Barcelona GSE Graduate School of Economics

Master Degree in Specialized Economic Analysis

"How Data Science, Algorithms and Machine Learning Can Help Antitrust Agencies? A Collusion Perspective"

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ABSTRACT IN ENGLISH (100 words)**:**

Regarding competition policy, there has been a concern about the use of algorithms to commit anticompetitive acts. As it is proven, the use of algorithms facilitates agreements, but at the same time, authorities may have incentives to invest in equipment to pursue this conduct. But recent literature has been leaving aside important issues: (i) the complexities to sanction cartels that use algorithms, and (ii) the specific benefit that algorithms can bring to authorities. These difficulties are not arguments for agencies to not use algorithms. In fact, the use of them can be important trough investigation process, obtaining results automatically, giving a better allocation of resources, developing continuous information and constantly monitor markets that have information available online.

ABSTRACT IN CATALAN (100 words)

En política de competències, hi ha hagut una preocupació sobre l'ús d'algoritmes per cometre actes anti-competitius. Com s'ha comprovat, l'ús d'algoritmes facilita els acords, però al mateix temps, les autoritats poden tenir incentius per invertir en equips que els ajudin a perseguir aquestes activitats. Però els estudis recents han deixat de costat problemes importants: (i) les complexitats per sancionar càrtels que utilitzen algoritmes, i (ii) el benefici específic que els algoritmes poden aportar a les autoritats. Aquestes dificultats no justifiquen que les agències no usin algoritmes. De fet, la seva utilització pot ser important al llarg dels processos d'investigació obtenint resultats automàticament, assignant els recursos de millor manera, desenvolupant una informació contínua i monitoritzant constantment els mercats que tenen la informació disponible en línia.

KEYWORDS IN ENGLISH (3)**:**

Anti-competitive acts   Algorithms   Authorities

KEYWORDS IN CATALAN (3)**:**

Actes anti-competitius   Algoritmes   Autoritats

HOW DATA SCIENCE, ALGORITHMS AND MACHINE LEARNING CAN HELP ANTITRUST AGENCIES? A COLLUSION PERSPECTIVE.

by

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Chapter 1

Introduction

Through the last decade, the development of big data has been characterized by the increase of storage capacities, processing power and specially the availability of data. Regarding this last point, we have different types of data that firms can use to develop their business as Web data[1], Text data[2], Time and location data[3], Smart grid and sensor data[4] and Social network data[5].

Given this data availability, there has been an exponential development in data science building a number of algorithms based on statistical models that are available for data scientists to create analytic platforms. Which algorithm is chosen is based on the goals that have been established beforehand, just as a statistician chooses the appropriate statistical model based on the problem to be solved. Although there are many algorithms, these methods, Classification, Regression, and Similarity Matching are the fundamental principles on which many of the algorithms used in data science rely. Some algorithms were developed to address business problems. Some were developed to augment algorithms in use for other purposes, or to have them perform somewhat differently, to tune them to a business environment.

[1]Customer level web behaviour data such as page views, searches, reading reviews, purchasing, can be captured. They can enhance performance in areas such as next best offer, churn modelling, customer segmentation and targeted advertisement.

[2]Email, news, Facebook feeds, documents, etc) is one of the biggest and most widely applicable types of big data. The focus is typically on extracting key facts from the text and then use the facts as inputs to other analytic process (for example, automatically classify insurance claims as fraudulent or not.

[3]GPS and mobile phone as well as Wi-Fi connection makes time and location information a growing source of data. At an individual level, many organizations come to realize the power of knowing when their customers are at which location. Equally important is to look at time and location data at an aggregated level. As more individuals open up their time and location data more publicly, lots of interesting applications start to emerge. Time and location data is one of the most privacy-sensitive types of big data and should be treated with great caution.

[4]Sensor data are collected nowadays from cars, oil pipes, windmill turbines, and they are collected in extremely high frequency. Sensor data provides powerful information on the performance of engines and machinery. It enables diagnosis of problems more easily and faster development of mitigation procedures.

[5]Within social network sites like Facebook, LinkedIn, Instagram, it is possible to do link analysis to uncover the network of a given user. Social network analysis can give insights into what advertisements might appeal to given users. This is done by considering not only interests the customers have personally stated, but also knowing what it is that their circle of friends or colleagues has an interest in.

That is the case regarding algorithms and competition policy, where recently has been formalized a concern by the OECD about the potential use of firm's algorithms to commit anti-competitives acts. In fact, the European Union antitrust chief Margrethe Vestager has said that using algorithms to follow competitors' prices for online goods and services could lead to more effective cartels. She also warned of higher fines if companies use software tools to enforce their cartels more strictly.

In theory, the use of algorithms would facilitate the potential agreement among competitors due to certain factors. For example, the increase in the speed of reaction to changes in competitors' variables would decrease the present value of the deviation. On the other hand, the probability of detection or sanction by the competition agency would also decrease given that the potential behaviour would be under the influence of tacit collusion, increasing the present value of the collusive benefit. Therefore, from the policy competition perspective there exist incentives to invest in equipment and human capital to detect and pursue this kind of conduct.

But until now, the recent literature has mentioned in a general way the potential risks that the use of algorithms in anticompetitive matter can bring, leaving aside and not considering two important related issues: (i) the series of technical and legal complexities to sanction cartels that use algorithms as a method of communication, and (ii) the specific benefit that algorithms can bring to the authorities to sanction cartels and trough the different processes related to investigations. For this reason, and the purpose of this document, is to clarify and be a bit more precise in how the competition agencies should focus the use of their resources considering two potential conflicting effects already mentioned, such as the incentives to improve the detection capacity of these kind of cartels and the complexities to effectively sanctioning this type of anticompetitive conduct.

The paper continues with a resume of the most recent literature in chapter 2, following chapter 3 with a mathematical proof to explain the firm's incentives in the presence of algorithms. In chapter 4 we explain our main hypothesis, finishing with the last chapter of recommendations and conclusions.

Chapter 2

Literature review

Collusion is an agreement between two or more parties, to limit open competition by deceiving, misleading, or defrauding others of their legal rights, or to obtain an objective forbidden by law typically by defrauding or gaining an unfair market advantage. One of the most recent ways that literature and practitioners have being discussing that firms can collude could be the design of processes or sets of rules supported by computers that facilitate anti-competitive price as an equilibrium in a secretive manner. For example, a pricing algorithm instructs the computer to set the price of an item for sale, and can be written to rely on competitors prices and demographic or other information about the customer. In the literature, artificial intelligence or machine learning could lead to outcomes not explicitly given in the programming.

According to the last, Merah(2015) concludes that automated pricing via algorithmic processing leads to pricing above the competitive level, either via tacit collusion. In the same path, Ezrachi & Stucke(2015) mention that pricing algorithms can spread tacit collusion beyond duopolies to markets with five or six large firms.

Salcedo(2016) in an a formal scientific approach to algorithmic collusion, considering that each duopolist (algorithmic agent) adapts its strategies based on the other concludes that collusion is "inevitable", but the model rests on several overly restrictive assumptions. Additionally, Waltman & Kaymak(2006) proofs that assuming cournot oligopoly and adopt reinforcement learning could exist collusion between algorithmic agents, but the model is not robust to small perturbations. Finally, Ittoo & Petit (2017) remain in lack of understanding of whether current artificial intelligence technology holds the capabilities that entitle algorithmic pricing agents to autonomously enter intocl tacitly collusive strategies without human intervention.

Thus, it is possible to conclude that recent literature mentions the possibility that the use of algorithms can lead to anticompetitive behaviours closer to tacit collusion than to hard cartels. Also, in cases that algorithms promote hard cartels, models are sensitive to small disturbances. It is important to mention that the literature related to this topic is relatively scarce given the recent interest by the academy as well as on the part of the competition agencies.

Chapter 3

Theory

Based on the model of the OECD (2017)[1] we explain how firms can change their incentive to collude while they incorporate algorithms as facilitators to monitor competition and accelerate the speed of reaction, but also considering firms' perception of antitrust authorities. As it is mentioned in OECD (2017)[2], in a perfectly transparent market where firms interact repeatedly and the retaliation lag tends to zero, collusion can always be sustained as an equilibrium strategy. Considering the last, our objective is to include a parameter that reflects the vision that firms have over competition authorities, seeing from a simple theoretical point of view how firms' incentives changes with this new parameter.

In order to incorporate the fact that algorithms allow companies to adapt their strategies very fast, the inter-temporal stream of profits is represented in continuous time, in opposition to the economic literature where collusion is commonly model in discrete time. Additionally we include the exogenous parameter ρ, which represent the strength or ability of the competition authorities to pursue cartels from the firm's point of view. We include the parameter ρ as a plus factor with the interest rate, then if firms wants to collude they have to considerer a higher opportunity cost. The parameter ρ does not represent the probability to detect cartels, it is just how firms take into consideration the option to collude if they see a competent antitrust authority from a financial perspective.

Collusion is a super game Nash equilibrium if, at any moment of time, the present discount value of the profits of an arbitrary firm under the collusive path is greater than or equal to the present discount value of the profits that the firm would earn by deviating from collusion. The value of the collusive and deviate path is given by:

[1] OECD (2017), *Algorithms and Collusion: Competition Policy in the Digital Age* www.oecd.org/competition/algorithms-collusion-competition-policy-in-the-digital-age.htm.

[2] Following the appendix on OECD (2017), the formal demonstration applies standard techniques usually used in the literature to analyse the relevant factors for collusion (see, for instance, Ivaldi et al. 2003).

$$V^m = \int_0^\infty e^{-r\rho t}\Pi_t^m dt \tag{3.1}$$

$$V^d = \int_0^{T+L} e^{-r\rho t}\Pi_t^d dt + \int_{T+L}^\infty e^{-r\rho t}\Pi_t^c dt \tag{3.2}$$

Where Π_t^m is the firm profit at moment t under collusion, r is the discount rate and ρ sees how competent the competition agency is, with a value equal to 1 if it is perfectly competent and 0 when it is not competent. Additionally, Π_t^d is the profit from deviating from collusion, Π_t^c is the payoff under competition, T is the number of periods that it takes for other firms to detect a deviation and L is the time lag required to react to the deviation. Accordingly to the last, T is a measure of the transparency in the market and L is a measure of the velocity of response or of the frequency of interaction.

For simplification, the profit functions are considered to be constant over time. Then, we could face a incentive compatibility constraint (ICC) of:

$$\int_0^\infty e^{-r\rho t}\Pi^m dt \geq \int_0^{T+L} e^{-r\rho t}\Pi^d dt + \int_{T+L}^\infty e^{-r\rho t}\Pi^c dt \tag{3.3}$$

$$\Pi^m \int_0^\infty e^{-r\rho t} dt \geq \Pi^d \int_0^{T+L} e^{-r\rho t} dt + \Pi^c \int_{T+L}^\infty e^{-r\rho t} dt \tag{3.4}$$

Solving the integrals and taking into consideration that $\int e^{-r\rho t}dt = \frac{-e^{-r\rho t}}{r\rho}$,

$$\Pi^m \left[\lim_{t\to\infty}\left(\frac{-e^{-r\rho t}}{r\rho}\right) - \left(\frac{-e^{-r\rho(0)}}{r\rho}\right)\right] \geq \Pi^d \left[\frac{-e^{-r\rho(T+L)}}{r\rho} - \left(\frac{-e^{-r\rho(0)}}{r\rho}\right)\right] + \Pi^c \left[\lim_{t\to\infty}\left(\frac{-e^{-r\rho t}}{r\rho}\right) - \left(\frac{-e^{-r\rho(T+L)}}{r\rho}\right)\right] \tag{3.5}$$

Solving the limits and the algebra we obtained,

$$\Pi^m \left[0 - \left(\frac{-1}{r\rho}\right)\right] \geq \Pi^d \left[\frac{-e^{-r\rho(T+L)}}{r\rho} - \left(\frac{-1}{r\rho}\right)\right] + \Pi^c \left[0 - \left(\frac{-e^{-r\rho(T+L)}}{r\rho}\right)\right] \tag{3.6}$$

$$\Pi^m \left[\frac{1}{r\rho}\right] \geq \Pi^d \left[\frac{-e^{-r\rho(T+L)}}{r\rho} + \left(\frac{1}{r\rho}\right)\right] + \Pi^c \left[\frac{e^{-r\rho(T+L)}}{r\rho}\right] \tag{3.7}$$

$$\Pi^m \geq \Pi^d \left[1 - e^{-r\rho(T+L)}\right] + \Pi^c \left[e^{-r\rho(T+L)}\right] \tag{3.8}$$

$$\Pi^m \geq \Pi^d - \left[\Pi^d - \Pi^c\right] e^{-r\rho(T+L)} \tag{3.9}$$

$$\Pi^m \geq \Pi^d - \left[\frac{\Pi^d - \Pi^c}{e^{r\rho(T+L)}} \right] \tag{3.10}$$

As a preliminary result, it is possible to conclude from equation (3.10) that transparency and velocity of interaction facilitate collusion. Indeed, when T and L fall (given constant ρ), the right-hand side of equation (3.10) also falls, relaxing the incentive compatibility constrain and thus making collusion more likely. The opposite happens with ρ, the more competent is the antitrust authority, more difficult is to sustain collusion[3]. As a conclusion, there exist incentives for firms to reduce T or L that can be done using algorithms, but at the same time, the presence of a strong competition authority dismiss the incentive to collude. Therefore, just following theoretical models, antitrust authorities should invest to be more competent as a sign to the market and firms should implement mechanisms to reduce T or L. But, it is this feasible and worthless for both agents in a practical way?

[3]If the parameter ρ is equal to 1, meaning that the antitrust authority is completely competent from the firm's perspective, we obtain exactly the same result as the OECD shows in their appendix. The intuition behind this result is that if firms observe that the competition authority is perfectly competent, the incentives for collusion are based exclusively on how to reduce T and L. On the other hand, when ρ is 0 or the company believes that the authority competition is not competent, the incentives of collusion are based exclusively on the collusive profits and deviations, without considering T and L. Indeed, if the authority is not competent, as a firm I have no incentive to invest in making endogenously collusion more profitable, then I will be only interested in the benefits I can get.

Chapter 4

Development

First of all, it is important to define the concept of algorithm. As it is mentioned in OECD (2017), an algorithm "*is an unambiguous, precise, list of simple operations applied mechanically and systematically to a set of tokens or objects (e.g., configurations of chess pieces, numbers, cake ingredients, etc.). The initial state of the tokens is the input; the final state is the output*"[1].

Accordingly to the last, our main focus is related in how algorithms can promote collusion. Ezrachi & Stucke (2016) mention four potential scenarios where algorithms can help firms to collude, being the first of them called Messenger, where humans agreeing to collude and using computers to execute their will. They use computers to assist in implementing, monitoring, and policing the cartel or to facilitate information exchange and signaling.

The second scenario they mention is called Hub and Spoke. In this case they consider the use of a single pricing algorithm to determine the market price charged by numerous users. In this framework, a cluster of similar vertical agreements with many of the industries competitors may give rise to a classic hub-and-spoke conspiracy, whereby the algorithm developer, as the hub, helps orchestrate industry-wide collusion, leading to higher prices.

At the third scenario they explain the concept of The Predictable Agent. The old patterns to collude are shifting to a world where pricing algorithms act as predictable agents and continually monitor and adjust to each others prices and market data. In this new context, there is no collusive agreement among executives. Each firm unilaterally adopts its own pricing algorithm, which sets its own price. The result is algorithm-enhanced conscious parallelism or known as tacit collusion.

Finally, the most challenging collusion scenario is called Digital Eye. The computers,

[1] Wilson, R. A. and F. C. Keil (1999), The MIT Encyclopedia of the Cognitive Sciences, MIT Press.

in learning by doing, determine independently the means to optimize profit. Artificial intelligence operating in enhanced market transparency leads to an anticompetitive outcome, with no evidence of any anticompetitive agreement or intent. In the end, we may think the markets, driven by these technologies, are competitive. And yet, were not benefiting from this virtual competition.

The most recent academic theory tells us that the use of algorithms could positively affect the probability of collusion. But is it possible that competition agencies can succeed in pursuing this type of collusion?. To answer this question, it is important to consider what scenario we are in. If we are dealing with cases of Messenger or Hub and Spoke, the current techniques of persecuting cartels should be effective since collusion is based on agreement between people, with the only difference that the implementation and verification tool of the agreement is an algorithm.

For example, at the U.K in 2016, the Competition and Markets Authority (CMA) busted a cartel between firms selling posters on Amazon of entertainment icons. Managers at Trod, a Birmingham company, and GB eye, based in Sheffield, agreed not to undercut each other, similar to a minimum price setting. To implement their commitment across a wide range of products in real time, they used an algorithm that would monitor their purported competitor's prices and set their own price accordingly[2]. In this case, the hardcore evidence to prove the cartel could be obtain by doing a dawn-raid or by the leniency program. The convoluted scenarios are the Predictable Agent and Digital Eye, where the sanction or persecution by the figure of cartel becomes more complex due to a series of complexities that we will explain in the following section.

(i) Difficulties to sanction cartels using algorithms

First of all, one of the main problems related to sanction cartels refers to forms of anticompetitive co-ordination which can be achieved without any need for an explicit agreement, but which competitors are able to maintain by recognising their mutual interdependence, mostly known as tacit collusion. Related to our scenarios, Predictable Agent and Digital Eye are the most feasible framework where competition authorities would not be able to sanction as the figure of cartels because there would not be any hardcore evidence. Accordingly to

[2]Source: http://eu-competitionlaw.com/the-cma-obtains-its-first-disqualification-undertaking-from-a-director-for-competition-law-infringement/. Last visit [05/24/2018].

the last, Šmejkal (2017) mention that it will be necessary to look for parallels behaviours between the robot and the employee and thus consider the responsibility of the employer undertaking for their acts against competition. Also, Harrington (2017) argues that collusion by software programs which choose pricing rules without any human intervention is not in violation of section 1 of the Sherman Act, but he offers a path towards making collusion by autonomous agents unlawful.

Additionally, from the firm's perspective there are several technological impediments. Accordingly to Ittoo & Petit (2017) many obstacles to arrive to an effective collusion are: (i) the joint optimization of actions where firms knowledge of their competitors' strategy has to be sufficient; (ii) Data Issue, for example the creation of payoff matrices is complicated where information may not be observable and (iii) Modeling the environment.

(ii) Incentive to invest

As has been demonstrated in Section 3, the competition agency has incentives to invest in order to show itself towards the market as a competent authority. The question that one must ask from the competition authority's view is where within the process of research analysis this investment must be carried out in order to be more competent. Initially, and has already being stated, there exist a legal complexity sanctioning cartels when we are under problematic scenarios, so investing additionally in sanctioning this type of behaviour would not have any positive outcome. The interesting issue is that there exist spaces to use tools such as big data, algorithms and machine learning within the process of research analysis and not only in the approach of a particular sanction.

In general, investigations through antitrust authorities are initiated by complaint of natural or legal persons (public or private entities) or ex officio. Usually, competition agencies segment the research process into three blocks with different timming depending on each legislation, which are: (1) Analysis of the admissibility of the complaint, (2) Research and (3) Closure of Research. The first is related to diligences where the authority request information and cite to declare to any person who could have knowledge of the denounced fact. The Research period has the same faculties as Admissibility, but these ones are obligatory for the investigated. Finally, authorities could file the investigation if they find lack of merit or they arrange a commitment to change behaviour that makes additional actions unnecessary.

The admissibility and investigation stages are intense moments in the request and data handling. Therefore, the use of these tools can be important thinking on saving time in the development of databases, obtaining results almost automatically and giving better allocation of human resources. Additionally, these same tools can help to develop continuous information and constantly monitor markets that have information available online. In general, the availability of economic data for the preparation of autonomous research by competition agencies is scarce, so web scraping and data analysis using algorithms can be a relevant tool. Also, collecting this information can help antitrust authorities to compare the requested data against the available one.

For example, related if antitrust agencies are using this kind of new tools, the most known example is the BRIAS from the Korean Fair Trade Commission. This algorithm is an automatic quantitative analysis system that predicts the probability of bid rigging, by analysing large amounts of bidding data from public agencies in Korea. Since 2013, the system collected bidding information from 51 central government agencies, 246 local governments and 26 public enterprises. The system is designed to quantify the possibility of bid rigging by weighting different types of information, such as the rate of successful bids, the number of companies that participated in the auction and the bid prices[3].

Such has been the relevance that this topic has taken in the antitrust world that, and in the same line of strengthening the research lines, recently have been created within the competition authorities exclusive units in the handling of data with exclusive responsibilities. In fact, on 6 December 2017, the CMA announced that would be appointing a chief data officer in an effort to strengthen enforcement of companies using digital tools to flout competition rules[4]. With this, the UK's antitrust regulator will have a specialized unit up and running this summer to focus on tackling companies' ilicit use of algorithm, machine

[3]The BRIAS operates in three phases, from the gathering of the data and input, to the generation of the results: In a first phase, BRIAS collects all bid-related data and information concerning large scale bidding contracts awarded by central and local administrations. All data and information are collected within 30 days of the tender award. In a second phase, the system analyses the data received and it automatically generates scores on the likelihood of bid rigging, by assessing each relevant factor for the analysis. To each of these factors a weighted value is assigned and the scores of each evaluation item are then added up. In a last step, the bidding opportunities are screened by BRIAS according to their score. Tenders which score above a certain threshold are flagged by the system for in-depth review by the Korean Fair Trade Commission (KFTC). Source: OECD (2017), *Algorithms and Collusion: Competition Policy in the Digital Age* www.oecd.org/competition/algorithms-collusion-competition-policy-in-the-digital-age.htm.

[4]See: Simon Zekaria UK antitrust agency's chief data officer will be the first in Europe, head says. Available at: https://www.mlex.com/GlobalAntitrust/DetailView.aspx?cid=943465&siteid=337.

learning and big data.

Antitrust agencies and algorithms, machine learning and data science

Agency	Summary
1. Korean Fair Trade Commission (KFTC)	The BRIAS is an automatic quantitative analysis system that predicts the probability of bid rigging, by analysing large amounts of bidding data from public agencies in Korea.
2. Competition and Markets Authority UK	CMA will have a specialized unit to focus on tackling companies' ilicit use of algorithm, machine learning and big data.

Chapter 5

Conclusions and recommendations

The development of big data has been characterized by the increase of several variables as storage capacities, processing power and data availability. Regarding to the last, there has been an exponential development in data science building a number of algorithms based on statistical models that are available for data scientists to create analytic platforms. That is the case regarding algorithms and competition policy, where recently has been formalized a concern by the OECD about the potential use of firms algorithms to commit anticompetitive acts.

As it was proven, the use of algorithms would facilitate potential agreements among competitors mainly due to the increasing speed of reaction to changes in competitors' variables, decreasing the present value of the deviation. But at the same time, antitrust authorities may have incentives to invest in equipment and human capital to detect and pursue this kind of conduct. Recent literature have been leaving aside two important related issues: (i) the series of technical and legal complexities to sanction cartels that use algorithms as a method of communication, and (ii) the specific benefit that algorithms can bring to the authorities to sanction cartels and trough the different processes related to investigations.

Related to the difficulties to sanction this type of cartels, it is important to make clear distinctions within the different types of algorithms. To pursue cartels, the problematic scenarios under our definitions are the Predictable Agent and Digital Eye, due to the no existence of hardcore evidence to prove the cartel and the no sanctioning law related to tacit collusion. According to the last, a more in-depth analysis of the legality of tacit collusion is needed as well as the development of new economic theories taking into account the differences of algorithms as collusion tools, as well as the differences between the different types of algorithms. Last but not least, in those same analysis should also be included the technological impediments for firms to collude using algorithms.

But these difficulties and development needs are not arguments for competition agencies to do not use these tools. In fact, the use of these tools can be important trough the investigation process thinking about saving time in the development of databases, obtaining results automatically and giving better allocation of human resources. Additionally, these same tools can help to develop continuous information and constantly monitor markets that have information available online. Finally, and in the same line of strengthening the research lines, competition authorities should invest in exclusive operating units in the handling of data with exclusive responsibilities.

Bibliography

[1] Šmejkal, V. (2017). *Cartels by robots current antitrust law in search of an answer.* InterEULawEast : journal for the international and european law, economics and market integrations, 4(2). doi:10.22598/iele.2017.4.2.1.

[2] Salcedo, B. (2015), *Pricing Algorithms and Tacit Collusion*, Working Paper, November.

[3] Oxera (2017), *When algorithms set prices: winners and losers*, Discussion Paper, June 19.

[4] OECD (2017), *Algorithms and Collusion: Competition Policy in the Digital Age* www.oecd.org/competition/algorithms-collusion-competition-policy-in-the-digital-age.htm

[5] Gomes, A. (2017). *Disruptive innovation, Big Data and Algorithms* [Powerpoint slides]. Retrieved from http://www.sic.gov.co/sites/default/files/documentos/092017/antonio-ferreira-gomes-disruptive-innovation-big-data-and-algorithms.pptx.

[6] Harrington Jr, Joseph E., *Developing Competition Law for Collusion by Autonomous Price-Setting Agents* (August 22, 2017). Available at SSRN: https://ssrn.com/abstract=3037818 or http://dx.doi.org/10.2139/ssrn.3037818

[7] Ezrachi, Ariel and Stucke, Maurice E., *Artificial Intelligence & Collusion: When Computers Inhibit Competition* (April 8, 2015). Oxford Legal Studies Research Paper No. 18/2015; University of Tennessee Legal Studies Research Paper No. 267. Available at SSRN: https://ssrn.com/abstract=2591874 or http://dx.doi.org/10.2139/ssrn.2591874

[8] Ittoo, A and Petit, N. (2017). *Algorithmic Pricing Agents and Tacit Collusion A Technological Perspective* [Powerpoint slides]. Retrieved from http://www.crids.eu/fichiers/slides-conference-20-octobre-2017/slides-nicolas-petit-et-ashwin-ittoo.

[9] Ittoo, Ashwin and Petit, Nicolas, *Algorithmic Pricing Agents and Tacit Collusion: A Technological Perspective* (October 2, 2017). Available at SSRN: https://ssrn.com/abstract=3046405 or http://dx.doi.org/10.2139/ssrn.3046405

[10] Ezrachi, A (2017). *Virtual Competition*, Session Information and Documentation at OECD Competition Policy Roundtable. Retrieved from http://www.slideshare.net/OECD-DAF/algorithms-and-collusion-oecd-competition-division-june-2017-oecd-discussion

[11] Capobianco, A, Gonzaga, P and Nyesö, A (2017). *Algorithms and Collusion*, Session Information and Documentation at OECD Competition Policy Roundtable.

Retrieved from http://www.slideshare.net/OECD-DAF/algorithms-and-collusion-oecd-competition-division-june-2017-oecd-discussion

[12] Ritter, C (2017). *Algorithms and Collusion*, Session Information and Documentation at OECD Competition Policy Roundtable. Retrieved from http://www.slideshare.net/OECD-DAF/algorithms-and-collusion-oecd-competition-division-june-2017-oecd-discussion

[13] Gal, M (2017). *Do algorithms facilitate coordination?*, Session Information and Documentation at OECD Competition Policy Roundtable. Retrieved from http://www.slideshare.net/OECD-DAF/algorithms-and-collusion-oecd-competition-division-june-2017-oecd-discussion

[14] Salcedo, B. Pricing algorithms and tacit collusion International Conference on Game Theory (SBU, 2016), Pennsylvania Economic Theory conference (poster, PSU, 2016), Cornell University, Warwick University, Collegio Carlo Alberto, CU-PSU Macro workshop (Cornell, 2015)

[15] Waltman, Ludo and Kaymak, U., *A Theoretical Analysis of Cooperative Behavior in Multi-Agent Q-Learning* (February 1, 2006). ERIM Report Series Reference No. ERS-2006-006-LIS. Available at SSRN: https://ssrn.com/abstract=880523

[16] S Mehra, *Antitrust and the Robo-Seller: Competition in the Time of Algorithms*, 100 Minnesota Law Review, 2015.

[17] Ivaldi, M., B. Jullien, P. Rey, P. Seabright and J. Tirole (2003), *The Economics of Tacit Collusion*, Final Report for DG Competition, European Commission, http://ec.europa.eu/competition/mergers/studies_reports/the_economics_of_tacit_collusion_en.pdf.

[18] Ezrachi, A. and M. E. Stucke (2016), *"Virtual Competition: The Promise and Perils of the Algorithm- Driven Economy"*, Harvard University Press, United States.