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Sovereign Default: The Role of Expectations*

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Abstract

In the standard model of sovereign default, as in Aguiar and Gopinath (2006) or Arellano (2008), default is driven by fundamentals alone. There is no independent role for expectations. We show that small variations of that model are consistent with multiple interest rate equilibria, similar to the ones found in Calvo (1988). For distributions of output that are commonly used in the literature, the high interest rate equilibria have properties that make them fragile. Once output is drawn from a distribution with both good and bad times, however, it is possible to have robust high interest rate equilibria.

Keywords: Sovereign default; multiple equilibria; good and bad times.
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1 Introduction

Are sovereign debt crises caused by bad fundamentals alone, or do expectations play an independent role? The literature on sovereign debt crises is ambiguous on the role of expectations. In a model with rollover risk, Cole and Kehoe (2000) have shown that expectations can play a role that is strengthened by bad fundamentals. Using a different mechanism, Calvo (1988) also shows that there are multiple interest rate equilibria. The reason is that, although interest rates may be high because of high default probabilities, it is also the case that high interest rates induce high default probabilities. This gives rise to equilibria with high rates/likely default and low rates/unlikely default. In contrast with the results in those models, in the standard quantitative model of sovereign default, as in Aguiar and Gopinath (2006) or Arellano (2008), a single low interest rate equilibrium is computed.

In this paper, we take the model of Aguiar and Gopinath (2006) and Arellano (2008), who build on Eaton and Gersovitz (1981), and make minor changes to the modeling choices concerning the timing of moves by debtors and creditors as well as the actions that they may take. In doing so, we are able to produce both high and low interest rate equilibria. The reason for the multiplicity is the one identified by Calvo (1988).

The model is a small open economy with a random endowment. For the distributions of the endowment that are commonly used in this literature, the expectations driven, high interest rate equilibria have features that make them fragile. They lie on the wrong side of a Laffer-type curve, meaning that the comparative statics are implausible, may be unstable, and may be refined away. We go on to consider bimodal distributions with good and bad times. With those distributions, there are low and high interest rate equilibria that are equally robust. We interpret the bimodal distributions as reflecting the likelihood of relatively long periods of stagnation, in line with the evidence in Kahn and Rich (2007), among others.

There is a large literature extending Calvo (1988) and Cole and Kehoe (2000) in directions other than the ones we are concerned with in this paper.[1] Closer to our work is Lorenzoni and Werning (2013), which we discuss below.

2 A two-period model

We study a two-period endowment economy populated by a representative agent that can borrow from a continuum of risk-neutral foreign creditors.[2] The utility function of

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[1]See, for example, Bocola and Dovis (2015), Conesa and Kehoe (2012), and Corsetti and Dedola (2016), among others.

[2]An agent with deep pockets that could affect the probabilities of default could induce uniqueness. That would be the role of a lender of last resort, such as the European Central Bank in the context of the recent European debt crisis.

the representative agent is assumed to be strictly increasing and strictly concave and to satisfy Inada conditions. The initial wealth is assumed to be equal to ω. In the second period, the agent receives endowment y, which follows a stochastic process with density $f(y)$ and corresponding cumulative distribution function $F(y)$. Low initial wealth and enough discounting induce the agent to borrow. In period one, the representative agent can borrow b in a noncontingent bond in international financial markets. The risk-neutral gross international interest rate is R^*. In period two, after observing the realization of the endowment, the borrower decides whether to pay the debt in full or default. If there is default, the penalty consumption is equal to y^d.

The timing of moves is as follows. In the first period, each creditor $i \in [0,1]$ offers funds at the gross interest rate R_i. The borrower moves next and picks the level of debt $b = \int_0^1 b_i di$, where b_i is the amount borrowed from each creditor.[3] The borrower's best response is to borrow from the low interest rate lenders first. In order for lenders to make zero profits in equilibrium, the interest rates they charge will have to be the same, $R_i = R$. We focus on symmetric outcomes where if $R_i = R_j$, then $b_i = b_j$. Then, $b_i = b$ for all $i \in [0,1]$, so $\int_0^1 b_i R_i di = Rb$.

In the second period, the borrower decides to default if and only if $U(y - bR) \leq U(y^d)$, so the default threshold for output is given by

$$y \leq y^d + bR.$$

The probability of default is then $F[y^d + bR]$.

Since creditors are risk neutral, in order for them to be indifferent between lending at R or at the risk-free rate R^*, the expected return from lending to the borrower must be the same as R^*, so

$$R^* = R\left[1 - F\left(y^d + bR\right)\right]. \tag{1}$$

This defines a locus of points (b, R) that can be interpreted as a supply curve of funds. The mapping from debt levels to interest rates is a correspondence because, in general, for each level of debt there are multiple interest rates that satisfy equation (1). Multiple functions can be selected with the points of the correspondence. We call those functions interest rate schedules.

The optimal choice of debt by the borrower is the one that maximizes utility:[4]

$$U(\omega + b) + \beta\left[F(y^d + bR)U(y^d) + \int_{y \geq y^d + bR} U(y - bR)f(y)dy\right]. \tag{2}$$

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[3]This timing specification assumes that the lenders move before the borrower. The standard assumption in the literature is that the borrower moves first, facing an interest rate schedule. We discuss the implications of this latter timing in Section 3.

[4]As is standard in the literature, we impose an upper bound on the level of debt. Absent this restriction, the optimal choice would be to borrow an arbitrarily large amount and default with probability one.

The marginal condition, for an interior solution, is

$$U'(\omega + b) = R\beta \int_{y \geq y^d + bR} U'(y - bR) f(y) \, dy. \qquad (3)$$

The optimal choice of debt for a given interest rate defines a locus of points (b, R) that can be interpreted as a demand curve for funds. The possible equilibria will be the points at which the demand curve intersects the supply curve described in (1).

An equilibrium in this economy is an interest rate $\widetilde{R}$ and a debt level $\widetilde{b}$ such that (i) given $\widetilde{R}, \widetilde{b}$ maximizes (2); and (ii) the arbitrage condition (1) is satisfied.

2.1 Multiple equilibria

In order to analyze the supply curve defined implicitly by (1), we define the function for the expected return on the debt,

$$h(R; b) = R \left[1 - F \left(y^d + bR \right) \right],$$

which in equilibrium must be equal to the riskless rate, R^*.

Notice that the function h is the product of two terms, R and $\left[1 - F \left(y^d + bR \right) \right]$ that is decreasing in R. Whether the function h is increasing or decreasing in R depends on which term dominates. For various commonly used distributions, such as the normal, the first term dominates for low levels of R and the second dominates for high levels of R, so that the function h is concave.[5] In this case there are at most two finite solutions of equation (1) for R, given b.

In Figure 1, the curve for $h(R; b)$ is depicted against R, for the cumulative normal distribution,[6] for three levels of debt. An increase in b shifts the curve downward so the solutions for R are closer to each other. For the low interest rate solutions on the left, an increase in b increases the rate, whereas for the high interest rate solutions on the right, an increase in b decreases the rate. To see this, notice that an increase in b increases the probability of default, therefore reducing the expected return. When interest rates are high, the expected return responds negatively to an increase in the rate because of the effect on the probability of default. The way to increase the expected return $h(R; b)$ back to R^* is by lowering the interest rate. It can also be seen from the figure that if b is high enough, there are no solutions.

The curve of the expected return h as a function of R can be interpreted as a Laffer-type curve, with the high interest rate solutions lying on the "wrong" side of the curve.

Figure 2 plots the solutions for R in equation (1) for each level of debt. This is a supply curve with two monotonic schedules. For lower values of the interest rate, there

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[5]The function $h(R; b)$ does not need to be concave everywhere. We explore this below.
[6]The distributions we consider are truncated below to avoid negative values for the endowment.

is a schedule that is increasing in b (solid blue line). There is also a decreasing schedule for higher values of the interest rate (dotted red line).

Figure 3 depicts the demand curve (dashed black line) and the supply curve (solid blue and dotted red lines) for the normal distribution and constant relative risk aversion utility. As can be seen, there are multiple equilibria.

The points on the high interest rate, decreasing schedule have particularly striking properties. Not only does the interest rate go down with the level of debt, b, but also the gross service of the debt, Rb, decreases with the level of debt, b. For those high rates, creditors jointly benefit from lowering interest rates because of their effect on the probabilities of default. For this reason, equilibria on the decreasing schedule can be refined away.[7] Those equilibria on the decreasing schedule may also be unstable. These are all features of the high interest rate equilibria for the distributions considered in Calvo (1988). As it turns out, even if the multiplicity in Calvo (1988) can be refined away, there is still multiplicity that is robust. This is the content of the next section.

2.1.1 A distribution with good and bad times

We now consider the case in which the endowment is drawn from a bimodal distribution, with good and bad times. One example can be constructed by combining two normal distributions, one with a high mean (good times) and one with a low mean (bad times), with nature choosing the distribution to draw from with some probability.

If the means of the two distributions are sufficiently far apart relative to the standard deviations, then equation (1) has four solutions for some debt levels, as Figure 4 shows. In order to understand the shape of the h function, it is important to note that the expected return h is decreasing in R if the cumulative distribution F is sensitive enough to changes in R, so that the second term dominates. That is going to happen for relatively low levels of R because for the bimodal distribution, there is mass at low levels of output. This explains the first decreasing part of the curve. For higher levels of R, the cumulative distribution F is relatively insensitive to changes in R because intermediate levels of the endowment are relatively unlikely to happen with a bimodal distribution. This explains the second increasing part of the curve.

For low enough debt levels, there are only two solutions, so there is only one increasing schedule. But for intermediate levels of debt, the equation has four solutions and therefore there are two increasing schedules. This means that, even if equilibria on decreasing schedules are ruled out, the model still exhibits multiplicity. If debt is very high, there is again a single increasing schedule. Thus, multiplicity depends on fundamentals and does not arise if there is either too little or too much debt.

The supply curve is plotted in Figure 5. The demand curve is the dashed black

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[7]See Ayres et al (2015) for a possible refinement.

line in the same figure. The demand is discontinuous in this case, since the maximum problem in (2) has two interior local maxima because of the bimodal distribution. As the interest rate changes, the relative value of utility between the two local maxima changes. Multiplicity only arises if the demand is high enough, so the resulting equilibrium level of debt is high.

2.2 Current debt versus debt at maturity

We now consider an alternative game in which the timing of moves is as before, but now the borrower chooses the value of debt at maturity, which we denote by a rather than by the amount borrowed, b. Are there still multiple equilibria in this setup? The answer is yes. This is a relevant question because in the models of Calvo (1988) and Arellano (2008), the assumption of whether the borrower chooses b or a is key to having uniqueness or multiplicity of equilibria.

We maintain the assumption that the creditors move first and offer to lend at gross interest rate R. The borrower moves next and picks the level of debt at maturity a. In the second period, the borrower defaults if and only if $y \leq y^d + a$. Arbitrage in international capital markets implies that

$$R^* = R \left[1 - F \left(y^d + a \right) \right]. \tag{4}$$

The locus of points (a, R) defined by (4), which we interpret as a supply curve of funds, is monotonically increasing (which is not the case for the supply curve in b and R defined in (1)).

The marginal condition of the borrower's utility maximization problem is

$$U' \left(\omega + \frac{a}{R} \right) = R\beta \int_{y \geq y^d + a} U'(y - a) f(y) \, dy, \tag{5}$$

where $\frac{1}{R}$ is the price of one unit of a as of the first period.

The locus of points (a, R) defined by (5) is a demand curve for funds. Again, this demand curve has multiple intersection points with the supply curve. Provided the choice of a is interior, those points are the solutions to the system of two equations, (4) and (5). Those are the same two equations (1) and (3) that determine the same equilibrium outcomes for R and b for $a = Rb$.

Figure 6 plots the supply curves for (b, R) and (a, R) defined in (1) and (4), respectively. It also plots the demand curves defined in (3) and (5). With the timing assumed so far, whether the borrower chooses debt net or gross of interest is irrelevant.

3 Timing of moves and multiplicity: Related literature

The timing of moves assumed above, with the creditors moving first, amounts to assuming that the borrower takes the current price of debt as given. The more common assumption in the literature is that the borrower moves first, choosing debt levels b or a, and facing a schedule of interest rates as a function of those levels of debt, depending on whether the choice is b or a, respectively.

Suppose the schedule the borrower faces is a function of a, corresponding to the supply curve derived from (4) and depicted in the right-hand panel of Figure 6. This is a monotonically increasing function. Since the borrower can choose a, the borrower is always going to choose in the low interest rate (and low a) part of the schedule. The equilibrium is unique.[8] These are the assumptions in Aguiar and Gopinath (2006) and Arellano (2008).

Suppose now, as in Calvo (1988), that the borrower is offered an interest rate schedule as a function of b, selected from the correspondence defined in (1), which can be the low rate increasing schedule or the high rate decreasing one (left-hand panel of Figure 6) .[9] Then by picking b, the borrower is not able to select the equilibrium outcome.

In summary, the assumption on the timing of moves is a key assumption in generating multiple equilibria. If the creditors move first, there are multiple equilibrium interest rates and debt levels, and they are the same equilibria whether the borrower chooses current debt or debt at maturity. If the borrower moves first, however, and chooses debt at maturity, there is a single equilibrium. Choosing debt at maturity amounts to picking the probability of default and therefore the interest rate as well. Finally, if the borrower moves first and chooses the current level of debt, given an interest rate schedule, then the equilibrium will depend on the schedule, and there are multiple such schedules.[10]

Our timing assumption, with the creditors moving first, is analogous to the timing assumption in Bassetto (2005) that generates multiple Laffer curve equilibria. In Bassetto, if households move first, choosing how much labor to supply, there are both low and high tax rate equilibria. If the government were to move first and pick the tax, however, there would be a single low tax equilibrium.[11]

Lorenzoni and Werning (2013) assume the standard timing and also assume that the interest rate schedule is a function of b, as in Calvo (1988). They argue against the

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[8]See Auclert and Rognlie (2016) for a uniqueness proof.

[9]Any other combination of those two schedules is also possible.

[10]In the model in Eaton and Gersovitz (1981), the borrower moves first, so it is key whether the equilibrium schedule is in b or a. In our notation, they consider a schedule in b, $R(b)$. They assume that $R(b)b$ cannot go down when b goes up. This amounts to excluding decreasing schedules by assumption. See proof of Theorem 3 in Eaton and Gersovitz (1981).

[11]Bassetto (2005) convincingly argues that the assumption that the government is a large agent is unrelated to the timing of the moves. See also Farhi and Tirole (2012) for similar timing assumptions.

assumption that the government may pick the debt level a. They devise a game in which the period is divided into an infinite number of subperiods. The government chooses end of period debt a, but cannot commit not to reissue within the period. In that game there are still multiple equilibria. The possibility to always reissue is as if the borrower is moving second, after the creditors, as in our timing. Another way to relate the game in Lorenzoni and Werning to our timing assumption is to notice that the government that cannot commit not to reissue is going to compete with its future self, as in the durable good monopoly. In equilibrium, the borrower behaves as a price taker, just as the borrower in our simple game with our timing assumptions.

4   Concluding remarks

In models with sovereign debt, interest rates are high because default probabilities are high. The object of this paper is to investigate conditions under which the reverse is also true: that default probabilities are high because interest rates are high. This means that there can be equilibrium outcomes in which interest rates are unnecessarily high and in which policy arrangements can bring them down. This exploration is motivated by the recent sovereign debt crisis in Europe,[12] but it is also motivated by a literature that does not appear to be consensual in this respect. Indeed, although Eaton and Gersovitz (1981) claim that there is a single equilibrium, Calvo (1988), using a similar structure, shows that there are both high and low interest rate equilibrium schedules. Aguiar and Gopinath (2006) and Arellano (2008) modify an assumption on the choice of debt (debt at maturity rather than current debt) by the borrower and find a single equilibrium. We show that small changes in timing assumptions and actions of agents can clarify the reasons for the apparently conflicting results.

Assumptions on whether the country chooses the debt net of interest payments or gross of those payments, or whether the borrower moves first or the creditors do, are not assumptions that can be obtained directly from data. Bond auctions are for announced quantities of discount bonds, but those quantities are revised many times. So it is not clear whether the choice at those auctions is for current debt or debt at maturity. Even if quantities of discount bonds were not revised, there are multiple auctions in a reference period, and how many auctions there are is a choice variable. In some auctions, lenders place price-quantity schedules, but those quantities are the intended purchases of the lender, not the aggregate quantities. Instead, schedules in the models in which the borrower moves first have the interest rate be a function of the aggregate quantity. Auction data could in principle be used to build a downward-sloping schedule in the aggregate

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[12]See De Grauwe and Ji (2012) on the poor correlation between spreads and fundamentals during the European sovereign debt crisis.

quantities, and those data are available at least for discriminatory price auctions.[13] If offers are to be ranked from high to low price, with corresponding total quantities, the schedule will be downward sloping. Can this schedule correspond to the equilibrium schedule in a model in which the borrower moves first? The equilibrium data produced by the model do not show a schedule, just a point, since homogeneous creditors offer the same price. Heterogeneous beliefs could explain a downward-sloping schedule, but that would be the case independently of the timing assumed.

With our timing assumptions, there are both high and low interest rate equilibria. For commonly used distributions of output, the high interest rate equilibria are fragile. That is not the case, however, if the stochastic process for output is bimodal, meaning that, with relatively high probability, output can be either very high or very low. Possible empirical content are the long periods of growth followed by long stagnations that can be found in the data. In the unraveling of the sovereign debt crisis in Europe, for some of the countries particularly exposed to it, such as Portugal, Spain, or Italy, the hypothesis that those economies were then facing a long period of stagnation seems plausible.

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[13]We thank Mark Aguiar for raising some of these issues.

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A Figures

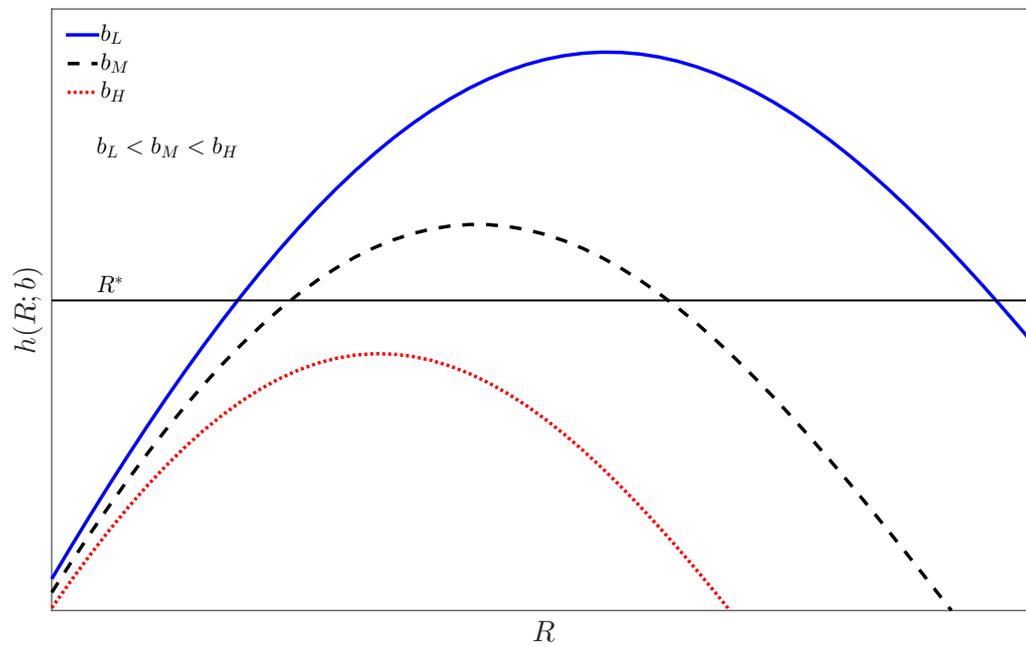Figure 1: Expected return $h(R; b)$
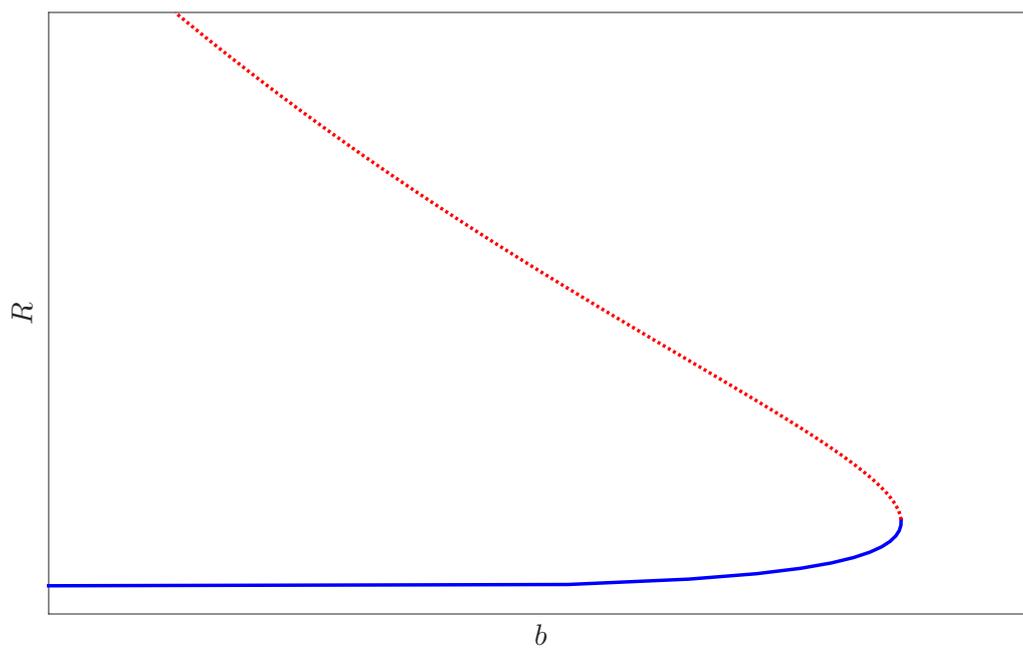

Figure 2: Interest rate schedule $R(b)$

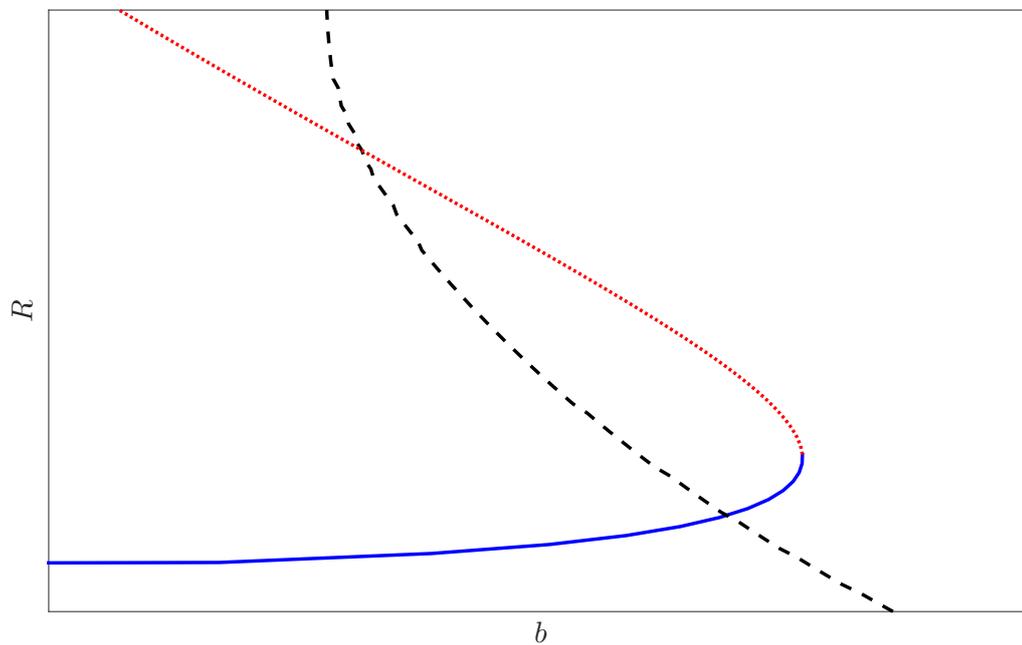Figure 3: Equilibrium borrowing and interest rate

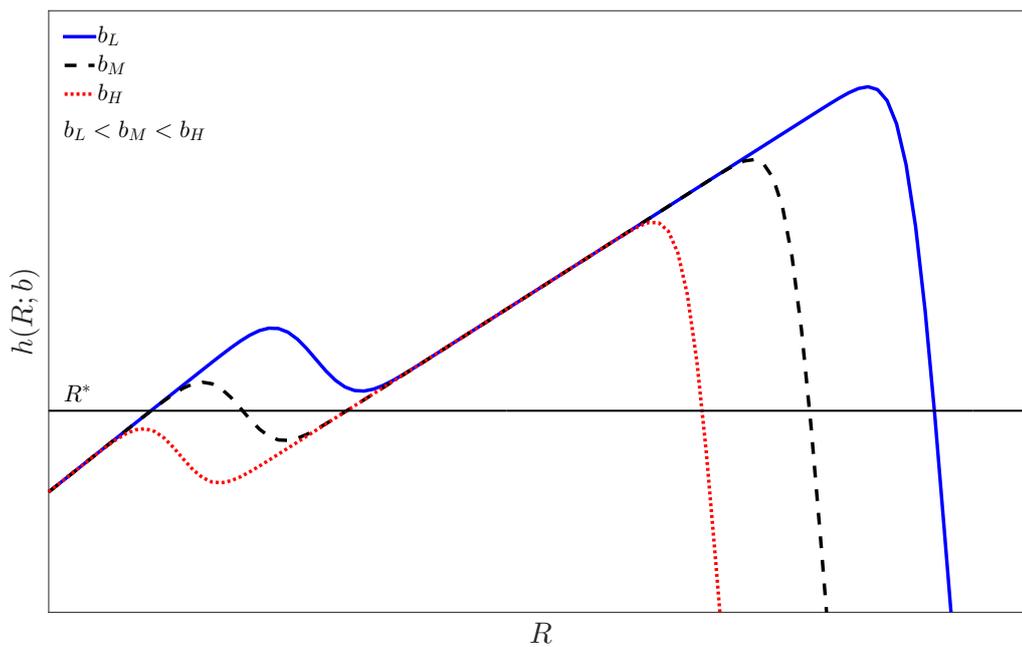Figure 4: Expected return for the bimodal distribution $h(R; b)$

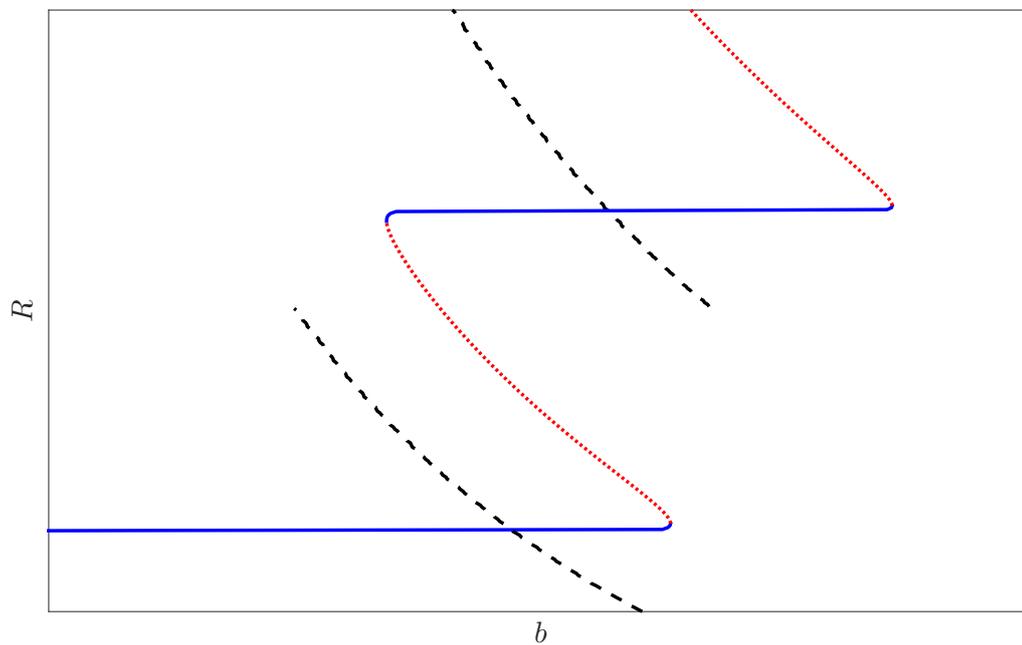Figure 5: Equilibrium borrowing and interest rate for the bimodal distribution

Figure 6: Equilibrium outcomes with choice of b or a