibria in which with some probability, the continuation trading equilibrium is efficient trade forever, and with the complementary probability it is inefficient trade forever.
costs, then the quantity of assets produced would be pro-cyclical. That is, while \( \Delta (Z_t) \) determines quality (which is lower when sentiments are strong) the expected price level determines quantity (which is higher when sentiments are strong). The prediction of lower quality during expansionary periods is consistent with evidence from credit markets (Mian and Sufi 2009, Greenwood and Hanson 2013, Krishnamurthy and Muir 2017).

IV. Discussion

In this section we discuss applications, the potential for amplification, and alternative specifications of the model.

A. Applications

Our model is intentionally stylized and we abstract from institutional features of specific markets. Therefore, it may be useful to provide several concrete interpretations of the model and discuss the implications of our results. By doing so, we also hope to demonstrate that all sentiment equilibria exhibit certain properties that generate testable implications for specific applications of the model.

Capital Reallocation.—Perhaps the most natural application of the model is the reallocation of existing capital among firms, which by some estimates accounts for a quarter of total investment by firms (Eisfeldt and Rampini 2006). Within this context, the agents are firms with the technology to operate capital to generate consumption...
goods and the assets should be interpreted as units of capital. Capital is heterogeneous in quality; all else equal, higher quality capital generates more consumption goods. The idiosyncratic shocks in our model can be interpreted as firm-specific productivity shocks. In the first-best outcome, capital is always immediately reallocated to the most productive firms, in which case aggregate output is constant over time. However, in order to reallocate capital from one firm to another, the two firms must agree to transact. And because capital is heterogeneous—all else equal, higher quality capital generates more of the consumption good—and its quality is privately observed by the firm employing it, such transactions do not necessarily materialize (i.e., if the market is illiquid).

In sentiment equilibria, aggregate output and aggregate productivity will be at (below) the first-best level in good (bad) states. In particular, the aggregate output is given by

\[ Y_t = \int \int u(\theta, \omega) \gamma_t(k, m) dk dm, \]

where \( \gamma_t(k, m) \) is the indicator for firm \( m \) operating capital unit \( k \) at date \( t \). Aggregate productivity is just rescaled output since the mass of capital units is fixed. Sentiment equilibria exhibit fluctuations in aggregate output and productivity, despite the absence of aggregate shocks to fundamentals. By Propositions 4 and 5, periods of high (and low) output must be sufficiently persistent and transitions between high and low output states must be stochastic, which gives rise to (endogenous) business-cycle dynamics driven by rational changes in expectations about the future state of the economy.

Moreover, in good states, all firms operating capital have high productivity whereas in bad states some firms with low productivity operate capital. At the same time, capital reallocation is higher in good states than in bad ones. Thus, sentiment equilibria exhibit properties in line with stylized facts documented by Eisfeldt and Rampini (2006); capital reallocation is pro-cyclical while the dispersion in productivity is countercyclical.

**New Investment with Financial Frictions.**—Rather than gains from trade arising from reallocating existing capital among firms, suppose instead that the gains arise from a difference in investment opportunities. To be more specific, interpret the agents in our model as entrepreneurs who can either manage existing projects or start new ones. All entrepreneurs are equally good at managing existing projects. But, in order to start a new project, an entrepreneur must have a new idea, which arrives randomly (the idiosyncratic shock). Due to frictions in financial markets, in order for an entrepreneur to turn her new idea into a project, she must sell her existing project. When they arrive, all new ideas are equally good; however, once an entrepreneur invests in a new idea and creates a project, its quality is realized and privately observed by the entrepreneur. Naturally, high quality projects create more of the consumption good. The efficient outcome is for all new ideas to be undertaken. However, an entrepreneur managing a high-quality project may decide not to undertake a new idea due to adverse selection in the market for existing projects.
This interpretation of the model is similar to that in the work by Eisfeldt (2004), Kurlat (2013), and Bigio (2015). One important difference is that project quality is persistent in our model whereas it is short-lived in theirs. Indeed, as we discuss in Section IVC, some degree of persistence in project quality is necessary for sentiment equilibria to exist. Thus, whereas the aforementioned literature has shown that adverse selection can amplify aggregate shocks, we show that it can, in fact, be the source of aggregate shocks. More specifically, in sentiment equilibria, both investment and growth will be driven by the market sentiment, which must evolve stochastically (Proposition 4). All new ideas will be undertaken in good states, but some will be foregone in bad states. Due to the (necessary) persistence of sentiments, periods of high or low investment and growth will persist in waves but will eventually end with a shift in the sentiment.

Because sentiments are persistent, not only will existing projects be more liquid in good states, but entrepreneurs will also find investing in new projects more profitable in good states because these projects are expected to trade more efficiently in the future. This has interesting implications when investment opportunities are not identical. For example, suppose the entrepreneur privately observes \( q \in [0,1] \), which corresponds to the probability that the idea will result in a good project if undertaken and suppose the distribution of \( q \) is i.i.d. Then, the quantity of investment will be higher in good states, but both the average quality of new investment and the return on new investment will be higher in bad states (similar to Proposition 6).

**Real Estate.**—For a third and final application of the model, consider a local real estate market. The assets are residential homes within a particular area and agents are households. Homes are heterogeneous in quality, which is privately observed by the household who owns and occupies it. The flow payoff corresponds to the utility or consumption value a household experiences from living in the home. All households experience a higher flow value from occupying a high-quality home, but whether the household is a good fit for a home (i.e., whether \( \omega = h \)) may change over time due to unforeseen changes in jobs, preferences, or family composition (the idiosyncratic private value shock). In the efficient outcome, all households who are not a good fit immediately sell their homes to households with a higher flow value. Of course, due to the adverse selection problem, a household owning a high-quality home that is not a good fit may choose to continue to live in the home.

Within this context, the predictions of any sentiment equilibrium are as follows. First, prices and transaction volume are positively correlated, and they are negatively correlated with time-to-sale. In good times, prices and volume are high and owners with a reason to move do so quickly. Conversely, in bad times, prices and volume are low and \((H,l)\)-households delay the sale of their home until market conditions improve. Second, real estate prices exhibit fluctuations even in the absence of aggregate shocks. These predictions are consistent with numerous empirical examinations of real-estate markets (see Mayer 2011 for a survey of this literature). Large movements in housing prices are difficult to explain based on fundamentals and, therefore, have often been interpreted by many as “bubbles” driven by non-rational agents (Scheinkman and Xiong 2003, Barberis et al. 2016). The time-series of prices in our sentiment equilibria exhibit similar behavior (see Figure 4) but obtain within a rational expectations framework.
B. Sentiments, Uncertainty, and Amplification

Our framework can be used to shed light on the recently growing body of empirical work, which documents that various measures of micro- and macro-level uncertainty tend to rise in bad times (e.g., Bloom et al. 2018).

Consider a modification of the sentiment process in Section IIB that still takes two values $Z_t \in \{b, g\}$, but where the good state is now more persistent than the bad state (i.e., $\Pr(Z_{t+1} = g|Z_t = g) > \Pr(Z_{t+1} = b|Z_t = b) \geq 1/2$). Next, consider a path of the sentiments such that prior to date $t$, sentiments have been good ($Z_t = g$), thus the market has been liquid and the assets allocated efficiently. But, at date $t$, sentiments change ($Z_t = b$), the market becomes illiquid, and some assets are allocated inefficiently. Observe that such a shift in equilibrium play manifests itself as a combination of a first- and a second-moment shock to economic activity.

To see this more concretely, consider our interpretation of assets as units of capital and asset owners as firms operating these units of capital (see Section IV A). Here, the shift in equilibrium play results in both a fall in the level of total output and TFP, and a rise in the cross-sectional dispersion of TFP across firms; in the good state all units of capital are operated by high productivity firms whereas in bad states some units are also operated by low productivity firms. Further, because the bad state is less persistent, uncertainty about future output and TFP also rises in bad times. Such a negative co-movement between first and second moments is broadly in line with the findings in Bloom et al. (2018). What is interesting is that, in our model, this co-movement can arise endogenously, even if there are no aggregate shocks to fundamentals.

Naturally, we could also generate these effects with small shocks to fundamentals rather than pure sunspots. Indeed, as discussed in Manuelli and Peck (1992), the early sunspot literature was motivated by the idea that small shocks to fundamentals are not very different from sunspots. They show that, in an overlapping generations endowment economy with money, small shocks to fundamentals can serve as the coordination device for different monetary equilibria. Our model can be extended to allow for aggregate shocks to fundamentals, which can then serve as the coordination device for agents’ expectations regarding the future market conditions. Of course, as we have highlighted, these shocks will need to be persistent enough in order to constitute an equilibrium.

To illustrate this point, suppose that the flow payoff of assets is a function of some observable aggregate state $X_t \in \{b, g\}$, which follows an observable Markov process. Concretely, consider the case where in state $X_t = g$ the flow payoff to a $(\theta, \omega)$-owner is $(1 + \varepsilon)u(\theta, \omega)$, whereas in state $X_t = b$ it is $(1 - \varepsilon)u(\theta, \omega)$ for some $\varepsilon \in (0, 1)$. Note that, when $\varepsilon = 0$, we are back to our baseline setup without aggregate shocks. It is therefore straightforward to show that, for $\varepsilon$ small enough, $X_t$ can serve as the coordination device for a sentiment equilibrium provided that it is sufficiently persistent. Such an equilibrium will display an amplification of fundamental shocks: although the shocks have a small direct effect on payoffs, they change expectations about future market conditions and have a large impact on

\[^{22}\text{Of course, both states need to be sufficiently persistent for the candidate to be part of equilibrium.}\]
equilibrium prices and allocations. Finally, note that we could also obtain amplification by feeding small aggregate shocks to other model primitives, which change the equilibrium set (see discussion of Figure 2).

C. Alternative Specifications

**Persistent Private Value Shocks.**—We have assumed that the idiosyncratic private value shocks are independent over time. We made this assumption in order to focus on the forward looking nature of the equilibrium and the role of sentiments. If we introduce persistence into the idiosyncratic shocks, then the equilibrium will depend not only on the agents’ expectations about the future but also on the history of play. This is due to the fact that with positively correlated shocks, the joint distribution of \((\theta, \omega)\) among asset owners is not necessarily stationary. When shocks are positively correlated over time, liquidity in the past is bad for liquidity today. A lot of trade yesterday implies most of the gains from trade have been realized and there is little reason to trade today.

However, the agents’ concern about future market conditions and the intertemporal coordination problem stemming from it would still be present. Indeed, most of our results can be generalized to an environment with persistence in the idiosyncratic shocks. One notable difference is that, deterministic transitions between good and bad states can be part of an equilibrium when shocks are persistent (i.e., Proposition 4 no longer holds). As Maurin (2016) has shown in an environment with search, it is possible to create deterministic trading cycles. They are characterized by a few periods of illiquidity followed by one period of liquidity, and so on. During the periods of illiquidity, the average pool of potential sellers improves (i.e., the fraction of \((H, l)\)-owners increases). Eventually, the pool quality is sufficiently high that buyers are willing to offer a pooling price. Immediately after, there are few \((H, l)\)-owners in the economy and thus the market is again illiquid until enough \((H, l)\)-owners have accumulated.

**Quality Persistence and Durability.**—We have also assumed that asset quality is (perfectly) persistent and that assets do not depreciate. While both of these assumptions can be relaxed, some degree of each is needed for the existence of sentiment equilibria. To see why some quality persistence is necessary, consider the expected difference in continuation values when asset quality can switch from one period to the next:

\[
\Delta(z) = E\{V(\theta', \omega', Z_{t+1})|\theta = H, Z_t = z\} - E\{V(\theta', \omega', Z_{t+1})|\theta = L, Z_t = z\}.
\]

Fixing the agents’ expectations about the future, the less persistent is asset quality, the smaller is \(\Delta(z)\). As a result, expectations about future market conditions play a less important role in determining whether the market is liquid today. In the extreme when asset quality is i.i.d., \(\Delta(z) = 0\) regardless of agents’ expectations about the future and there is no scope for sentiments.

A simple way to capture asset depreciation is by incorporating a Poisson arrival at which the asset fully depreciates or matures. It is not difficult to show that this extension of the model in which assets depreciate with probability \(\rho\) each period is
isomorphic to our model without depreciation and with a discount factor $\delta (1 - \rho)$. Thus, a higher rate of depreciation has the same effect as a decrease in $\delta$; faster depreciation reduces both $\pi$ and $\hat{\pi}$ (see Figure 1) as well as the wedge between them (Proposition 3).

**Competition.**—We have assumed that buyers are competitive and, hence, all rents go to the sellers. This assumption is convenient, but not necessary. Our main results also extend to a setting where buyers have some (or all) of the bargaining power. When bidding for an asset, buyers would still take into account that they may want to resell the asset in the future. Their expectations of future market conditions would continue to play a role in determining how aggressively they bid. As a result, the intertemporal coordination problem we have highlighted, which is key for the existence sentiment equilibria, remains present. The only qualitative change is that, with some bargaining power, the buyers may forego trade with the high-quality asset owners (even when they can break even by doing so), in order to extract rents from the low quality ones. The implication of less than perfect competition is that it becomes more difficult to sustain efficient trade and easier to sustain inefficient trade; therefore, the thresholds $\hat{\pi}$ and $\pi$ in Theorem 1 (or Figure 1) would be strictly higher than under perfect competition.

**Information Structure.**—Finally, we have assumed that the idiosyncratic shocks are privately observed. Relative to the alternative environment in which these shocks are publicly observable, this assumption implies that the severity of the adverse selection problem is larger. With publicly observable idiosyncratic shocks, $(L, h)$-owners will be unable to pool with owners of high quality assets and are effectively excluded from the market. This improves the pool of traded assets, but leaves the mechanism underlying sentiment equilibria unchanged. Essentially, all of the results in Section II hold if $\hat{\pi}$ is replaced by $\pi$. Since $\hat{\pi} < \pi$, this implies that the thresholds characterizing the set of equilibria in Theorem 1 are strictly lower than with unobservable private value shocks.

**V. Conclusions**

We study a dynamic market in which asset owners have private information about their asset quality and experience shocks to the private value of ownership, generating repeated gains from trade. The interaction of adverse selection with resale concerns gives rise to an intertemporal coordination problem that can sustain multiple self-fulfilling equilibria. We construct sentiment equilibria in which agents expectations about future liquidity vary over time and affect equilibrium prices and allocations. In sentiment equilibria, the price is equal to the expected fundamental value, yet prices display large fluctuations due to changes in sentiments, resembling behavior that is often interpreted as “bubbles.” Thus, sentiments cannot be separated from fundamentals; both are essential for determining asset valuations. More broadly, we have argued that sentiments, liquidity, and prices are intrinsically connected even when agents are fully rational.

Unlike static coordinations problems, the dynamics of sentiment equilibria are disciplined by model parameters. Notably, the sentiment process on which agents
coordinate must be both stochastic and sufficiently persistent. When asset production is endogenous, our model predicts that the quality of assets produced is lower in good times than in bad times; furthermore, we show that for a wide range of parameters any equilibrium must involve sentiments. Finally, we discuss the predictions of our theory within the context of capital reallocation, new investment, and real estate.

APPENDIX

PROOF OF PROPOSITION 1:
See text.

PROOF OF PROPOSITION 2:
From the zero profit condition (4), the equilibrium price must satisfy

\[(A1) \quad P_t = E\{v_\theta + \delta V_{t+1}(\theta, \omega')| (\theta, \omega) \in \Gamma_t(P_t)\} \geq E\{v_\theta + \delta V_{t+1}(\theta, \omega')| \theta = L\},\]

where the right-hand side is equal to the value of the \((L, h)\)-owner if she were to keep the asset for a period. Thus, it must be that \((L, h) \in \Gamma_t(P_t)\) and \(V_t(L, h) = P_t\).

On the other hand, the \((L, l)\)-owner has a weakly lower value than the \((L, h)\)-owner since the quality of her asset is the same, but the payoff she derives while keeping it is lower. Hence, in equilibrium we must also have \((L, l) \in \Gamma_t(P_t)\) and \(V_t(L, l) = P_t\).

Finally, \(V_t(H, \omega) \geq P_t\) holds trivially since the owner always has the option to trade at the equilibrium price, and we have that \((H, h) \not\in \Gamma_t(P_t)\) because the low quality assets always trade and thus

\[(A2) \quad P_t = E\{v_\theta + \delta V_{t+1}(\theta, \omega')| (\theta, \omega) \in \Gamma_t(P_t)\} < v_H + \delta E_t\{V_{t+1}(\theta, \omega')| \theta = H\} = V_t(H, h),\]

i.e., buyers cannot attract the \((H, h)\)-owner without making losses in expectation. ☐

PROOF OF THEOREM 1:
That there can at most be two types of non-sentiment equilibria follows from Proposition 2, which shows that there are only two possibilities depending on whether the \((H, l)\)-owner trades or not.

Efficient Trade Equilibrium.—The equations (6), (7), and (8) characterize the equilibrium owner values and asset price in candidate efficient trade equilibria. Since this system is linear, if an efficient trade equilibrium exists, there is only one of its kind. Moreover, this equilibrium exists if and only if inequality (10) is satisfied. Thus, combining (6) through (10), the efficient trade equilibrium exists if and only if

\[(A3) \quad (c_H - \hat{\pi} v_H - (1 - \hat{\pi}) v_L) + \delta(1 - \hat{\pi}) \frac{(1 - \lambda)(1 - \hat{\pi})(v_H - v_L)}{1 - \delta(1 - \hat{\pi})(1 - \lambda)} \leq 0,\]
where $\hat{\pi} \equiv \lambda \pi / (\lambda \pi + 1 - \pi)$. The left-hand side is strictly decreasing in $\pi$, positive at $\pi = 0$ and negative at $\pi = 1$. Hence, the threshold $\overline{\pi} \in (0, 1)$ exists, is unique, and the efficient trade equilibrium exists if and only if $\overline{\pi} \geq \bar{\pi}$.

**Inefficient Trade Equilibrium.**—Equations (12), (13), and (14) characterize the equilibrium owner values and asset price in candidate inefficient trade equilibria. Since this is a system of linear equations, if an inefficient trade equilibrium exists, there is only one of its kind. Moreover, this equilibrium exists if and only if inequality (15) is satisfied. Thus, by combining (12) through (15), the inefficient trade equilibrium exists if and only if

$$
(A4) \quad 0 \leq (c_H - \hat{\pi} v_H - (1 - \hat{\pi}) v_L) + \delta(1 - \hat{\pi}) \frac{(1 - \lambda)(v_H - v_L) + \lambda(c_H - v_L)}{1 - \delta},
$$

where $\hat{\pi} \equiv \lambda \pi / (\lambda \pi + 1 - \pi)$. The right-hand side is strictly decreasing in $\pi$, positive when $\pi = 0$ and negative when $\pi = 1$. Hence, the threshold $\bar{\pi} \in (0, 1)$ exists, is unique, and the inefficient trade equilibrium exists if and only if $\bar{\pi} \leq \bar{\pi}$.

**Existence and Multiplicity.**—Next, we show that $\overline{\pi} < \bar{\pi}$, which will establish that an equilibrium exists and that the two equilibria coexist whenever $\pi \in (\overline{\pi}, \bar{\pi})$.

From (A3) and (A4), we have that $\overline{\pi} < \bar{\pi}$ if and only if

$$
(A5) \quad \frac{(1 - \lambda)(1 - \hat{\pi})(v_H - v_L)}{1 - \delta(1 - \hat{\pi})(1 - \lambda)} \bigg|_{\pi = \overline{\pi}} < \frac{(1 - \lambda)(v_H - v_L) + \lambda(c_H - v_L)}{1 - \delta},
$$

but this inequality holds because, for any $\pi < 1$

$$
\frac{(1 - \lambda)(1 - \hat{\pi})(v_H - v_L)}{1 - \delta(1 - \hat{\pi})(1 - \lambda)} \leq \frac{(1 - \lambda)(v_H - v_L)}{1 - \delta(1 - \lambda)} < \frac{(1 - \lambda)(v_H - v_L) + \lambda(c_H - v_L)}{1 - \delta},
$$

where we used that $c_H > v_L$.

Finally, we have shown in the text that the asset prices are strictly higher in the efficient trade equilibrium. But, since the asset prices are higher, it must be that the $(L, \omega)$-owners are better off, the $(H, l)$-owner is better off by revealed preference, and the $(H, h)$-owner is better off since she becomes an $(H, l)$-owner with positive probability. Thus, the efficient trade equilibrium Pareto dominates the inefficient trade equilibrium.

**PROOF OF PROPOSITION 3:**

Consider the expressions defining the thresholds $\overline{\pi}$ and $\bar{\pi}$:

$$
(A6) \quad (c_H - \hat{\pi} v_H - (1 - \hat{\pi}) v_L) + \delta(1 - \hat{\pi}) \frac{(1 - \lambda)(v_H - v_L) + \lambda(c_H - v_L)}{1 - \delta} = 0,
$$
and

\[ (A7) \quad (c_H - \hat{\pi} v_H - (1 - \hat{\pi}) v_L) + \delta(1 - \hat{\pi}) \left( \frac{1 - \lambda}{1 - \delta(1 - \hat{\pi})} \right) \left( v_H - v_L \right) = 0, \]

where in both cases the left-hand side is decreasing in \( \pi \), since \( \hat{\pi} \) is increasing in \( \pi \).

First, note that

\[ \lim_{\delta \to 0} \hat{\pi} = \lim_{\delta \to 0} \pi = \frac{c_H - v_L}{v_H - v_L} + \left( 1 - \frac{c_H - v_L}{v_H - v_L} \right) \cdot \lambda, \]

and thus the equilibrium becomes generically unique as \( \delta \to 0 \).

Second, observe that

(i) thresholds \( \pi \) and \( \hat{\pi} \) coincide as \( \delta \to 0 \),

(ii) \( \frac{(1 - \lambda)(1 - \hat{\pi})(v_H - v_L)}{1 - \delta(1 - \hat{\pi})(1 - \lambda)} \bigg|_{\pi = \hat{\pi}} < \frac{(1 - \lambda)(v_H - v_L) + \lambda(c_H - v_L)}{1 - \delta} \quad \text{(see proof of Theorem 1), and} \]

(iii) \( \frac{(1 - \lambda)(1 - \hat{\pi})(v_H - v_L)}{1 - \delta(1 - \hat{\pi})(1 - \lambda)} \) is decreasing in \( \pi \). Therefore, \( \hat{\pi} \) is increasing faster in \( \delta \) than \( \pi \), and so the wedge \( \hat{\pi} - \pi \) is increasing in \( \delta \). \( \blacksquare \)

**PROOF OF PROPOSITION 4:**

Let \( \kappa(\hat{\pi}) \equiv \hat{\pi} v_H + (1 - \hat{\pi}) v_L - c_H \). If \( z_i \in Z_1 \), then by the same logic as in the construction of the efficient trade equilibrium, it must be that

\[ \kappa(\hat{\pi}) \geq \delta(1 - \hat{\pi}) \left( V(H, \omega', Z_{t+1}) - V(L, \omega', Z_{t+1}) \big| Z_t = z_i \right). \]

\[ \equiv \Delta(z_i) \]

Similarly, if the market is illiquid at date \( t + 1 \) w.p. 1 then it must be that for all \( j \) such that \( q_{ij} > 0 \),

\[ \kappa(\hat{\pi}) \leq \delta(1 - \hat{\pi}) \left( V(H, \omega', Z_{t+2}) - V(L, \omega', Z_{t+2}) \big| Z_{t+1} = z_j \right). \]

\[ \equiv \Delta(z_j) \]

Thus, we must have that \( \Delta(z_j) \geq \Delta(z_i) \). Thus, let us assume that this is the case.

We will now show that \( \Delta(z_i) > \Delta(z_j) \) for all \( j \) such that \( q_{ij} > 0 \), implying a contradiction. Because trade is inefficient w.p. 1 in the next period starting from \( Z_t = z_i \), and because \( \Delta(z_j) \geq \Delta(z_i) \),

\[ \Delta(z_i) = (1 - \lambda) v_H + \lambda c_H - v_L + \delta E \{ \Delta(Z_{t+1}) \big| Z_t = z_i \} \]

\[ \geq (1 - \lambda) v_H + \lambda c_H - v_L + \delta \Delta(z_i) \]

\[ = \frac{(1 - \lambda) v_H + \lambda c_H - v_L}{1 - \delta}. \]
On the other hand, because there is positive probability of efficient trade at some point in the future starting from $Z_{t+1} = z_j$,

$$\Delta(z_j) < \frac{(1 - \lambda)v_H + \lambda c_H - v_L}{1 - \delta}.$$ 

Therefore, $\Delta(z_i) > \Delta(z_j)$ for all $j$ such that $q_{ij} > 0$.  

PROOF OF PROPOSITION 5:

Let $N$ denote the number of elements in $\mathcal{Z}$. Let $I_Z$ denote the $N \times N$ identity matrix and $I_z$ be the $N \times 1$ vector of ones. Next, let $I_Z$ ($I_z$) be the matrix which coincides with $I_N$ except that it has zeros on the diagonal entries that correspond to the states $z \in \mathcal{Z}_0$ ($z \in \mathcal{Z}_1$). It is straightforward to construct the candidate sentiment equilibrium prices $p = \{p(z)\}_{z \in \mathcal{Z}}$ and values $V(\theta, \omega) = \{V(\theta, \omega, z)\}_{z \in \mathcal{Z}}$ from equations (2) through (5) as follows:

\begin{align}
(A8) \quad & V(H, l) = I_{Z_1} \cdot p + I_{Z_0} \cdot (c_H \cdot 1_Z + \delta Q(\lambda V(H, l) + (1 - \lambda) V(H, h))), \\
(A9) \quad & V(H, h) = v_H \cdot 1_Z + \delta Q(\lambda V(H, l) + (1 - \lambda) V(H, h)), \\
(A10) \quad & p = I_{Z_1} \cdot \left(\frac{\pi V(H, h)}{v_L} + (1 - \pi)(v_L \cdot 1_Z + \delta Qp)\right) + I_{Z_0} \cdot (v_L \cdot 1_Z + \delta Qp).
\end{align}

Thus, in order to establish the result, we only need to check that there are no profitable deviations in all states $z \in \mathcal{Z}$:

(i) There are no profitable deviations for the owners if and only if

\begin{equation}
(A11) \quad I_{Z_1} \cdot \left(c_H \cdot 1_Z + \delta Q(\lambda V(H, l) + (1 - \lambda) V(H, h))\right) \leq I_{Z_1} \cdot p,
\end{equation}

i.e., $\forall z \in \mathcal{Z}_1$, the $(H, l)$-owner prefers to trade than keep her asset.

(ii) There are no profitable deviations for the buyers if and only if

\begin{equation}
(A12) \quad I_{Z_0} \cdot \left(\frac{\pi V(H, h)}{v_L} + (1 - \pi)(v_L \cdot 1_Z + \delta Qp)\right) \leq I_{Z_0} \cdot V(H, l),
\end{equation}

i.e., $\forall z \in \mathcal{Z}_0$, the buyers cannot make positive profits by attracting the $(H, l)$-owner.

Next, as in the text, define

\begin{equation}
(A13) \quad \Delta(z) \equiv \mathbb{E}\{V(H, \omega', Z_{t+1}) - V(L, \omega', Z_{t+1}) | Z_t = z\}.
\end{equation}

Using the equilibrium prices and values above to solve for $\Delta \equiv \{\Delta(z)\}_{z \in \mathcal{Z}}$, we get

\begin{equation}
(A14) \quad \Delta = Q \cdot M \cdot v,
\end{equation}
where
\[
M = \left[ I_Z - \left( I_{Z_1} \cdot (1 - \lambda) \cdot (1 - \hat{\pi}) + I_{Z_0} \right) \cdot \delta Q \right]^{-1},
\]
\[
v = I_{Z_1} \cdot (1 - \lambda) \cdot (1 - \hat{\pi}) \cdot (v_H - v_L) \cdot 1_Z \\
+ I_{Z_0} \cdot \left( (1 - \lambda) \cdot v_H + \lambda \cdot c_H - v_L \right) \cdot 1_Z.
\]

After some algebra, the conditions for no profitable deviations become
\[
I_{Z_1} \cdot \delta (1 - \hat{\pi}) \Delta \leq I_{Z_1} \cdot \left( \hat{\pi} v_H + (1 - \hat{\pi}) v_L - c_H \right) \cdot 1_Z,
\]
and
\[
I_{Z_0} \cdot \delta (1 - \hat{\pi}) \Delta \geq I_{Z_0} \cdot \left( \hat{\pi} v_H + (1 - \hat{\pi}) v_L - c_H \right) \cdot 1_Z,
\]
which establishes the result.

PROOF OF THEOREM 2:

If \( \pi \in (\hat{\pi}, \bar{\pi}) \), then by construction the binary symmetric sentiment equilibrium exists whenever \( \rho \geq \bar{\rho} \) (see Corollary 1). On the other hand, suppose that a sentiment equilibrium exists, and the sentiment process is \( Z_t \) that takes values in set \( \mathcal{Z} \).

For any given \( z \in \mathcal{Z} \), we can express \( \Delta(z) \) recursively as follows:
\[
\Delta(z) = \sum_{z' \in \mathcal{Z}_1} \Pr(Z_{t+1} = z'|Z_t = z) \cdot \left( (1 - \lambda) \cdot (1 - \hat{\pi}) \cdot (v_H - v_L) + \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi}) \cdot \Delta(z') \right) \\
+ \sum_{z' \in \mathcal{Z}_0} \Pr(Z_{t+1} = z'|Z_t = z) \cdot \left( (1 - \lambda) \cdot v_H + \lambda \cdot c_H - v_L + \delta \cdot \Delta(z') \right).
\]

Since \( (1 - \lambda) \cdot v_H + \lambda \cdot c_H - v_L > (1 - \lambda) \cdot (1 - \hat{\pi}) \cdot (v_H - v_L) \), and because the Markov chain is irreducible, \( \forall z \in \mathcal{Z} \):
\[
\Delta(z) < (1 - \lambda) \cdot (1 - \hat{\pi}) \cdot (v_H - v_L) + \delta \cdot \sum_{z' \in \mathcal{Z}} \Pr(Z_{t+1} = z'|Z_t = z) \cdot \Delta(z') \\
\leq \frac{(1 - \lambda) \cdot v_H + \lambda \cdot c_H - v_L}{1 - \delta} = \Delta_{IT}.
\]

Analogously, \( \forall z \in \mathcal{Z} \), we have
\[
\Delta(z) > (1 - \lambda) \cdot (1 - \hat{\pi}) \cdot (v_H - v_L) + \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi}) \cdot \sum_{z' \in \mathcal{Z}} \Pr(Z_{t+1} = z'|Z_t = z) \cdot \Delta(z') \\
\geq \frac{(1 - \lambda) \cdot (1 - \hat{\pi}) \cdot (v_H - v_L)}{1 - \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi})} = \Delta_{ET}.
\]

Thus, the requirement that there be no profitable deviations for the owners is
\[
\frac{\kappa(\hat{\pi})}{\delta(1 - \hat{\pi})} \geq \max_{z \in \mathcal{Z}} \Delta(z) \Rightarrow \frac{\kappa(\hat{\pi})}{\delta(1 - \hat{\pi})} > \Delta_{ET}.
\]
whereas the requirement that there be no profitable deviations for the buyers is

\[
(A20) \quad \frac{\kappa(\hat{\pi})}{\delta(1 - \hat{\pi})} \leq \min_{z \in \mathcal{Z}_0} \Delta(z) \Rightarrow \frac{\kappa(\hat{\pi})}{\delta(1 - \hat{\pi})} < \Delta^{IT}.
\]

But these are equivalent to requiring that $\pi \in (\bar{\pi}, \hat{\pi})$ (see Section IIA).

PROOF OF COROLLARY 1:

From Proposition 5, a binary symmetric sentiment equilibrium exists if and only if

\[
\Delta(g) \leq \frac{\hat{\kappa}(\hat{\pi})}{\delta(1 - \hat{\pi})} \leq \Delta(b).
\]

We show that this is equivalent to $\pi \in (\bar{\pi}, \hat{\pi})$ and $\rho \geq \bar{\rho}$ for some $\bar{\rho} \in [1/2, 1)$. We will do this in two steps:

(i) We show that $\Delta(g) \leq \Delta(b)$ if and only if $\rho \geq 1/2$. This immediately implies that a candidate $\bar{\rho}$ must be greater than $1/2$.

(ii) We show that $\Delta(g)$ is decreasing and $\Delta(b)$ is increasing in $\rho$ for $\rho \geq 1/2$, and that $\lim_{\rho \to 1} \Delta(g) = \Delta^{ET}$ and $\lim_{\rho \to 1} \Delta(b) = \Delta^{IT}$. The existence of threshold $\bar{\rho}$ then follows from the fact that $\pi \in (\bar{\pi}, \hat{\pi})$ is equivalent to $\Delta^{ET} < \hat{\kappa}(\hat{\pi})/(\delta(1 - \hat{\pi})) < \Delta^{IT}$.

For (i), define $\alpha \equiv (1 - \lambda) \cdot (1 - \hat{\pi}) \cdot (v_H - v_L)$ and $\beta \equiv (1 - \lambda) \cdot v_H + \lambda \cdot c_H - v_L$, where note that $\alpha < \beta$. We can express $\Delta(g)$ and $\Delta(b)$ as follows:

\[
(A21) \quad \Delta(g) = \rho \cdot (\alpha + \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi}) \cdot \Delta(g)) + (1 - \rho) \cdot (\beta + \delta \cdot \Delta(b)),
\]

\[
(A22) \quad \Delta(b) = (1 - \rho) \cdot (\alpha + \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi}) \cdot \Delta(g)) + \rho \cdot (\beta + \delta \cdot \Delta(b)).
\]

Combine (A21) and (A22) to get

\[
\Delta(b) - \Delta(g) = (1 - 2\rho) \cdot (\alpha - \beta + \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi}) \cdot \Delta(g) - \delta \cdot \Delta(b)).
\]

Clearly, $\Delta(b) = \Delta(g)$ if $\rho = 1/2$. Next, if $\rho < 1/2$, then

\[
\Delta(b) - \Delta(g) < (1 - 2\rho) \cdot (\alpha - \beta - \delta \cdot (\Delta(b) - \Delta(g))),
\]

and thus $\Delta(b) < \Delta(g)$. But, if $\rho > 1/2$, then:

\[
\Delta(b) - \Delta(g) > (1 - 2\rho) \cdot (\alpha - \beta - \delta \cdot (\Delta(b) - \Delta(g))),
\]

and thus $\Delta(b) > \Delta(g)$.  

For (ii), differentiate (A21) and (A22) with respect to \( \rho \) to get

\[
0 = \left(1 - \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi})\right) \cdot \frac{d\Delta(g)}{d\rho} + (1 - \delta) \cdot \frac{d\Delta(b)}{d\rho},
\]

\[
\frac{d\Delta(g)}{d\rho} = \frac{(\alpha + \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi}) \cdot \Delta(g)) - (\beta + \delta \cdot \Delta(b))}{1 - \rho \cdot \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi}) + (1 - \rho) \cdot \delta \cdot \frac{1 - \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi})}{1 - \delta}}.
\]

Thus, since from (i) we have \( \Delta(g) \leq \Delta(b) \) for \( \rho \geq 1/2 \), it follows that \( d\Delta(g)/d\rho < 0 < d\Delta(b)/d\rho \) for \( \rho \geq 1/2 \). Finally, it is clear that as \( \rho \to 1 \), \( \Delta(g) \to \alpha / (1 - \delta \cdot (1 - \lambda) \cdot (1 - \hat{\pi})) = \Delta_{ET} \) and \( \Delta(b) \to \beta / (1 - \Delta) = \Delta_{IT} \).

**PROOF OF PROPOSITION 6:**

Follows from (i) the first order condition in (21), (ii) \( c'' \geq 0 \), and (iii) the fact that \( \Delta(z_1) < \Delta(z_0) \) for any \( z_0 \in Z_0 \) and \( z_1 \in Z_1 \).

**PROOF OF PROPOSITION 7:**

(i) In the efficient trade equilibrium, production optimality requires

\[
c'(\pi) = \hat{\delta} \cdot \Delta_{ET},
\]

where \( \hat{\delta} = \delta(1 - \mu) \), and recall that

\[
\Delta_{ET} = \frac{(1 - \hat{\pi})(1 - \lambda)(v_H - v_L)}{1 - \hat{\delta}(1 - \hat{\pi})(1 - \lambda)},
\]

which depends on the actual quality of assets \( \pi \) and is decreasing in \( \pi \). Since \( c(\cdot) \) is convex, this defines a unique candidate quality \( \pi \) of assets produced. From Theorem 1, therefore, efficient trade equilibrium exists if and only if \( \pi \geq \pi^\bar{\pi} \) or equivalently

\[
c'(\pi^\bar{\pi}) \leq \hat{\delta} \Delta_{ET}|_{\pi=\pi^\bar{\pi}}.
\]

(ii) In the inefficient trade equilibrium, production optimality requires

\[
c'(\pi) = \bar{c} \equiv \hat{\delta} \cdot \Delta_{IT},
\]

and recall that

\[
\Delta_{IT} = \frac{(1 - \lambda) v_H + \lambda c_H - v_L}{1 - \hat{\delta}},
\]

which is independent of \( \pi \). Again, this defines a unique candidate quality \( \pi \) of assets produced. From Theorem 1, therefore, inefficient trade equilibrium exists if and only if \( \pi \leq \pi^\bar{\pi} \) or equivalently

\[
c'(\pi^\bar{\pi}) \geq \hat{\delta} \Delta_{IT}.
\]

(iii) It follows immediately that any equilibrium must feature sentiments if both \( c'(\pi^\bar{\pi}) > \bar{c} \) and \( c'(\bar{\pi}) < \bar{c} \). Next, we prove existence of sentiment equilibrium.
Consider a simple sentiment process $Z_t$ that takes values in $\mathcal{Z} = \{g, b\}$, with $\Pr(Z_{t+1} = g | Z_t) = \gamma \in (0, 1)$ for all $t$ and $Z_t$. It is straightforward that in such a candidate equilibrium $\Delta(g) = \Delta(b) = \Delta$, where

$$\Delta = \frac{\gamma \cdot (1 - \hat{\pi})(1 - \lambda)(v_H - v_L) + (1 - \gamma) \cdot ((1 - \lambda)v_H + \lambda c_H - v_L)}{1 - \gamma \cdot \hat{\delta}(1 - \hat{\pi})(1 - \lambda) - (1 - \gamma) \cdot \hat{\delta}},$$

and note that $\Delta \uparrow \Delta^{IT}$ as $\gamma \downarrow 0$ and $\Delta \downarrow \Delta^{ET}$ as $\gamma \uparrow 1$.

Production optimality requires that the quality of assets satisfy

(A23) \[ c'(\pi) = \hat{\delta} \cdot \Delta. \]

And, from Proposition 5, this candidate is an equilibrium if and only if

(A24) \[ \frac{\kappa(\hat{\pi})}{\hat{\delta}(1 - \hat{\pi})} = \Delta. \]

Observe that (a) for $\gamma$ close to 1, by our assumption that $c'(\pi) > \xi$ and continuity we have that: $c'(\pi) = \hat{\delta} \cdot \Delta \Rightarrow \kappa(\hat{\pi})/\left(\hat{\delta}(1 - \hat{\pi})\right) < \Delta$, and (b) for $\gamma$ close to zero, by our assumption that $c'(\pi) < \bar{c}$ and continuity we have that: $c'(\pi) = \hat{\delta} \cdot \Delta \Rightarrow \kappa(\hat{\pi})/\left(\hat{\delta}(1 - \hat{\pi})\right) > \Delta$. Thus, by continuity, there exist $\pi$ and $\gamma$ such that both (A23) and (A24) hold and the candidate is an equilibrium. ■

REFERENCES


