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Downward Wage Rigidity and Wage Restraint

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Abstract

The combination of downward nominal wage rigidity and pegged exchange rate creates an externality which leads to excessive wage inflation (Schmitt-Groh_e and Uribe, 2016). This paper re-examines this result assuming that wage setters are forward looking, hence endogenously restrain wage increases facing downward wage rigidity, as in Elsby (2009). In this case, wage inflation is either excessively high or excessively low compared to the social optimum: while wages increase too strongly following demand shocks, they rise by too little following Balassa-Samuelson-type technology shocks. Applying the model to euro area countries, I document excessively high wage inflation rates in the euro periphery, but excessively low rates in the euro core, in the pre-crisis period.

Keywords: downward nominal wage rigidity, currency peg, unemployment, euro crisis, unit labour costs, real exchange rate, wage restraint

JEL-Codes: E24, E32, F41

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1 Introduction

The sharp rise in wages in the euro periphery before the Great Recession, followed by the prolonged slump and slowly declining wages in the aftermath of the Great Recession, have reinvigorated concerns about the harmfulness of downward nominal wage rigidity for macroeconomic adjustment (Baldwin and Giavazzi, 2015). These concerns have received theoretical support by a new class of currency-peg open economy models in which downward nominal wage rigidity is put at center stage. Most influentially, Schmitt-Grohé and Uribe (2016) show that aggregate demand booms entail excessive wage inflation as economic agents fail to internalize the social cost of their own contribution to wage growth, exacerbating unemployment in subsequent recessions.[1]

However, the stark rise in wages in the euro periphery stands in contrast to low rates of wage inflation (i.e., wage moderation) in the euro core in the pre-crisis period, notably in Germany. Moreover, intra-euro imbalances are frequently blamed on the *interaction* of these two effects: as labor costs in the two groups of countries have diverged, so have their real exchange rates and hence their "competitiveness".[2] On the theory side, Schmitt-Grohé and Uribe (2016) cannot explain wage moderation as they assume perfect labor market competition. Clearly, once wage setting is static, the unemployment effects of downward nominal wage rigidity remain uninternalized by assumption. In contrast, as stressed by Elsby (2009), once wage setters are forward-looking they internalize downward nominal wage rigidity in future periods. As they act to restrain current wage inflation, the unemployment effects of the rigidity in future periods may then remain small.

This paper studies wage inflation across euro area countries through the lens of a Schmitt-Grohé and Uribe (2016)-type model in which wage setters are assumed to be forward looking. The main theoretical finding is that the model can generate excessive wage inflation, but equally excessive wage *moderation*. Precisely, compared to social optimum, forward-looking wage setters may restrain current wage inflation by too little, but equally *too strongly*, facing wage-inflationary pressure. On the applied side, I use this insight to assess wage developments across euro area countries before the euro zone crisis. Using country-level data on real exchange rates and tradable-sector TFP assembled in Berka et al. (2017), I find some evidence for the above-cited wisdom: wages increased too strongly in the euro periphery, but by *too little* in the euro core during the pre-crisis period.

[1] Other recent papers building on the Schmitt-Grohé and Uribe (2016) framework include Na et al. (2018), Kuvshinov et al. (2016), Schmitt-Grohé and Uribe (2017) and Bianchi et al. (2018).

[2] See, for example, the recent discussion about Germany in The Economist (Economist, 2017). A recent article highlighting the asymmetry in wage developments across euro area countries is Gilchrist et al. (2016). The VoxEU book by Baldwin and Giavazzi (2015) discusses causes and consequences of the euro area crisis.

To build intuition about the economic mechanism, in a first part of the paper I study a textbook neoclassical labor market with downward nominal wage rigidity. Precisely, I assume that wages can not be lower than a fraction of their level from the last period, following Schmitt-Grohé and Uribe (2016). In this framework I derive the following condition for efficiency:[3] in periods when the wage rigidity is slack, the real wage must lie *below* the household's marginal rate of substitution between consumption and leisure. In a labor market diagram, this corresponds to a right-ward shift of labor supply, which keeps wage inflation in check hence reduces the potential for unemployment in the following periods. I interpret this as "wage restraint". Wage restraint smooths working hours over the cycle, which raises the average amount of output available for consumption.

Wage restraint arises because risk and nominal rigidities interact, as in Basu and Bundick (2017). It depends on characteristics of the economy such as shock volatility, but also on the economy's current state. First, wage restraint is negatively related to (expected) price inflation: as price inflation erodes the *real* wage, any rigidity on the *nominal* wage is effectively dampened—a "greasing the wheels" effect (Tobin, 1972). Second, it varies with the *elasticity of labor demand*. Intuitively, if whilst in a boom, the unemployment effect of the rigidity in the next recession is expected to be large (a counter-cyclical elasticity of labor demand), the extent of wage restraint in the boom must also become larger.

These insights carry over to a second part of the paper, in which I construct a model of a two-sector (tradable and non-tradable) small open economy characterized by downward nominal wage rigidity and a pegged nominal exchange rate. As the exchange rate peg rules out the "greasing the wheels" effect, wage rigidity is particularly potent in this environment.[4] Production occurs in both sectors, and the relative price between tradables and non-tradables pins down the real exchange rate. Any nominal wage rise (the same across sectors as perfect mobility of labor is assumed) entails a real appreciation by virtue of the Balassa-Samuelson effect. Wage formation is forward looking, as households organize into labor unions.[5] Upward pressure on wages arises from two sources. First, I assume that technology shocks in the tradable sector may drive wage inflation from the "supply" side. Second, preference shocks may drive wage inflation from the "demand" side.

[3] The efficient allocation is the one which maximizes household welfare, subject to downward nominal wage rigidity being in place. Therefore, the efficient allocation is not first best, as this would be the allocation without wage rigidity. Strictly speaking, I therefore characterize a "constrained-efficient" outcome, as in Lorenzoni (2008) or Bianchi (2011, 2016).

[4] Reducing price inflation (possibly to eliminate an "inflation bias") is a classical argument for joining into a nominal anchor, that is, for fixing the nominal exchange rate (Alesina and Barro, 2002).

[5] The formalism follows the literature on centralized wage setting (e.g., Alesina and Perotti 1997 and Guzzo and Velasco 1999), and in the limit as union size shrinks to zero, nests the case of monopolistic competition in the labor market considered in Benigno and Ricci (2011).

The model's key insights are as follows. First, in line with Elsby (2009), the unions restrain wage inflation in expansions in order to reduce unemployment in recessions. Second, in spite of the aforesaid, the *extent* of wage restraint is not socially efficient.[6]

To see the intuition, consider a technology shock in the tradable sector. This shock drives up wages, as well as shifts labor from the non-tradable into the tradable sector where the technology innovation occurs. Because the labor demand elasticity is larger in this sector than in the non-tradable sector, this shock temporarily raises the aggregate labor demand elasticity hence makes this elasticity *pro-cyclical*.[7] By the logic above, the extent of wage restraint in the efficient allocation therefore *declines*. In contrast, because what matters for the demand faced by the unions is their own (rather than the aggregate) labor demand elasticity, they restrain wage inflation heavily following the shock, hence giving rise to a wage increase that is *excessively small*. The opposite holds following demand shocks, as these draw labor into the non-tradable sector thereby making the aggregate labor demand elasticity counter-cyclical. Therefore following these shocks, in line with the results in Schmitt-Grohé and Uribe (2016), wage inflation rates are excessively *large*.

In brief, the model predicts excessive wage inflation following demand shocks, but excessive wage moderation following supply shocks. I use this insight to assess the differential in wage developments across euro area countries before the euro zone crisis. To do so, I feed the model with country-level data on tradable-sector TFP as well as intra-euro real exchange rates from 1995-2007. The data is taken from Berka et al. (2017) and, as explained there, can be interpreted not only in the time series, but also in the *cross section*. I am using the model to back out the unobserved series for demand shocks during the sample period, which allows me to construct a real exchange rate counter factual.

Results are the following. Wage restraint was too strong in the euro core in general. For example, the Dutch real exchange rate was too weak throughout the entire sample period, whereas the German and the Austrian real exchange rates depreciated *too quickly* until the start of the crisis, albeit both starting from a too appreciated level. In contrast, the Spanish real exchange appreciated too quickly in the years preceeding the euro zone crisis, even though in the early years after 1995 the Spanish real exchange rate has been *too weak*. Overall, I view my findings as a rough confirmation of the perceived-wisdom hypothesis (stated earlier

[6] Two other sources of inefficiency are the following. First, the unions charge monopolistic wage mark-ups. Second, the planner exploits (an equivalent of) the terms-of-trade externality, by which domestic working hours can be reduced without a corresponding drop in consumption (Corsetti and Pesenti, 2001).

[7] This follows directly from the Balassa-Samuelson effect. As nominal wages rise, the price of non-tradables rises relative to the price of tradables. As a result, firms in the tradable sector face a larger increase in their real wage (nominal wage divided by the sales price of their output) than firms in the non-tradable sector. But this means that hours are reduced by more in the tradable sector, hence that labor demand in this sector is relatively steeper.

above), even though the picture in its details is more nuanced.

I also compute permanent consumption losses relative to the allocation where wage restraint is efficient. I find that consumption losses are disproportionate in Spain and Ireland, amounting to more than 3% in about one tenth of the periods. The model makes this prediction, even though the crisis has not materialized in sample (recall that the data ends in 2007). Finally, I also compute consumption losses relative to first best—the allocation without downward wage rigidity to begin with. I show that the *additional* welfare loss is negligible. Hence, I confirm the central prediction of Elsby (2009) in a dynamic stochastic general equilibrium environment: welfare costs of downward wage rigidity *per se*, once internalized in the appropriate way, cannot be expected to be substantial.

Related literature.—The paper contributes to the literature on the macroeconomic implications of downward wage rigidity. In currency pegs, downward nominal wage rigidity is particularly powerful, because the long-run price level is pinned down by the nominal anchor country (Corsetti et al., 2013; Farhi and Werning, 2012).[8] While the wage rigidity is hardwired in my analysis, other articles study ways to overcome it directly. For example, "fiscal devaluations" consider internal devaluation via time-varying taxes (Farhi et al., 2014).

The paper adds to our understanding of the determinants of the euro area crisis. Gilchrist et al. (2016) point to financial frictions as explaining the differential wage developments across the euro core and periphery. Kuvshinov et al. (2016) explore how wage rigidity and the zero lower bound may have paralyzed real exchange rate adjustment following a period of deleveraging in the euro periphery. In my model, real exchange rate "misalignment" results from inefficient wage restraint by wage setting agents.

On the theory side, my paper adds to the growing literature studying the interaction between risk and nominal rigidities. As in Basu and Bundick (2017), wage restraint in my analysis arises due to this interaction, and would be absent in a linearized version of my model. Another contribution in this spirit is Fernández-Villaverde et al. (2015).

A final strand of related literature links wage bargaining centralization and macroeconomic efficiency. One part of the Calmfors and Driffill (1988) hypothesis is that larger labor unions internalize the feedback of their action into the economy which may be welfare improving. Evidence for a positive link between union-size and employment is contained in Blanchard and Wolfers (2000); Daveri and Tabellini (2000). In my model I find that larger-sized unions make wage restraint more efficient, yet that welfare may be reduced due to larger unions' charging larger monopolistic mark-ups.

[8] A resolution is temporary inflation in the nominal anchor country: Schmitt-Grohé and Uribe (2013) and Fahr and Smets (2010) make this case for the euro zone, arguing that price inflation in the euro core may help overcome downward nominal wage rigidity in the euro periphery.

2 Intuition

This section describes a textbook neoclassical labor market that is characterized by downward nominal wage rigidity (henceforth DNWR). Specifically, as in Benigno and Ricci (2011), Schmitt-Grohé and Uribe (2016) and others, I assume that wages can not be lower than a fraction of their level from the previous period. The goal of this analysis is to demonstrate that, with this friction in place, efficient wage dynamics are characterized by what may be called "wage restraint"—the fact that in boom periods, wages should increase by strictly less than if they were completely downward-flexible.

An economy is populated by a representative household who derives utility from consumption and dis-utility from work

$$\mathcal{U}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{c_t - V(h_t)\} \tag{2.1}$$

where V is positive, increasing and convex and where $\beta < 1$. A representative firm produces consumption goods from labor $y_t = a_t F(h_t)$, where F is positive, increasing and concave, and where a_t is the firm's level for productivity (exogenous and stochastic). The firm hires labor according to a labor demand function:[9]

$$a_t F'(h_t) = \frac{w_t}{p_t} \tag{2.2}$$

that is, by equating its marginal product to the real wage which it takes as given. The price level p_t is exogenous and stochastic. In contrast, the nominal wage w_t is endogenous, can adjust freely upwards, but is restricted in its ability to fall. Formally, I assume that

$$w_t \geq \gamma w_{t-1}; \quad \gamma < 1, \tag{2.3}$$

such that the nominal wage may fall by at most $(1-\gamma)$ percent per period. The model is closed by assuming that the household consumes the output produced in each period (market clearing condition)

$$c_t = a_t F(h_t), \tag{2.4}$$

such that there is no savings technology. This assumption is without loss of generality, because the household's consumption utility is linear.

To understand the structure of this economy, consider any nominal wage path $\{w_t\}$ that satisfies DNWR, equation (2.3), at all times. Such a path, given that $\{p_t\}$ is exogenous,

[9] "Labor demand" is the function $h(w_t/p_t)$ implicitly defined in equation (2.2). Given the assumptions imposed on F, labor demand is positive, but strictly falling in w_t/p_t. The *elasticity* of labor demand is the elasticity of $h(w_t/p_t)$ with respect to the the real wage w_t/p_t.

implies a path for hours $\{h_t\}$ from the firm's labor demand (2.2). This in turn implies a path for consumption $\{c_t\}$ from market clearing (2.4), and thereby a certain (expected) life-time utility for the household, $\mathcal{U}_0$, from (2.1).

It follows that an equation is missing, an equation that pins down a *unique* such path $\{w_t\}$. Clearly, the missing equation is a labor supply curve. The labor supply curve that maximizes household utility can be obtained as follows[10]

Definition 1. [EFFICIENT ALLOCATION] *The allocation that maximizes household utility $\mathcal{U}_0$ is the solution to the following problem*

$$\mathcal{W}(w_{t-1}) = \max \{a_t F(h_t) - V(h_t) + \beta E_t \mathcal{W}(w_t)\}$$

subject to (2.2) *and* (2.3)*, for given exogenous* $\{a_t, p_t\}$*.*

Technically, this is a constrained dynamic optimization problem as in Lorenzoni (2008) and Bianchi (2011, 2016).

I solve this problem in the Appendix A, which contains all proofs and derivations. By attaching a multiplier to equation (2.3), the solution is as follows

Proposition 1. [LABOR SUPPLY EFFICIENT ALLOCATION] *The labor supply curve in the efficient allocation is given by the following expression*

$$w_t^r + \gamma E_t \beta \left(\frac{\varepsilon_{t+1}}{\varepsilon_t} \frac{h_{t+1}}{h_t} \frac{w_t^r}{w_{t+1}^r} \frac{p_t}{p_{t+1}} \right) \psi_{t+1} = V'(h_t) + \psi_t, \tag{2.5}$$

where $\psi_t \geq 0$, and where $\varepsilon_t > 0$ denotes the (negative of the) elasticity of labor demand with respect to the real wage $w_t^r \equiv w_t/p_t$ at time t.

The labor supply curve (2.5), along with the labor demand curve (2.2), the complementary slackness condition, $\psi_t(w_t - \gamma w_{t-1}) = 0$, and the constraints $\psi_t \geq 0$ and $w_t \geq \gamma w_{t-1}$, pin down the efficient equilibrium in this labor market.

Wage restraint.—As is well known, if the wage rigidity binds ($\psi_t > 0$), households are being "pushed off" their labor supply curve as hours worked fall below the level which they would like to supply. Yet, labor supply curve (2.5) shows that, in the efficient allocation, the converse holds as well: in periods when the wage rigidity is slack ($\psi_t = 0$), but binding in the next period with a positive probability ($\psi_{t+1} > 0$ in some states of the world), households should work *more* in the current period, as this keeps wages low. In other words, the real

[10] As mentioned earlier, throughout the paper, "efficient" refers to efficiency conditional on DNWR being in place, and therefore reflects "second best". This terminology is in line with New Keynesian analysis of optimal policy. For instance, any New Keynesian account of optimal monetary policy is, in fact, optimality conditional on the assumed price or wage (typically, Calvo-style) rigidity.

wage must fall short of the household's marginal rate of substitution between consumption and leisure.[11] This may be interpreted as "wage restraint". In a labor market diagram, it corresponds to a shift of labor supply to the right.

The purpose of wage restraint is to reduce *variability* in hours, as well as to the raise the average *level* of hours—the first excessively high, the second excessively low in the presence of DNWR. Both increases welfare from equation (2.1): a higher average level of hours translates into a higher average level of output available for consumption. In turn, a lower variability of hours raises consumption even further, as well as reduces the dis-utility of labor supply. This is because hours enter household utility via the two concave functions F and $-V$. It should be stressed that these arguments are completely independent of the traditional motive for consumption smoothing.[12]

There is no wage restraint if wages are not rigid ($\gamma = 0$), or if shocks are sufficiently small for DNWR to be never binding in equilibrium ($\psi_t = 0$ at all times)—the conventional condition for efficiency whereby the real wage equates the marginal rate of substitution. Conversely, wage restraint is more pronounced as the labor market is more rigid and as shock volatility is larger. The interaction between risk and nominal rigidity is key, as in Basu and Bundick (2017) in the context of price rigidities.[13] Moreover, wage restraint depends on the cycle. First, it is stronger as the utility loss associated with a drop in hours, as measured by multiplier ψ_{t+1}, is expected to be larger. A second effect arises through a *stochastic discount factor*: $\beta(\varepsilon_{t+1}/\varepsilon_t)(h_{t+1}/h_t)(w_t^r/w_{t+1}^r)(p_t/p_{t+1}) > 0$.

The discount factor declines in (expected) price inflation. This is a classical "greasing the wheels" effect: price inflation erodes the real wage, such that any rigidity on the nominal wage is effectively dampened (Tobin, 1972). Furthermore, the stochastic discount factor depends on the slope of labor demand in the current versus the next period. To see this, note that $\varepsilon_t h_t/w_t^r$ is just the slope of labor demand at time t. Hence, the discount factor (and therefore the extent of wage restraint) increases in the slope of labor demand in the next period, but falls in the slope of labor demand in the current period.

This channel is important, as it will be the main driver of my results in the open economy model below. The intuition is as follows: if labor demand is steep in the next period (h_{t+1}

[11] The marginal dis-utility of labor supply $V'(h_t)$ corresponds to the households' marginal rate of substitution between consumption and leisure in this model, because utility is linear in consumption. Otherwise, the corresponding condition would be $w_t/p_t \leq V'(h_t)/U'(c_t)$, where $U'(c_t)$ is the marginal utility of consumption (see the open economy model in Section 3 below, which has a curved consumption utility).

[12] This can be seen by recognizing that consumption utility is linear, such that the elasticity of substitution between consuming today and in the future is plus infinity.

[13] The model from this section can be easily solved numerically, by using value function iteration on the expression in Definition 1. Therefore, in the Appendix B, I present a numerical example. The numerical example confirms that wage restraint depends on *shock volatility*: a rise in volatility *per se* implies more pronounced wage restraint.

moves strongly with changes in w^r_{t+1}), a large drop in hours ensues if the wage rigidity binds in the next period. As a result, wage restraint today should become stronger. On the other hand, if labor demand is steep in the *current* period, wage restraint *itself* is highly distortive. This is because, by keeping wages low, working hours are far *above* their frictionless level in the *current* period. Intuitively, a trade-off between "under-employment" in the next versus "over-employment" in the current period arises, which the stochastic discount factor resolves in the most efficient way.

The previous arguments imply that wage restraint moves with the elasticity of labor demand over the cycle. For example, if the elasticity is large precisely when the rigidity binds (which occurs in downturns, that is, if the elasticity is counter-cyclical), wage restraint must become stronger. The opposite holds if the elasticity is pro-cyclical. In the one-sector model studied here, a cyclicality of the labor demand elasticity does not naturally arise. For example, if we specialize to the production function $F(h) = h^\alpha$, the elasticity is $\varepsilon_t = 1/(1-\alpha)$—a constant. In contrast, in a multi-sector economy where labor flows between sectors, the share of labor employed in each sector determines the (aggregate) labor demand elasticity. This will be the case in my open economy model, to which I turn next.

3 The model

This section lays down the open economy model which will be studied in the rest of the paper. I consider an open economy that is small enough to not affect variables in the rest of the world, that has a fixed nominal exchange rate vis-à-vis the rest of the world (a currency peg), and that faces DNWR, following Schmitt-Grohé and Uribe (2016). Three key features distinguish my analysis. First, the country produces both tradables and non-tradables, and labor may flow freely across both sectors. Second, the labor market is imperfectly competitive, implying that wage setting is forward-looking (Elsby, 2009). Third, wage inflation may be driven from the supply side, following technology shocks in the tradable sector, but also from the demand side, following preference shocks.[14] In what follows I describe the economic environment under "laissez-faire". Echoing my analysis of Section 2, below in Section 4 I will contrast this allocation to the allocation where wage dynamics are efficient.

[14] Both channels have been emphasized to have played a role in determining real exchange rates before the euro crisis. The analysis in Berka et al. (2017) is based on the technology shocks, supply narrative. In contrast, Martin and Philippon (2017) provide an example of the demand narrative.

3.1 Households

There is a continuum of identical households, indexed $i \in [0,1]$. Each household maximizes its objective

$$\mathcal{U}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{ Z_t U(A(c_t^T(i), c_t^N(i))) - V(h_t(i)) \}$$

subject to the period budget constraint

$$p_t^T c_t^T(i) + p_t^N c_t^N(i) + E_t s_{t,t+1} d_{t+1}(i) = w_t(i) h_t(i) + \Phi_t^T + \Phi_t^N + d_t(i),$$

and subject to a no-Ponzi constraint.

Here, Z_t is the demand shock and $U = (c^{1-\kappa})/(1-\kappa)$ is consumption felicity with risk aversion coefficient $\kappa > 1$. Consumption is an aggregate $c = A(c^T, c^N) = [\omega (c^T)^{1-1/\zeta} + (1-\omega)(c^N)^{1-1/\zeta}]^{1/(1-1/\zeta)}$ where "T" and "N" distinguish tradable and non-tradable consumption. In turn, $\omega \in (0,1)$ is a consumption share parameter, and $\zeta > 0$ is the elasticity of substitution between the two consumption types. Cross-border state contingent claims are denoted $d_{t+1}(i)$ and have a discount factor $s_{t,t+1}$. Households receive a wage income $w_t(i)h_t(i)$ and firm profits from both sectors, Φ_t^T and Φ_t^N.

I assume that the law of one price holds for tradable goods, such that $p_t^T = \bar{e} p_t^{T,*}$, where $\bar{e}$ is the fixed nominal exchange rate and $p_t^{T,*}$ is the price of tradable goods in the rest of the world. For ease of exposition, I normalize the level of the fixed exchange rate to unity, and I also assume that the price of tradables in the rest of the world is constant and equal to unity. As a result, also $p_t^T = 1$.[15]

First order conditions with respect to state contingent claims link the marginal utility of tradable consumption to its rest-of-the-world counterpart: $Z_t U_{c_t^T}(i) = U^*_{c_t^{T,*}}$—the "Backus Smith condition" (Backus and Smith, 1993).[16] I assume the rest of the world is in steady state throughout, hence I let $U^*_{c_t^{T,*}} = U^*_{c^{T,*}}$ be a constant. Moreover, throughout the paper, I am using the following parametric assumption

Assumption 1. *The intertemporal consumption elasticity $1/\kappa$ and the intratemporal consumption elasticity ζ, coincide: $1/\kappa = \zeta$.*

This assumption, which is known as "Cole-Obstfeld condition" (Cole and Obstfeld, 1991), is useful as it simplifies substantially my algebraic expressions. Most importantly, it allows me

[15] Frictions in the tradable-goods sector such as consumption home bias, would allow for short-run departures of the price of tradable consumption from unity (that is, changes in the terms of trade). However this would not affect that, under a fixed nominal exchange rate, tradable goods' prices in the *long run* are pinned down by the nominal anchor country (Corsetti et al., 2013). Therefore, the mechanisms described in this paper would continue to hold in this extended environment.

[16] In the general case, also the (tradable-goods) real exchange rate would appear in this equation. However, because of my normalizations above, $p_t^{T,*} \bar{e} / p_t^T = 1$.

to derive this economy's labor demand elasticity in closed form, which is useful for interpreting wage restraint (see Section 4.2, equation (4.3)).[17]

As a result of Assumption 1 the Backus-Smith equation can be written as

$$Z_t \omega (c_t^T)^{-1/\zeta} = U^*_{c^{T,*}}. \tag{3.1}$$

such that c_t^T moves one-by-one with the demand shifter Z_t. Note that, in equation (3.1), I have already omitted household index i by anticipating that, in equilibrium, all households will behave identically.

First order conditions with respect to tradable and non-tradable consumption link their marginal utilities to their relative price: $U_{c_t^N}/U_{c_t^T} = p_t^N$, where I have used that $p_t^T = 1$. This can be written as

$$\frac{1-\omega}{\omega} \left(\frac{c_t^N}{c_t^T} \right)^{-1/\zeta} = p_t^N. \tag{3.2}$$

Finally, labor supply is discussed below, as households are organized into labor unions which take the labor supply decision on their behalf.

3.2 Firms

Firms hire labor, produce either tradable or non-tradable consumption goods, and maximize profits. They are price takers in the goods market, but face imperfectly substitutable worker types in the labor market. The worker types are bundled by using a CES-type technology, with elasticity of substitution $\theta > 1$. Hence aggregate labor is $h_t = (\int_0^1 h_t(i)^{(\theta-1)/\theta} di)^{\theta/(\theta-1)}$, implying labor demand curves $h_t(i) = (w_t(i)/w_t)^{-\theta} h_t$ with corresponding wage index $w_t = (\int_0^1 w_t(i)^{1-\theta} di)^{1/(1-\theta)}$. Aggregate labor splits into labor employed by firms operating in the tradable and non-tradable sector

$$h_t = h_t^T + h_t^N. \tag{3.3}$$

Profit maximization by firms in the tradable sector, $\Phi_t^T = \max\{a_t F(h_t^T) - w_t h_t^T\}$, where a_t is the technology shock, implies a labor demand curve

$$a_t F'(h_t^T) = w_t. \tag{3.4}$$

Profits in the non-tradable sector are $\Phi_t^N = \max\{p_t^N F(h_t^N) - w_t h_t^N\}$, implying that

$$p_t^N F'(h_t^N) = w_t. \tag{3.5}$$

Both labor demand curves must always hold with equality, because an excess labor demand can always be healed if wages are unrestricted in their ability to rise. Below I will specialize

[17] I experimented numerically with relaxing Assumption 1, and found that the quantitative difference, under plausible model parameters, is small. In the Appendix A, I present all model equilibrium conditions in the case where Assumption 1 is relaxed.

to the functional form $F(h) = h^{\alpha_j}$, $\alpha_j \in (0,1)$, where I allow parameter α_j to depend on the sector in which labor is employed: $j \in \{T, N\}$.

3.3 Unions

To allow wage formation to be forward looking, I assume that the labor market is imperfectly competitive. Moreover, I consider a rich specification of labor supply, as I allow households to be organized into (an arbitrary number of) labor unions. In terms of formalism, I follow a well-established literature, focusing on the effects of varying degrees of wage bargaining centralization (Calmfors and Driffill (1988); see also Alesina and Perotti (1997), Guzzo and Velasco (1999), Cukierman and Lippi (1999), and Gnocchi (2009)).

There is a total number of $J \geq 1$ unions in the economy. All households are assumed to be organized into one of those unions, and unions are of equal size. Hence union $j \in \{1, ..., J\}$ has its members in the interval $\mathcal{I}(j) \equiv J^{-1}[j-1, j]$. Because households are symmetric, each union j will choose the same wage $w_t(i)$ for all of its members $i \in \mathcal{I}(j)$. We therefore write $w_t(j)$ for the members of union j. Note that, as union size shrinks to zero ($J \to \infty$), this formulation nests the case of monopolistic competition (Benigno and Ricci, 2011). Following the above-cited literature, the number $1/J$ can be interpreted as the economy's degree of wage bargaining centralization.

The problem of the individual union j is summarized in the following Definition.

Definition 2. [UNION PROBLEM] *The problem of individual union $j \in \{1, ..., J\}$ is to solve the following problem*

$$\mathcal{W}(w_{t-1}(j)) = \max \left\{ Z_t U_{c_t^T}(j) w_t(j) h_t(j) - V(h_t(j)) + \beta E_t \mathcal{W}(w_t(j)) \right\}$$

subject to the set of constraints

$$
\begin{aligned}
&i) && w_t(j) \geq \gamma w_{t-1}(j) \quad \text{(downward wage rigidity)}\\
&ii) && h_t(j) = (w_t(j)/w_t)^{-\theta}(h_t^T + h_t^N) \quad \text{(union-type labor demand)}\\
&iii) && w_t = (\textstyle\sum 1/J \; w_t(j)^{1-\theta})^{1/(1-\theta)} \quad \text{(aggregate wage)}\\
&iv) && a_t F'(h_t^T) = w_t, \quad \text{(labor demand tradable sector)}\\
&v) && ((1-\omega)/\omega)(F(h_t^N)/c_t^T)^{-1/\zeta} F'(h_t^N) = w_t \quad \text{(labor demand non-tradable sector)}
\end{aligned}
$$

by respecting Backus-Smith condition $Z_t U_{c_t^T}(j) = U_{c^{T,*}}$, for given wages $\{w_t(-j)\}$ and for the given exogenous $\{Z_t, a_t\}$.

The objective of each union is to maximize the (utility-weighted) wage bill of its members, net of the dis-utility of working, and subject to wages being restricted in their ability to fall.

This is hence where DNWR enters the model: it is assumed to arise at the *household* level, then, in equilibrium, passes through to the aggregate level. Notice also that each union takes the wages of the other unions as given (denoted $w_t(-j)$), such that the equilibrium aggregate wage can be understood as the outcome of a Bertrand game. This includes the wages set by the other unions in future periods, which otherwise the individual union could manipulate by changing the aggregate wage through its own wage setting decision.[18]

To understand the constraints in Definition (2), note that the unions have market power over the wage of their members if the labor types are imperfectly substitutable ($\theta < \infty$). As a result, the unions' problem is dynamic, subject to downward wage rigidity (condition i), and subject to union-type labor demand curves (condition ii). If unions are small ($J \to \infty$), this completes the unions' problem, because their effect on the aggregate wage and hence on the economy more generally is negligible. However, if unions have positive mass ($J < \infty$), they understand their contribution to the aggregate wage (condition iii).[19] The aggregate wage w_t matters for the unions, because an increase in w_t reduces the demand for aggregate labor $h_t = h_t^T + h_t^N$ via conditions iv)-v) and thereby, indirectly, the demand for union-type labor $h_t(j)$, via h_t appearing in condition ii).

Specifically, as the aggregate wage w_t rises, the firms in the tradable sector hire less hours as their marginal product of labor $a_t F'(h_t^T)$ must rise (condition iv). The same occurs in the non-tradable sector (condition v), yet, hours fall by strictly *less* due to a rise in the relative price between tradable and non-tradable consumption p_t^N, equation (3.2).[20] This follows from the Balassa-Samuelson effect. As wages rise, the relative price p_t^N must increase. As a result, the sales price of non-tradables rises faster than the sales price of tradables (in fact, the sales price of tradables does not rise at all due to our earlier assumptions). By implication, hours fall by less in the non-tradable sector than in the tradable sector following a wage rise. This channel will be taken up again in my analysis below.

In equilibrium, all unions behave identically such that index j disappears and only aggregates matter. The following characterizes aggregate labor supply.

Proposition 2. [LABOR SUPPLY] *Assume that each individual union behaves as detailed*

[18] Hence for tractability, I abstract from dynamic strategic interaction (e.g. Maskin and Tirole, 2001). Without this assumption, the labor supply curve (3.6) can not be derived in closed form. Also, the numerical solution procedure would complicate significantly, because finding the Markov-perfect equilibrium of the Bertrand game would involve solving a fixed point problem. Note that in the case of either very large or very small unions, the solution under dynamic strategic interaction coincides with the equilibrium studied here.

[19] The integral $w_t = (\int_0^1 w_t(i)^{1-\theta} di)^{1/(1-\theta)}$ can be written as $w_t = (\sum_{j \in \{1,\dots,J\}} 1/J \; w_t(j)^{1-\theta})^{1/(1-\theta)}$ once we recognize that $w_t(i) = w_t(j)$ for all $i \in \mathcal{I}(j)$, as argued above.

[20] Here it is used that $c_t^N = F(h_t^N)$ in equilibrium, that is, all non-tradable output must be consumed within the same period. See Section 3.4 below, the summary of equilibrium conditions.

in Definition 2. In this case, the aggregate labor supply curve is given by

$$Z_t U_{c_t^T} w_t \left(\frac{\tilde{\varepsilon}_t - 1}{\tilde{\varepsilon}_t}\right) + \gamma E_t \beta \left(\frac{\tilde{\varepsilon}_{t+1}}{\tilde{\varepsilon}_t}\frac{h_{t+1}}{h_t}\frac{w_t}{w_{t+1}}\right)\psi_{t+1} = V'(h_t^T + h_t^N) + \psi_t \tag{3.6}$$

where $\psi_t \geq 0$*, and where* $\tilde{\varepsilon}_t > 0$ *denotes the (negative of the) union-specific elasticity of labor demand with respect to the wage* w_t*, written out in equation* (4.2) *below.*[21]

3.4 Equilibrium definition

For given exogenous $\{Z_t, a_t\}$ and an initial condition $w_{-1} > 0$, an equilibrium is a sequence for endogenous variables $\{c_t^T, c_t^N, h_t^T, h_t^N, h_t, w_t, p_t^N, \psi_t\}$ such that equations (3.1)-(3.6), along with the definition for $\tilde{\varepsilon}_t$ given in equation (4.2) below, market clearing $F(h_t^N) = c_t^N$, the complementary slackness condition $\psi_t(w_t - \gamma w_{t-1})$, and the constraints $\psi_t \geq 0$ and $w_t \geq \gamma w_{t-1}$, are all satisfied.

4 Efficiency

I now turn to the main theoretical results of my analysis. First, I show that, while wage restraint is an integral part of the equilibrium under laissez-faire, this equilibrium is generally not efficient. Second, I show that this implies that wages may rise too strongly, but also by too little, in boom periods under laissez-faire.

4.1 The efficient benchmark

I define the efficient benchmark for the open economy model introduced in the last section, echoing Section 2. This benchmark allows me to assess the inefficiencies which characterize the private allocation. Specifically, the efficient benchmark distills the optimal dynamics for wages, while leaving all other behavioral and technological constraints in the private sector unchanged. For example, the efficient allocation takes as given the consumption/saving choice of the private sector, as captured by condition (3.1).[22]

To derive the efficient allocation, note that, first, the planner chooses the same individual wage for all the unions, $w_t(j) = w_t \forall j \in \{1, ..., J\}$. This is because the unions are symmetric and the bundling technology is of the CES type. This also implies that $h_t(j) = h_t \forall j \in$

[21] The attentive reader may notice that in Section 2, the elasticity ε_t was expressed with respect to the *real* wage w_t/p_t, while in the present context, it appears to be expressed with respect to the *nominal* wage w_t. This is, however, not correct. In the current environment the numeraire is tradable consumption, the price of which was normalized to one. Hence, the elasticity is expressed with respect to w_t/p_t^T, which by setting $p_t^T = 1$ *coincides* with the nominal wage w_t.

[22] This is in contrast to Bianchi (2011), Benigno et al. (2013) and others, who, in their analyses of optimal capital controls, study the efficiency of the consumption/saving behavior of the private sector. In contrast, in my analysis I study the efficiency of the *wage setting* behavior of the private sector.

$\{1, ..., J\}$, that is, the planner also allocates the same amount of union-type labor. Next, the social planner solves the following problem

Definition 3. [EFFICIENT ALLOCATION] *The allocation that maximizes household utility* $\mathcal{U}_0$ *is the solution to the following problem*

$$\mathcal{W}(w_{t-1}) = \max \{Z_t U(A(c_t^T, c_t^N)) - V(h_t) + \beta E_t \mathcal{W}(w_t)\}$$

subject to equations (3.1)-(3.5)*, market clearing* $c_t^N = F(h_t^N)$*, and downward nominal wage rigidity* $w_t \geq \gamma w_{t-1}$*, for given exogenous* $\{Z_t, a_t\}$*.*

While the unions in their maximization problem also respect private-sector equilibrium conditions (in particular, the Backus-Smith condition (3.1) as well as the labor demand curves, recall Definition 2 above), the key difference between the unions and the planner is that the former maximize the utility-weighted wage bill, whereas the latter directly maximizes household utility. I obtain the following condition for efficient labor supply

Proposition 3. [LABOR SUPPLY EFFICIENT ALLOCATION] *The labor supply curve in the efficient allocation is given by the following expression*

$$Z_t U_{c_t^T} w_t \left(\frac{\varepsilon_t^N}{\varepsilon_t} \frac{h_t^N}{h_t^T + h_t^N} \right) + \gamma E_t \beta \left(\frac{\varepsilon_{t+1}}{\varepsilon_t} \frac{h_{t+1}}{h_t} \frac{w_t}{w_{t+1}} \right) \psi_{t+1} = V'(h_t^T + h_t^N) + \psi_t \tag{4.1}$$

where $\psi_t \geq 0$*, and where* $\varepsilon_t > 0$ *(*$\varepsilon_t^N > 0$*) denotes the (negative of the) labor demand elasticity (in the non-tradable sector) with respect to the wage* w_t*, written out in the main text in equation* (4.3) *below.*

The equilibrium definition in the efficient allocation is identical as in Section 3.4, except that labor supply curve (3.6) has to be replaced by labor supply curve (4.1).

4.2 Wage restraint

Start with the observation that the equilibrium under laissez-faire is characterized by wage restraint: a wedge term appears in labor supply curve (3.6), determined by the utility loss expected to be suffered from DNWR in the next period. Therefore, in line with Elsby (2009), the unions restrain wage increases in booms to reduce the potential for unemployment in recessions. However, by inspecting labor supply in the efficient allocation (4.1), we also note that wage restraint under laissez-faire is inefficient.

The difference arises from different stochastic discount factors entering the expected utility loss. Plainly, the unions therefore assess the risk associated with the wage rigidity differently than the social planner. More precisely, different labor demand elasticities enter the stochastic discount factors: $\tilde{\varepsilon}_t \neq \varepsilon_t$ in general.

Recall the role of the elasticity of (aggregate) labor demand ε_t in shaping wage restraint in the efficient allocation, the discussion in Section 2. Instead, $\tilde{\varepsilon}_t$ corresponds to the elasticity of the labor demand curve that is faced by the *individual union*. To understand this difference, it helps to write out the elasticity $\tilde{\varepsilon}_t$ as

$$\tilde{\varepsilon}_t = \left(1 - \frac{1}{J}\right)\theta + \frac{1}{J}\varepsilon_t. \tag{4.2}$$

It therefore corresponds to a *weighted average* between the elasticity of substitution between the different labor types θ, and the elasticity of aggregate labor demand, ε_t, with the weight determined by the size of the unions $1/J$. Intuitively, as union grow small, $1/J \to 0$, $\tilde{\varepsilon}_t \to \theta$, reflecting monopolistic competition in the labor market. In contrast, as unions grow large $1/J \to 1$, $\tilde{\varepsilon}_t \to \varepsilon_t$, reflecting monopolistic labor supply.

The mechanism underlying this inefficiency is not new. It corresponds to an externality due to strategic complementarity (e.g., Woodford, 2003), here due to strategic complementarity in wages. The wages set by the unions are strategic complements, because by lowering the wage of its members thereby lowering the wage index, the individual union creates an incentive for the other unions to also lower the wage of their members—because their members have now become relatively more expensive. This effect is not internalized by the individual union. In contrast, were the unions to coordinate on this complementarity, their collective wage choice would be the same as under the efficient allocation.[23]

To characterize this inefficiency further, we compute explicitly the (aggregate) labor demand elasticity ε_t as follows

$$\varepsilon_t = \left(1 - \frac{h_t^N}{h_t^T + h_t^N}\right)\varepsilon_t^T + \frac{h_t^N}{h_t^T + h_t^N}\ \varepsilon_t^N \tag{4.3}$$

where $\varepsilon_t^T = 1/(1-\alpha_T)$ is the labor demand elasticity in the tradable, $\varepsilon_t^N = 1/(1-\alpha_N+\alpha_N/\zeta)$ in the non-tradable sector. Hence, the aggregate labor demand elasticity is a *weighted average* between the elasticity of labor demand in the two sectors, where the weight is determined by the share of labor that is employed in the respective sector.[24]

It follows that ε_t is time-varying, to the extent that shocks shift labor across the two sectors over the business cycle. Moreover, it follows that the individual labor demand elasticity $\tilde{\varepsilon}_t$ reflects this cyclicality only to a limited extent, and not at all if unions are small (as in this

[23] In less technical terms, the individual union perceives the effect on the aggregate wage of its compressing a wage increase individually, to be small. It ignores that, by compressing a wage increase individually, it creates an incentive for the other unions to also compress a wage increase. Therefore, in equilibrium, its effect on the aggregate wage may in fact be large.

[24] Notice that the labor demand elasticities in both sectors are constants, such that ε_t is time-varying only because, following shocks, labor is being shifted across sectors. This is a consequence of Assumption 1. Without this assumption, ε_t^N would be time-varying, too—see the Appendix A.

case $\tilde{\varepsilon}_t = \theta$, see above). It remains to be determined whether this implies that wage restraint under laissez-faire is generally too weak, too strong, or whether this depends on the kind of shocks which hit the economy.

To answer this question, first note that $\varepsilon^T > \varepsilon^N$ unless the degree of substitutability between tradables and non-tradables ζ is unreasonably large.[25] In words, labor demand is *steeper* in the tradable than in the non-tradable sector. The intuition was given in Section 3.3, however is repeated here for convenience. Following a wage increase, the relative price of non-tradables vis-à-vis tradables rises by virtue of the Balassa-Samuelson effect—the strength of the relative price rise measured by ζ (the lower ζ, the more pronounced the relative price effect). As a result, the real wage (the nominal wage w_t divided by the goods' sales price in the respective sector) rises by more for firms in the tradable sector, leading them to curb employment to a stronger extent.

Therefore, expansionary shocks that push labor into the tradable sector make ε_t pro-cyclical. This happens for technology shocks a_t in this sector, as labor is drawn to this sector which becomes relatively more productive. From the discussion in Section 2, we recall that a pro-cyclical labor demand elasticity requires wage restraint to become weaker. Because the labor demand elasticity $\tilde{\varepsilon}_t$ reflects this pro-cyclicality only to a limited extent, wage restraint under laissez-faire, following these shocks, is *too strong*.

In sum, following technology (supply) shocks, wage restraint is too strong under laissez-faire, implying that wage increases are excessively small. This qualifies results in Schmitt-Grohé and Uribe (2016), according to which DNWR in currency pegs implies that, necessarily, wage increases are excessively *large*. Nonetheless, under demand shocks Z_t, my results conform with the findings in their paper: as demand shocks draw labor to the non-tradable sector, wage increases under laissez-faire are excessively large.

4.3 Other sources of inefficiency

By comparing labor supply curves (3.6) and (4.1), we note two other differences (sources of inefficiency) that are briefly discussed next. These inefficiencies are subordinate to my analysis of wage restraint, because they would arise even if wages were not downward rigid. Precisely, note that the (utility-adjusted) wage both under laissez-faire and in the efficient allocation is multiplied by a mark-up, $\mathcal{M}_t \equiv (\tilde{\varepsilon}_t - 1)/\tilde{\varepsilon}_t < 1$ under laissez-faire, $\mathcal{M}^e_t \equiv (\varepsilon^N_t/\varepsilon_t)(h^N_t/h_t) < 1$

[25] Empirically, elasticity ζ is found to be below unity, such that tradables and non-tradables are complementary (see the cross-country estimates in Stockman and Tesar (1995); see Schmitt-Grohé and Uribe (2016) and Bianchi (2011) and the references therein). While the relative size of the elasticities ε^T and ε^N also depends on the labor share parameters α_T and α_N in the two sectors, even assuming extreme values such as $\alpha_T = 0.5$ and $\alpha_N = 1$ would put an upper bound $\zeta = 2$ above which the inequality $\varepsilon^T > \varepsilon^N$ would no longer hold—clearly above the empirically plausible range for this parameter.

in the efficient allocation.[26]

The mark-up $\mathcal{M}_t$ is well understood: it corresponds to a monopolistic wage mark-up as a result of the unions' exercising their market power. This distorts the equilibrium as wages are pushed above the households' marginal rate of substitution between consumption and leisure. Through the elasticity $\tilde{\varepsilon}_t$, the size of the mark-up depends on union size, which I discuss more thoroughly in Section 5 below.

In contrast, by $\mathcal{M}_t^e < 1$, a wage mark-up also characterizes the efficient allocation. Therefore, interestingly, (some) monopolistic mark-ups may actually make the laissez-faire equilibrium more efficient. The reason for this is as follows. The planner internalizes that, by virtue of complete asset markets, labor can be reduced in the tradable sector without insofar reducing tradable consumption—the latter being pinned down in Backus-Smith condition (3.1). As a result, the planner tolerates a wage somewhat above the households' marginal rate of substitution, as this reduces working hours in the tradable sector thereby reducing the dis-utility of work, while only by little reducing consumption (namely, only non-tradable consumption falls).[27] The unions do not internalize this effect as they maximize the households' wage bill, rather than the households' utility from consumption.[28]

5 Decentralization

Wage dynamics under laissez-faire are inefficient. Before applying this insight to wage developments in euro area countries, in this section I briefly discuss (policy) options for decentralizing the efficient allocation; discussing, in turn, capital controls, (time-varying) income subsidies, and the industrial organization of the labor market.

5.1 Capital controls

One prominent way to address externalities in open economies is by using capital controls. In fact, they are the prime tool for macro prudential policy in the context of credit frictions (e.g. Benigno et al., 2013; Bianchi, 2011; Mendoza, 2002). Moreover, in Schmitt-Grohé and

[26] The fact that $\mathcal{M}_t^e < 1$ follows both from $\varepsilon_t^N < \varepsilon_t$, recall the discussion in the previous subsection, and from the fact that necessarily $h_t^N < h_t$ because of $h_t^T > 0$.

[27] The well known terms-of-trade externality (Corsetti and Pesenti, 2001; De Paoli, 2009) relies on a similar effect. By reducing hours, a small open economy can appreciate its real exchange rate, thereby maintaining a high level of consumption via imports despite the drop in domestic production.

[28] Notice that, in addition to their effect on the level for wages, the two terms $\mathcal{M}_t$ and $\mathcal{M}_t^e$, because they are time-varying, also impact the *cyclicality* of wages. This is in addition to the effect on the cyclicality of wages induced by wage restraint. As it turns out, taking this into account only re-enforces my conclusions. For example, a technology shock, by increasing ε_t, moves $\mathcal{M}_t$ closer to unity such that wage inflation is reduced in the laissez-faire allocation. At the same time, this shock pushes $\mathcal{M}_t^e$ farther below unity, such that wage inflation is increased in the efficient allocation.

Uribe (2016) capital controls are also shown to be effective in the context of DNWR, which is not a credit but a nominal friction.

One benefit of my analysis is to explicitly describe the efficient allocation conditional on DNWR. As a result, the source of inefficiency which stems from DNWR is directly revealed, compare labor supply curves (3.6) and (4.1). This shows that, in my context, capital controls cannot be used to decentralize the efficient allocation, even though both allocations have different implications for the current account.[29]

For this reason, and because the benefits of capital controls are well understood, here I abstract from an in-depth analysis of capital controls.[30] Rather, in what follows I ask about ways to address the inefficiency *directly in the labor market.*

5.2 Time-varying income subsidies

One way to achieve decentralization is by using time-varying subsidies. For example, imagine that the households' wage bill is subsidized by τ_t (paid for by lump-sum taxes). In this case, the labor supply curve (3.6) needs to be replaced by

$$Z_t U_{c_t^T} w_t (1+\tau_t)\mathcal{M}_t + \gamma E_t \beta \left(\frac{\tilde{\varepsilon}_{t+1}}{\tilde{\varepsilon}_t} \frac{h_{t+1}}{h_t} \frac{w_t}{w_{t+1}} \right) \psi_{t+1} = V'(h_t^T + h_t^N) + \psi_t, \tag{3.6'}$$

while all other equations remain unaffected by the policy intervention. It follows that, in a period where the wage rigidity is slack ($\psi_t = 0$), a labor income subsidy

$$(1+\tau_t) = \frac{1}{\mathcal{M}_t} \left(\left[Z_t U_{c_t^T} w_t \right]^{-1} (\mathcal{W}_t^e - \mathcal{W}_t) + \mathcal{M}_t^e \right) \tag{5.4}$$

decentralizes the efficient allocation. For better readability, here I have written short for wage restraint $\mathcal{W}_t \equiv \gamma E_t \beta (\tilde{\varepsilon}_{t+1}/\varepsilon_t)(h_{t+1}/h_t)(w_t/w_{t+1})\psi_{t+1} \geq 0$ in the decentralized, $\mathcal{W}_t^e \equiv \gamma E_t \beta (\varepsilon_{t+1}/\varepsilon_t)(h_{t+1}/h_t)(w_t/w_{t+1})\psi_{t+1} \geq 0$ in the efficient allocation.

If the wage rigidity is not expected to be binding in the next period, the income subsidy corrects for the monopolistic mark-up, net of the mark-up effect in the efficient allocation (recall Section 4.3), such that $(1+\tau_t) = \mathcal{M}_t^e/\mathcal{M}_t$. Instead, if wage restraint is stronger in the efficient than in the decentralized allocation $\mathcal{W}_t^e - \mathcal{W}_t > 0$ (following demand shocks, recall Section 4.2), the subsidy needs to be raised even further, as this raises the households'

[29] By implementing optimal capital controls, the resulting allocation could only be "third best", whereas the efficient allocation described here is "second best". Recall that "first best" is the allocation without downward wage rigidity in the first place.

[30] There is another, more applied, reason for which I do not analyze capital controls any further. In the context of the euro area crisis, proposals to introduce capital controls in the euro zone have been consistently met with the deepest skepticism, on the grounds that restricting the mobility of international capital would undermine the single monetary system and therefore the unity of the euro zone. Even the case of temporary capital controls has been fiercely debated, and is not clear to be compatible with EU law (see for example the debate on capital controls in Cyprus, Markets Insight, 2013).

after-tax income without a corresponding increase in the wage. Conversely, following supply shocks the subsidy would need to be reduced.

The downside of using this scheme is that the income subsidy would need to be time varying. In particular, while the subsidy can be easily used to make the steady state efficient, the whole idea of wage restraint is to curb variation over the cycle. It follows that implementing wage restraint via such subsidies would require them to be equally varying with the cycle. Therefore, even though such subsidies are a good policy tool in theory, in practice they may be difficult to actually implement.[31]

5.3 The industrial organization of the labor market

As Section 4 has made clear, the inefficiency characterizing wage restraint is closely linked with union size $1/J$, or better, the degree of "bargaining centralization" (Calmfors and Driffill, 1988). Therefore, one way to decentralize the efficient allocation (or at least, to come close to decentralization) may be to assess the industrial organization of the labor market.

In this respect, my analysis is closely linked with the literature that studies the benefits of increased bargaining centralization, which traces back to Calmfors and Driffill (1988). This literature argues that countries may benefit from having more centralized labor markets—despite larger unions having more market power—because larger unions may better internalize the effects of their actions and hence moderate their wage claims in the economy. Empirical evidence for this mechanism is discussed in the related literature section.

In my analysis, these considerations gain new relevance in a dynamic stochastic general equilibrium context of DNWR and wage restraint. I have shown earlier that $\tilde{\varepsilon}_t \to \varepsilon_t$ as $1/J \to 1$, such that wage restraint becomes more efficient indeed as unions grow larger-sized. Intuitively, this benefit arises because larger unions internalize the externality arising from strategic complementarity.

From this does not follow, however, that more centralization (necessarily) raises *household welfare*. Namely, against this stands a negative effect which stems from larger-sized unions charging larger monopolistic mark-ups. This channel is well understood. As highlighted in Guzzo and Velasco (1999), the effective elasticity between labor types decreases as the number of unions falls due to reduced labor market competition, prompting the unions to exercise their market power to a greater extent. In the open economy context discussed here, the mark-up channel is particularly powerful, because the aggregate labor demand elasticity ε_t

[31] Another issue is the following: if time-varying subsidies are available, they could be used to eliminate DNWR outright. Indeed, while this would require an even higher degree of cyclicality, by subsidizing labor heavily in recessions, the effects of downward wage rigidity can be eliminated entirely (see Schmitt-Grohé and Uribe (2016), but also the fiscal devaluations literature more generally: Farhi et al. (2014)).

is particularly small. To see this, recall that ε_t is determined in equation (4.3). If the share of employment in the tradable sector is 25 percent, if the labor share α_j in both sectors is two thirds, and if the elasticity between tradables and non-tradables is $\zeta = 0.7$ (see Section 6 below for a discussion of these parameters), then equation (4.3) reveals that $\varepsilon \approx 1.23$. It follows that $(\tilde{\varepsilon} - 1)/\tilde{\varepsilon} \approx 0.187$ as union size $1/J$ approaches one. But this corresponds to a wage mark-up of more than 400 percent. Clearly, a monopolistic distortion of this size dominates any dynamic gains from more efficient wage restraint.

My analysis thus suggests that very high degrees of centralization are likely to be detrimental for welfare, whereas an intermediate degree (or a very low degree, if the mark-up effect always dominates the wage-restraint effect) may be best for welfare. This is in contrast to the original finding in Calmfors and Driffill (1988), who have argued that very large unions can be best for welfare. However, it is in line with later contributions in this literature, highlighting that an intermediate degree of centralization may be welfare optimal (e.g., Guzzo and Velasco, 1999). In the Appendix B, I verify this result numerically. I show that this model produces a U-shaped wage centralization curve, such that an intermediate degree of centralization is best for welfare. However, recall that even with this best possible degree of centralization, this allocation remains strictly inferior to the efficient benchmark.

6 Quantitative results

I now use the model to assess the efficiency of wage developments of euro area countries before the euro crisis. For the empirical part, I use country-level data on tradable-sector TFP and real exchange rates from 1995-2007, assembled in Berka et al. (2017).

6.1 Numerical algorithm, parameters, and policy functions

I first discuss the model's numerical solution algorithm and its accuracy, and how I fix some of the model's parameters. I also show policy functions for endogenous variables against the two shocks to provide intuition on shock identification, as well as to corroborate the theoretical discussion from Section 4.

Solution algorithm.—The model features an occasionally binding constraint, making numerical analysis challenging. Moreover, unlike in most models of occasionally binding constraints, the multiplier associated with the constraint appears explicitly in a conditional expectation—see equations (3.6) and (4.1). As a result, conventional methods of dealing with occasionally binding constraints, such as Guerrieri and Iacoviello (2015), fail in this environment. Instead, I develop a fixed point iteration (FPI) algorithm which solves the model fully non-linearly. The algorithm relies on iteration directly over the conditional expectations.

More details on the algorithm are in the Appendix C.

To assess the accuracy of the FPI algorithm, I solve the efficient allocation by using a standard value function iteration (VFI) approach, based on an extensive grid such that the outcome can be considered very precise. The value function approach is feasible, because the efficient allocation has a recursive representation (recall Definition 3). I then compare the resulting policy functions to those obtained from using FPI. The result is shown in the Appendix B—the policy functions are close to identical. This is reassuring regarding the accuracy of the solution of the equilibrium under laissez-faire, for which a recursive representation (and hence a VFI approach) is not available.

Parameters.—Below I will apply the model to a set of euro area countries. In this analysis, I will use the same behavioral and technology parameters for each country, while allowing these countries to face a different stochastic structure. I am making this assumption because most of the model's parameters are standard, do not differ much across countries, and can be easily deduced from other studies. The same strategy is pursued in Berka et al. (2017) in their DSGE model which they calibrate to countries in the euro zone.[32]

The data I employ is annual, hence I set $\beta = 0.96$ for the time-discount factor. I specialize to the conventional functional form $V(h) = h^{1+\varphi}/(1+\varphi)$ for the dis-utility of labor supply, and I set the standard value $\varphi = 2$ for the (inverse) Frisch elasticity. This value is also used in Eggertsson et al. (2014) in their open economy business cycle model which they calibrate to the euro zone. I set the marginal utility of tradable consumption abroad $U^*_{c^{T,*}} = 0.82$ such that, in steady state, the current-account-to-GDP ratio is zero, $1 - c^T_{ss}/y^T_{ss} = 0$. This implies that ω measures the share of tradable in total consumption in steady state. I thus set $\omega = 0.25$, as well as the intratemporal consumption elasticity $\zeta = 0.7$ such that the consumption types are complements, both in line with the calibration for euro zone countries in Berka et al. (2017). Note that $\zeta = 0.7$ implies a risk aversion parameter $\kappa = 1/0.7 \approx 1.43$ from Assumption 1. It is thus within the ballpark of risk aversion parameters used in open economy business cycle models (Bianchi (2011) uses a value of 2). I follow Schmitt-Grohé and Uribe (2016) and set the labor share in the tradable and the non-tradable sector $\alpha_T = 0.5$ versus $\alpha_N = 0.75$, such that the labor share in the latter is slightly higher. Concerning DNWR, I set $\gamma = 0.96$ such that wages can fall by at most one percent per quarter, again following estimates in Schmitt-Grohé and Uribe (2016) for euro zone countries. I set $\theta = 5$ for the elasticity between labor types, in line with the calibration to the euro zone in Galí (2011) and Galí and Monacelli (2016). I also set $J = \infty$, such that the unions operate under monopolistic competition. This is a relevant benchmark, as this case has been explored in

[32] At the same time, this eliminates a degree of freedom by which different parameter choices across countries could be used to explain their different wage experiences.

β	γ	ζ	κ	ω	φ	$U^*_{c^T,*}$	α_T	α_N	θ	J
0.96	0.96	0.7	1/0.7	0.25	2	0.82	0.5	0.75	5	∞

Table 1: Parameters. Details on the shock processes are provided in the text.

earlier studies (see for example Benigno and Ricci (2011)). Furthermore, a more appropriate value for $1/J$ is hard to determine, because the degree of bargaining centralization lacks a direct empirical counterpart.[33] Larger values for bargaining centralization $1/J$ are discussed in the Appendix B. The parameters used are summarized in Table 1.

Policy functions.—I model the two shocks Z_t and a_t as a first-order bivariate autoregressive process in logs, as in Bianchi (2011)

$$\log([a_t, Z_t]') = \boldsymbol{\rho} \times \log([a_{t-1}, Z_{t-1}]') + u_t, \tag{6.1}$$

where $u_t \sim \mathcal{N}([0,0]', \boldsymbol{\Sigma})$, with $\boldsymbol{\rho}$ and $\boldsymbol{\Sigma}$ conformable matrices. Both $\boldsymbol{\rho}$ and $\boldsymbol{\Sigma}$ will be estimated on a by-country basis below. To illustrate the effects of the shocks on endogenous variables, and to corroborate the discussion from Section 4, here I assume that both shocks are independent with an autocorrelation of 0.95, and have innovations with variance 0.001.[34] Figure 1 shows the policy functions for the real exchange rate, a CPI-based measure in the left panels, and since I focus on wage dynamics, a unit-labor-cost-based measure in the right panels. I plot both against the technology (supply) shock a_t in the upper two panels, the demand shock Z_t in the lower two panels.

Both figures distinguish the equilibrium under laissez-faire (dashed-dotted in black) and the efficient allocation (solid in blue). They also highlight the region where DNWR binds under laissez-faire (the shaded area in gray). As expected, a positive shock to either demand or to technology in the tradable sector appreciates (for both measures) the real exchange rate. This is because both shocks increase the domestic nominal wages (not shown). Interestingly, both shocks have quite different implications when negative: whereas the real exchange rate (ULC-based) depreciates following negative demand shocks, it actually *appreciates* following

[33] For example, trade union density by country as provided by the OECD is a related concept, however does not strictly map into $1/J$. Trade union density measures the ratio of wage and salary earners that are trade union members, divided by the total number of wage and salary earners.

[34] Hence the shock process used in Figure 1 is

$$\boldsymbol{\rho} = \begin{pmatrix} 0.95 & 0 \\ 0 & 0.95 \end{pmatrix}, \qquad \boldsymbol{\Sigma} = \begin{pmatrix} 0.001 & 0 \\ 0 & 0.001 \end{pmatrix}.$$

To implement this process numerically, I use the discretization routine developed in Gospodinov and Lkhagvasuren (2014), who extend the method of Rouwenhorst (1995) to vector auto regressions. Their method is superior to the more common Tauchen-algorithm, if the shock process has a very high persistence. This happens to be the case in my empirical analysis, where for some countries the shocks are estimated to be close to unit root. More details on the numerical implementation are in the Appendix C.

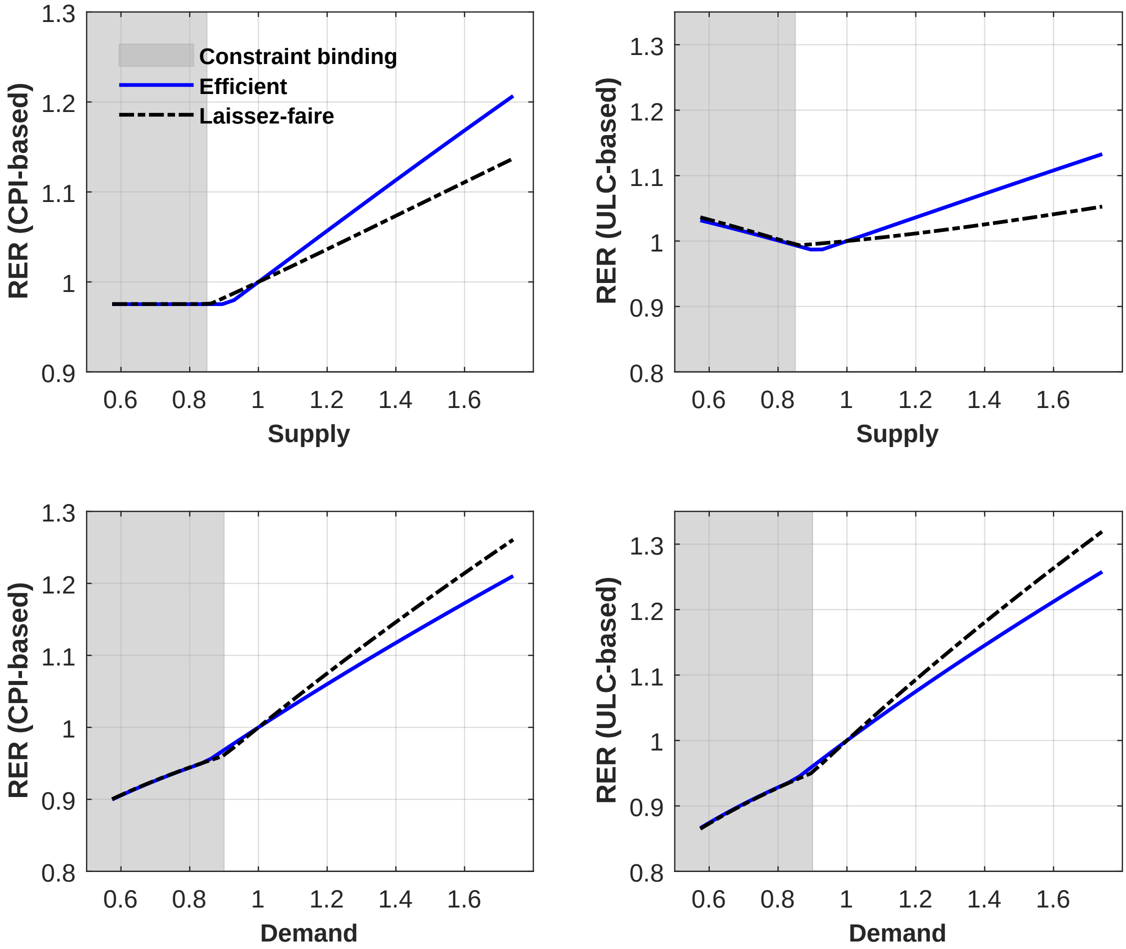

Figure 1: Policy function for the real exchange rate (RER). The CPI-based measure is defined as cpi_t/cpi_{ss}, where the consumer price index is $cpi_t \equiv (\omega + (1-\omega)(p_t^N)^{1-\zeta})^{1/(1-\zeta)}$. The ULC-based measure is $(w_t h_t/rgdp_t)/(w_{ss}h_{ss}/rgdp_{ss})$, where real GDP is $rgdp_t \equiv (y_t^T + p_t^N y_t^N)/cpi_t$. The subscript "ss" indicates the steady state of a variable. The respective other shock and the endogenous state variable w_{t-1} are set to their steady state values.

negative technology shocks when DNWR is binding. This is because in this case, real GDP declines, but not so the domestic nominal wages.

The most noteworthy result arises, however, once we compare the responses under laissez-faire and the efficient allocation. Along the lines of the discussion in Section 4, the slope of the real exchange rate is too small under laissez-faire following technology (supply) shocks, but indeed too large following demand shocks.

6.2 Euro area wage dynamics

In their paper, Berka et al. (2017) collect tradable-sector TFP and real exchange rate data for euro area countries from 1995-2007. As explained there, these data can be interpreted in the time series, but also in the *cross section*. This is because all variables are expressed relative to an EU15 average. As a result, these data are already de-trended and therefore can be used directly in my analysis.

In my analysis, I use these data to assess the efficiency of wage dynamics of euro countries in the 1995-2007 period.[35] The countries are Belgium (BE), Germany (GER), Spain (SPA), France (FRA), Ireland (IRE), Italy (ITA), Netherlands (NET), Austria (AUS), and Finland (FIN). I follow a two-step procedure. First, for each euro country, for the given tradable-sector TFP data $\{a_t\}$, and for the given set of fixed parameters discussed above, I estimate the series of demand shocks $\{Z_t\}$ so as to match the observed data series $\{rer_t\}$, where rer_t denotes the (CPI-based) domestic real exchange rate (as in Figure 1 above). Second, I simulate the efficient allocation on $\{a_t, Z_t\}$ to obtain counter factual efficient estimates of the respective euro area country's real exchange rate.

To obtain the series $\{Z_t\}$ from the series of observables $\{rer_t, a_t\}$, in the general case one has to use a non-linear filter. This is because of unobserved states, here the lagged level for wages w_{t-1}, which matter for the evolution of endogenous variables. I sidestep this issue by making the following assumption. I assume that, in the initial period of the sample, the wage rigidity has not been binding in any country. Economically, I am therefore assuming that initially, none of the countries has experienced (severe) downward pressure on nominal wages vis-à-vis the newly-created nominal anchor. This assumption appears justified, given that at the inception of the euro, exchange rates have been set to minimize the countries' distance to purchasing power parity (e.g., Berka et al., 2017).[36]

By denoting the solution for the real exchange rate $rer(w_{t-1}, a_t, Z_t)$, this implies that $rer(a_0, Z_0)$ is not a function of w_{-1}—because under occasionally binding constraints, the lagged level for w_{t-1} matters only in periods where the constraint is binding. Next, because a_0 is observed, this function can be inverted from rer_0 into Z_0, because of the monotonicity of the real exchange rate in demand shocks (see Figure 1). Finally, I use the policy function for nominal wages $w(a_0, Z_0)$ to obtain the lagged level for wages w_0 in the first period, and so forth in the next periods, such that in the rest of the sample the unobserved state w_{t-1} is

[35] Extending the sample to the crisis period (beyond 2007) is not feasible, because the tradable-sector TFP data is (currently) not available beyond 2007. Including more euro countries, such as Greece or Portugal, is not feasible because the tradable-sector TFP data is not available for those countries.

[36] While the euro has been incepted not before 1999, these countries have been part of the European Exchange Rate Mechanism (ERM) in between 1995-1999. While not exactly fixed, intra-euro nominal exchange rate movements during this period have been small (Berka et al., 2017).

	$\boldsymbol{\rho}$				$\boldsymbol{\Sigma}$			
	ρ_{11}	ρ_{12}	ρ_{21}	ρ_{22}	σ_{11}^2	σ_{12}^2	σ_{21}^2	σ_{22}^2
BE	0.8853	-0.4106	0.0450	0.4175	0.00023	0.00002	0.00002	0.00139
GER	0.6829	-0.7754	0.0065	0.7355	0.00028	-0.00035	-0.00035	0.00173
SPA	1.0020	-0.0912	0.0526	0.9703	0.00039	-0.00026	-0.00026	0.00227
FRA	0.6678	0.0785	-0.0089	0.7111	0.00041	0.00004	0.00004	0.00121
IRE	1.0084	0.2860	-0.0515	0.7775	0.00219	0.00017	0.00017	0.00557
ITA	-	-	-	-	-	-	-	-
NET	0.9527	-0.6293	0.0020	0.3600	0.00048	-0.00026	-0.00026	0.00125
AUS	0.7110	0.4548	-0.1413	0.9946	0.00026	0.00013	0.00013	0.00112
FIN	-	-	-	-	-	-	-	-

Table 2: Estimated shock processes for euro area countries. No estimates are available for Italy and Finland, see Figure 2 and the explanations in the text.

in fact no longer unobserved.

By using this procedure, I define a mapping from the solution of the model into a sequence of shocks $\{a_t, Z_t\}$. Furthermore, because of the assumed autoregressive structure of the shocks described earlier, I can use standard econometric methods to obtain the two matrices $\boldsymbol{\rho}$ and $\boldsymbol{\Sigma}$ characterizing the shock processes. However, there is one last complication. Because the model is highly non-linear and wage setting is forward looking, the solution of the model *itself* depends on the underlying structure of the shocks $\boldsymbol{\rho}$ and $\boldsymbol{\Sigma}$. If we denote the solution of the model $\boldsymbol{\Omega}$, we therefore end up with a fixed point problem

$$\boldsymbol{\Omega} : (\boldsymbol{\rho}, \boldsymbol{\Sigma}) \mapsto (\boldsymbol{\rho}, \boldsymbol{\Sigma}). \tag{6.2}$$

This fixed point has to be obtained numerically. Further details on how I obtain this fixed point are provided in the Appendix C.

Shock series.—The estimated shock series $\{a_t, Z_t\}$ are shown in Figure 2, and summarized in Table 2. Recall that the supply shocks (blue solid) correspond to tradable-sector TFP data, whereas the demand shocks (dashed-dotted in black) are estimated to match the respective country's real exchange rate. It turns out that no estimates for $\{Z_t\}$ can be obtained for Italy and Finland, because the process $\{a_t\}$ for these countries is estimated to have an explosive eigenvalue (this can be understood from inspecting $\{a_t\}$ for these countries in Figure 2).[37]

[37] Most likely, this is due to the sample being comparatively short. As mentioned above, the data is already de-trended such that deviations of variables from their mean are expected to die out in a long enough sample.

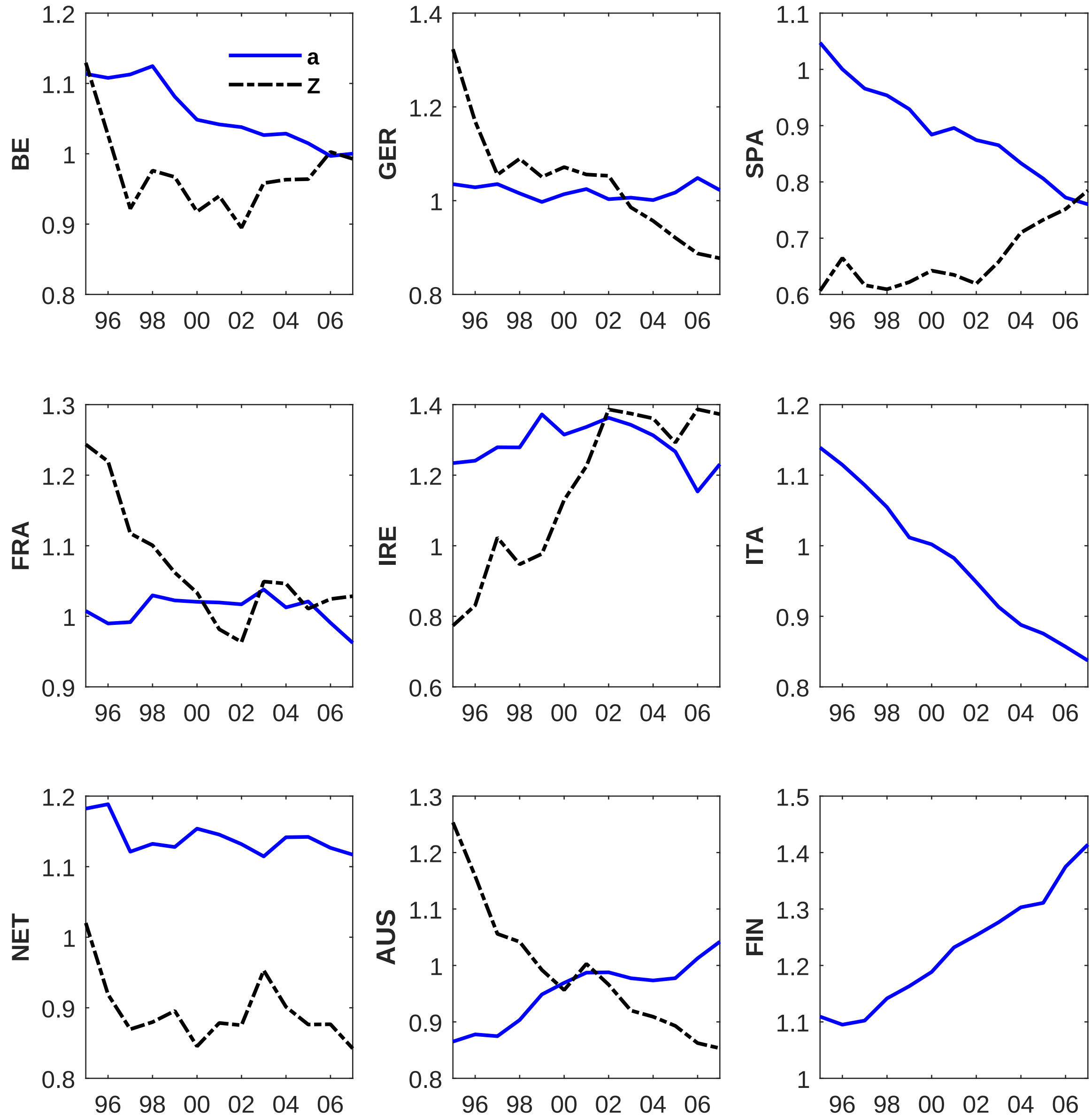

Figure 2: Shock processes for euro area countries. Technology shocks $\{a_t\}$ are used as model input, demand shocks $\{Z_t\}$ are estimated. Years on the horizontal axis, from 1995-2007. The shocks are expressed relative to an EU15 average (for example, a value of 1.2 indicates 20 percent above the EU15 average). No estimates for $\{Z_t\}$ are available for Italy and Finland, as $\{a_t\}$ for these countries is estimated to be explosive (see the figure).

Spain, in turn, is a knife-edge case as its shocks are very close to unit root. Overall, the estimated shocks are large and persistent, and often times negatively correlated.

Efficient dynamics.—Turn now to the main result, the evolution of real exchange rates of euro area countries in the efficient allocation. In Figure 3, black dashed lines correspond to the real exchange rate data rer_t (used as model input), whereas blue solid lines are the counter factual. I discuss each country in turn.

First, Belgium (BE) had a real exchange rate that was too *weak* throughout the entire sample. This effect was particularly pronounced around the time where the euro was incepted, and died out slightly before the start of the crisis. Recall that, in this model, the difference between the efficient allocation and laissez-faire arises because of inefficient wage restraint. Therefore, wage restraint was too strong in Belgium, reflecting the dominance of supply over demand shocks throughout the sample, recall Figure 2.

For Germany (GER) a nuanced picture emerges. Initially, Germany had a very strong real exchange rate relative to the rest of the euro area. In turn, the well-known fact that German wages increased slower than in other euro countries before the crisis, is reflected by the fact that its real exchange rate constantly depreciates until 2007—albeit starting from a high level. The counter factual delivers two insights. First, the German real exchange rate was *too strong* at the beginning of the sample. Second, there was too much wage restraint in Germany until 2007 (which confirms conventional wisdom), reflected by the fact that the real exchange rate in the efficient allocation depreciates *more slowly* than happened in actuality. Yet, because of the initial level effect, the cumulative impact of too-strong wage restraint in Germany until 2007, is comparably small.

The stark real appreciation in Spain (SPA) after 2002 is only partly detected as inefficient, because the Spanish real exchange rate was too weak from the beginning of the sample until about 2004, reflecting a relatively high tradable-sector technology (see Figure 2). However, after 2004 the result flips, because the real appreciation is not matched by further technological advances. As a result, in the efficient allocation the real exchange rate hardly appreciates after 2004. It follows that, in the years before the crisis, wage restraint in Spain should indeed have been *stronger*.[38] However, as a mirror image to Germany, the effect is weaker than expected because of the Spanish real exchange rate having initially been too weak.

A similar pattern emerges for Ireland (IRE), whereas in the case of France (FRA) the real exchange rate movements are found to be close-to efficient. Another clear case of too-strong wage restraint is found in the Netherlands (NET): throughout the sample, the Dutch real exchange rate is found to be too weak. Finally, the behavior of Austria (AUS) matches very

[38] Unfortunately, the sample ends in 2007, such that it remains unclear whether this trend continues until the height of the euro zone crisis.

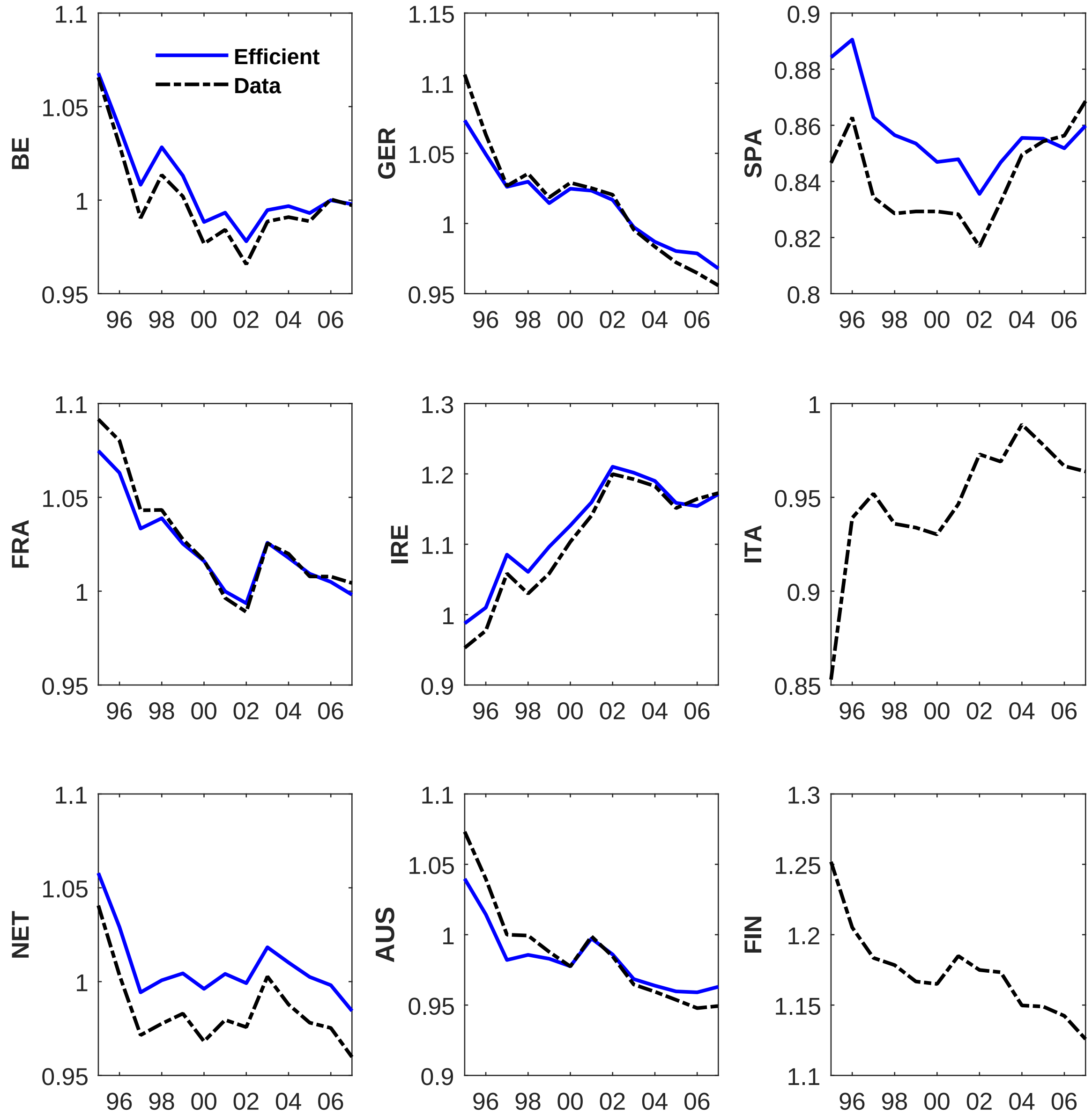

Figure 3: Real exchange rates (CPI-based)—data and counter factual. Real exchange rates are in levels relative to an EU15 average. For example, a value of 1.2 indicates that the country is 20 percent more expensive than the EU15 average. Years on the horizontal axis, from 1995-2007. No estimates are available for Italy and Finland, see Figure 2.

closely the experience of Germany.

In sum, I find evidence for the "perceived-wisdom hypothesis"—the fact that some euro core countries have seen too little wage inflation before 2007, whereas some euro periphery countries have seen too much of it—although the picture that emerges is more nuanced. For example, the fact that the Spanish real exchange rate appreciated heavily is inefficient only at the very end of the sample, whereas the fact that German wage moderation was inefficient comes out less pronounced than is commonly thought.

6.3 Welfare effects

I conclude the empirical section by discussing welfare losses for the euro area countries. Note first that, by the arguments made above, welfare losses under laissez-faire arise differently depending on whether wage restraint is inefficiently strong or inefficiently weak. In the former case, welfare losses arise during *expansions*, as wages and therefore consumption are *too low* during these periods. In the latter case, instead, welfare losses arise during *contractions*, due to the wage rigidity becoming binding too often (and too severely) which creates unemployment, thereby generates an output loss and a loss in consumption.[39]

I report losses in terms of permanent consumption, which is the (negative of the) percentage increase in aggregate consumption in each period that is necessary for households to be indifferent between staying under laissez-faire and moving to the efficient equilibrium. Formally, by denoting $\mathbf{s}_t \equiv (w_{t-1}, a_t, Z_t)$ the state of the economy, the per-period consumption loss in percent $\lambda^{\%}(\mathbf{s}_t) \geq 0$ is defined by

$$E_t \sum_{j=0}^{\infty} \beta^j \{Z_{t+j} U(A_{t+j} \times (1 + \lambda^{\%}(\mathbf{s}_t)/100)) - V(h_{t+j})\} \stackrel{!}{=} \mathcal{W}(\mathbf{s}_t), \tag{6.3}$$

where the value function $\mathcal{W}$ is welfare in the efficient equilibrium (Definition 3), and where A_{t+j} and h_{t+j} are (policy functions for) aggregate consumption and aggregate working hours in the laissez-faire equilibrium at time $t+j$. In the Appendix A, I derive a formula to compute $\lambda^{\%}(\mathbf{s}_t)$ conveniently. Through its dependence on the state of the economy $\mathbf{s}_t$, the loss $\lambda^{\%}(\mathbf{s}_t)$ has a stationary distribution. I compute this stationary distribution numerically for each country and report key statistics.

Table 3 shows the results. The first three columns are the 10-percentile, median, and the 90-percentile, respectively, of the stationary distribution for the euro area countries under consideration. The median loss is about 0.7 percent of permanent consumption for all

[39] More precisely, unemployment itself does not generate the welfare loss, but the fact that if working hours fall, the amount of production hence consumption must also be reduced. This implies that welfare losses would be bigger if households suffered additionally from unemployment (rather than perceiving drops in h_t as additional leisure).

	$\lambda^{\%}(\mathbf{s}_t)$			$\lambda^{\%}(\mathbf{s}_t)$ "efficient - first best"		
	10%	median	90%	10%	median	90%
BE	0.4774	0.6922	0.9717	0.0001	0.0001	0.0001
GER	0.2701	0.6926	1.4155	0.0001	0.0001	0.0003
SPA	0.0260	0.6955	3.3298	0.0001	0.0001	0.0016
FRA	0.5483	0.6922	0.8591	0.0001	0.0001	0.0001
IRE	0.0207	0.6331	3.3705	0.0001	0.0002	0.0206
NET	0.3006	0.6923	1.3823	0.0001	0.0001	0.0001
AUS	0.4603	0.6922	0.9903	0.0001	0.0001	0.0001

Table 3: Consumption loss per period, in percent, by country. The first three columns are the 10-percentile, median, 90-percentile of the stationary distribution of $\lambda^{\%}(\mathbf{s}_t)$, respectively. The last three columns are the consumption loss between the efficient allocation and the allocation under "first best"—that is, the efficient allocation under no wage rigidity: $\gamma = 0$.

countries. However, two countries stand out: Spain and Ireland. Given that their stochastic structure has been estimated to be highly volatile and persistent (recall Table 2), both have a 90-percentile consumption loss of more than 3 percent. Therefore, even though the crisis has not materialized in sample, the model identifies those countries with the more severe crisis after 2008, as those countries with the highest chance of running into a severe loss in terms of permanent consumption.

One last interesting statistic is the consumption loss between the efficient allocation and the allocation under *first best*. Clearly, first best corresponds to the efficient allocation where it is additionally imposed that $\gamma = 0$—wages are not downwardly rigid. This loss therefore isolates the adverse effects of DNWR *per se*. The last three columns in Table 3 show the result. Strikingly, I find that for all countries the consumption loss between the efficient allocation and first best is very close to zero—it is slightly positive only for Spain and Ireland. This verifies the main result in Elsby (2009) in a dynamic stochastic general equilibrium analysis: once wage setters are forward looking and wage restraint is efficient, the macroeconomic effects of DNWR cannot be expected to be substantial.

7 Conclusion

A longstanding issue in macroeconomics has been the possible dis-employment effects of downward nominal wage rigidity (DNWR). This issue has resurfaced with the recent euro euro area crisis, where some countries, especially those in the periphery of the euro zone, have experienced a slow decline in their nominal wages despite large-scale unemployment. This concern has been supported on the theoretical front, as Schmitt-Grohé and Uribe (2016) have shown that DNWR in currency pegs generates an externality by which wages rise too quickly in expansions, exacerbating unemployment in contractions.

This paper adds to this debate by studying the implications of introducing forward-looking wage formation into a Schmitt-Grohé and Uribe (2016)-type model of DNWR. In this case, I find that wage increases can be too large, but equally too *small*, in economic expansions. Moreover I characterize that this depends on the kinds of shocks which hit the economy: excessive wage inflation results from demand shocks, whereas excessive wage moderation results from technology shocks in the tradable sector.

I use these insights to apply the model to euro area country data. This reflects a perceived wisdom hypothesis: while euro periphery wage inflation is commonly perceived as too high before 2007, euro core wage inflation is commonly perceived as too *low* before 2007—the resulting imbalances exacerbating the subsequent euro zone crisis. I find some evidence for the perceived wisdom hypothesis, yet quantitatively, it comes out less pronounced than is commonly thought.

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A Appendix: Analytical derivations

The Appendix A contains analytical derivations, as well as the proofs of all propositions.

A.1 Assumption 1

Here I show the implications of Assumption 1 for Backus-Smith condition (3.1). In the general case, this condition is $Z_t U_{c_t^T} = U^*_{c_t^{T,*}}$, as written in the text. Hence, we need to evaluate the derivative $U_{c_t^T}$. From the utility function $U(A(c^T, c^N))$, this is

$$U_{c_t^T} = U'((A(c_t^T, c_t^N)))A_{c^T}(c_t^T, c_t^N)$$

and with the functional forms imposed

$$U_{c_t^T} = (A(c_t^T, c_t^N))^{-\kappa}[\omega(c_t^T)^{1-1/\zeta} + (1-\omega)(c_t^N)^{1-1/\zeta}]^{1/(1-1/\zeta)-1}\omega(c_t^T)^{-1/\zeta}$$

By recognizing that $A(c^T, c^N) = [\omega(c^T)^{1-1/\zeta} + (1-\omega)(c^N)^{1-1/\zeta}]^{1/(1-1/\zeta)}$ the term $[\cdot]^{1/(1-1/\zeta)-1}$ can be written as $(A(c_t^T, c_t^N))^{1/\zeta}$. Therefore the whole expression becomes

$$U_{c_t^T} = (A(c_t^T, c_t^N))^{1/\zeta-\kappa}\omega(c_t^T)^{-1/\zeta}.$$

We see that under $1/\kappa = \zeta$ the first term drops out such that we obtain

$$U_{c_t^T} = \omega(c_t^T)^{-1/\zeta}$$

as claimed in the main text.

Notice that, without Assumption 1, tradable consumption c_t^T is endogenous, as it moves with $c_t^N = F(h_t^N)$, and therefore with the wage. This matters in the proof of Proposition 3 below, and for the derivation of the labor demand elasticity in Section A.5 below.

A.2 Proof of Proposition 1

The maximization problem is

$$\mathcal{W}(w_{t-1}) = \max \{a_t F(h_t) - V(h_t) + \beta E_t \mathcal{W}(w_t)\}$$

subject to the two constraints

$$i) \qquad w_t^r = a_t F'(h_t)$$

$$ii) \qquad w_t \geq \gamma w_{t-1}$$

where $w_t^r \equiv w_t/p_t$, for given exogenous $\{a_t, p_t\}$.

To solve this problem, attach multipliers λ_t to constraint i) and $\tilde{\psi}_t \geq 0$ to constraint ii). Set up the Lagrangian

$$\mathcal{L} = a_t F(h_t) - V(h_t) + \beta E_t \mathcal{W}(w_t) + \lambda_t (w_t^r - a_t F'(h_t)) + \tilde{\psi}_t (w_t - \gamma w_{t-1}).$$

The first order conditions are

$$\partial h_t \qquad a_t F'(h_t) - V'(h_t) - \lambda_t (\partial w_t^r / \partial h_t) = 0$$

$$\partial w_t^r \qquad \beta E_t \frac{\partial \mathcal{W}(w_t)}{\partial w_t^r} + \lambda_t + \tilde{\psi}_t p_t = 0$$

and the Envelope condition is

$$\frac{\partial \mathcal{W}(w_{t-1})}{\partial (w_{t-1}^r)} = -\gamma \tilde{\psi}_t p_{t-1}.$$

Combine these equations and use condition i) to obtain

$$w_t^r + \frac{\partial w_t^r}{\partial h_t} (\tilde{\psi}_t - \beta \gamma E_t \tilde{\psi}_{t+1}) p_t = V'(h_t).$$

Now define $\psi_t \equiv -(\partial w_t^r / \partial h_t) \tilde{\psi}_t p_t \geq 0$ to arrive at

$$w_t^r + \gamma E_t \beta \frac{\partial h_{t+1} / \partial w_{t+1}^r}{\partial h_t / \partial w_t^r} \frac{p_t}{p_{t+1}} \psi_{t+1} - \psi_t = V'(h_t)$$

Multiply and divide by w_t^r / h_t and define the labor demand elasticity $\varepsilon_t \equiv -(\partial h_t / \partial w_t^r) \times (w_t^r / h_t)$ to rewrite this as

$$w_t^r + \gamma E_t \beta \left(\frac{\varepsilon_{t+1}}{\varepsilon_t} \frac{h_{t+1}}{h_t} \frac{w_t^r}{w_{t+1}^r} \frac{p_t}{p_{t+1}} \right) \psi_{t+1} - \psi_t = V'(h_t),$$

which is equation (2.5) in the main text.

A.3 Proof of Proposition 2

To prove Proposition 2, it will be convenient to first rewrite the union-type labor demand curves such that $w_t(j)$ appears isolated on one side of the curve, not also indirectly through aggregator w_t. I do this in the following. Thereafter, I use the rearranged union type labor demand curves to solve the union problem in Definition 2.

A.3.1 Rewrite the labor demand curves

Rearrange the union-type labor demand curve as

$$\sum_{j=1}^{J} \frac{1}{J} w_t(j)^{1-\theta} = w_t(j)^{1-\theta} \left(\frac{h_t(j)}{h_t}\right)^{\frac{1-\theta}{\theta}}$$

$$\Leftrightarrow w_t(j)^{1-\theta} \left(\frac{1}{J} - \left(\frac{h_t(j)}{h_t}\right)^{\frac{1-\theta}{\theta}}\right) = -\frac{J-1}{J} \sum_{\neq j} \frac{1}{J-1} w_t(i)^{1-\theta}$$

$$\Leftrightarrow w_t(j) = \left(\frac{J-1}{J}\left(\left(\frac{h_t(j)}{h_t}\right)^{\frac{1-\theta}{\theta}} - \frac{1}{J}\right)^{-1}\right)^{\frac{1}{1-\theta}} w_t(-j)$$

such that

$$w_t(j) = \left(\frac{J}{J-1}\left(\frac{h_t(j)}{h_t}\right)^{\frac{1-\theta}{\theta}} - \frac{1}{J-1}\right)^{-\frac{1}{1-\theta}} w_t(-j)$$

where I define the wage index that includes all individual wages except for $w_t(j)$ as

$$w_t(-j) \equiv \left(\frac{1}{J-1} \sum_{\neq j} w_t(i)^{1-\theta}\right)^{1/(1-\theta)}.$$

Here $\sum_{\neq j}$ indicates summation over all indices $i \in \{1, ..., J\}$ except for index j. Solving for $h_t(j)$ yields

$$h_t(j) = \left(\frac{J-1}{J}\left(\frac{1}{J-1} + \left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta)}\right)\right)^{\frac{\theta}{1-\theta}} h_t.$$

A.3.2 Solve the problem of the unions

With the rewritten labor demand curve, the problem in Definition 2 can be reformulated as follows. Maximize

$$\mathcal{W}(w_{t-1}(j)) = \max \left\{Z_t U_{c_t^T} w_t(j) h_t(j) - V(h_t(j)) + \beta E_t \mathcal{W}(w_t(j))\right\}$$

subject to the sequence of constraints

$$i) \qquad w_t(j) \geq \gamma w_{t-1}(j)$$

$$ii) \qquad h_t(j) = \left(\frac{J-1}{J}\left(\frac{1}{J-1} + \left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta)}\right)\right)^{\frac{\theta}{1-\theta}} h_t(w_t)$$

$$iii) \qquad w_t = \left(\sum_{j=1}^{J} \frac{1}{J} w_t(j)^{1-\theta}\right)^{\frac{1}{1-\theta}}.$$

In this expression, $h_t = h_t^T + h_t^N = h_t(w_t)$ depends on w_t through the two labor demand curves, as detailed in Definition 2. The Lagrangian for this problem is

$$\begin{aligned}\mathcal{L} = Z_t U_{c_t^T} w_t(j) \left(\frac{J-1}{J} \left(\frac{1}{J-1} + \left(\frac{w_t(j)}{w_t(-j)} \right)^{-(1-\theta)} \right) \right)^{\frac{\theta}{1-\theta}} h_t(w_t) \\ - V \left(\left(\frac{J-1}{J} \left(\frac{1}{J-1} + \left(\frac{w_t(j)}{w_t(-j)} \right)^{-(1-\theta)} \right) \right)^{\frac{\theta}{1-\theta}} h_t(w_t) \right) \\ + \beta E_t \mathcal{W}(w_t(j)) + \tilde{\psi}_t(j)(w_t(j) - \gamma w_{t-1}(j)).\end{aligned}$$

In a first step, we compute the partial derivative of $h_t(w_t)$ with respect to $w_t(j)$ (recall that w_t depends on $w_t(j)$ for $1/J > 0$)

$$\begin{aligned}\frac{\partial}{\partial w_t(j)} h_t(w_t(w_t(j))) &= \frac{\partial}{\partial w_t(j)} w_t \times \frac{\partial}{\partial w_t} h_t(w_t) \\ &= -\frac{1}{J} \left(\frac{w_t(j)}{w_t} \right)^{-\theta} \times \Omega_t\end{aligned}$$

where I have denoted $\Omega_t \equiv -(\partial h_t / \partial w_t)$ the (negative of the) derivative of aggregate labor demand h_t with respect to the aggregate wage w_t. In a second step, we compute the derivative of the product $w_t(j)h_t(j)$ with respect to $w_t(j)$, but by keeping aggregate hours h_t constant (recall that $h_t(j)$ is defined in condition ii) above; also, note that the product $w_t(j)h_t(j)$ appears in the Lagrangian). I obtain the following

$$\begin{aligned}&\left[\frac{\partial}{\partial w_t(j)} w_t(j) \left(\frac{J-1}{J} \left(\frac{1}{J-1} + \left(\frac{w_t(j)}{w_t(-j)} \right)^{-(1-\theta)} \right) \right)^{\frac{\theta}{1-\theta}} \right] h_t \\ &= \left[(\cdot)^{\frac{\theta}{1-\theta}} - \theta w_t(j) \, (\cdot)^{\frac{\theta}{1-\theta}-1} \frac{J-1}{J} \left(\frac{w_t(j)}{w_t(-j)} \right)^{-(1-\theta)-1} \frac{1}{w_t(-j)} \right] h_t \\ &= h_t(j) - \theta \frac{J-1}{J} \left(\frac{h_t(j)}{h_t} \right)^{\frac{2\theta-1}{\theta}} h_t \left(\frac{J}{J-1} \left(\frac{h_t(j)}{h_t} \right)^{\frac{1-\theta}{\theta}} - \frac{1}{J-1} \right) \\ &= h_t(j) - \theta \left(h_t(j) - \frac{1}{J} \left(\frac{h_t(j)}{h_t} \right)^{\frac{2\theta-1}{\theta}} h_t \right) \\ &= h_t(j) \left(1 - \theta \left(1 - \frac{1}{J} \left(\frac{h_t(j)}{h_t} \right)^{\frac{\theta-1}{\theta}} \right) \right),\end{aligned}$$

where I use "$(\cdot)$" as shorthand notation for the first (large) round bracket that appears in the first line.

In a third step, consider the derivative of the product $w_t(j)h_t(j)$ with respect to $w_t(j)$ by only considering the variation in $h_t(w_t)$

$$
\begin{aligned}
& w_t(j)\left(\frac{J-1}{J}\left(\frac{1}{J-1}+\left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta)}\right)\right)^{\frac{\theta}{1-\theta}}\frac{\partial}{\partial w_t(j)}h_t(w_t) \\
&= -w_t(j)\left(\frac{J-1}{J}\left(\frac{1}{J-1}+\left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta)}\right)\right)^{\frac{\theta}{1-\theta}}\frac{1}{J}\left(\frac{w_t(j)}{w_t}\right)^{-\theta}\Omega_t \\
&= -\frac{1}{J}w_t(j)\frac{h_t(j)}{h_t}\left(\frac{w_t(j)}{w_t}\right)^{-\theta}\Omega_t,
\end{aligned}
$$

where I have used the results on the impact of $w_t(j)$ on $h_t(w_t)$ from above. Similarly, we obtain the derivative of $V(h_t(j))$ with respect to $w_t(j)$

$$
\begin{aligned}
& \frac{\partial}{\partial w_t(j)}V\left(\left(\frac{J-1}{J}\left(\frac{1}{J-1}+\left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta)}\right)\right)^{\frac{\theta}{1-\theta}}h_t(w_t)\right) \\
&= V'(h_t(j))\left(-\theta\frac{h_t(j)}{w_t(j)}+\theta\frac{1}{J}\left(\frac{h_t(j)}{h_t}\right)^{\frac{2\theta-1}{\theta}}\frac{h_t}{w_t(j)}-\frac{1}{J}\frac{h_t(j)}{h_t}\left(\frac{w_t(j)}{w_t}\right)^{-\theta}\Omega_t\right) \\
&= V'(h_t(j))\left(-\theta\frac{h_t(j)}{w_t(j)}\left(1-\frac{1}{J}\left(\frac{h_t(j)}{h_t}\right)^{\frac{\theta-1}{\theta}}\right)-\frac{1}{J}\frac{h_t(j)}{h_t}\left(\frac{w_t(j)}{w_t}\right)^{-\theta}\Omega_t\right).
\end{aligned}
$$

In a fourth step, consider the partial derivative

$$
\begin{aligned}
& \frac{\partial}{\partial w_t(j)}\ \beta E_t\mathcal{W}(w_t(j))+\tilde{\psi}_t(j)(w_t(j)-\gamma w_{t-1}(j)) \\
&= \beta E_t\frac{\partial}{\partial w_t(j)}\mathcal{W}(w_t(j))+\tilde{\psi}_t(j),
\end{aligned}
$$

which, once combined with the Envelope condition

$$\frac{\partial}{\partial w_{t-1}(j)}\mathcal{W}(w_{t-1}(j))=-\gamma\tilde{\psi}_t(j)$$

becomes

$$=\tilde{\psi}_t(j)-\beta\gamma E_t\tilde{\psi}_{t+1}(j).$$

We have computed all derivatives which are necessary to take the first order condition of the Lagrangian with respect to $w_t(j)$. By putting all pieces together and by using that, in the symmetric equilibrium, $w_t(j)=w_t$, $h_t(j)=h_t$, and $\tilde{\psi}_t(j)=\tilde{\psi}_t$, I obtain

$$V'(h_t)\left(\frac{J-1}{J}\theta+\frac{1}{J}\Omega_t\frac{w_t}{h_t}\right)=Z_tU_{c_t^T}w_t\left(\frac{J-1}{J}\theta+\frac{1}{J}\Omega_t\frac{w_t}{h_t}-1\right)-\frac{w_t}{h_t}(\tilde{\psi}_t-\beta\gamma E_t\tilde{\psi}_{t+1})$$

Define now the elasticity

$$\tilde{\varepsilon}_t\equiv\frac{J-1}{J}\theta+\frac{1}{J}\varepsilon_t$$

where $\varepsilon_t \equiv \Omega_t w_t / h_t$ denotes the elasticity of aggregate labor demand, to write the labor supply curve as

$$Z_t U_{c_t^T} w_t \left(\frac{\tilde{\varepsilon}_t - 1}{\tilde{\varepsilon}_t} \right) - \frac{1}{\tilde{\varepsilon}_t} \frac{w_t}{h_t} (\tilde{\psi}_t - \beta\gamma E_t \tilde{\psi}_{t+1}) = V'(h_t).$$

Lastly, define $\psi_t \equiv (\tilde{\psi}_t w_t)/(\tilde{\varepsilon}_t h_t) \geq 0$ to rewrite this as

$$Z_t U_{c_t^T} w_t \left(\frac{\tilde{\varepsilon}_t - 1}{\tilde{\varepsilon}_t} \right) + \gamma E_t \beta \left(\frac{\tilde{\varepsilon}_{t+1}}{\tilde{\varepsilon}_t} \frac{h_{t+1}}{h_t} \frac{w_t}{w_{t+1}} \right) \psi_{t+1} - \psi_t = V'(h_t)$$

which is equation (3.6) in the main text.

A.4 Proof of Proposition 3

The problem of the planner can be written as a Lagrangian

$$\mathcal{L} = Z_t U(A(c_t^T(h_t^N), F(h_t^N))) - V(h_t) + \beta E_t \mathcal{W}(w_t) + \lambda_t (w_t - w_t(h_t)) + \tilde{\psi}_t (w_t - \gamma w_{t-1}),$$

subject to Backus-Smith condition (3.1) (in the general case, making c_t^T a function of h_t^N, see the discussion in Section A.1), and subject to equations (3.2)-(3.5). Note that c_t^T, h_t^N, h_t^N and therefore $h_t = h_t^T + h_t^N$ all depend on the wage w_t through the demand curves and the Backus-Smith condition, which are constraints in the maximization. In the Lagrangian, the fact that the wage depends on labor demand is captured in the constraint with multiplier λ_t. The first order conditions are

$$\partial h_t^T \qquad -V'(h_t) - \lambda_{1,t} \frac{\partial w_t}{\partial h_t^T} = 0$$

$$\partial h_t^N \qquad Z_t (U_{c_t^T} \frac{\partial c_t^T}{\partial h_t^N} + U_{c_t^N} F'(h_t^N)) - V'(h_t) - \lambda_{2,t} \frac{\partial w_t}{\partial h_t^N} = 0$$

$$\partial w_t \qquad \beta E_t \frac{\partial \mathcal{W}(w_t)}{\partial w_t} + \lambda_{1,t} + \lambda_{2,t} + \tilde{\psi}_t = 0$$

and the Envelope condition is

$$\frac{\mathcal{W}(w_{t-1})}{\partial w_{t-1}} = -\gamma \tilde{\psi}_t.$$

Now use that $p_t^N = U_{c_t^N}/U_{c_t^T}$ (equation (3.2)) as well as $p_t^N F'(h_t^N) = w_t$ (labor demand curve (3.5)) to combine the previous equations as

$$Z_t U_{c_t^T} \left(\frac{\partial c_t^T}{\partial h_t^N} + w_t \right) \frac{\partial h_t^N}{\partial w_t} - V'(h_t) \frac{\partial h_t}{\partial w_t} + (\tilde{\psi}_t - \gamma\beta E_t \tilde{\psi}_{t+1}) = 0,$$

where we have used that $h_t = h_t^T + h_t^N$ and hence that $\partial h_t / \partial w_t = \partial h_t^T / \partial w_t + \partial h_t^N / \partial w_t$. If we now define $\psi_t \equiv \tilde{\psi}_t / (-\partial h_t / \partial w_t) \geq 0$ we can rewrite this further as

$$Z_t U_{c_t^T} \left(\frac{\partial c_t^T}{\partial h_t^N} + w_t \right) \left(\frac{\varepsilon_t^N}{\varepsilon_t} \frac{h_t^N}{h_t} \right) + \gamma E_t \beta \left(\frac{\varepsilon_{t+1}}{\varepsilon_t} \frac{h_{t+1}}{h_t} \frac{w_t}{w_{t+1}} \right) \psi_{t+1} - \psi_t = V'(h_t) \qquad \text{(A.1)}$$

where $\varepsilon_t \equiv -(\partial h_t / \partial w_t) \times (w_t / h_t)$ and $\varepsilon_t^N \equiv -(\partial h_t^N / \partial w_t) \times (w_t / h_t^N)$. If we impose Assumption 1, $\partial c_t^T / \partial h_t^N = 0$, and this equation collapses to equation (4.1).

A.5 Labor demand elasticity

Here I derive the labor demand elasticities ε_t, ε_t^N and ε_t^T, see equation (4.3) in Section 4. Recall that the labor demand curves are

$$a_t \alpha_T (h_t^T)^{\alpha_T - 1} = w_t \tag{A.2}$$

$$\alpha_N \frac{(1-\omega)}{\omega} \frac{(h_t^N)^{\alpha_N - 1 - \alpha_N/\zeta}}{(c_t^T)^{-1/\zeta}} = w_t, \tag{A.3}$$

where the first equation is equation (3.4), the second is the combination of equations (3.5) and (3.2). From this we may easily compute ε_t^T

$$-\frac{\partial h_t^T}{\partial w_t} = \frac{1}{1-\alpha_T} \frac{h_t^T}{w_t}$$

such that $\varepsilon_t^T \equiv -(\partial h_t^T/\partial w_t) \times (w_t/h_t^T) = 1/(1-\alpha_T)$.

The slope $\partial h_t^N/\partial w_t$ is more involved. This is because, in the general case, c_t^T depends on h_t^N because of the Backus-Smith condition (3.1), see Section A.1. If we take the derivative with respect to h_t^N on both sides of equation (A.3) we get

$$-\frac{\partial w_t}{\partial h_t^N} = (1 - \alpha_N + \alpha_N/\zeta) \frac{w_t}{h_t^N} - \frac{1}{\zeta} \frac{w_t}{c_t^T} \frac{\partial c_t^T}{\partial h_t^N}.$$

Under Assumption 1, $\partial c_t^T/\partial h_t^N = 0$ such that ε_t^N collapses to $\varepsilon_t^N \equiv -(\partial h_t^N/\partial w_t) \times (w_t/h_t^N) = 1/(1-\alpha_N + \alpha_N/\zeta)$, as claimed in the main text. The general case is discussed below.

To compute the aggregate labor demand elasticity, recall that $h_t = h_t^T + h_t^N$, such that $\partial h_t/\partial w_t = \partial h_t^T/\partial w_t + \partial h_t^N/\partial w_t$. From this follows that

$$\begin{aligned}
\varepsilon_t \equiv \frac{\partial h_t}{\partial w_t} \frac{w_t}{h_t} &= \left(\frac{\partial h_t^T}{\partial w_t} + \frac{\partial h_t^N}{\partial w_t} \right) \frac{w_t}{h_t^T + h_t^N} \\
&= \frac{\partial h_t^T}{\partial w_t} \frac{w_t}{h_t^T} \frac{h_t^T}{h_t^N + h_t^T} + \frac{\partial h_t^N}{\partial w_t} \frac{w_t}{h_t^N} \frac{h_t^N}{h_t^N + h_t^T} \\
&= \left(1 - \frac{h_t^N}{h_t^T + h_t^N} \right) \varepsilon_t^T + \frac{h_t^N}{h_t^T + h_t^N} \, \varepsilon_t^N,
\end{aligned}$$

which is equation (4.3).

To compute $\partial c_t^T/\partial h_t^N$ in the general case without Assumption 1 (note that this derivative also appears explicitly in equation (A.1)), we use the Backus-Smith equation (3.1) in the general case, derived in Section A.1. Taking derivative with respect to h_T^N yields

$$\begin{aligned}
&\left(\frac{1}{\zeta} - \kappa \right) \frac{U^*_{c^T,*}}{A(c_t^T, F(h_t^N))} \left(A_{c^T} \frac{\partial c_t^T}{\partial h_t^N} + A_{c^N} F'(h_t^N) \right) - \frac{1}{\zeta} \frac{U^*_{c^T,*}}{c_t^T} \frac{\partial c_t^T}{\partial h_t^N} = 0 \\
\Leftrightarrow &\left(\frac{1}{\zeta} - \kappa \right) \left(A_{c^T} \frac{\partial c_t^T}{\partial h_t^N} + A_{c^N} F'(h_t^N) \right) - \frac{1}{\zeta} \frac{A(c_t^T, F(h_t^N))}{c_t^T} \frac{\partial c_t^T}{\partial h_t^N} = 0 \\
\Leftrightarrow &\left(\frac{1}{\zeta} - \kappa \right) \left(\omega (c_t^T)^{-1/\zeta} \frac{\partial c_t^T}{\partial h_t^N} + (1-\omega)(F(h_t^N))^{-1/\zeta} F'(h_t^N) \right) - \frac{1}{\zeta} \frac{A(c_t^T, F(h_t^N))^{1-1/\zeta}}{c_t^T} \frac{\partial c_t^T}{\partial h_t^N} = 0
\end{aligned}$$

such that

$$\left(\frac{1}{\zeta}-\kappa\right)\zeta(1-\kappa)\left(\omega(c_t^T)^{-1/\zeta}\frac{\partial c_t^T}{\partial h_t^N}+(1-\omega)\alpha_N(h_t^N)^{-\alpha_N/\zeta+\alpha_N-1}\right)-\frac{A^{1-\kappa-1/\zeta}}{c_t^T}\frac{\partial c_t^T}{\partial h_t^N}=0,$$

from which $\partial c_t^T/\partial h_t^N$ is implicitly defined.

A.6 Welfare

Here I show how to compute equation (6.3) conveniently, see Section 6.3. Denote $A^{lf}(\mathbf{s}_t)$ the policy function for aggregate consumption under laissez-faire in state $\mathbf{s}_t \equiv (w_{t-1}, a_t, Z_t)$, and equivalently for $h^{lf}(\mathbf{s}_t)$. The state under laissez-faire evolves as $\mathbf{s}_{t+1}^{lf}(\mathbf{s}_t) = (w^{lf}(\mathbf{s}_t), a_{t+1}, Z_{t+1})$, where $w^{lf}(\mathbf{s}_t)$ is the policy function for aggregate wages.

Define welfare implied by this allocation via

$$\mathcal{W}^{lf}(\mathbf{s}_t) = Z_t U(A^{lf}(\mathbf{s}_t)) - V(h^{lf}(\mathbf{s}_t)) + \beta E_t \mathcal{W}^{lf}(\mathbf{s}_{t+1}^{lf}(\mathbf{s}_t)).$$

In turn, welfare in the efficient allocation can be computed as

$$\mathcal{W}^{e}(\mathbf{s}_t) = Z_t U(A^{e}(\mathbf{s}_t)) - V(h^{e}(\mathbf{s}_t)) + \beta E_t \mathcal{W}^{e}(\mathbf{s}_{t+1}^{e}(\mathbf{s}_t))$$

with the corresponding policies having superscript e. Note that, by definition of efficiency, it must be the case that $\mathcal{W}^{lf}(\mathbf{s}_t) \leq \mathcal{W}^{e}(\mathbf{s}_t)$ in all states $\mathbf{s}_t$.

The permanent consumption equivalent is a function $\lambda(\mathbf{s}_t)$ multiplying consumption policy $A^{lf}(\mathbf{s}_t)$ such that $\mathcal{W}^{lf}(\mathbf{s}_t) = \mathcal{W}^{e}(\mathbf{s}_t)$ in all states $\mathbf{s}_t$. We may therefore write

$$\mathcal{W}^{e}(\mathbf{s}_t) = Z_t U(\lambda(\mathbf{s}_t) A^{lf}(\mathbf{s}_t)) - V(h^{lf}(\mathbf{s}_t)) + \beta E_t \mathcal{W}^{e}(\mathbf{s}_{t+1}^{lf}(\mathbf{s}_t)),$$

where I have inserted function $\lambda(\mathbf{s}_t)$ and replaced value function $\mathcal{W}^{lf}$ by $\mathcal{W}^{e}$. Importantly, the policy $\mathbf{s}_{t+1}^{lf}(\mathbf{s}_t)$ from laissez-faire still appears in the argument of the expectation of the value function on the right hand side. Hence, the state still evolves as under laissez-faire, and only the utility derived from consumption is altered by function $\lambda(\mathbf{s}_t)$.

If we use that function U is of the CRRA type (and thus homothetic), this equation can be further rewritten as

$$\mathcal{W}^{e}(\mathbf{s}_t) = \lambda(\mathbf{s}_t)^{1-\kappa} Z_t U(c^{lf}(\mathbf{s}_t)) - V(h^{lf}(\mathbf{s}_t)) + \beta E_t \mathcal{W}^{e}(\mathbf{s}_{t+1}^{lf}(\mathbf{s}_t)).$$

Rearranging and solving for $\lambda(\mathbf{s}_t)$ I obtain

$$\lambda(\mathbf{s}_t) = \{[\mathcal{W}^{e}(\mathbf{s}_t) - \beta E_t \mathcal{W}^{e}(\mathbf{s}_{t+1}^{lf}(\mathbf{s}_t)) + V(h^{lf}(\mathbf{s}_t))]/(Z_t U(c^{lf}(\mathbf{s}_t)))\}^{1/(1-\kappa)},$$

which can be solved numerically. The gain in percent follows from $\lambda(\mathbf{s}_t)$ via

$$\lambda^{\%}(\mathbf{s}_t) = (\lambda(\mathbf{s}_t) - 1) * 100\%.$$

B Appendix: Additional Figures

The Appendix B contains additional figures.

B.1 Neoclassical labor market with DNWR

Here I complement the insights in Section 2 by solving the model numerically. I assume the model is quarterly frequency, hence I set $\beta = 0.99$ and $\gamma = 0.99$. I set gross price inflation $p_t/p_{t-1} = 1$. Hence I assume that p_t is not stochastic. Zero net inflation is without loss of generality, because in this model, higher trend inflation is isomorphic to a lower γ. The labor share α is set to two thirds. As in the open economy model, the Frisch elasticity is set to $\varphi = 2$. As for the technology shocks, I assume an autoregressive process as follows

$$\log(a_t) = 0.95 \times \log(a_{t-1}) + 0.02 \times u_t.$$

The parameters used are summarized in Table 4.

Parameter		*Value assigned*
β	Time discount factor	0.99
p_t/p_{t-1}	Gross price inflation	1
γ	Downward nominal wage rigidity	0.99
α	Labor share	0.66
φ	Inverse Frisch elasticity of labor supply	2

Table 4: Parameters used for numerical example.

I solve the model using value function iteration, see Definition 1. The result is shown in Figure 4. The left panel shows the policy function for the nominal wage against technology shocks. I contrast the efficient allocation (blue solid) to an allocation where $w_t^r = V'(h_t)$ in periods when the wage rigidity is slack (dashed dotted in black), which I call "Perfect competition", because this would be the allocation under a perfectly competitive labor market (see Schmitt-Grohé and Uribe (2016)).

The difference between the two allocations can be made more stark in the following way. Define an "endogenous wage mark-down" $\mathcal{M}_t$ as $w_t^r = (1 + \mathcal{M}_t/100)V'(h_t)$. Then, the mark-down is negative whenever there is wage restraint in the efficient allocation, but positive whenever the wage rigidity binds, as wages rise above the marginal rate of substitution. In the right panel of Figure 4, I plot $\mathcal{M}_t$ only in the region where it is negative.[40]

[40] The kink in the mark-down policy is the point where the constraint starts to bind in the efficient allocation. As wages cannot be lowered to the left of this point, the mark-down quickly shrinks to zero. "Constraint

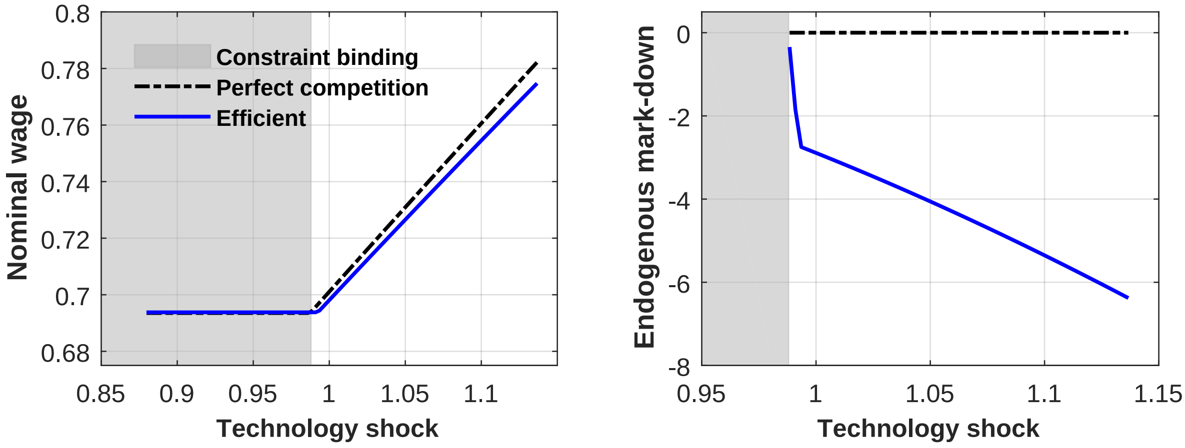

Figure 4: Policy function for nominal wages w_t and the mark-down $\mathcal{M}_t$ in the technology shock a_t. The other state variable w_{t-1} is kept at its steady state.

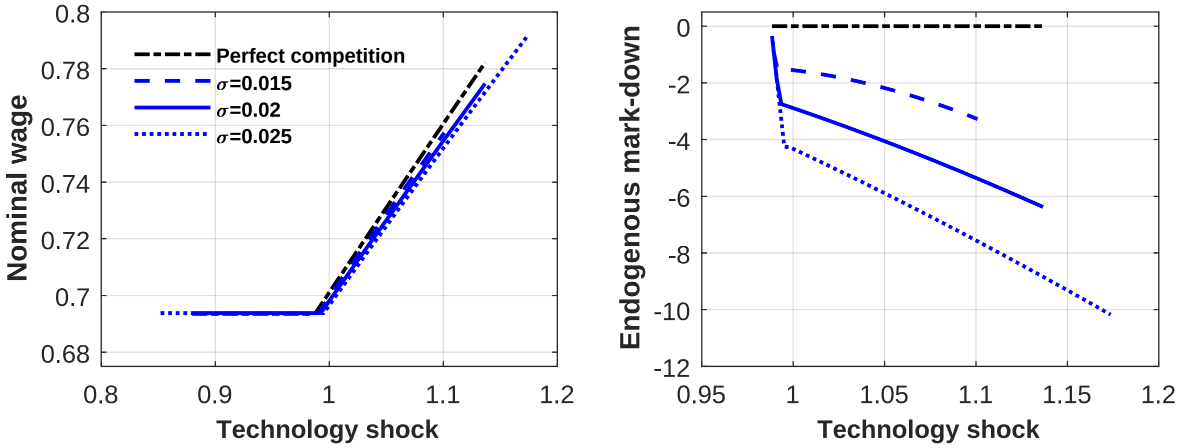

Figure 5: Robustness against alternate values for shock volatility, σ. The blue lines denote the efficient allocation. Details as in Figure 4.

As expected, wages rise in a "boom", yet are restricted in their ability to fall in a "bust". Note that the mark-down becomes more negative, or equivalently, wage restraint becomes stronger, as the boom is larger. This reflects that larger technology shocks generate stronger wage-inflationary pressure, such that, in the following period, there is a higher chance that a negative shock will make the wage rigidity bind. Therefore, more wage restraint in the boom period is required.

Figure 5 shows how results change with a different shock *volatility*. We note that as

binding" is the region where also the allocation under $w_t^r = V'(h_t)$ becomes constrained (which occurs at slightly lower technology shocks than in the efficient allocation, see Figure 4 left panel).

shock volatility is increased to $\sigma = 0.025$, the efficient wage mark-down reaches more than 10 percent when shocks are large. In contrast, the policy functions under perfect competition are not affected by shock volatility. This verifies that, under DNWR, wage restraint operates through economic uncertainty, as in the literature on time-varying risk (Basu and Bundick, 2017; Fernández-Villaverde et al., 2015).

B.2 Comparing FPI and VFI

As explained in Section 6.1, I develop a fixed point iteration (FPI) algorithm to solve the model fully non-linearly. The algorithm is described in more detail in the Appendix C. To assess the accuracy of the algorithm, here I compare policy functions obtained from FPI and, alternatively, value function iteration (VFI) for the efficient allocation. This is feasible, because the efficient allocation has a recursive representation (see Definition 3).

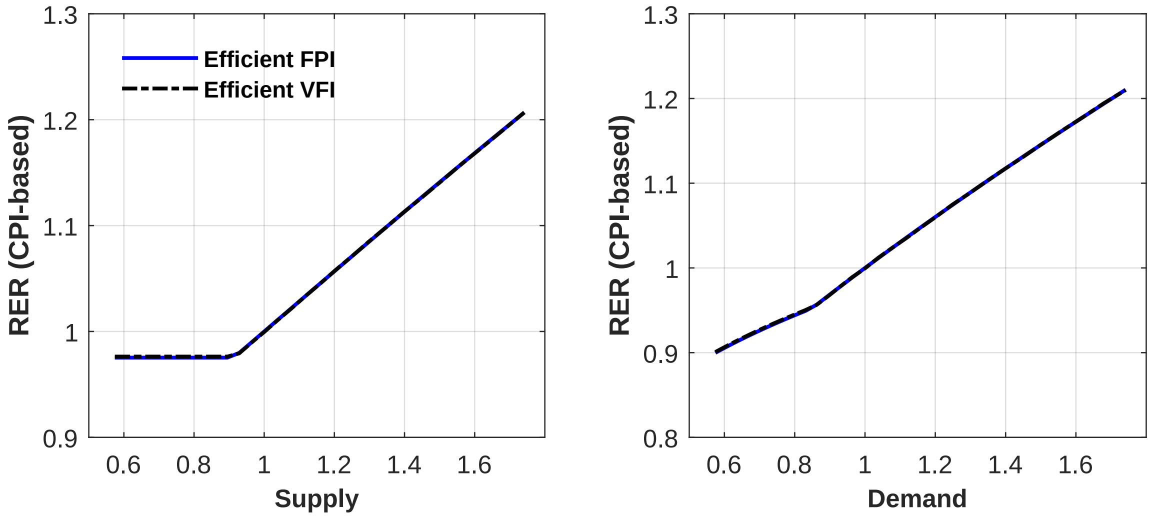

Figure 6: Solving the efficient allocation using fixed point iteration (FPI, blue solid) versus value function iteration (VFI, black dashed)—see the discussion in Section 6.1. Shown are policy functions for the CPI-based real exchange rate cpi_t/cpi_{ss}, where $cpi_t \equiv (\omega + (1-\omega)(p_t^N)^{1-\zeta})^{1/(1-\zeta)}$, against the supply shock a_t and the demand shock Z_t, as in Figure 1.

The result is shown in Figure 6: policy functions are close to identical. The value function iteration is based on an extensive grid for the endogenous state variable w_{t-1}, composed of 500 grid points. Moreover, the shocks a_t and Z_t are discretized using the method in Gospodinov and Lkhagvasuren (2014), extending the method in Rouwenhorst (1995), by using 31 grid points each. The convergence criterion is set at $1e-7$.

B.3 The quantitative impact of 1/J

In my quantitative application in Section 6, I have used a value of $1/J = 0$, such that unions operate under monopolistic competition. To complement my findings there, and to corroborate my discussion from Section 5, here I show an example of a wage bargaining centralization curve in this model.

These curves have been popularized by Calmfors and Driffill (1988). They correspond to average welfare (or average welfare loss) plotted against the degree of bargaining centralization $1/J$. In my analysis, the relevant metric is the median permanent consumption equivalent, given in equation (6.3).

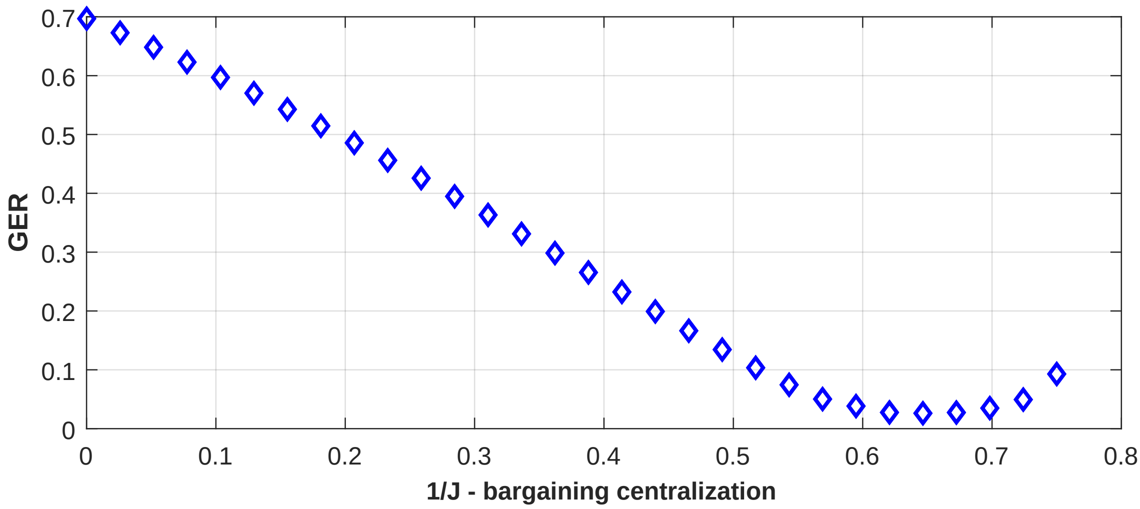

Figure 7: Bargaining centralization curve. Median permanent consumption loss in percent, plotted against union size $1/J$, for the case of Germany.

Figure 7 shows this curve at the example of Germany; qualitatively, results are similar for the other countries. As it turns out, the median welfare loss is *decreasing* initially in union size, whereas for $1/J$ large enough, the loss increases again. As explained in Section 5, an intermediate degree of bargaining centralization is therefore optimal in this model, as for example in Guzzo and Velasco (1999).

C Appendix: Numerical implementation

The Appendix C contains details on the numerical implementation of the model.

C.1 Solving the equilibrium under laissez-faire

Here I provide details on the fixed point iteration (FPI) algorithm to solve the equilibrium under laissez-faire. The relevant system of first order conditions is repeated for convenience

$$Z_t \omega (c_t^T)^{-1/\zeta} = U^*_{c^{T,*}} \tag{C.1}$$

$$\frac{1-\omega}{\omega}\left(\frac{(h_t^N)^{\alpha_N}}{c_t^T}\right)^{-1/\zeta} = p_t^N \tag{C.2}$$

$$a_t \alpha_T (h_t^T)^{\alpha_T - 1} = w_t \tag{C.3}$$

$$p_t^N \alpha_N (h_t^N)^{\alpha_N - 1} = w_t \tag{C.4}$$

$$(h_t^T + h_t^N)^{\varphi} + \psi_t = U^*_{c^{T,*}} w_t \left(\frac{\tilde{\varepsilon}_t - 1}{\tilde{\varepsilon}_t}\right) + \gamma E_t \beta \left(\frac{\tilde{\varepsilon}_{t+1}}{\tilde{\varepsilon}_t} \frac{h_{t+1}^T + h_{t+1}^N}{h_t^T + h_t^N} \frac{w_t}{w_{t+1}}\right) \psi_{t+1} \tag{C.5}$$

$$\tilde{\varepsilon}_t = \left(1 - \frac{1}{J}\right)\theta + \frac{1}{J}\varepsilon_t \tag{C.6}$$

$$\varepsilon_t = \left(1 - \frac{h_t^N}{h_t^T + h_t^N}\right)\frac{1}{1-\alpha^T} + \frac{h_t^N}{h_t^T + h_t^N}\,\frac{1}{1-\alpha^N + \alpha^N/\zeta}. \tag{C.7}$$

along with the conditions $\psi_t \geq 0$, $w_t \geq \gamma w_{t-1}$ and $\psi_t(w_t - \gamma w_{t-1}) = 0$.

The iteration proceeds as follows. Construct a grid for the state $\mathbf{s}_t = (w_{t-1}, a_t, Z_t)$. Set $i = 0$ (first iteration). Guess a function $\Lambda^i(\mathbf{s}_t) = \gamma E_t \beta \left(\frac{\tilde{\varepsilon}_{t+1}}{\tilde{\varepsilon}_t} \frac{h_{t+1}^T + h_{t+1}^N}{h_t^T + h_t^N} \frac{w_t}{w_{t+1}}\right)\psi_{t+1}$ (I guess a matrix of zeros initially). Assume the constraint is slack everywhere $\psi(\mathbf{s}_t) = 0$. Given that c_t^T is exogenous from (C.1), at each grid point, equations (C.2)-(C.7) form a system of six equations in six unknowns $(h_t^N, p_t^N, h_t^T, w_t, \tilde{\varepsilon}_t, \varepsilon_t)$. Solve this system for w_t at each grid point, which results in a function $w(\mathbf{s}_t)$. Notice that for this step, a non-linear equation solver is required given that the system cannot be solved for w_t in closed form.

Check that $w(\mathbf{s}_t) \geq w^-(\mathbf{s}_t)$, the latter the function for w_{t-1} that is flat in the shocks, the identity function in w_{t-1}. At the grid points where the check fails, update w_t by setting it to γw_{t-1}. This results in a function $w(\mathbf{s}_t)$ where the wage rigidity is respected. Use this function to obtain functions $h^T(\mathbf{s}_t)$ from (C.3), thereafter $h^N(\mathbf{s}_t)$ and $p^N(\mathbf{s}_t)$ from (C.2) and (C.4), $\varepsilon(\mathbf{s}_t)$ from (C.7), $\tilde{\varepsilon}(\mathbf{s}_t)$ from (C.6) and finally, $\psi(\mathbf{s}_t)$ from (C.5).

Set $i = 1$ (second iteration). By using these policy functions, update the guess for $\Lambda^i(\mathbf{s}_t)$. Repeat until convergence of $\Lambda^i(\mathbf{s}_t)$. For the algorithm, I use 300 grid points for w_{t-1}, and 31 grid points for each of the shocks. I discretize the shocks using the method described in Gospodinov and Lkhagvasuren (2014).

C.2 Solving the fixed point in the estimation in Section 6.2

I obtain this fixed point by iteration. Given $(\boldsymbol{\rho}, \boldsymbol{\Sigma})$, I compute the model solution $\boldsymbol{\Omega}(\boldsymbol{\rho}, \boldsymbol{\Sigma})$. I am using this model solution to obtain the shock series $\{a_t, Z_t\}$ as described in Section

6.2. I am using a vector auto regression toolbox in order to obtain an estimate $(\boldsymbol{\rho}', \boldsymbol{\Sigma}')$ from this process. Finally, I use these to obtain a new model solution $\boldsymbol{\Omega}(\boldsymbol{\rho}', \boldsymbol{\Sigma}')$. If the updating (dampening) parameter on $\boldsymbol{\rho}$ is sufficiently small, this iteration always converges in my application. In contrast, no dampening parameter can be used for $\boldsymbol{\Sigma}$, as a weighted average of two covariance matrices need not again be a covariance matrix (that is, the resulting matrix may not be positive definite).