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Search-Based Endogenous Asset Liquidity and the Macroeconomy

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Abstract

We develop a search-theory of asset market liquidity which gives rise to endogenous financing constraints in an otherwise standard dynamic general equilibrium model. Asset liquidity describes the ease of issuance and re-saleability of private financial claims for a certain price. We model asset liquidity as an outcome of the participation margins of buyers and sellers on an asset market, where financial intermediaries implement a costly search-and-matching process. Limited market liquidity of private claims creates a role for liquid assets, such as fiat money, to ease financing constraints. We show that endogenising liquidity is essential to generate positive co-movement between asset (re)saleability and asset prices. When the capacity of the asset market to channel funds to entrepreneurs deteriorates, investment falls while the hedging value of liquid assets increases, driving up liquidity premia. Our model, thus, demonstrates that shocks to the intermediation capacity of financial markets can be an important source of flight-to-liquidity dynamics and macroeconomic fluctuations, matching key business cycle characteristics of the U.S. economy.

Keywords: endogenous asset liquidity; asset search frictions; financing constraints; general equilibrium
Jel codes: E22; E44; G11

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Acknowledgments

1 Introduction

Asset market liquidity describes the ease with which financial claims can be traded and their price impact. The frictions associated with asset transactions also motivate the demand for highly liquid assets, such as fiat money or government bonds, to hedge future liquidity risks. Investors thus demand a premium for bearing liquidity risks.

Empirical evidence points to *countercyclical* variation in both liquidity premia and portfolio shifts towards highly liquid assets.[1] Prices across a wide range of financial assets, on the other hand, are *procyclical and volatile.* As observed, for instance, during the 2007-2009 financial crisis, deteriorating asset market liquidity is also a key feature of economic downturns precipitated by a meltdown in the financial sector.[2]

How can these dynamic interactions between the macroeconomy and asset markets be captured in a standard macro model? We address this issue by developing a search theory of asset market liquidity which gives rise to *endogenous* financing constraints in general equilibrium. Money can exist in equilibrium to relax financing constraints. This simple search theory captures typical asset market imperfections including participation costs, transaction costs, imperfect competition, and funding constraints observed in micro-level data (see e.g., Vayanos and Wang, 2013). In addition, our theory also accounts for the extensive margin of asset liquidity, i.e., the numbers of trades, and its relationship with asset prices as in the data. The model embeds a two-way feedback between asset liquidity and macroeconomic dynamics, which gives rise to endogenous amplification of financial and real shocks.

Our theory of endogenous asset liquidity and financing constraints contrasts with existing works on dynamic macro-financial interactions, which often involve *exogenous* shocks operating directly on agents' financing constraints. In particular, well-known general equilibrium models such as Kiyotaki and Moore (2012) and Jermann and Quadrini (2012) introduce a negative shock to exogenous asset saleability which tightens firms' financing constraints, but does not reduce the optimal amount of investment. As a result, assets that remain liquid record higher prices while the economy slides into recession. The counterfactual response of asset prices to financial shocks in these frameworks has been documented by Shi (2015).

To avoid this asset pricing anomaly in response to financial shocks, we model asset

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[1]Studies by Huberman and Halka (2001), Chordia, Roll, and Subrahmanyam (2001), Chordia, Sarkar, and Subrahmanyam (2005), and Naes, Skjeltorp, and Odegaard (2011) assert that market liquidity is procyclical and highly correlated across asset classes such as bonds and stocks in the US.

[2]Dick-Nielsen, Feldhütter, and Lando (2012) identify a structural break in the market liquidity of corporate bonds at the onset of the sub-prime crisis. The liquidity component of spreads of all but AAA rated bonds increased and turnover rates declined, making refinancing more difficult. Similarly, the liquidity of commercial paper slumped as reported by Anderson and Gascon (2009), with money market mutual funds, the main investors in the commercial paper market, shifting to highly liquid and secure government securities. Finally, Gorton and Metrick (2012) show that haircuts on repo markets increased strongly during the crisis, thus undermining the use of financial paper as collateral repo transactions.

saleability that depends on asset price directly. Liquidity of privately issued financial assets is a function of the participation margins of buyers and sellers, which is characterised by a costly search-and-matching process. Illiquidity of private financial claims creates a role for money (liquid assets), which provide insurance against future financing constraints. Privately issued assets must thus pay an endogenous liquidity premium over money.

We find that the cyclical behaviour of asset prices and the liquidity premium depend on the underlying drivers of macroeconomic fluctuations. Following a persistent fall in aggregate productivity, asset prices and investment fall. Since there is less investment need, financing constraints are less tight, and the hedging value of money and, hence, the liquidity premium fall. Following adverse shocks to intermediation costs in asset markets, however, both asset saleability and asset prices fall simultaneously, thereby tightening firms' financing constraints endogenously and amplifying the initial shocks. Tighter financing constraints increase the hedging value of money, such that the liquidity premium rises. Our framework, thus, shows that shocks to the intermediation costs on asset markets are essential to jointly capture the countercyclical liquidity premium, procyclical and volatile asset prices, and large fluctuations of macroeconomic variables as observed in the data.

Mechanism. Consider an economy where both money and privately issued financial claims circulate. The latter are backed by the cash flow from physical capital, which is owned by households and rented to final goods producers. All household members are endowed with a portfolio of money and private claims. In each period, household members are temporarily separated and face idiosyncratic investment risks. Some become workers, others entrepreneurs. Only entrepreneurs have investment opportunities for capital goods creation.

Entrepreneurs can finance investment using their fully liquid cash balances, and they can tap into private asset markets by issuing new financial claims on their investment projects and by liquidating their existing portfolio of private assets. Private claims (both new and old) are only partially liquid. They are traded on a search market where intermediaries offer costly matching services for buy and sell orders. Only a fraction of quoted orders is successfully matched each period. This fraction is endogenously determined by the participation intensity on either side of the market. For instance, the more buy orders are posted relative to sell orders, the easier it is to match a sell order. This limited saleability of financial claims captures the quantity dimension of asset liquidity.

Intermediaries determine the transaction price in successful matches by maximizing the total match surplus, similar to the bargaining process in the labour search literature (Mortensen and Pissarides, 1994; Shimer, 2005). As the match surpluses of buyers and sellers depend on the ease with which private claims can be traded, the transaction price also depends on the relative asset supply and demand conditions, thus linking the quantity with

the price dimension of asset liquidity. Finally, the intermediation search cost drives a wedge between this transaction price and the effective purchase and sale prices.[3]

In the steady state, fiat money is valued for its liquidity service as long as intermediation costs are large enough. Private assets then need to pay a liquidity premium over money in order to compensate investors for carrying liquidity risks. The higher the premium, the lower is the price of private assets. However, if intermediation costs become too large, sellers stop offering their assets for sale, such that private asset markets break down and only money circulates. Therefore, our endogenous asset liquidity framework embeds the inherent fragility of the financial sector.

Dynamic and Empirical Properties. We consider two types of *persistent* exogenous shocks: an aggregate productivity (TFP) shock and a shock to the intermediation costs. The latter captures any generic disruption in the financial sector that affects the cost of providing intermediation services and, therefore, the provision of private liquidity.

Negative TFP shocks decrease the return to capital, make investment into capital goods less attractive, and hence crowd out investors from the search market. Adverse intermediation cost shocks (that increase search costs) make investment into liquid assets more attractive as a hedge against future financing constraints. This reduces investors' incentives to post costly buy orders.

In either case, the fall in demand on the asset market exceeds that of supply, such that sell orders have a lower chance of being matched with a buy order. Hence, the saleability of financial claims decreases. At the same time, the asset price falls as the demand effect dominates the supply effect. Limited saleability and lower asset prices jointly tighten entrepreneurs' financing constraints, such that fewer resources are transferred to entrepreneurs in the aggregate. Real investment thus falls and economic activity contracts.

While both shocks generate procyclical asset saleability and prices, only adverse intermediation cost shocks induce a persistent flight to liquidity, manifested in a higher liquidity premium. Negative TFP shocks depress the expected return on capital, thereby exerting downward pressure on the profitability of future investment projects. Therefore, investors have a weak incentive to hedge against future financing constraints. Adverse intermediation cost shocks, however, do not affect the quality of investment projects as such. Investors strongly value the hedging service provided by money and rebalance their asset portfolios accordingly. More active portfolio rebalancing increases asset price volatility.

To confront the model implications for the cyclical properties of asset market liquidity

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[3]A financial system is often characterised by multiple layers of financial intermediaries, such as custodians, dealers, or market makers (Shen, Wei, and Yan, 2015). These provide costly brokerage, clearing, and settlement services to attenuate trading fictions. Our costly search-and-matching framework is designed to capture these generic participation costs arising from financial services.

with the data, we construct an illiquidity difference measure as a proxy for the liquidity premium. This measure is based on Amihud (2002), taking into account both volumes and prices to quantify the degree of asset liquidity. Intermediation cost shocks are able to match the countercyclical illiquidity difference measure and procyclical, but volatile, asset prices (in addition to the dynamics of macroeconomic variables) in the U.S. data. TFP shocks, on the other hand, generate a strongly procyclical liquidity premium, and procyclical, but insufficiently volatile, asset prices. Liquidity premia thus emerge as a potential discriminant between financial sector and productivity shocks.

Our paper's main message can be summarized as follows: when both private assets and fiat money co-exist, the price of private assets can fall with adverse financial shocks (either permanent or temporary) if the increase of the liquidity premium dominates the contraction of asset supply. Falling asset prices, in turn, tighten financing constraints and generate large aggregate fluctuations as in the data.

Relation to Literature on Macroeconomics with Financial Frictions. By studying intermediation cost shocks which affect the financial market directly, we complement the literature on financial shocks. Recent contributions such as Jermann and Quadrini (2012) and Jaccard (2013) identify financial shocks as an important source of business cycles. Gazzani and Vicondoa (2016) provide evidence that liquidity shocks in secondary sovereign debt markets can have potent real effects on firms' financing constraints.

As regards the analysis of the impact of financial shocks on macroeconomic dynamics, our framework is related to Kiyotaki and Moore (2012) (henceforth KM) and Shi (2015). These studies propose models with *exogenous* differences in the market liquidity between private claims and government-issued assets to study the macroeconomic impact of liquidity shocks. Del Negro, Eggertsson, Ferrero, and Kiyotaki (2016) extend the KM framework with the zero lower bound on nominal interest rates to simulate unconventional monetary policy in response to an exogenous liquidity crisis.[4] Ajello (2012) studies exogenous bid-ask spread shocks within a New Keynesian framework and KM frictions.

Nevertheless, our search-theory of asset market liquidity differs conceptually from these studies. First, as we endogenously link asset saleability with asset prices and bid-ask spreads, asset liquidity in our framework *jointly* captures the speed of asset transactions and the associated costs and price impact. Second, models with exogenous asset market liquidity ignore the feedback effects transmitted from the real economy to the financial system, while this paper features a two-way feedback between financial markets and macroeconomic conditions. Finally, the above papers, are plagued by the asset pricing anomalies critiqued by

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[4]The macroeconomic impact of the liquidity freeze during the 2007-2009 financial crisis is also studied by Radde (2015). Kara and Sin (2013) show that market liquidity frictions induce a trade-off between output and inflation stabilization off the zero lower bound that can be attenuated by quantitative easing measures.

Shi (2015), much in contrast to our framework.

Relation to Literature on Asset Market Liquidity. The empirical literature emphasises trading delays as well as volatile market depth, trading volumes and bid-ask spreads as salient features of asset markets (Bao, Pan, and Wang, 2011; Gavazza, 2011). As these features emerge naturally from search frictions, the latter are a logical starting point for a theory of asset liquidity. The pioneering work of Duffie, Gârleanu, and Pedersen (2005) applies search theory to model trading frictions on over-the-counter (OTC) markets.

This framework has been extended to include general asset holdings (Lagos, Rocheteau, and Weill, 2011), liquidity provision (Weill, 2007), and markets for a wide range of financial assets, such as asset-backed securities, corporate bonds, federal funds, private equity and housing (Duffie, Gârleanu, and Pedersen, 2007; Ashcraft and Duffie, 2007; Feldhutter, 2011; Wheaton, 1990; Ungerer, 2012). Rocheteau and Weill (2011) provides an extensive survey on search theory and asset market liquidity. Meanwhile, Den Haan, Ramey, and Watson (2003), Wasmer and Weil (2004), and Petrosky-Nadeau and Wasmer (2013) have emphasised the role of search frictions in credit markets and their impact on aggregate dynamics.[5]

An alternative approach to endogenising asset liquidity focuses on information frictions, such as the adverse selection models in Eisfeldt (2004) and Guerrieri and Shimer (2014). But these studies do not consider the feedback effects of liquidity fluctuations on production. Two notable exceptions are Kurlat (2013) and Bigio (2015), who extend KM with endogenous resaleability through adverse selection. However, both papers ignore the role of liquid assets, which may be owed to insufficient tractability. Besides tractability reasons, search can capture the extensive margin of asset liquidity, i.e., the numbers of trades, and its relationship with asset prices. This feature allows us to link the model with empirical measures of liquidity premia as described above.

Search-theoretic models of money, such as Shi (1995), Trejos and Wright (1995), Lagos and Wright (2005), Rocheteau and Wright (2005), and Guerrieri and Lorenzoni (2009) have highlighted the importance of money for transaction purposes on anonymous search markets. The framework has been extended to analyse asset liquidity and pricing with multiple types of assets (Lester, Postlewaite, and Wright, 2012), privately created liquid assets (Lagos and Rocheteau, 2008), trading delays with market makers (Lagos and Zhang, 2016), and bank-deposits (Williamson, 2012). Rocheteau (2011) shows that the trading restrictions from the money-search framework can be derived from tractable microfoundations exploiting the relative information-sensitivity of different financial assets.

Our model differs from these studies in that private claims are subject to search frictions

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[5]Further, Kurmann and Petrosky-Nadeau (2006) study search frictions associated with physical capital in a macroeconomic setting. As shown in Beaubrun-Diant and Tripier (2013), search frictions also help explain salient business cycle features of bank lending relationships.

themselves, rather than serving to overcome search frictions on the goods market. More importantly, we consider endogenous supply of financial assets tied to physical investment and production in a standard business cycle model. The model thus features liquidity and financing constraints on primary and secondary asset markets,[6] liquid assets as the lubricant of investment financing, and feedback effects between asset liquidity and the real economy.

In this regard, our paper is related to Yang (2014) and Cao and Shi (2014), which apply search theory to asset or capital markets. In Yang (2014), TFP shocks can generate co-movement between asset liquidity and prices similar to our model. However, we show that asset prices are not sufficiently volatile in response to TFP shocks compared to the data. In addition, TFP shocks generate a pro- rather than countercyclical liquidity premium. Cao and Shi (2014) emphasize capital reallocation, while we focus on asset pricing implications of asset liquidity and liquidity premium.

Finally, while sharing similarities, this paper differs along important dimensions from Cui and Radde (2016). First, the latter introduces directed search and intermediation chains on asset markets in contrast to the random search approach used here. Second, the latter model exhibits equilibrium multiplicity, thereby compromising its tractability. Third, Cui and Radde (2016) does not offer insights into the dynamic behaviour of asset liquidity including the distinct role of financial shocks as an important source of business cycles.

2   The Model

Time is discrete and infinite ($t = 0, 1, 2, ...$). The economy has three sectors: final goods producers, households, and financial intermediaries. There is a continuum of identical households (with measure one) and each household has a continuum of members. Similar to Shi (2015) and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2016), we use the big family structure to simplify aggregation.

2.1   Timing

Each period is divided into four sub-periods:

The Household's Decision Period. Aggregate shocks to productivity (A_t) and intermediation costs (κ_t) are realized. Types are still unknown and all members in a representative household *equally divide* the household's assets. The household instructs its members on their type-contingent decisions.

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[6]In this sense, we complement the studies of cyclical capital reallocation, such as Eisfeldt and Rampini (2006) and Cui (2013).

The Production Period. Each member receives a status draw, becoming an entrepreneur with probability χ and a worker otherwise. The type-draw is independent across members and over time. An entrepreneur has investment projects but no labour endowment, while a worker has a unit of labour endowment but no investment project. Both groups are temporarily separated during each period and there is no consumption risk insurance among them. Competitive firms rent aggregate capital stock K_t and hire aggregate labour N_t from households to produce output (consumption goods) according to a standard Cobb-Douglas production function:

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha},$$

where $\alpha \in (0,1)$ and A_t measures exogenous aggregate productivity. The profit-maximizing rental rate and wage rate are thus

$$r_t = \alpha A_t \left(\frac{K_t}{N_t}\right)^{\alpha-1} \quad \text{and} \quad w_t = (1-\alpha)A_t \left(\frac{K_t}{N_t}\right)^\alpha. \tag{1}$$

The Investment Period. Entrepreneurs use their return from capital and liquid assets, and seek further external funding to finance *scalable* investment projects, which can transform one unit of consumption goods into one unit of capital stock. Liquid assets, traded on a spot market, are fiat money in *fixed supply* with $\bar{B}$. Money is accepted by everyone in the economy, and the price is thus determined by the demand for liquid assets. Entrepreneurs sell private claims to the cash flow from their investment projects through intermediaries to workers (in exchange for consumption goods). The asset market, on which such claims are traded, is characterized by search frictions. Financial intermediaries implement a costly matching process in the asset market and determine the transaction price.

The Consumption Period. After investment, agents of both types consume. Then, they return to their households with their assets and pool these assets together.

2.2   A Representative Household

Let c_t^i denote the consumption of an individual entrepreneur, and c_t^n and n_t denote the consumption and hours worked of an individual worker.

Preferences. The household aggregates the utility of consumption and the dis-utility of labour supply from all its members according to

$$\mathbb{E}_0 \sum_{t=0}^\infty \beta^t \{\chi u(c_t^i) + (1-\chi)u(c_t^n) - (1-\chi)h(n_t)\},$$

where $\beta \in (0,1)$ is the household's discount factor, the expectation is taken over aggre-

gate shocks (A_t, κ_t), $u(.)$ is a standard strictly increasing and concave utility function of consumption, and $h(.)$ captures the dis-utility derived from labour supply $n_t \in [0, 1]$.

Balance Sheet. Households can invest into nominal and fully liquid assets (money) with a nominal price P_t. Physical capital (K_t^h) held by households is rented to final goods producers, earning a rental return r_t. Entrepreneurs can issue equity claims to the future returns on newly created capital. Following Kiyotaki and Moore (2012), we normalize one unit of issued assets to be a claim on the stream of future returns on one unit of investment at time t, which amounts to: r_t, $(1 - \delta)r_{t+1}$, $(1 - \delta)^2 r_{t+2}$, ...

In general, a household has three kinds of assets: liquid assets (B_t); financial claims on other households' return on capital (S_t^O); and own physical capital (K_t^h). These assets are financed by net worth and private financial claims issued to outside investors (S_t^I), backed by a fraction of the households' own physical capital. This financing structure gives rise to the beginning-of-period balance sheet in Table 1. It turns out that, besides liquid assets B_t, we only need to keep track of net private financial claims S_t, defined as

$$
\begin{aligned}
S_t &\equiv \text{ financial claims on other households' capital } + \text{ unissued capital} \\
&= S_t^O + (K_t^h - S_t^I).
\end{aligned}
$$

To see this, let q_t be the transaction price of a private financial claim. Both existing claims to capital (i.e., S_t^O) and the fraction of the capital stock on which no financial claims have been written yet (i.e., $K_t^h - S_t^I$) could be offered on the search market, and both the saleable parts are valued at q_t. Therefore, we can focus on S_t alone.

Table 1: A Household's Balance Sheet

Assets		Liabilities	
liquid assets holding	B_t/P_t	financial claims issued	$q_t S_t^I$
financial claims-			
on other households' capital	$q_t S_t^O$		
capital	$q_t K_t^h$	net worth	$q_t S_t + B_t/P_t$

Asset Accumulation. Let S_t^j and B_t^j denote net private financial claims and money held by entrepreneurs $(j = i)$ or workers $(j = n)$. Let S_{t+1}^j and B_{t+1}^j denote the end-of-period asset positions at t. Since all financial assets are equally divided, the fraction of private claims held by entrepreneurs and workers corresponds to their respective population shares, i.e., $S_t^i = \chi S_t$, $S_t^n = (1 - \chi)S_t$, $B_t^i = \chi B_t$, and $B_t^n = (1 - \chi)B_t$.

To simplify, we use recursive notation and omit time t in subscripts from now on. The

size of end-of-period net private financial claims satisfies

$$S_{+1}^j = (1 - \delta)S^j + I^j - M^j, \tag{2}$$

where I^j is physical investment, and M^j corresponds to the quantity of private claims sold by group j members. When M^j is negative, group j members are buying private claims.

Workers' Constraints. The household delegates purchases of private claims to workers, because they earn a wage rate w and do not have investment opportunities ($I^n = 0$). Therefore, workers post bid quotes (i.e., purchase orders) of size V through financial intermediaries to acquire new or old private claims at a unit search cost κ. On the search market, each bid is matched with an ask quote by financial intermediaries with an endogenous probability $f \in [0, 1]$, such that an individual buyer expects to purchase an amount fV (or sell $M^n = -fV$). Workers' flow-of-funds constraint in terms of consumption goods reads

$$C^n + \kappa V + q f V + \frac{B_{+1}^n}{P} = wN + r(1 - \chi)S + \frac{(1 - \chi)B}{P}, \tag{3}$$

where $C^n = (1 - \chi)c^i$ and $N = (1 - \chi)n$. That is, labour income, the return on private claims, and money are used to finance consumption, search costs, and the new acquisition of private claims and money. Finally, workers cannot issue money:

$$B_{+1}^n \geq 0. \tag{4}$$

Entrepreneurs' Constraints. In order to finance new investment ($I^i > 0$), entrepreneurs can use return on their claims on capital and liquid assets; they can also post ask quotes (i.e., sales quotes) of a size U, backed by private financial claims, for sale at the unit search cost κ. These assets include existing net private claims $(1 - \delta)\chi S$, plus claims on new investment I^i:

$$U \leq (1 - \delta)\chi S + I^i. \tag{5}$$

U is bounded from above, because entrepreneurs may not respect the delivery of assets after receiving payments. Therefore, intermediaries ensure that all quotes are backed by capital; if entrepreneurs default, intermediaries can seize the assets. Ask quotes are matched with bid quotes with the - endogenously determined - probability $\phi \in [0, 1]$. Therefore, entrepreneurs expect to sell $M^i = \phi U$ units of financial claims, and their flow-of-funds constraint is

$$C^i + \kappa U + I^i - q\phi U + \frac{B_{+1}^i}{P} = r\chi S + \frac{\chi B}{P}, \tag{6}$$

where $C^i = \chi c^i$. That is, the returns on private claims and money are used to finance

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consumption, search costs, physical investment not funded by the revenue from asset issuance and reselling, and end-of-period money holdings. Similar to workers, entrepreneurs cannot issue money

$$B_{+1}^i \geq 0. \tag{7}$$

The Household's Problem. Let $J(S, B; \Gamma)$ be the value of the representative household with net private financial claims S, money holdings B, given the aggregate state variable $\Gamma \equiv (K, \bar{B}; A, \kappa)$. Since at the end of period t, workers and entrepreneurs reunite to share their stocks of private claims and money, we have

$$S_{+1} = S_{+1}^i + S_{+1}^n = (1 - \delta)S + I^i - \phi U + fV, \tag{8}$$

$$B_{+1} = B_{+1}^i + B_{+1}^n. \tag{9}$$

where we know end-of-period asset holdings $S_+^i = (1 - \delta)\chi S + I^i - \phi U$ and $S_+^n = (1 - \delta)(1 - \chi)S + fV$ from equation (2). The value $J(S, B; \Gamma)$ satisfies the following Bellman equation

$$J(S, B; \Gamma) = \max_{\left\{N, C^i, C^n, I^i, U, V, B_{+1}^i, B_{+1}^n\right\}} \left\{ \chi u \left(\frac{C^i}{\chi} \right) + (1 - \chi) u \left(\frac{C^n}{1 - \chi} \right) - (1 - \chi) h \left(\frac{N}{1 - \chi} \right) \right.$$
$$\left. + \beta \mathbb{E} \left[J(S_{+1}, B_{+1}; \Gamma_{+1}) | \Gamma \right] \right\}, \quad \text{subject to} \quad (3) - (9).$$

2.3 Asset Search and Matching

Unlike money, privately issued financial assets typically have heterogeneous characteristics and may only attract investors with specific knowledge or investment strategies. Finding a counterpart to trade private financial assets can, therefore, be difficult. Financial intermediation helps address these information asymmetries and trading frictions in order to channel funds from savers to borrower. Even when conducted through decentralised markets, this process typically involves the costly services of financial agents. Costs related to financial intermediation may arise, for instance, from brokerage and settlement services offered by dealers and market makers on OTC markets, legal and advertising costs related to IPOs or fees collected by rating agencies.

In our framework, we capture these features through the costly search-and-matching process described below, where financial intermediaries operate the matching technology, determine the transaction price in successful matches, and settle the trade, thus performing the functions of financial agents in the market-based intermediation process.[7]

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[7]Alternatively, the intermediation process could be regarded as bank-based, with financial intermediaries being interpreted as banks offering costly screening and monitoring services and channeling funds from

Financial Intermediaries. Intermediaries collect bid quotes V from workers and ask quotes U from entrepreneurs. They verify that ask quotes are fully backed by capital. Then, they implement the matching technology against a participation - or intermediation - cost of κ per unit of the quoted quantities paid by buyers and sellers. Only a fraction of bid and ask quotes are successfully matched.

Search and Matching. The technology operated by financial intermediaries takes the form of a matching function, which is concave and homogeneous of degree 1 in (U, V) space:

$$M(U, V) = \xi U^\eta V^{1-\eta}, \tag{10}$$

where ξ captures matching efficiency and $\eta \in [0, 1]$ is the elasticity w.r.t. ask quotes. Let $\theta = V/U$ denote asset market tightness from buyers' perspective. Then,

$$\phi = \frac{M(U, V)}{U} = M(1, \theta) \quad \text{and} \quad f = \frac{M(U, V)}{V} = M(\theta^{-1}, 1)$$

are the probability that one unit of the ask quotes can be sold and the probability that one unit of the bid quotes can be purchased. Since ϕ_t also represents the fraction of financial assets that can be sold *ex post* in a given period, we refer to ϕ_t as *asset saleability.*

The above Cobb-Douglas matching function reflects the fact that as asset market tightness θ increases, it becomes easier for the sellers to find potential buyers (i.e., ϕ increases), whereas buyers have more difficulty in finding appropriate investment opportunities (i.e., f decreases). The opposite is true, when θ goes to zero.

Asset Prices. For simplicity, we refer to the price of private claims as "the" asset price. The transaction price of private claims is determined by a bargaining process, which sellers and buyers delegate to financial intermediaries. Therefore, once a unit of assets offered for sale is matched to a buy quote, intermediaries offer a price q_t to both parties. This price is chosen to maximizing the total surplus of the trade by bargaining on behalf of each side. As the amount of matched assets M^j is predetermined at the point of bargaining, buyers and sellers interact at the margin.

Buyers' and Sellers' Marginal Surpluses. Denote by J^n and J^i the marginal transaction surpluses of individual workers (buyers) and entrepreneurs (sellers). A buyer's surplus amounts to

$$J^n(S, B; \Gamma) = -u'\left(\frac{C_t^n}{1 - \chi}\right) q + \beta \mathbb{E}\left[J_S(S_{+1}, B_{+1}; \Gamma_{+1})|\Gamma\right], \tag{11}$$

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depositors to borrowers as in De-Fiore and Uhlig (2011). In the interest of tractability and to preserve the generic nature of the intermediation process, we refrain, however, from modeling financial intermediaries' balance sheets more explicitly.

which consists of the resources sacrificed today to acquire an additional unit of private claims and the value of this additional unit of asset holdings to its household tomorrow.[8]

Similarly, a seller's surplus is the marginal value to the household of an additional match for entrepreneurs. However, the sellers' surplus differs from that of buyers', because, unlike workers, entrepreneurs need to implement physical investment after a successful transaction. To see this, we express ask quotes as $U = e\left[(1-\delta)\chi S + I^i\right]$, where $e \in [0,1]$ according to the financing constraint (5). Then, a seller's surplus is given by

$$
\begin{aligned}
J^i(S, B; \Gamma) \;=\; & u'\left(\frac{C^i}{\chi}\right) q - \beta \mathbb{E}\left[J_S(S_{+1}, B_{+1}; \Gamma_{+1})|\Gamma\right] \\
& + e^{-1}\phi^{-1}\left[-u'\left(\frac{C^i}{\chi}\right) + \beta \mathbb{E}\left[J_S(S_{+1}, B_{+1}; \Gamma_{+1})|\Gamma\right]\right]
\end{aligned}
\tag{12}
$$

The first line in equation (12) mirrors the buyers' surplus with inverted signs: the first term is the marginal value of the additional resources *obtained* through the transaction, while the second captures the *loss* to the household on account of the reduction of tomorrow's asset holdings. The second line reflects the fact that entrepreneurs need to implement the households' investment plan. Moreover, financial intermediaries monitor the implementation of these investment plans in order to ensure that ask quotes are ex ante credibly backed by capital. Given the matching technology (10), entrepreneurs sell claims amounting to $M^i = \phi U = \phi e\left[(1-\delta)\chi S + I^i\right]$. Hence, entrepreneurs need to invest $\partial I^i/\partial M^i = e^{-1}\phi^{-1}$ per unit of resources obtained in a successful match, which reduces marginal consumption today by $e^{-1}\phi^{-1}$ while increasing tomorrow's equity position also by $e^{-1}\phi^{-1}$.

Importantly, the asymmetry in buyers' and sellers' surpluses reflects the fact that the model features endogenous supply on the asset market, which is a key innovation of the search framework. For a detailed derivation, see Appendix A.1.

"Bargaining". Note that all members within the groups of buyers and sellers are homogeneous, such that the type-specific valuations are identical in all matched pairs. Then, intermediaries set a price q to maximize

$$
\max_q \{(J^i)^\omega (J^n)^{1-\omega}\}
\tag{13}
$$

where $\omega \in (0,1)$ is the fraction of the surplus that goes to sellers. This set-up is similar to bilateral (generalized) Nash bargaining between buyers and sellers over the match surplus. In the bilateral bargaining case, ω is the bargaining power of sellers. Our set-up differs, however,

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[8]Note that search market participation costs are already sunk at the bargaining stage. However, search costs are not ignored since households take them into account when determining optimal asset posting decisions by workers and entrepreneurs.

in that intermediaries bargain on behalf of household members, such that household members do not think they can influence the asset price.[9]

2.4 Recursive Competitive Equilibrium

Having described the environment, we proceed with the equilibrium definition. A **recursive competitive equilibrium** consists of a mapping of state variables $(K, A, \kappa) \to (K_{+1}, A_{+1}, \kappa_{+1})$ and equilibrium objects that are functions of the state variables: a value function $J(S, B, \Gamma)$, policy functions for consumption, investment, labour, and portfolio choices $\{C^i, C^n, N, I^i, e, U, V, B^i_{+1}, B^n_{+1}\}$, asset market features $\{\theta, \phi, f\}$, and a collection of prices $\{P, q, w, r\}$, such that the following conditions are satisfied:

(i) final goods producers' optimality conditions in (1) hold;

(ii) given prices, the value function and the policy functions solve the representative household's decision problem, with e defined as $e \equiv U / [(1 - \delta)\chi S + I^i]$;

(iii) $\theta \in [0, +\infty)$ and the asset price q solves (13);

(iv) the capital market clears: $K_{+1} = (1 - \delta) K + I^i$ and $S = K$; the search market "clears": $\phi = M(1, \theta)$ and $f = M(\theta^{-1}, 1)$; and the money market clears: $B_{+1} = B = \bar{B}$;

(v) aggregate productivity A and intermediation costs κ follow some exogenous processes.

To verify that Walras' Law holds, we notice that entrepreneurs' and workers' budget constraints (3) and (6), together with (9) and the equilibrium conditions in (iv), imply the aggregate resource constraint

$$C + I + \kappa(V + U) = AK^\alpha N^{1-\alpha} = Y, \tag{14}$$

where aggregate consumption is $C = C^i + C^n$ and aggregate investment is $I = I^i$. For accounting purposes, gross investment expenditure is defined as $I + \kappa(V + U)$, of which only (net) investment I adds to the aggregate capital stock at time $t + 1$.

3 Equilibrium Characterisation

Money (public liquidity) is fully saleable, while private claims (private liquidity) are only partially convertible into consumption goods in each period due to the costly search-and-matching process. As a result, the equilibrium described in the previous section admits different types, which can be distinguished by the activity of asset markets and the kind of financial assets that circulate.

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[9]In this sense, our price setting is similar to the wage determining process in Ravn (2008) and Ebell (2011), where individual workers come to bargain on behalf of their respective households. That is, households take asset prices as given and the strategic bargaining issue from Rauch (2000) and Zhu (2008) do not arise.

One polar case is autarky, i.e., an equilibrium in which neither private claims nor money exist.[10] We restrict our attention to the more realistic case of a non-autarky economy in which at least one type of financial claims circulates. For ease of exposition, we first characterise the economy with $\kappa > 0$ and with both private and public liquidity. We defer the discussion of the conditions for an equilibrium featuring the co-existence of both types of assets versus the existence of only one type to Section 4.2.

3.1 Simplified Household's Constraints

Simplified Flow-of-Funds Constraints. It is convenient to use the notion of *effective* buy and sell prices. We define the effective buy (or bid) price per unit of private claims as

$$q^n \equiv q + \frac{\kappa}{f}, \tag{15}$$

where q captures the transaction price and κ/f represents search costs per transaction (scaled by the probability of encountering a matching ask quote f). Symmetrically to the bid price, we define the effective sell (or ask) price of a unit of private financial assets as

$$q^i \equiv q - \frac{\kappa}{\phi}. \tag{16}$$

Note that q^i is also equal to Tobin's q: the ratio of the market value of capital to its replacement cost (i.e., unity).

When $\kappa > 0$, the ask price is below the transaction price. Hence, entrepreneurs not only face constraints regarding the quantity of private claims that can be issued and resold, but also have to sell at a discount due to the intermediation cost κ/ϕ when liquidating financial claims. One may further intrepret $q^n - q^i = \kappa(f^{-1} + \phi^{-1})$ as the bid-ask spread. Therefore, our quantity measure of asset liquidity ϕ is closely linked with the bid ask spread, a price measure of asset liquidity.

Notice that (15) and (16) are helpful to transform the budget constraints (3) and (6). By using the definitions of the effective prices (15) and (16), together with the laws of motion of private asset positions (2) and $M^n = -fV$ and $M^i = \phi U$, we can rewrite the workers' flow-of-funds constraint (3) and the entrepreneurs' flow-of-funds constraint (6) as

$$C^n + q^n S^n_{+1} + \frac{B^n_{+1}}{P} = wN + [r + (1-\delta)q^n](1-\chi)S + \frac{(1-\chi)B}{P}, \tag{17}$$

---

[10]Such a complete breakdown of financial transactions on the private asset market may be self-fulfilling. For instance, when one party of the market does not participate, the other party would expect this inaction and stay out of the search market, validating the initial non-participation decision of their counterparts.

$$C^i + I^i + q^i \left[S^i_{+1} - (1-\delta)\chi S - I^i \right] + \frac{B^i_{+1}}{P} = r\chi S + \frac{\chi B}{P}. \tag{18}$$

We can further simplify (18) by substituting out investment. Again, we use $U = e[(1 - \delta)\chi S + I^i]$, where $e \in [0, 1]$ is the fraction of total assets quoted for sale. Following equation (2), the evolution of entrepreneurs' private asset holdings then becomes

$$S^i_{+1} = (1 - e\phi) \left[(1-\delta)\chi S + I^i \right]. \tag{19}$$

By using (19), we can express investment as $I^i = S^i_{+1}/(1 - e\phi) - (1 - \delta)\chi S$. Finally, substituting investment out of equation (18), entrepreneurs' flow-of-funds constraint becomes

$$C^i + q^r S^i_{+1} + \frac{B^i_{+1}}{P} = r\chi S + (1-\delta)\chi S + \frac{\chi B}{P}, \tag{20}$$

$$\text{where} \quad q^r \equiv \frac{1 - e\phi q^i}{1 - e\phi}. \tag{21}$$

The left-hand side (LHS) of (20) captures entrepreneurs' spending on consumption goods and holdings of private claims and money, while the right-hand side (RHS) represents entrepreneurial (total) *net worth* including rental income from capital, the value of existing financial claims, and the real value of money. On the LHS, end-of-period private asset holdings are valued at q^r, which is the *effective replacement cost* of private claims to entrepreneurs: for every unit of investment, entrepreneurs sell a fraction $e\phi$ of claims on the search market at price q^i, thus obtaining outside funding amounting to $e\phi q^i$. Hence, they only need to finance the "down-payment" $(1 - e\phi q^i)$ per unit of investment out of their own net worth. Moreover, entrepreneurs retain only a fraction $(1 - e\phi)$ of all claims created against an additional unit of investment as inside equity, such that they need $(1 - e\phi q^i)/(1 - e\phi)$ to acquire one unit of next-period private claims. The smaller is q^r, the larger is the amount of private claims S^i_{+1}, which entrepreneurs can bring back to their family.

In sum, (17) and (20) are the simplified budget constraints. Next, we turn to the discussion of financing constraints.

Simplified Financing Constraints. Workers cannot create new financial claims as they lack investment projects. Moreover, they would incur losses if they sold their existing stock of private financial assets due to the costs involved in divesting assets via the search market. Therefore, the accumulation of financial assets - including private claims - on behalf of the household is delegated to workers. The workers' financing constraint (4) is thus slack.

Entrepreneurs face two financing constraints (5) and (7), i.e., $e \leq 1$ and $B^i_{+1} \geq 0$. Again, $e \leq 1$ follows from the convenient expression $U = e[(1-\delta)\chi S + I]$. To understand these

constraints, it is instructive to back out aggregate investment $I \equiv I^i$ from (19) and (20):

$$I = \frac{\left[r + e\phi q^i(1-\delta)\right]\chi S + \frac{\chi B}{P} - C^i - \frac{B^i_{+1}}{P_t}}{1 - e\phi q^i} \leq \frac{\left[r + \phi q^i(1-\delta)\right]\chi S + \frac{\chi B}{P} - C^i}{1 - \phi q^i}. \quad (22)$$

To invest in new capital stock, entrepreneurs' liquid net worth $[r + e\phi q^i(1-\delta)]\chi S + \chi B/P$, net of consumption and newly purchased liquid assets, can be leveraged at $(1 - e\phi q^i)^{-1}$. If both financing constraints bind with equality, the RHS of (22) determines the upper bound on investment I.[11] That is, the two constraints $e \leq 1$ (or $U \leq (1-\delta)\chi S + I$) and $B^i_{+1} \geq 0$ imply a constraint on investment I.

Next, we check whether entrepreneurs' financing constraints are binding. A necessary condition for *private* claims to exist is that the replacement cost is bounded above by the internal cost of creating private claims, i.e., $q^r \leq 1$. Otherwise, entrepreneurs would finance investment fully out of internal funds. In equilibrium, the assumption of nonzero search costs and $q^r \leq 1$ imply, by definition, that $q^n > q > q^i \geq 1$. In other words, the ask price q^i on the search market (weakly) exceeds entrepreneurs' internal cost of investment, such that the issuance of financial claims against new investment yields non-negative profits.

Therefore, the representative household will prompt entrepreneurs to spend whatever net worth they are not consuming on creating new financial claims. These entrepreneurs sell as many private financial assets as possible and divest their entire stock of money holdings, i.e., $e = 1$ (or $U = (1-\delta)\chi S + I$) and $B^i_{+1} = 0$. That is, entrepreneurs' financing constraints are binding and investment is bounded from above as in (22).

3.2 The Household's Optimal Decisions

Now, we know that the household maximizes $J(S, B; \Gamma)$ subject to (17), (20), (8), (9), by choosing labour supply N, consumption C^i and C^n, $e = 1$, total private financial claims S^i_{+1} and S^n_{+1}, and liquid assets $B^i_{+1} = 0$ and B^n_{+1}. These choices are discussed in the following.

Labour Choice. The first-order condition for labour from this optimisation problem is

$$u'\left(\frac{C^n}{1-\chi}\right)w = h'\left(\frac{N}{1-\chi}\right), \quad (23)$$

which is a standard intra-period optimality condition. It requires that the marginal gain of extra consumption goods from earning wages equal the marginal dis-utility from working.

Portfolio Choice and Risk Sharing. We now turn to the optimal portfolio choices for

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[11]Note that $\partial I/\partial e > 0$, as a higher fraction of assets posted on the search market increases the amount of outside funding (numerator effect), thereby increasing the leverage ratio, i.e. decreasing the denominator.

private financial claims and money. The first-order conditions for S_{+1}^i and S_{+1}^n read

$$q^n u' \left(\frac{C^n}{1-\chi} \right) = q^r u' \left(\frac{C^i}{\chi} \right) = \beta \mathbb{E} \left[J_S(S_{+1}, B_{+1}; \Gamma_{+1}) | \Gamma \right], \tag{24}$$

where J_S denotes the partial derivative of J w.r.t. S. Then, the allocation of consumption between entrepreneurs and workers satisfies

$$u' \left(\frac{C^i}{\chi} \right) = \rho u' \left(\frac{C^n}{1-\chi} \right), \tag{25}$$

where ρ is inversely related to risk-sharing among household members and measures the impact of financing frictions on consumption risk sharing:

$$\rho \equiv \frac{q^n}{q^r}. \tag{26}$$

To see this, suppose that idiosyncratic risks can be fully insured, as in a standard real business cycle (RBC) model. In this case, entrepreneurs are not financing constrained and can implement the first-best investment schedule, such that the market price of private claims equals its internal replacement cost. This would imply that $q = q^i = q^r = q^n = 1$. In such an unconstrained economy, entrepreneurs do not need to restrain themselves and are able to implement the same consumption level as workers. Therefore, full insurance implies $\rho = 1$. In contrast, in an economy where idiosyncratic labour income and investment risks are not fully insurable, entrepreneurs cannot finance the first-best investment schedule. The market price of private assets remains above its replacement cost, such that $\rho > 1$. Because of the concavity of $u(.)$, we have $C^i/\chi < C^n/(1-\chi)$, i.e., entrepreneurs consume less than workers in order to expand investment.

Next, using equation (25), we know from the envelope condition that J_S satisfies

$$J_S(S, B; \Gamma) = u' \left(\frac{C^n}{1-\chi} \right) \left[\chi \rho \left(r + 1 - \delta \right) + (1 - \chi) \left(r + (1 - \delta) q^n \right) \right]. \tag{27}$$

Combining equations (24) and (27) yields the asset pricing formula (Euler equation) for private claims

$$\mathbb{E} \left[\frac{\beta u' \left(\frac{C_{+1}^n}{1-\chi} \right)}{u' \left(\frac{C^n}{1-\chi} \right)} \left[\chi \rho_{+1} r_{+1}^{ni} + (1 - \chi) r_{+1}^{nn} \right] | \Gamma \right] = 1, \tag{28}$$

where the term $\chi \rho_{+1} r_{+1}^{ni} + (1 - \chi) r_{+1}^{nn}$ in the expectations operator captures the expected

return on private claims from the perspective of the household, and

$$r_{+1}^{ni} \equiv \frac{r_{+1} + (1-\delta)}{q^n} \text{ and } r_{+1}^{nn} \equiv \frac{r_{+1} + (1-\delta)q_{+1}^n}{q^n}$$

denote the returns from an individual worker's perspective for the case of becoming an entrepreneur next period $\left(r_{+1}^{ni}\right)$ or staying a worker $\left(r_{+1}^{nn}\right)$. These returns reflect the fact that the unit price of private claims for workers in the current period is q^n, while the value of one unit of private claims next period is 1 for entrepreneurs and q_{+1}^n for workers according to the budget constraints (17) and (20).

To understand the return from the perspective of the household $\chi\rho_{+1}r_{+1}^{ni} + (1-\chi)r_{+1}^{nn}$, recall that an entrepreneur's marginal utility of consumption is ρ_{+1} times that of a worker. If a period-t worker becomes an entrepreneur at time $t+1$ (which happens with probability χ), the household's return on holding one unit of private assets is $\rho_{+1}r_{+1}^{ni}$, since the household values each unit of next-period resources in the hands of entrepreneurs at ρ_{+1}. If the worker does not change type at time $t+1$ (which happens with probability $1-\chi$), the return on private claims is r_{+1}^{nn}. Therefore, the return from the perspective of the household is $\chi\rho_{+1}r_{+1}^{ni} + (1-\chi)r_{+1}^{nn}$.

Following similar steps, we derive another asset pricing formula for money:

$$u'\left(\frac{C^n}{1-\chi}\right)\frac{1}{P} = \beta\mathbb{E}\left[J_B\left(S_{+1}, B_{+1}; \Gamma_{+1}\right)|\Gamma\right] = \beta\mathbb{E}\left[u'\left(\frac{C_{+1}^n}{1-\chi}\right)\frac{\chi\rho_{+1} + 1 - \chi}{P_{+1}}|\Gamma\right],$$

where J_B denotes the partial derivative of J w.r.t. B. Therefore,

$$\mathbb{E}\left[\frac{\beta u'\left(\frac{C_{+1}^n}{1-\chi}\right)}{u'\left(\frac{C^n}{1-\chi}\right)}\left[\chi\rho_{+1}\frac{1}{\Pi_{+1}} + (1-\chi)\frac{1}{\Pi_{+1}}\right]|\Gamma\right] = 1 \text{ where } \Pi_{+1} \equiv \frac{P_{+1}}{P_t}. \tag{29}$$

Note that the return on holding money is the inverse of inflation Π_{+1}, while the return on money from the household's point of view is $(\chi\rho_{+1} + 1 - \chi)/\Pi_{+1}$. The return accruing to a future entrepreneur is, again, adjusted by ρ_{+1}.

3.3 The Asset Price

Bargaining Solution. Financial intermediaries determine the asset price to maximize the total match surplus of buyers and sellers. The sufficient and necessary first-order condition associated with the bargaining problem (13) is

$$\omega J^n(S, B; \Gamma) = (1-\omega)J^i(S, B; \Gamma). \tag{30}$$

By using the household's optimality condition for asset holdings (24) and the risk-sharing condition (25), we can derive an analytical solution for the asset price:

Proposition 1:

Suppose that private claims exist. The bargaining solution for the asset price simplifies to

$$\rho = \frac{\omega}{1-\omega}\theta. \tag{31}$$

Alternatively, (31) *can be solved for the asset price as*

$$q = \frac{\rho\left(1+\frac{\kappa}{\omega}\right) - \frac{\kappa}{f}}{1 + (\rho-1)\phi}. \tag{32}$$

Proof. See Appendix B.1. $\qquad\square$

Proposition 1 is our main analytical result linking the asset price with search costs and asset saleability.[12] Importantly, the equilibrium on the market for private claims is not simply determined by a market clearing condition and the Euler equation for these assets, as it requires the asset price and asset market tightness (or asset saleability ϕ) to be pinned down simultaneously. The bargaining solution (31) solves this issue by establishing a relationship between asset saleability and the asset price.[13]

Market Participation. Equation (31) is, in fact, a participation condition, which is similar to the entry conditions commonly found in the asset search literature (Rocheteau and Weill, 2011; Vayanos and Wang, 2007). To be specific, if the Euler equation for private assets determines the asset price, then demand and supply conditions as captured by asset market tightness θ need to be such that (31) is satisfied in order to induce individual agents to participate in the market. We can see the participation decision by rewriting (31) as

$$\frac{(1-\omega)q^n M}{\omega q^r M} = \theta = \frac{\kappa V}{\kappa U},$$

where the LHS captures the ratio of the valuation of asset transactions by buyers and sellers, weighted by their respective bargaining weights; the RHS is the ratio of participation costs of buyers (κV) and sellers (κU). In sum, buyers and sellers increase their ask and bid sizes until the ratio of gains and costs from participation are equal on either side of the market.[14]

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[12]Although we do not solve explicitly for asset market tightness θ, ask size V, and bid size U, these could be easily backed out from (31) with the laws of motion of S^i and S^n.

[13]As a comparison, in a traditional asset pricing model, the Euler equation of the investors will determine the asset price, given their consumption profiles. Assets have full liquidity and $\phi = 1$.

[14]One can interpret V as the amount of capital inflow into the asset market, which is akin to the concept of "funding liquidity" in Brunnermeier and Pedersen (2009). As $\theta = \xi^{\frac{1}{\eta-1}}\phi^{\frac{1}{1-\eta}}$ is also related to market

20

3.4 A Summary

From the above characterisation, our costly-search model captures many aspects of asset market frictions. First, there are participation costs and transaction costs, and the total costs are determined by sell and buy orders U and V. The endogenous participation decisions, unlike free entry conditions, feature imperfect competition within the buyers' group or the sellers' group. Finally, because financing constraints are tied to asset saleability, the search theory also captures the funding condition in asset markets.

The final step to characterise the equilibrium is to eliminate the type specific asset positions, and derive a single household budget constraint. We sum over the type-specific budget constraints (17) and (20), multiplying the latter by ρ; we further eliminate S^i_{+1}, S^n_{+1}, B^i_{+1}, and B^n_{+1}, by using $e = 1$ and $B^i_{+1} = 0$; we finally use (8) and (9), and replace S by K:

$$
\begin{aligned}
\rho C^i + C^n + q^n K_{+1} + \frac{B_{+1}}{P} \; = \; & wN + [\chi\rho + (1-\chi)] \, rK \\
+ \; & [\chi\rho + (1-\chi)q^n] \, (1-\delta)K + [\chi\rho + (1-\chi)] \frac{B}{P}. \quad (33)
\end{aligned}
$$

This household budget constraint and the investment equation (22) represent the original workers' and entrepreneurs' budget constraints (17) and (20).

The recursive competitive equilibrium can then be summarized as a function (K_{+1}, I, ϕ, q, P) of the aggregate states (K, A, κ), which satisfies the optimality conditions for the rental and wage rates in (1), the household's budget constraints and optimality conditions (22), (33), (23), (25), (28), and (29), financial market equilibrium conditions (15), (16), (26), and (32), and capital accumulation $K_{+1} = (1-\delta)K + I$, given $B_{+1} = \bar{B}$ and the law of motion of the exogenous shocks A and κ. See Appendix A.2 for a collection of these conditions.

These equations, involving macroeconomic quantities, the nominal price level, as well as asset saleability and the asset price, imply rich macro-financial interactions.

4 The Dimensions of Asset Liquidity

Having characterised the equilibrium conditions, we now turn to its implications for asset liquidity. Our *notion of asset liquidity* has three dimensions: i) the speed at which an asset can be converted into consumption goods; ii) the cost incurred during the conversion; iii) the price impact of trading the asset. All three dimensions interact and jointly affect the liquidity of private claims as argued below, whereas liquid assets are traded on a frictionless

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liquidity ϕ_t, our framework thus provide the two-way interaction between "funding liquidity" and "market liquidity" in an otherwise standard general equilibrium setting.

spot market and can be converted into consumption goods instantly and costlessly with minimal price impact.

4.1 Liquidity Premium, Asset Saleability, and the Asset Price

Liquidity Premium. Importantly, when $\rho > 1$, the asset pricing formulae (28) and (29) imply that private claims carry a liquidity premium, which compensates investors for impediments to trading these assets. For simplicity, we illustrate the liquidity premium by focusing on deterministic steady-state values. Let $\bar{x}$ be the steady-state value of a variable x.

When money is valued, $\bar{\Pi} = 1$ in the steady state. From the optimality condition for money holdings (29), we must have

$$\bar{\rho} = \rho^* \equiv \chi^{-1}[\beta^{-1} - (1 - \chi)] > 1,$$

where ρ^* is a *parameter* that denotes the degree of risk-sharing in the steady state in which money is valued. As shown in the first-order condition (25), individual entrepreneurs are financing constrained and consume less than workers if $\bar{\rho} > 1$. Then, the real interest rate on liquid assets $\bar{\Pi}^{-1} = 1$ is lower than the rate of time preference β^{-1} in such a constrained economy.[15] By providing a liquidity service, money mitigates financing constraints when workers become entrepreneurs and is, therefore, valued by them.

We define the premium that private financial assets carry as the difference between the returns on private claims and money

$$\Delta^{LP} \equiv \mathbb{E}\left[\chi r^{ni}_{+1} + (1 - \chi) r^{nn}_{+1} | \Gamma_t\right] - \mathbb{E}\left[\Pi^{-1}_{+1} | \Gamma\right].$$

In the steady state without aggregate risks, a positive wedge $\bar{\Delta}^{LP}$ between the returns on private claims and money reflects pure liquidity frictions. $\bar{\Delta}^{LP}$ is thus the liquidity premium.

Proposition 2:
Suppose that the economy is in the steady state and that both private claims and money exist. Then, $\bar{r}^{nn} > 1$ and money provides a liquidity service in the neighbourhood around the steady state. The steady state liquidity premium amounts to

$$\bar{\Delta}^{LP} = \left[1 - (\rho^*)^{-1}\right] (\bar{r}^{nn} - 1) (1 - \chi) > 0.$$

Proof. See Appendix B.2. □

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[15] Although we focus on fiat money, similar results obtain in an economy where the government issues interest-bearing securities as shown in Cui (2016) .

Proposition 2 has profound implications for the interaction between asset liquidity and the underlying state of the economy. To illustrate these interactions, we first focus on the impact of a permanent increase in search costs κ on asset prices and asset market liquidity in the long-run and, secondly, on the dynamic effect of aggregate shocks on the price and quantity dimensions of asset liquidity.

Long-run Impact of Search Costs. Consider the steady state of an economy, in which money is valued. Then, we know from the previous discussion that $\rho^* = \chi^{-1}[\beta^{-1} - (1-\chi)] > 1$, and Proposition 1 implies that search market tightness is pinned down by the equilibrium level of risk-sharing, $\bar{\theta} = (1-\omega)\omega^{-1}\rho^*$. Since both asset saleability $\phi = M(1, \theta)$ and purchase probability $f = M(\theta^{-1}, 1)$ are functions of asset market tightness θ only, Proposition 1 also implies that the equilibrium relationship between q and κ can be directly determined:

Corollary 1:

Suppose that the economy is in the steady state and that both private claims and money co-exist. Then, asset market tightness satisfies $\bar{\theta} = \frac{1-\omega}{\omega}\rho^*$. Suppose, further, that intermediation costs κ increase permanently. If the new equilibrium still features the co-existence of both types of assets, then the increase of κ in the steady state

1. *does not affect asset saleability $\bar{\phi}$ or the purchase probability $\bar{f}$ and, hence, increases the bid-ask spread $\bar{\Delta}^s \equiv \bar{q}^n - \bar{q}^i = \kappa(\bar{\phi}^{-1} + \bar{f}^{-1})$; it increases the bid price $\bar{q}^n$, i.e. $\frac{\partial \bar{q}^n}{\partial \kappa} > 0$, while it decreases the ask price $\bar{q}^i$, i.e., $\frac{\partial \bar{q}^i}{\partial \kappa} < 0$;*

2. *increases the liquidity premium $\bar{\Delta}^{LP}$;*

3. *decreases the asset price $\bar{q}$, i.e., $\frac{\partial \bar{q}}{\partial \kappa} < 0$, if and only if*

$$M(1, \bar{\theta}) = M\left(1, \frac{1-\omega}{\omega}\rho^*\right) < 1 - \omega. \tag{A1}$$

Proof. See Appendix B.3. $\qquad \square$

An increase of search costs always raises the bid-ask spread and liquidity premium. Note that sellers will never be able to recover the increased intermediation costs fully, such that the ask price $\bar{q}^i$ always decreases with κ. The questions is whether the asset price $\bar{q}$ can fall with κ. (A1) implies that the answer is positive.

To understand (A1), note that an increase in κ implies that entrepreneurs have to spend more resources to engage in private asset transactions, such that their financing constraints tighten. As a result, the supply of private claims on the search market and aggregate investment fall, while the marginal product of capital (MPK) increases, exerting upward

pressure on the asset price q. At the same time, demand for private claims will fall. If money circulates, higher search costs drive buyers into the money market, pushing up the liquidity premium and putting downward pressure on q.

The latter effect dominates the former if the demand side is more sensitive to changes in κ than the supply side, i.e., when assumption (A1) is satisfied $M(1, \bar{\theta}) < 1 - \omega$ and the market is relatively tight.

Liquidity Dynamics. In the presence of temporary shocks to κ, asset saleability fluctuates with the asset price q, linking the quantitative and price dimensions of asset liquidity. The asset price co-moves positively with asset saleability, as long as the latter is sufficiently small.

Corollary 2:

Suppose that the economy starts from the steady state with both private claims and money and suppose (A1) holds. In response to a temporary increase of κ, q can drop together with ϕ if

$$\left[M\left(1, \frac{(1-\omega)\rho^*}{\omega}\right) \right]^{\frac{1}{1-\eta}} \leq \frac{(1-\omega)\xi^{\frac{1}{1-\eta}}}{2\omega(1-\eta)} \left[1 - 2\eta + \frac{\eta}{M\left(1, \frac{(1-\omega)\rho^*}{\omega}\right)} \right]. \qquad (A2)$$

When $\eta = \frac{1}{2}$, the above sufficient condition simplifies to $\xi^{1/3} \left(\frac{1-\omega}{\omega}\right)^{1/6} (\rho^*)^{1/2} \leq 2^{-1/3}$.

Proof. See Appendix B.4. □

Again, this result reflects the reaction of supply relative to demand on the asset search market. On the one hand, a drop in asset saleability due to a lower demand from investors tightens entrepreneurs' financing constraints, which reduces the supply of private claims and aggregate investment, but raises the MPK, thus exerting upward pressure on the asset price q. On the other hand, given the existence of the money market, investors rebalance towards money to hedge against future liquidity risks. This, again, raises the liquidity premium of private claims and pushes down q.

Proposition 2 shows that the demand (or liquidity-premium) effect dominates the supply (or MPK) effect under assumption (A2). Our model can thus generate simultaneous falls in the asset price and asset saleability, thereby endogenously tightening financing constraints through both the quantity and price dimensions of liquidity.

In sum, the cost, the quantity, and the price aspects of asset liquidity are linked through the participation decisions of sellers and buyers on the asset search market and jointly give rise to the liquidity premium Δ^{LP}. These results above highlight the importance of modeling asset saleability as an *endogenous* market outcome, rather than an exogenous constraint.

Remark: In Shi (2015) and Kiyotaki and Moore (2012), asset saleability ϕ is an ex-

ogenous parameter and constrains entrepreneurs, such that asset demand is only a (fixed) clearing factor. When ϕ falls, the asset supply schedule shifts to the left, while demand is not directly affected. Therefore, a drop of ϕ pushes up the asset price q in these models. In our model, a drop of ϕ reflects a simultaneous left-shift of both asset supply *and* demand. The main message of our paper can thus be summarized as follows: when financial markets are less liquid, asset prices fall as long as the increase of the liquidity premium dominates the fall of asset supply. This result is reflected both in the steady state and dynamic properties of the model.

Remark: The drop of both ϕ and q reduces aggregate investment $I = \frac{\left[r+\phi q^i(1-\delta)\right]S^i + \frac{B^i}{P} - C^i}{1-\phi q^i}$ in (22) via two channels. First, it reduces the saleable part of existing assets, thus shrinking the numerator. Second, it tightens the financing constraints and restricts entrepreneurs' ability to leverage, thus increasing the denominator. These effects are at the heart of the macro-financial interactions discussed in the numerical simulations in Section 6.

4.2   The Existence of Private and Public Liquidity

The circulation of different types of assets is important, since the liquidity premium only matters when both assets exist. When both types of assets exist, a higher degree of search frictions drives demand away to liquid assets and asset price can fall. In contrast, when only private claims circulate a higher degree of search frictions does not lead to a fall of the asset price q or asset saleability ϕ, simply because investors lack an alternative asset class to rebalance their portfolios toward.

As shown below, the different types of equilibrium are closely linked to the severity of liquidity frictions parameterised by intermediation costs κ. The modified equilibrium conditions can be found in Appendix A.3. Intuitively, one would conjecture the existence of two thresholds for the steady state level of intermediation costs, which separate these types of equilibrium and characterise the existence of private and public liquidity.

First, between the two thresholds, private claims and money co-exist, and we know that search market tightness $\bar{\theta} = \frac{1-\omega}{\omega}\rho^*$ in the private asset market is uniquely determined. Also, asset saleability $\bar{\phi} = M(1,\bar{\theta})$ is unique. Because of this feature, our asset search model does not feature multiple stationary equilibria when private claims and money co-exist, in contrast to, e.g., Rocheteau and Wright (2013).

Second, private claims will only be created if intermediation costs are not too large. To see this, note that the ask price for private claims must (weakly) exceed the internal replacement cost, $\bar{q}^i = \bar{q} - \frac{\kappa}{\phi} \geq 1$ for private claims to exist. Otherwise, entrepreneurs would only resort to internal financing. Moreover, $\bar{q}$ is bounded from below by zero and must be bounded from above as total resources of buyers are limited. Therefore, a threshold $\kappa = \kappa_2$

must exist such that $\bar{q}^i = 1$. Any value of intermediation costs in excess of this threshold would push the ask price $\bar{q}^i$ to below unity, and entrepreneurs would prefer to self-finance.

Third, for public liquidity (money) to exist, intermediation costs cannot be too small. Otherwise, money would not be valued, because private claims would provide sufficient liquidity by themselves and dominate the return of money (which is $\bar{\Pi}^{-1} = 1$).

Proposition 3:

Suppose that the economy is in the steady state. Then, there are three types of non-autarky equilibrium, depending on the level of intermediation costs κ:

1. *Non-monetary equilibrium. For $\kappa \in [0, \kappa_1)$, public liquidity is not valued, such that only private liquidity exists. In this case, we have $\bar{q}^n > \bar{q} > \bar{q}^i \geq 1 \geq \bar{q}^r$. Moreover, risk sharing $\bar{\rho}$ satisfies $1 < \bar{\rho} < \bar{\rho}^*$, but deteriorates with κ, i.e., $\frac{\partial \bar{\rho}}{\partial \kappa} > 0$. Asset saleability increases with κ, i.e., $\frac{\partial \bar{\phi}}{\partial \kappa} > 0$; asset price $\bar{q}$ can increase or decrease with κ even if (A1) holds.*

2. *Co-existence. Both types of financial assets exist if and only if κ satisfies $\kappa \in [\kappa_1, \kappa_2]$, where κ_2 and κ_1 are defined as*

$$\kappa_2 \equiv \frac{\rho^* - 1}{\frac{1-\omega}{\omega}\rho^* + 1} M\left(1, \frac{1-\omega}{\omega}\rho^*\right), \quad \kappa_1 \equiv \max\{0, \tilde{\kappa}_1\}, \quad \tilde{\kappa}_1 = H(\chi, \beta, \delta, \alpha, \xi, \eta, \omega)$$

and H is some non-linear function specified in Appendix B.5. Asset prices satisfy $\bar{q}^n > \bar{q} > \bar{q}^i \geq 1 \geq \bar{q}^r$ and the degree of risk-sharing is given by $\bar{\rho} = \rho^* > 1$; The asset saleability $\bar{\phi} = M\left(1, \frac{1-\omega}{\omega}\rho^*\right)$.

3. *Pure monetary equilibrium. For $\kappa \in [\kappa_2, +\infty)$, private liquidity is not issued, such that only public liquidity exists. In this case, $\bar{\rho} = \rho^* > 1$ and $\bar{\phi} = 0$.*

Proof. See Appendix B.5. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ □

When $\kappa < \kappa_1$, financial frictions are less severe, thus implying improved risk-sharing than in the region of $\kappa \geq \kappa_1$. Therefore, $\bar{\rho} < \rho^*$. An increase of κ within the region $[0, \kappa_1)$ impairs risk-sharing, such that $\bar{\rho}$ increases while asset saleability $\bar{\phi}$ needs to rise in order to encourage sellers' participation in the search market. The asset price $\bar{q}$ can also increase, reflecting the shortage of supply. In such a non-monetary equilibrium, intermediation costs act like capital-adjustment costs. Notice that $\bar{\phi}$ is flat when $\kappa_1 \leq \kappa \leq \kappa_2$ and is zero when $\kappa \geq \kappa_2$. Therefore, endogenising asset market liquidity implies a *non-linear and non-monotone* relationship between search frictions and asset saleability.

Figure 1: **Different Types of Equilibrium**. The different types of equilibrium are differentiated by the level of intermediation costs κ and the degree of risk-sharing ρ. The latter measures the difference between the marginal utility of consumption of an entrepreneur relative to a worker.

RBC	Private liquidity	Private and public liquidity	Public liquidity	Autarky

$\rho = 1$	$1 < \rho < \rho^*$	$\rho = \rho^*$	$\rho = \rho^*$	$\rho > \rho^*$
$\kappa = 0$	$\kappa \in [0, \kappa_1)$	$\kappa \in [\kappa_1, \kappa_2]$	$\kappa \in [\kappa_2, \infty)$	$\kappa \in (\kappa_2, \infty)$

Notice further that there are two equilibria at the upper threshold κ_2. When $\kappa_1 \leq \kappa \leq \kappa_2$, $\bar{q}^i$ falls with κ to $\bar{q}^i = 1$. When $\kappa \geq \kappa_2$, entrepreneurs are strongly financing constrained, but the benefits of outside financing cannot compensate the costs of transacting private financial assets anymore. Therefore, in this region, the entrepreneurs value capital at $\bar{q}^i = 1$. At $\kappa = \kappa_2$, both the case in which entrepreneurs participate in the financial market and the opposite case could be equilibrium outcomes. However, more wealth will be accumulated if private claims circulate; once private claims cease to be traded, only low-yielding money provides liquidity, and the economy becomes less efficient. We will illustrate this dis-continuity with numerical examples in Section 5.

In summary, the level of intermediation costs κ and the degree of risk-sharing $\bar{\rho}$ jointly characterise different types of equilibrium as shown in Figure 1. The two polar cases in these dimensions are the equilibrium of a basic RBC model and the autarky equilibrium. When $\bar{\rho} = 1$, money is not valued, and the full-insurance economy resembles a RBC economy.[16][17] In contrast, an autarky economy features no risk-sharing due to the absence of any asset markets, such that $\bar{\rho} > \rho^*$.

5   Calibration

In order to illustrate the analytical results of our model, we calibrate the model to the U.S. economy using data on macroeconomic aggregates and financial markets. In Section 6, we use this calibrated version of the model to evaluate its dynamic features in response

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[16]In this case, the household budget constraint (33) becomes $C + K_{+1} = wN + [r + (1 - \delta)]K$, which resembles the budget constraint in a basic RBC model.

[17]The absence of intermediation costs, $\kappa = 0$, is not sufficient for the steady state asset price to be one. It only implies $q^n = q^i$. Entrepreneurs will still be financing constrained as there are uninsured labour income risks. Money may or may not be valued depending on whether $\tilde{\kappa}_1 \geq 0$.

to aggregate productivity and intermediation cost shocks. We choose a conventional CRRA utility function of consumption and a linear function for the dis-utility of labour:[18]

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \text{ and } h(n) = \mu n.$$

5.1 Targets

We calibrate the steady state to match several long-run characteristics of the U.S. economy. The parameters capturing the discount factor, the coefficient of relative risk aversion, and the depreciation rate of capital (β, σ, and δ), are set exogenously to standard values. The capital share of output α and the weight of labour supply μ in the utility function are set to target the investment-to-GDP ratio and working hours (Table 2). Note that GDP in the model corresponds to the sum of real private consumption (C_t) and real private investment ($I_t + \kappa_t U_t + \kappa_t V_t$). Using this definition, we obtain an investment-to-GDP ratio of about 20% based on quarterly data from 1971Q1 to 2014Q4 from the FRED data set.

Table 2: **Steady state calibration**

	Parameter	Baseline Value	Target/Source
Preferences and Production Technology			
Household discount factor	β	0.9850	Exogenous
Relative risk aversion	σ	2	Exogenous
Utility weight on leisure	μ	2.6904	Working time: 33%
Mass of entrepreneurs	χ	0.0540	Doms and Dunne (1998)
Depreciation rate of capital	δ	0.0250	Exogenous
Capital share of output	α	0.3750	Investment-to-GDP ratio: 20%
Search and Matching			
Supply sensitivity of matching	η	0.5000	Exogenous
Matching efficiency	ξ	0.2695	Saleability $\phi = 0.3000$
Bargaining weight	ω	0.5085	Tobins $q = 1.1500$
Search costs	κ	0.0216	Liquidity/GDP = 30.1%

There are four less conventional parameters $\{\xi, \eta, \kappa, \omega\}$ related to the asset search-market and one parameter χ that is related to idiosyncratic investment risks. The population share of entrepreneurs χ, can be interpreted as the fraction of firms, which adjust capital in each period. According to Doms and Dunne (1998), this fraction is about 20% annually in the U.S., which translates to a value of $\chi = 0.054$ at quarterly frequency (Shi (2015) also uses a similar value). ξ and η are not independent due to the constant returns to scale matching technology on the search market. Without loss of generality, we set $\eta = 0.5$ and calibrate ξ.

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[18]This dis-utility function facilitates the steady state solution. The main results are robust to a more complicated specification. See the discussion in the calculation of steady-state values in the Appendix.

We are then left with three parameters $\{\xi, \kappa, \omega\}$, which are calibrated to jointly match three targets: The asset price q captures Tobin's q (excluding transaction costs), which ranges from 1.1 to 1.21 in the U.S. economy according to COMPUSTAT data. We target a value of $q = 1.15$. Steady-state asset saleability ϕ is set to 0.30, which corresponds to the ratio of funds raised in the market to fixed investment in the U.S. flow-of-funds data.[19] Finally, as $\kappa = 0$ is likely to generate an equilibrium without the existence of money, we calibrate κ such that the ratio of liquid assets to GDP, i.e. the real value of public liquidity divided by GDP, $\frac{B}{PY}$, is 30.1% in the steady state. In the data, this number is identified as the ratio of the total amount of money-like assets, such as cash, checkable deposits, and short-term Treasury bills (all from the flow-of-funds data), divided by GDP.

These calibration targets imply that intermediation costs are only about 2% of total GDP, and the annualized liquidity premium is about 106 basis points in line with previous empirical studies (e.g., Chen, Lesmond, and Wei, 2007).

5.2 The Long-run Quantitative Impact of Intermediation Costs

With the above parameters' choice, the conditions set out in Corollary 1 and Proposition 3 are satisfied. Using this calibration, we trace steady-state intermediation costs κ over the positive domain to illustrate the quantitative impacts of κ on financial markets and macro variables (see Figure 2). The two critical thresholds for intermediation costs, which separate the non-autarky equilibria, are

$$\kappa_1 = 0.0054 \quad \text{and} \quad \kappa_2 = 0.0378.$$

Non-monetary Equilibrium. When $\kappa \in [0, 0.0054)$, only private claims exist as they provide sufficient liquidity to dominate money. Recall that the absence of intermediation costs ($\kappa = 0$) is not equivalent to a basic RBC economy, as investment opportunities and labour income risk are still not fully insurable. As a result, the steady-state capital stock of a model with search frictions characterizing financial markets is 7% lower than the RBC level even if $\kappa = 0$. Lower capital accumulation reduces the marginal product of labour, such that demand for labour drops. Lower factor input depresses output to about 82% of that in a basic RBC model, while consumption drops by 2%.

As κ increases from 0 to κ_1, ρ rises from 1.26 to $\rho^* = 1.282$. That is, the risk-sharing capacity of the economy deteriorates increasingly in intermediation costs as transferring funds via the search market becomes more costly. However, in this region intermediation costs are

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Figure 2: **Comparative statics**. Consumption, investment, and output are expressed as percentages of the corresponding quantities in a frictionless model, i.e., a basic RBC model (for details see Appendix A.4). The liquidity share of output is defined as L/Y and the intermediation-cost-to-output ratio as $\kappa(U+V)/Y$ (intermediation share of output). The bold vertical line indicates the calibrated level of intermediation costs κ, while the dash-dotted line represents the lower threshold κ_1 and the dashed line the upper threshold κ_2.
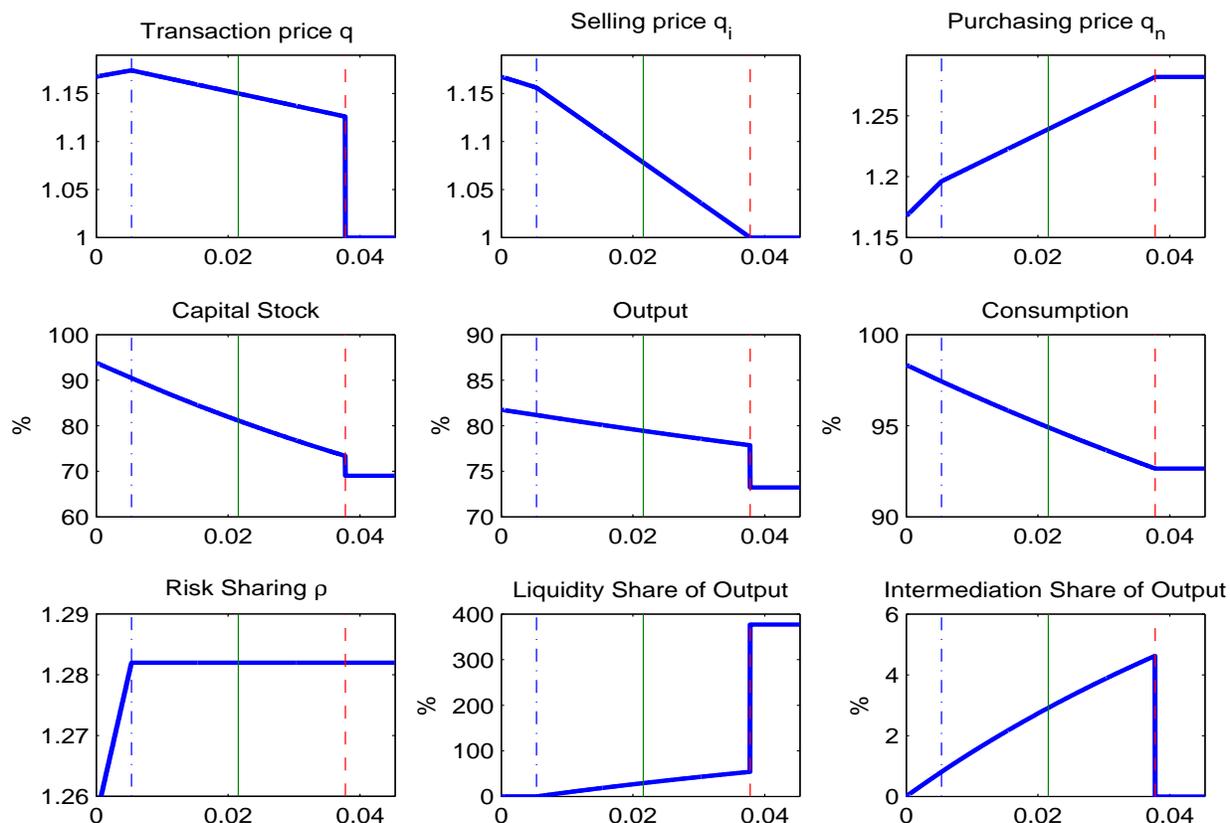
still small enough for money *not* to be valued, such that the liquidity share of output L/Y is zero. Asset saleability ϕ increases with ρ in order to encourage sellers' participation in the search market as captured by (31). Asset prices q and q^n increase with search costs, reflecting the tighter financing constraints. Thus, search costs act like investment adjustment costs that consume resources, in response to which investment, consumption, and production fall.

Private and Public liquidity. When $\kappa \in [0.0054, 0.0378]$, the liquidity of private claims - as captured by the combination of steady-state asset saleability and transaction costs - falls sufficiently, such that money is valued and circulates in addition to private claims. The hedging value of money increases with κ. The liquidity share of output thus increases monotonically with search costs from 0% to 54%. In contrast to the region $[0, \kappa_1)$, the equilibrium asset price q now decreases in intermediation costs, as demand for private claims falls more strongly than supply, reflecting the theoretical results in Corollary 1. That is, Assumption (A1) is satisfied.

As intermediation costs increase from κ_1 to κ_2, capital stock, output and consumption drop, respectively, by about 20, 8, and 6 percentage points compared to a frictionless economy. The under-accumulation of capital and the associated fall in production and consumption, are driven by two effects: first, agents rebalance their portfolios towards money as intermediation costs increase. However, money delivers a smaller return than capital. Second, higher trading costs imply a larger resource loss per transaction of private claims. Both effects reduce entrepreneurs' net worth, thus limiting their capacity to create new capital.

Note that the macroeconomic impact is brought about by a mere increase of intermediation costs from about 1% to 4.8% of output. The strong impact of intermediation costs on real allocations points to sizeable amplification.

Purely Monetary Equilibrium. When $\kappa \in [0.0378, +\infty)$, only money circulates and ρ is still equal to ρ^*. As the private asset market are shut down, entrepreneurs cannot finance investment projects by issuing or re-selling claims, i.e., $\phi = 0$. Therefore, financing constraints tighten abruptly when intermediation costs cross the threshold κ_2, inducing a downward jump in investment and the steady-state capital stock. Output declines by another 4.5% of the RBC level, as a result. Since money is the only available means for risk-sharing purposes, demand for money soars and the liquidity share of output increases to about 380%. Once private asset markets are inactive, intermediation costs are irrelevant for real allocations, such that the long-run equilibrium becomes invariant to them.

The dis-continuity at $\kappa = \kappa_2 = 0.0378$ reflects our previous discussion. The shutting down of private asset market leads to much lower capital accumulation and, therefore, reduces output significantly. Our calibration illustrates that even comparatively small increases in intermediation costs (less than twice the calibrated level) can lead to a complete shut-down

of asset markets. This finding highlights the inherent fragility of financial markets with endogenous participation similar to Gorton and Ordonez (2013).

6 Equilibrium Responses to Shocks

This section shows the model's dynamics after (1) TFP shocks, and (2) shocks to the intermediation capacity of financial markets, i.e., temporary changes of participation costs on the asset search market. Importantly, we find that these two types of shocks generate opposite liquidity premium dynamics; TFP shocks generate far less volatile asset price movements than that in the data.

In order to focus on variation in the liquidity premium (first-order effects) and abstract from the risk premium (second-order effects), we log-linearize the model.

6.1 TFP Shocks and Intermediation Cost Shocks

We consider a standard AR(1) process for aggregate productivity, i.e.,

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t^A, \quad 0 < \rho_A < 1$$

with i.i.d. $\epsilon_t^A \sim N(0, \sigma_A^2)$. We further introduce a shock to the cost of financial intermediation, which corresponds to a change in the participation costs

$$\log(1 + \kappa_t) = \rho_\kappa \log(1 + \kappa_{t-1}) + (1 - \rho_\kappa) \log(1 + \kappa) + \epsilon_t^\kappa, \quad 0 < \rho_\kappa < 1$$

with i.i.d. $\epsilon_t^\kappa \sim N(0, \sigma_\kappa^2)$. Rather than affecting the production frontier of the economy, this shock only impairs the capacity of the financial sector to intermediate funds between workers and entrepreneurs. Both in a market and a banking context, such an increase in intermediation costs may, for example, be triggered by rising uncertainty about counter-party risk. Such shocks unfold their effects through the endogenous response of asset saleability and prices, which affect entrepreneurs' financing constraints, investment, and production.

To compare productivity and intermediation cost shocks, the persistence and standard deviation of the underlying shock processes target the volatility (0.02) and first order correlation (0.91) of GDP's cyclical components (HP filtered with a smoothing coefficient of 1600). When using only productivity shocks, we have

$$\rho_A = 0.90 \text{ and } \sigma_A = 0.008.$$

When focusing on shocks to intermediation costs only, the exercise yields

$$\rho_\kappa = 0.82 \text{ and } \sigma_\kappa = 0.012.$$

We use these parameters in the subsequent numerical simulations. By design, both shocks will generate very similar aggregate output dynamics. We focus on the differences in the paths of other variables.

Negative TFP Shocks. Suppose an adverse productivity shock hits the economy at time 0 (see A_t in Figure 3). This shock depresses the marginal product of capital and its value to the household. Search for investment becomes less attractive and the amount of purchase orders from workers drops. The demand-driven fall is reflected in the endogenous drop in asset saleability ϕ, which amplifies the initial shock in two ways: (1) it reduces the quantity of assets that entrepreneurs are able to sell; (2) the asset price, falls - though only modestly - in line with Proposition 2. Both effects render private financial assets less liquid, thus tightening entrepreneurs' financing constraints. As a result, investment falls; consumption also falls because fewer resources are produced at the lower level of aggregate productivity.

In principle, money's liquidity service becomes more valuable to households when private claims' liquidity declines. However, in the case of a persistent TFP shock, lower expected returns to capital make *future* investment less attractive. This effect weakens the incentive to hedge against asset illiquidity for future investment. The former effect has a positive impact on the liquidity premium, while the latter has a negative impact.

In our calibration, the decline in the profitability of investment projects is sufficient for the liquidity premium to drop. Therefore, the demand for liquid assets falls, which is reflected in the decrease of their price $1/P_t$ on impact and, conversely, a surge in inflation $\Pi_t = P_t/P_{t-1}$. To the extent that TFP reverts back to the steady state, while asset liquidity is still subdued, hedging becomes more attractive which explains the relatively fast recovery of the liquidity premium.

Intermediation Shocks. Suppose an increase of intermediation costs hits the economy at time 0 (see the dynamics of κ in Figure 3). The output dynamics in this scenario are, by construction, similar to those of the productivity shock.

Realizing that market participation is more costly now and later, households seek to reduce their exposure to private claims, such that demand on the search market falls. On the supply side, financing-constrained entrepreneurs would still like to sell as many assets as possible in order to take full advantage of profitable investment opportunities. Therefore, asset demand on the search market shrinks relative to supply, reducing the likelihood for sellers' quotes to be matched with buyers' quotes and depressing asset saleability.

The sharp drop in asset liquidity tightens entrepreneurs' financing constraints substan-

Figure 3: **Impulse responses after a standard deviation shock to TFP or intermediation search costs at time 0**. Units of variables are percentage changes from their steady-state levels.
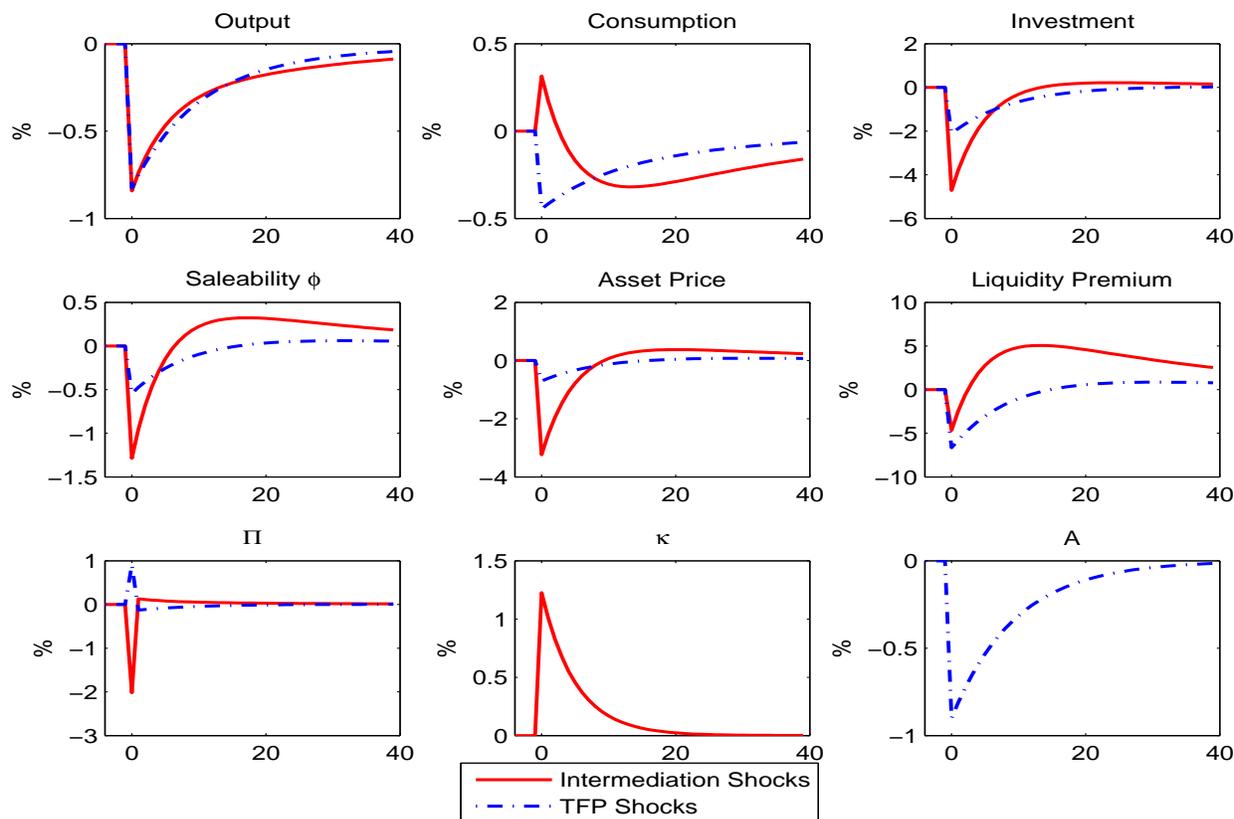
tially. Investment falls, and the MPK rises. But the liquidity premium dominates the increase of the MPK. Hence, q_t falls strongly and amplifies the initial shock by depressing entrepreneurs' net worth further. This effect is mirrored in a significant decline of investment activity, the impact response of which is about 6 times stronger than that of output.

As saving via the financial market becomes more expensive with higher intermediation costs, workers reduce their labour supply and consume slightly more after the initial shock. Entrepreneurs, on the other hand, have to cut back consumption significantly in view of tightly binding financing constraints. Given the small population share of entrepreneurs, aggregate consumption increases slightly initially, while output falls on impact because of the drop of labour hours. However, lower investment into the capital stock soon reduces the marginal product of labour and the wage rate. As labour income of workers falls, consumption persistently drops below the steady state.

While the intermediation cost shock depresses the demand for and the liquidity of private assets, it substantially increases their hedging value. To see this, note that future investment

remains profitable since the productivity of capital is not affected by the shock. To take advantage of future investment opportunities, households seek to hedge against the persistent illiquidity of private claims by expanding their holdings of public liquidity. The additional demand increases the real price of money, $1/P_t$, on impact, such that inflation $\Pi_t = P_t/P_{t-1}$ drops. Therefore, the liquidity premium initially falls due to the "flight to liquidity". However, once the real value of liquid assets has adjusted, the higher valuation of money relative to private assets leads to a persistent rise in the liquidity premium.

The faster accumulation of public liquidity relaxes future financing constraints, as entrepreneurs can finance more out of their stock of liquid assets and buyers have more resources to buy private claims. Both effects improve liquidity conditions on the private asset market. That is why both the asset price and asset saleability overshoot above the steady-state levels after about 3 years.

6.2   Business Cycle Statistics

The equilibrium dynamics confirm two key results: (1) In order to reconcile declining asset saleability with falling asset prices, the former must be an endogenous phenomenon. In other words, ϕ_t must be a consequence, rather than a cause of economic disturbances. (2) Both standard productivity and intermediation cost shocks affect the hedging value of liquid assets. However, only the latter generate a negative co-movement between the liquidity premium and aggregate output. In light of these findings, we further compare the data with the model's predictions for the cyclical behaviour of macroeconomic and financial variables.

We use the Wilshire 5000 price full cap index from 1971Q1-2014Q4 as a proxy for q, as it covers a vast universe of traded stocks. An aggregate measure of asset liquidity, on the other hand, is more difficult to construct due to the various dimensions of asset liquidity and associated measurement issues. For instance, transaction costs and trading delays depend on many factors such as the size of a trade relative to market depth, its timing, and the market structure.

In order to condense these characteristics of asset markets in a single indicator, we follow Naes, Skjeltorp, and Odegaard (2011), in choosing a simple and popular proxy for the illiquidity of private claims and money-like government issued assets suggested by Amihud (2002). This measure can be easily obtained from quarterly, monthly, or even daily data and is constructed as an illiquidity ratio (ILR) as follows:

$$ILR_{i,T} = \frac{\sum_{t=1}^{T} \frac{|R_{i,t}|}{VOL_{i,t}}}{D_T}$$

where D_T is the number of trading days within a time window T, $|R_{i,t}|$ is the absolute return

on day t for an asset i, and $VOL_{i,t}$ is the trading volume (in units of currency) on date t.

The ILR captures the price impact per volume unit of trades for a security i, thus combining the price and volume dimensions of asset liquidity. Liquid assets are traded in deep markets characterized by large transaction volumes and low price volatility. Therefore, liquid assets are associated with a low ILR, while the opposite is true for illiquid assets.

In our model, the different liquidity properties of privately and publicly issued assets are collapsed into the liquidity premium. As we do not directly observe the liquidity premium in the data, we construct an *illiquidity difference measure* as the empirical counterpart for the model-implied premium. This measure is computed as the difference between the illiquidity ratio for private claims ILR_T^P and the corresponding ratio for money-like assets ILR_T^M, i.e.,

$$ILR_T^D = ILR_T^P - ILR_T^M.$$

To calculate the illiquidity difference measure, we use data on stock prices, returns, and trading volume. We use quarterly averages of monthly data obtained from the Center for Research in Security Prices (CRSP). Furthermore, we obtain an aggregate illiquidity ratio ILR_T^P as the equally weighted average of cross-sectional $ILR_{i,T}$ measures. Note that we restrict attention to stocks listed at the NYSE to keep the sample as homogenous as possible. We use a similar strategy to compute the illiquidity ratio ILR_T^M for 3-month Treasury bills.

We de-trend all time series with the HP filter (with a smoothing coefficient of 1600). Asset prices tend to fall in recessions, while our illiquidity difference measure increases, possibly reflecting portfolio rebalancing towards liquid assets (Figure 4). Not surprisingly, the illiquidity difference measure correlates negatively with GDP (-0.67), while asset prices correlate positively with GDP (0.51).

Some key business cycle statistics of the model in comparison to the data are reported in Table 3, where only TFP shocks are considered. Similar to a basic RBC model, consumption and investment volatility, the correlation of macroeconomic variables with GDP, and first-order autocorrelations are roughly in line with the data. However, the liquidity premium and the asset price move too little in the model. Besides, the model-implied positive correlation between the liquidity premium and GDP (0.78) is at odds with the data (-0.67). All these statistics confirm the results gleaned from the impulse responses to TFP shocks.

As a comparison, Table 4 shows the relevant statistics associated with intermediation cost shocks as the only exogenous disturbance. Compared to the previous case, the volatility of both the liquidity premium and the asset price increase substantially. In addition, the volatility of investment is closer to the data, while consumption becomes more volatile.

Importantly, the model with shocks to intermediation costs successfully generates countercyclical liquidity premium (correlation with GDP: -0.50), mimicking the deterioration of

Figure 4: **Cyclical components of the illiquidity difference measure, asset prices, and GDP.** All series are cyclical components of HP filtered original series times 100. The shaded areas are NBER dated recessions.
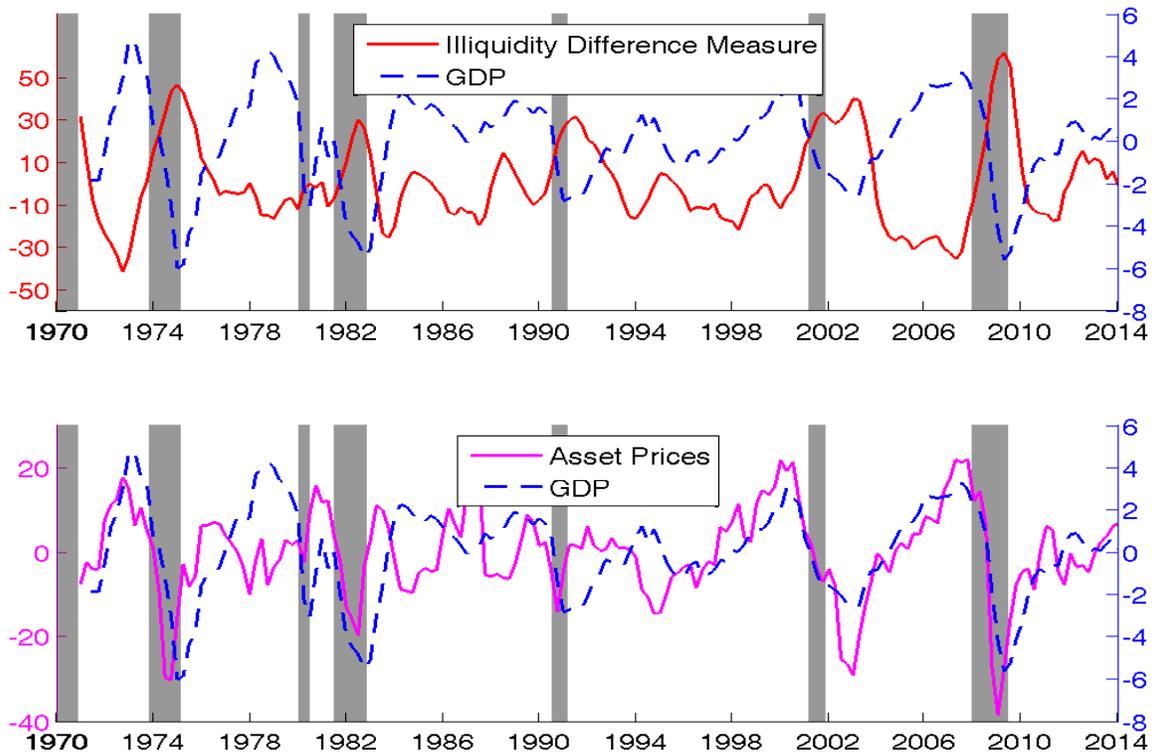

Table 3: **Business cycle statistics with only TFP shocks**

	Relative volatility $\frac{\sigma_x}{\sigma_y}$		Correlation $\rho(x,y)$		1st auto-correlation	
Variable x	Data	Model	Data	Model	Data	Model
Output	0.02	0.02	1.00	1.00	0.91	0.91
Consumption	0.58	0.66	0.93	0.98	0.89	0.94
Investment	3.21	2.26	0.95	0.98	0.86	0.89
Liquidity premium	9.31	6.62	-0.67	0.78	0.92	0.87
Asset Price	4.88	0.70	0.51	0.82	0.81	0.87

private assets' liquidity relative to publicly issued assets typically observed in recessions. As a result, the liquidity premium can serve as a discriminant between the sources of recessions. In addition, the asset price is more volatile, and its correlation with GDP (0.58) is closer to the data (0.51). Due to the overshooting of the asset price illustrated in Figure 3, this correlation is substantially smaller than that in the model with only TFP shocks (0.82) .

Table 4: **Business cycle statistics with only intermediation cost shocks**

	Relative volatility $\frac{\sigma_x}{\sigma_y}$		Correlation $\rho(x,y)$		1st auto-correlation	
Variable x	Data	Model	Data	Model	Data	Model
Output	0.02	0.02	1.00	1.00	0.91	0.91
Consumption	0.58	0.82	0.93	0.48	0.89	0.97
Investment	3.21	3.86	0.95	0.78	0.86	0.80
Liquidity premium	9.31	13.03	-0.67	-0.50	0.92	0.97
Asset Price	4.88	2.64	0.51	0.58	0.81	0.79

6.3 Discussion

We discuss a number of key determinants of these dynamics in greater detail in this section.

Hedging Value of Money. While rising intermediation costs increase the hedging value of money as shown in Section 6.1, TFP shocks may have an ambiguous effect on the incentive to hold cash. Persistently low productivity diminishes the return on capital, such that investment becomes less profitable and the willingness to hedge idiosyncratic investment risks shrinks. At the same time, low productivity depresses entrepreneurs' net worth, such that financing constraints become more binding. This effect should raise the hedging motive and the willingness to hold money. In the baseline experiment in Figure 3, the first effect dominates the second effect, such that the liquidity premium contracts strongly.

However, as both effects crucially depend on the persistence of the low-productivity spell, we illustrate the sensitivity of the net effect by varying the persistence of the productivity shock, by +10% and -10% compared to the baseline calibration. After changing the persistence of shocks, the equilibrium dynamics are similar to the baseline simulation (Figure 5). But different degrees of persistence alter the magnitude and the speed of the adjustment of macroeconomic and financial variables.

When the productivity process is more persistent than in the baseline scenario, agents anticipate that their net worth will be persistently lower, such that financing constraints will remain tight for longer. As a result, the hedging value of money rises, and the liquidity premium contracts less in the first few quarters after the initial shock. In fact, households curtail their consumption compared to the baseline scenario, in order to accumulate money holdings and buffer investment on impact. Thereafter, higher cash reserves help entrepreneurs finance investment and workers purchase private claims, which prevents output, investment, saleability, and the asset price from dropping as much as in the baseline. Naturally, the economy takes longer to revert back to the steady state from these less compressed levels due to the high persistence of the TFP shock.

Endogenous Asset Saleability. Kiyotaki and Moore (2012) and Shi (2015) consider aggre-

Figure 5: **Impulse responses after a standard deviation shock to TFP at time 0**.
The units of liquidity premium are annualized changes in basis points. Units of other variables are percentage changes from their steady-state levels.
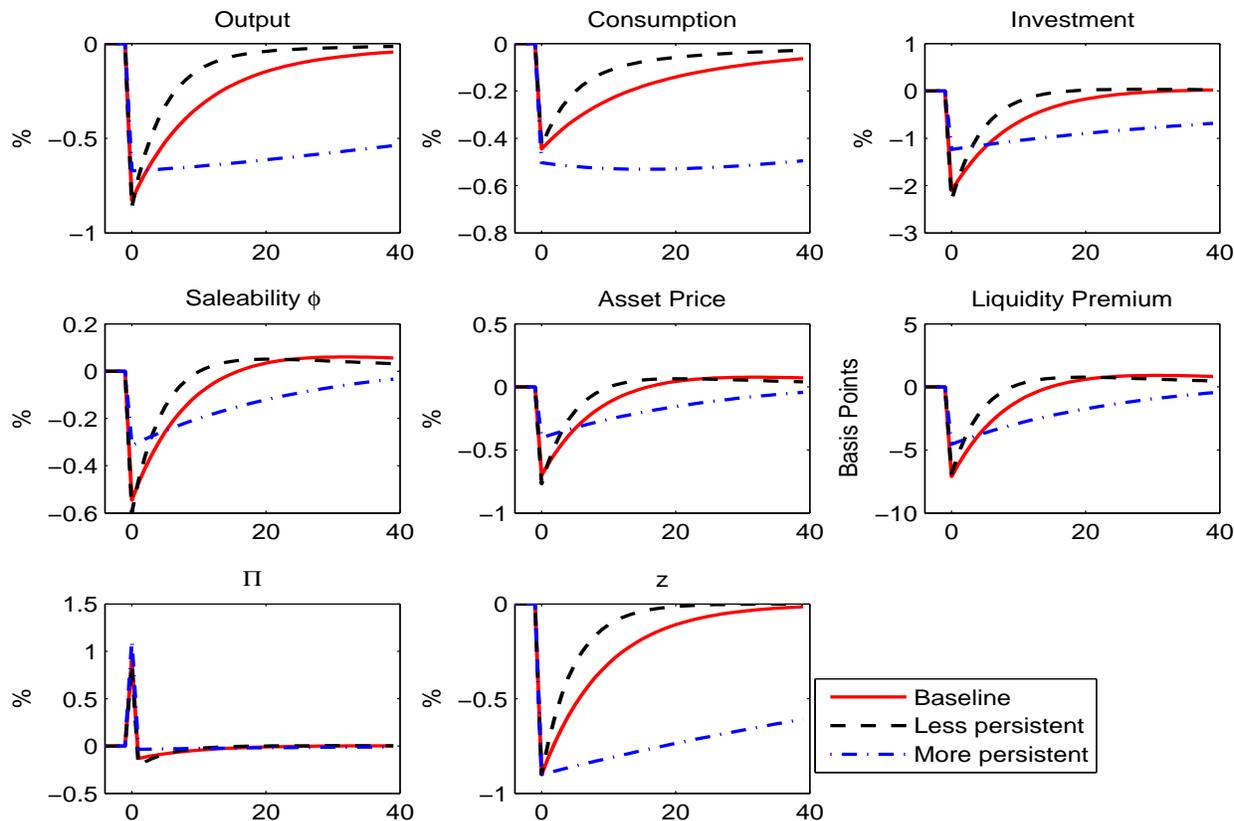
gate liquidity shocks in the form of an exogenous and persistent reduction of asset saleability ϕ. This shock mechanically depresses the supply of private assets on financial markets by tightening entrepreneurs' financing constraints. Demand for private claims, on the other hand, is hardly affected by such a shock as the return on capital goods does not fall. As supply contracts relative to demand on the asset market, such adverse aggregate liquidity shocks have the unrealistic feature of generating asset price booms. In other words, tighter financing constraints implies a higher value of Tobin's q.

In contrast, we demonstrate that financial shocks need to strongly affect the demand side of the asset market in order to overturn this anomaly in the reaction of the asset prices. In our model environment, higher intermediation costs directly deter buyers from participating in the asset search market. As a result the average size of their bid quotes declines. This slump in asset demand is amplified by the persistence of intermediation cost shocks: buyers, who perceive financial markets to be illiquid for an extended period, anticipate that holding additional private financial assets may constrain their own funding ability in the future,

thus becoming even less inclined to buy them. These demand side effects only occur with endogenous asset saleability.[20] That is why we consider *shocks to the financial sector*, instead of shocks to *the financing constraints* directly.

Notice that we assume the entry costs for buyers and sellers are identical. More generally, one could have different participation costs for buyers and sellers. Because sellers have investment opportunities, they will still offer all their assets on sale. In this case, the above results continue to hold as long as the costs levied on the buyers are large enough relative to sellers .[21]

Investment-specific Technological Shocks. The financial shocks considered here are different from productivity shocks, since they affect investment via financing constraints rather than directly reducing the production frontier of the economy. To see this, recall the goods market clearing condition (14)

$$C + I + \kappa (V + U) = Y.$$

Aggregate productivity shocks directly affect the RHS and then affect consumption and investment on the LHS, while intermediation cost shocks directly affect the investment-related costs $\kappa (V + U)$ on the LHS and then affect labour supply and output on the RHS. Such cost shocks may thus be interpreted as a particular form of non-linear investment-specific technology shocks (e.g., Greenwood, Hercowitz, and Huffman, 1988; Fisher, 2006; Primiceri, Justiniano, and Tambalotti, 2010), whose impact is amplified by their effect on endogenous market participation.

7   Conclusion

We endogenise asset liquidity in a macroeconomic model with search frictions. Endogenous fluctuations of asset liquidity are triggered by shocks that affect asset demand and supply on the search market either directly (intermediation cost shocks), or indirectly (productivity shocks). By tightening entrepreneurs' financing constraints, these shocks feed into real activity. Agents hedge endogenous financing constraints arising from assets market illiquidity with liquid assets. This idea harks back to Keynes' speculative motive for cash balances

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[20]An alternative financial market disturbance in our framework is a shock to the matching efficiency ξ. We check whether an efficiency problem occurring in the financial sector could induce co-movement between the asset price and asset saleability. An adverse efficiency shock, capturing, for example, a contagious bank run, impairs the intermediation capacity of the financial sector. Nevertheless, such a shock triggers an increase in the asset price as the supply reaction dominates. The detailed simulation results of the matching efficiency shock are available upon request.

[21]Details are available from the authors upon request.

([Keynes](), [1936]()) and Tobin's theory of risk-based liquidity preferences ([Tobin](), [1958](), [1969]()).

Our model is able to capture several dimensions of asset liquidity. In particular, when both private assets and money exist, we show that asset prices can positively co-move with asset saleability when the liquidity-premium effect dominates the marginal-product of-capital effect. The *endogenous* nature of asset liquidity is key to match this positive correlation, as adverse *exogenous* liquidity shocks would raise marginal product of capital and lead to asset price booms in recessions.

We also show that the liquidity service provided by intrinsically worthless liquid assets, is higher when financing constraints bind tightly. As a result, shocks to the cost of financial intermediation increase the hedging value of liquid assets, enabling our model to replicate the flight-to-liquidity dynamics measured by a countercyclical liquidity premium, thus matching U.S. business cycle features.

While it is straightforward to interpret our asset search framework as a model of market-based financial intermediation, it can also be seen as a short-cut to modeling bank-based intermediation: Financial intermediaries help channel funds from investors to suitable creditors in need of outside funding, which resembles a matching process. Adding further texture by explicitly accounting for intermediaries' balance sheets would open interesting interactions between liquidity cycles and financial sector leverage and maturity transformation.

Regarding government interventions, our framework suggests that, as in KM, open market operations in the form of asset purchase programs can have real effects by easing liquidity frictions. However, government demand may crowd out private demand due to congestion externalities in an endogenous liquidity framework. Therefore, future research could focus on the optimal design of conventional and unconventional monetary as well as fiscal policy measures in the presence of illiquid asset markets.

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Appendices

A Equilibrium Conditions

A.1 Buyers' and Sellers' Surpluses

First, we show the impact of marginal transaction on private claim accumulation. We rewrite (8) as a function of M^n and M^i,

$$S_{+1} = (1-\delta)S^n - M^n + (1-\delta)S^i + I^i - M^i = (1-\chi)(1-\delta)S - M^n + \frac{1-e\phi}{e\phi}M^i$$

since $M^i = \phi U = \phi e \left[\chi(1-\delta)S + I^i\right]$. Then, we know that the marginal transaction's impact on S_{+1}: $\partial S_{+1}/\partial M^n = -1$ and $\partial S_{+1}/\partial M^i = (1-e\phi)/e\phi$. Because M^n and M^i denote selling, the partial derivatives imply that workers bring back one unit of claims if the transaction is successful, while entrepreneurs bring back $(1-e\phi)/e\phi$ units of claims if the transaction is successful. The difference arises from the fact that entrepreneurs invest in physical capital after selling claims as instructed by the household.

Second, we show the impact of marginal transaction on consumption. For workers, we replace fV by $-M^n$ in the budget constraint (3)

$$C^n - qM^n = wN + r(1-\chi)S + \frac{(1-\chi)B}{P} - \frac{B_{+1}^n}{P} - \kappa V$$

At the time of bargaining, the costs of buy-quotes have been paid and the accumulation of liquid assets is finished. Then, it is straightforward to see that $\partial C^n/\partial M^n = q$. For entrepreneurs, we replace ϕU by M^i in the budget constraint (6) and use the fact that $I^i = M^i/e\phi - \chi(1-\delta)S$

$$C^i + \frac{M^i}{e\phi} - qM^i = r\chi S + \chi(1-\delta)S + \frac{\chi B}{P} - \frac{B_{+1}^i}{P_t} - \kappa U$$

We know that an additional successful match imply that they commit $\partial I^i/\partial M^i = e^{-1}\phi^{-1}$ units of physical investment, and it is straightforward to see that $\partial C^i/\partial M^i = q - e^{-1}\phi^{-1}$.

A buyer's surplus consists of the resources sacrificed today to acquire an additional unit of private assets and the value of this additional unit of asset holdings to its household tomorrow. Then, a buyer's surplus amounts to

$$
\begin{aligned}
J^n(S,B;\Gamma) &= -u'\left(\frac{C^n}{1-\chi}\right)\frac{\partial C^n}{\partial M^n} - \beta\frac{\partial S_{+1}}{\partial M^n}\mathbb{E}\left[J_S(S_{+1},B_{+1};\Gamma_{+1})|\Gamma\right] \qquad (34)\\
&= -u'\left(\frac{C^n}{1-\chi}\right)q + \beta\mathbb{E}\left[J_S(S_{+1},B_{+1};\Gamma_{+1})|\Gamma\right].
\end{aligned}
$$

Note that search market participation costs are already sunk at the bargaining stage. However, search costs are not ignored since households take them into account when determining optimal asset posting decisions by workers and entrepreneurs. Similarly, a seller's surplus is the marginal value to the household of an additional match for entrepreneurs. The difference here is due to the fact that entrepreneurs, unlike workers, need to implement physical investment after transaction again:

$$
\begin{aligned}
J^i(S,B;\Gamma) &= u'\left(\frac{C^i}{\chi}\right)\frac{\partial C^i}{\partial M^i} + \beta\frac{\partial S_{+1}}{\partial M^i}\mathbb{E}\left[J_S(S_{+1},B_{+1};\Gamma_{+1})|\Gamma\right] \qquad (35)\\
&= u'\left(\frac{C^i}{\chi}\right)\left(q - e^{-1}\phi^{-1}\right) + \frac{1-e\phi}{e\phi}\beta\mathbb{E}\left[J_S(S_{+1},B_{+1};\Gamma_{+1})|\Gamma\right]\\
&= u'\left(\frac{C^i}{\chi}\right)q - \beta\mathbb{E}\left[J_S(S_{+1},B_{+1};\Gamma_{+1})|\Gamma\right] + e^{-1}\phi^{-1}\left[-u'\left(\frac{C^i}{\chi}\right) + \beta\mathbb{E}\left[J_S(S_{+1},B_{+1};\Gamma_{+1})|\Gamma\right]\right]
\end{aligned}
$$

A.2 Recursive Competitive Equilibrium

Assume that private claims and money co-exist. We directly use $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ for exposition simplicity, but other utility functions will give similar features. Using the fact that total consumption $C = C^n + C^i$ and (25), we know $C^n = \rho^n C$ and $C^i = \rho^i C$, where

$$\rho^n \equiv \frac{1-\chi}{1-\chi+\rho^{-1/\sigma}\chi}, \quad \rho^i \equiv \frac{\chi\rho^{-1/\sigma}}{1-\chi+\rho^{-1/\sigma}\chi}$$

We further define the real liquidity as $L \equiv \frac{B}{P_{-1}}$ and substitute $S = K$. Given the aggregate state variables $\Gamma = (K, L, A, \kappa)$, we solve the equilibrium system

$$\left(K_{+1}, L_{+1}, C, I, N, \rho, \rho^i, \rho^n, \phi, q, q^i, q^n, r, w, \Pi\right)$$

together with the exogenous laws of motion of (A, κ). To solve for these 15 endogenous variables, we use the following 15 equilibrium conditions obtained from the main text:

1. The representative household's optimality conditions:

$$u'\left(\frac{\rho^n C}{1-\chi}\right) w = h'\left(\frac{N}{1-\chi}\right) \tag{36}$$

$$\rho^n \equiv \frac{1-\chi}{1-\chi+\rho^{-1/\sigma}\chi} \tag{37}$$

$$\rho^i \equiv \frac{\chi\rho^{-1/\sigma}}{1-\chi+\rho^{-1/\sigma}\chi} \tag{38}$$

$$1 = \beta\mathbb{E}_\Gamma\left[\frac{u'(\rho^n_{+1}C_{+1})}{u'(\rho^n C)}\frac{(\chi\rho_{+1}+1-\chi)r_{+1}+(1-\delta)\left(\chi\rho_{+1}+(1-\chi)q^n_{+1}\right)}{q^n}\right] \tag{39}$$

$$1 = \beta\mathbb{E}_\Gamma\left[\frac{u'(\rho^n_{+1}C_{+1})}{u'(\rho^n C)}\frac{(\chi\rho_{+1}+1-\chi)}{\Pi_{+1}}\right] \tag{40}$$

$$I = \frac{\left[(r+(1-\delta)\phi q^i)\chi K + \chi L\right] - \rho^i C}{1-\phi q^i} \tag{41}$$

2. Final goods producers:

$$r = \alpha A\left(\frac{K}{N}\right)^{\alpha-1} \quad \text{and} \quad w = (1-\alpha)A\left(\frac{K}{N}\right)^{\alpha} \tag{42}$$

3. Market clearing:

(a) The household's budget constraint:

$$
\begin{aligned}
\left(\rho\rho^i + \rho^n\right)C + L_{+1} + q^n K_{+1} &= wN + \frac{[\chi\rho+(1-\chi)]L}{\Pi} + [\chi\rho+(1-\chi)]rK \\
&+ [\chi\rho+(1-\chi)q^n](1-\delta)K
\end{aligned} \tag{43}
$$

(b) Capital accumulation: $K_{+1} = (1-\delta)K + I$

(c) Given the matching function $M(U, V) = \xi U^\eta V^{1-\eta}$

$$\phi = \xi\left[\frac{(1-\omega)\rho}{\omega}\right]^{1-\eta} \tag{44}$$

(d) Asset Prices

$$q = \frac{\rho\left(1+\frac{\kappa}{\omega}\right) - \frac{\kappa}{f}}{1+(\rho-1)\phi}, \quad q^i = q - \frac{\kappa}{\phi}, \quad q^n = q + \frac{\kappa}{\xi^{\frac{1}{1-\eta}}\phi^{\frac{\eta}{\eta-1}}} \tag{45}$$

(e) Liquid assets in fixed supply (note: $L = \frac{B}{P_{-1}}$ and $\Pi = \frac{P}{P_{-1}}$)

$$L_{+1} = \frac{L}{\Pi} \tag{46}$$

The following illustrates the steady-state values of 15 variables when both private claims and money exist. We also directly use $h(n) = \mu n$. In fact, no numerical solver is necessary because of this functional form. However, this is not a crucial assumption, and we will explain the reason after computing the stead-state values.

First, notice that market clearing for liquid assets implies that $\bar{\Pi} = 1$. Next, we use (40) to obtain

$$\bar{\rho} = \chi^{-1} \left[\beta^{-1} - (1 - \chi) \right], \tag{47}$$

and therefore

$$\bar{\rho}^n \equiv \frac{1 - \chi}{1 - \chi + \bar{\rho}^{-1/\sigma}\chi}, \quad \bar{\rho}^i \equiv \frac{\chi\bar{\rho}^{-1/\sigma}}{1 - \chi + \bar{\rho}^{-1/\sigma}\chi}. \tag{48}$$

With $\bar{\rho}$, we know that

$$\bar{\phi} = \xi \left[\frac{(1 - \omega)\bar{\rho}}{\omega} \right]^{1-\eta}. \tag{49}$$

Then, we can compute asset prices

$$\bar{q} = \frac{\bar{\rho}\left(1 + \frac{\kappa}{\omega}\right) - \frac{\kappa}{\bar{f}}}{1 + (\bar{\rho} - 1)\bar{\phi}}, \quad \bar{q}^i = \bar{q} - \frac{\kappa}{\bar{\phi}}, \quad \bar{q}^n = \bar{q} + \frac{\kappa}{\xi^{\frac{1}{1-\eta}}\bar{\phi}^{\frac{\eta}{\eta-1}}}. \tag{50}$$

From (39) and (42), we have

$$\bar{r} = \frac{\frac{\bar{q}^n}{\beta} - (1 - \delta)\left[\chi\bar{\rho} + (1 - \chi)\bar{q}^n\right]}{\chi\bar{\rho} + 1 - \chi}, \tag{51}$$

$$\bar{w} = (1 - \alpha)\left(\frac{\bar{r}}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}, \quad \bar{C} = \left(\frac{\bar{w}}{\mu}\right)^{1/\sigma} \frac{1 - \chi}{\bar{\rho}^n}. \tag{52}$$

Now, we need to solve real liquidity value $\bar{L}$ and capital stock $\bar{K}$. One can simplify (41) and (43) to be

$$\bar{\rho}^i \bar{C} + \bar{d}\bar{K} = \chi\bar{L},$$

$$(\bar{\rho}\bar{\rho}^i + \bar{\rho}^n)\bar{C} = \left[\frac{(1-\alpha)\bar{r}}{\alpha} + \bar{q}^n\left(\beta^{-1} - 1\right)\right]\bar{K} + \chi(\bar{\rho} - 1)\bar{L},$$

where we use the fact that $\bar{I} = \delta\bar{K}$ and

$$\bar{d} = \delta(1 - \bar{\phi}\bar{q}^i) - \chi(\bar{r} + (1 - \delta)\bar{\phi}\bar{q}^i). \tag{53}$$

As a result, we can solve real liquidity and capital stock as

$$\bar{L} = \frac{\left(\bar{\rho}\bar{\rho}^i + \bar{\rho}^n\right)\bar{d} + \bar{\rho}^i\left[\frac{(1-\alpha)\bar{r}}{\alpha} + \bar{q}^n(\beta^{-1} - 1)\right]}{\chi\left[\frac{(1-\alpha)\bar{r}}{\alpha} + \bar{q}^n(\beta^{-1} - 1) + (\bar{\rho} - 1)\bar{d}\right]}\bar{C}, \tag{54}$$

$$\bar{K} = \frac{1}{\frac{(1-\alpha)\bar{r}}{\alpha} + \bar{q}^n(\beta^{-1} - 1) + (\bar{\rho} - 1)\bar{d}}\bar{C}. \tag{55}$$

We finally express labour supply $\bar{N}$ and (physical) investment as a function of $\bar{K}$

$$\bar{N} = \left(\frac{\bar{r}}{\alpha}\right)^{\frac{1}{1-\alpha}}\bar{K} \text{ and } \bar{I} = \delta\bar{K}. \tag{56}$$

Remark: similar steps still go through with other types of $u(.)$ and $h(.)$. A different $u(.)$ only changes the computation of $\bar{\rho}^i$ and $\bar{\rho}^n$. A different $h(.)$ (especially a non-linear $h(.)$) modifies (52) to

$$\bar{C} = (u')^{-1} \left(\frac{h'\left(\frac{\bar{N}}{1-\chi}\right)}{\bar{w}} \right) \frac{1-\chi}{\bar{\rho}^n}.$$

Therefore, we need to guess a value of N and check whether the guess is correct by using (54), (55), and (56). For these reasons, the proofs later in Section B do not rely on a particular choice of $u(.)$ and $h(.)$.

A.3 Two Special Cases

When only money exists and when only private claims exist, we will show how to modify the equilibrium conditions (36)-(46) when both money and private claims exist. For $\kappa > 0$, we know that entrepreneurs are always financing constrained.

If only money exists, private claims does not circulate. Then, (39) is not included in the equilibrium conditions. (44) is modified to

$$\phi = 0,$$

and we should replace (45) to

$$q^i = q = 1, \quad q^n = \rho_t.$$

When computing the steady-state equilibrium, we continue solving (47)-(56) and modify (49) and (50) to

$$\bar{\phi} = 0, \quad \bar{q}^i = \bar{q} = 1, \quad \bar{q}^n = \bar{\rho}.$$

If only private claims circulate, money is not valued. Therefore, we keep all the equilibrium conditions (36)-(46), except the Euler equation for money holdings (40); we also need to add $L_{+1} = 0$. When computing the steady-state equilibrium, we solve (48)-(56) (note: ρ is not pinned down by (47) any more), and we pick a particular $\bar{\rho}$ which yields $\bar{L} = 0$.

A.4 A Basic RBC Model

We briefly describe the corresponding basic RBC model relative to our model. As is well known, one can solve the planner's solution to a RBC model. The planner maximizes

$$V(K; A) = \max_{C,N,K_{+1}} \left\{ \frac{C^{1-\sigma} - 1}{1-\sigma} - \mu N + \beta \mathbb{E}\left[V(K_{+1}; A_{+1})|A\right] \right\}$$

$$s.t. \quad C + K_{+1} = AK^\alpha N^{1-\alpha} + (1-\delta)K \tag{57}$$

where K is capital stock and A is aggregate productivity. The optimality conditions of labor supply and capital accumulation are

$$(C)^{-\sigma} (1-\alpha)A\left(K/N\right)^\alpha = \mu, \tag{58}$$

$$1 = \beta \mathbb{E}\left[\left(\frac{C_{+1}}{C}\right)^{-\sigma} \left[\alpha A_+ \left(K_+/N_+\right)^{\alpha-1} + 1 - \delta\right]|A\right]. \tag{59}$$

Therefore, equations (57), (58), and (59) solve (K_{+1}, C, N) given the state variables (K, A). To calculate the steady-state values, we substitute out capital-labour ratio K/N in (58) and (59), and we obtain

$$\frac{\bar{K}}{\bar{N}} = \left(\frac{\mu\bar{C}^\sigma}{1-\alpha}\right)^{\frac{1}{\alpha}}, \quad \bar{C} = \left(\frac{1-\alpha}{\mu}\right)^{\frac{1}{\sigma}} \left[\frac{\beta^{-1} - (1-\delta)}{\alpha}\right]^{\frac{\alpha}{(\alpha-1)\sigma}}.$$

Therefore, from the social resources constraint (57) and by using the capital labour-ratio, we derive capital and labour in the steady state as

$$\bar{K} = \left[\frac{\beta^{-1} - (1 - \delta)}{\alpha} - \delta\right]\bar{C}, \quad \bar{N} = \left[\frac{\beta^{-1} - (1 - \delta)}{\alpha} - \delta\right]\bar{C} / \left(\frac{\mu\bar{C}^\sigma}{1 - \alpha}\right)^{\frac{1}{\alpha}}.$$

B Proofs

B.1 Proposition 1

We first simplify the bargaining solution in (30) to

$$\frac{\omega}{\rho\left(q - \frac{1}{\phi}\right) + \frac{1 - \phi}{\phi}q^n} = \frac{1 - \omega}{q^n - q},$$

by using the first-order condition (24) and the risk-sharing condition (25) from the household. Then,

$$\omega\frac{\kappa}{f} = (1 - \omega)\left[\rho\left(q - \frac{1}{\phi}\right) + \frac{1 - \phi q^i}{\phi}\frac{(1 - \phi)q^n}{1 - \phi q^i}\right].$$

Using the definition $\rho \equiv \frac{q^n}{q^r} = \frac{(1 - \phi)q^n}{1 - \phi q^n}$, we further simplify the above identity to (31)

$$\omega\frac{\kappa}{f} = (1 - \omega)\rho\left(q - q^i\right) \iff \rho = \frac{\omega}{1 - \omega}\frac{\phi}{f}.$$

By using $\rho = \frac{(1 - \phi)q^n}{1 - \phi q^i} = \frac{(1 - \phi)\left(q + \frac{\kappa}{f}\right)}{1 - \phi q + \kappa}$ again, we solve q as

$$q = \frac{\rho(1 + \kappa) - (1 - \phi)\frac{\kappa}{f}}{1 + (\rho - 1)\phi} = \frac{\rho\left(1 + \frac{\kappa}{\omega}\right) - \frac{\kappa}{f}}{1 + (\rho - 1)\phi}, \tag{60}$$

where the second equality uses (31) again. □

B.2 Proposition 2

In the steady state, the two asset pricing formulae (40) and (39) become

$$\bar{\rho}\chi\bar{r}^{ni} + (1 - \chi)\bar{r}^{nn} = \beta^{-1}, \quad \bar{\rho}\chi\frac{1}{\bar{\Pi}} + (1 - \chi)\frac{1}{\bar{\Pi}} = \beta^{-1}. \tag{61}$$

Since $\bar{\Pi} = 1$, one knows that $\bar{\rho} = \rho^* = \left[\beta^{-1} - (1 - \chi)\right]/\chi > 1$. We will keep writing $1/\bar{\Pi}$ to highlight the return on money.

Since $\bar{q}^n > 1$, we know that $\bar{r}^{nn} = \frac{\bar{r} + (1 - \delta)\bar{q}^n}{\bar{q}^n} > \frac{\bar{r} + (1 - \delta)}{\bar{q}^n} = \bar{r}^{ni}$. As a result, $\bar{r}^{nn} > 1$; otherwise, the two asset pricing formulae in (61) cannot simultaneous hold. Further, rearranging (61), we have

$$\chi\bar{r}^{ni} = \frac{\beta^{-1} - (1 - \chi)\bar{r}^{nn}}{\bar{\rho}}, \quad \chi\frac{1}{\bar{\Pi}} = \frac{\beta^{-1} - (1 - \chi)\frac{1}{\bar{\Pi}}}{\bar{\rho}},$$

which are used to express the liquidity premium $\bar{\Delta}^{LP}$ as

$$\bar{\Delta}^{LP} = \chi\bar{r}^{ni} + (1 - \chi)\bar{r}^{nn} - \frac{1}{\bar{\Pi}} = \left(1 - \bar{\rho}^{-1}\right)\left(\bar{r}^{nn} - \frac{1}{\bar{\Pi}}\right)(1 - \chi). \tag{62}$$

Since $\bar{\rho} > 1$ and $\bar{r}^{nn} > 1$, we know that $\bar{\Delta}^{LP} > 0$. □

B.3 Corollary 1

First, when both private claims and money exist, $\bar{\rho} = \rho^*$ and $\bar{\theta} = \frac{1-\omega}{\omega}\bar{\rho}$ in the steady state. Then, we know that ρ, $\phi(\theta)$, and $f(\theta)$ are functions of parameters that are independent of search costs κ. We thus know that the spread $\bar{q}^n - \bar{q}^i = \kappa\left(\bar{\phi}^{-1} + \bar{f}^{-1}\right)$ increases with κ in steady states.

Second, $\bar{q}^n = \bar{q} + \frac{\kappa}{\bar{f}} = \frac{\bar{\rho}(1+\frac{\kappa}{\omega})+(\bar{\rho}-1)\kappa\bar{\theta}}{1+(\bar{\rho}-1)\bar{\phi}}$; using again $\bar{\theta} = \frac{1-\omega}{\omega}\bar{\rho}$, we know that $\bar{q}^n = \frac{\bar{\rho}[1+\kappa(1+\bar{\rho}(1-\omega)/\omega]}{1+(\bar{\rho}-1)\bar{\phi}}$ and therefore $\bar{q}^n$ increases with κ. As for liquidity premium, the key is to prove that $\bar{r}^{nn} = [\bar{r} + (1-\delta)\bar{q}^n]/\bar{q}^n$ increases with κ; (62) thus implies that $\bar{\Delta}^{LP}$ increases with κ. To see this, we can solve the steady-state level of MPK $\bar{r}$ from (51)

$$\frac{\bar{r}}{\bar{q}^n} = \frac{\beta^{-1} - (1-\chi)(1-\delta) - \chi\bar{\rho}(1-\delta)/\bar{q}^n}{\chi\bar{\rho} + 1 - \chi}.$$

As a result, $\bar{r}/\bar{q}^n$ increases with $\bar{q}^n$ and thus increases with κ. Given $\bar{r}^{nn} = \bar{r}/\bar{q}^n + 1 - \delta$, we know that $\bar{r}^{nn}$ increases with κ, and that is why the liquidity premium $\bar{\Delta}^{LP}$ is also an increasing function of κ.

Third, we prove that $\bar{q}^i = \bar{q} - \frac{\kappa}{\bar{\phi}} = \frac{\bar{\rho}+\kappa\left[\frac{\bar{\rho}}{\omega}-\bar{f}^{-1}-\bar{\phi}^{-1}-(\bar{\rho}-1)\right]}{1+(\bar{\rho}-1)\bar{\phi}}$ is a decreasing function of κ. This is because $\frac{\bar{\rho}}{\omega} - \frac{1}{\bar{f}} - \frac{1}{\bar{\phi}} - (\bar{\rho}-1) \leq 0$, which is equivalent to

$$\frac{\bar{\rho}(1-\omega)}{\omega} - \frac{1}{\bar{f}} \leq \frac{1-\bar{\phi}}{\bar{\phi}} \iff \frac{1}{\bar{f}}(\bar{\phi}-1) \leq \frac{1-\bar{\phi}}{\bar{\phi}},$$

where we use the relationship $\rho = \frac{\omega}{1-\omega}\frac{\phi}{f}$. The inequality is then satisfied for any $\bar{\phi} \in [0,1]$.

Finally, since $\bar{q} = \frac{\bar{\rho}(1+\frac{\kappa}{\omega})-\frac{\kappa}{\bar{f}}}{1+(\bar{\rho}-1)\bar{\phi}}$, we know that $\frac{\partial\bar{q}}{\partial\kappa} < 0$ is equivalent to

$$\frac{\bar{\rho}}{\omega} - \frac{1}{\bar{f}} < 0 \iff \bar{\phi} < 1 - \omega \iff M\left(1, \frac{1-\omega}{\omega}\rho^*\right) < 1 - \omega,$$

where we have used $\rho = \frac{\omega}{1-\omega}\frac{\phi}{f}$ again and $\phi = M(1,\theta)$. $\square$

B.4 Corollary 2

Using $\rho = \frac{\omega}{1-\omega}\theta$, $\theta = \phi/f$, and $\phi = M(1,\theta)$, we can express $\rho = \frac{\omega M^{-1}(\phi)}{(1-\omega)}$ and $f = \frac{\phi}{M^{-1}(\phi)}$. Therefore, q in (60) can be written as

$$q = \frac{\frac{\omega M^{-1}(\phi)(1+\frac{\kappa}{\omega})}{(1-\omega)} - \frac{\kappa M^{-1}(\phi)}{\phi}}{1 + \left[\frac{\omega M^{-1}(\phi)}{(1-\omega)} - 1\right]\phi} = \frac{M^{-1}(\phi)\left[(\omega+\kappa)\phi - (1-\omega)\kappa\right]}{\omega\phi^2 M^{-1}(\phi) + (1-\omega)\phi(1-\phi)} = \frac{\omega\phi + [\phi - (1-\omega)]\kappa}{\omega\phi^2 + (1-\omega)f(1-\phi)}.$$

If ϕ falls with κ, we know that the numerator $\omega\phi + [\phi - (1-\omega)]\kappa$ after the third equality sign has to fall because $\phi < 1 - \omega$. Then, we know that a sufficient condition for q to positively co-move with ϕ is that the denominator, $g(\phi) = \omega\phi^2 + (1-\omega)f(1-\phi)$, is a decreasing function of ϕ. Since

$$\begin{aligned}
\frac{dg(\phi)}{d\phi} &= 2\omega\phi - \frac{(1-\omega)\eta}{1-\eta}\xi^{\frac{1}{1-\eta}}\phi^{\frac{1}{\eta-1}}(1-\phi_t) - (1-\omega)\xi^{\frac{1}{1-\eta}}\phi^{\frac{\eta}{\eta-1}} \\
&= 2\omega\phi - \frac{(1-\omega)\eta}{1-\eta}\xi^{\frac{1}{1-\eta}}\phi^{\frac{1}{\eta-1}} - \frac{(1-\omega)(1-2\eta)}{1-\eta}\xi^{\frac{1}{1-\eta}}\phi^{\frac{\eta}{\eta-1}}
\end{aligned}$$

where we have used the fact that $f = \xi^{\frac{1}{1-\eta}}\phi^{\frac{\eta}{\eta-1}}$, we know $\frac{dg(\phi)}{d\phi} \leq 0$ is equivalent to

$$\phi^{\frac{1}{1-\eta}} \leq \frac{1-\omega}{2\omega(1-\eta)}\xi^{\frac{1}{1-\eta}}\left[1 - 2\eta + \frac{\eta}{\phi}\right].$$

If the economy starts with the steady state, $\bar{\phi} = M(1, \frac{(1-\omega)\rho^*}{\omega})$ and we thus have a sufficient condition with only exogenous parameters as in the main text. When $\eta = 1/2$, the above inequality becomes $\bar{\phi} =$

$\xi \left(\frac{1-\omega}{\omega}\rho^*\right)^{1/2} \leq \xi^{\frac{2}{3}} \left[\frac{1-\omega}{2\omega}\right]^{\frac{1}{3}}$, or $\xi^{1/3} \left(\frac{1-\omega}{\omega}\right)^{1/6} (\rho^*)^{1/2} \leq 2^{-1/3}$. $\quad\square$

B.5 Proposition 3

We first compute the two thresholds κ_2 and κ_1 and use a guess-and-verify strategy. We start by assuming that private claims and money co-exist so that all the steady-state equilibrium conditions (47)-(56) are satisfied.

First, we search for the threshold κ_2 that yields $\bar{q}^i \geq 1$ when $\kappa \leq \kappa_2$. Using the asset price derived in Proposition 1, the selling price $q^i = q - \frac{\kappa}{\phi}$, and the relationship $\rho = \frac{\omega}{1-\omega}\frac{\phi}{f}$, we know that

$$q^i = \frac{\rho(1+\frac{\kappa}{\omega}) - \frac{\kappa}{f} - \frac{\kappa}{\phi} - (\rho-1)\kappa}{1+(\rho-1)\phi} = \frac{\rho + \kappa(\phi-1)(\frac{1}{\phi}+\frac{1}{f})}{1+(\rho-1)\phi}. \tag{63}$$

Therefore, $q^i \geq 1$ is equivalent to

$$(1-\phi)\left(\rho - 1 - \frac{\kappa}{f} - \frac{\kappa}{\phi}\right) \geq 0 \iff \rho - 1 \geq \frac{\kappa}{f} + \frac{\kappa}{\phi} = \frac{\kappa(\theta+1)}{M(1,\theta)},$$

where we have used the fact that $\phi \in [0,1]$ together with the definition of f and ϕ. By using again the relationship $\rho = \frac{\omega}{1-\omega}\theta$, we can simplify the above condition to

$$\kappa \leq \frac{\rho - 1}{\frac{\omega}{1-\omega}\rho + 1} M\left(1, \frac{\omega}{1-\omega}\rho\right).$$

Notice that $\rho = \bar{\rho} = \rho^* = [\beta^{-1} - (1-\chi)]/\chi$ in the steady state (again, ρ^* is pinned down from the asset pricing formula of money in the steady state); further, $\frac{\rho-1}{\frac{\omega}{1-\omega}\rho+1}$ and $M(1, \frac{\omega}{1-\omega}\rho)$ are increasing functions of ρ. Therefore, we know that the threshold $\kappa_2 = \frac{\rho^*-1}{\frac{\omega}{1-\omega}\rho^*+1} M\left(1, \frac{\omega}{1-\omega}\rho^*\right)$. When $\kappa = \kappa_2$, $\bar{q}^i = 1$ in the steady-state economy when both prviate claims and money exist.

Second, we calculate the threshold κ_1 as a function of exogenous parameters. The key is to find κ_1 such that the real value of liquidity $\bar{L} = 0$. From (47) and (49), $\bar{\rho} = \rho^*$ and we know that $\bar{\phi}$ and $\bar{f}$ are *not* functions of κ in the steady state. In addition, we have calculated q^i in (63) and can also calculate q^n:

$$\bar{q}^i = \frac{\bar{\rho} + \kappa(\bar{\phi}-1)(\frac{1}{\bar{\phi}}+\frac{1}{\bar{f}})}{1+(\bar{\rho}-1)\bar{\phi}}, \quad \bar{q}^n = \bar{q}^i + \frac{\kappa}{\bar{\phi}} + \frac{\kappa}{\bar{f}} = \frac{\bar{\rho} + \kappa\left[\frac{\bar{\rho}}{\omega} + \frac{(\bar{\rho}-1)\bar{\phi}}{\bar{f}}\right]}{1+(\bar{\rho}-1)\bar{\phi}}.$$

As a result,

$$\bar{q}^i = c_{i1} + \kappa c_{i2}, \quad \bar{q}^n = c_{n1} + \kappa c_{n2}, \tag{64}$$

where coefficients

$$c_{i1} = \frac{\bar{\rho}}{1+(\bar{\rho}-1)\bar{\phi}}, \quad c_{i2} = -\frac{(1-\bar{\phi})(\frac{1}{\bar{\phi}}+\frac{1}{\bar{f}})}{\bar{\phi}\left[1+(\bar{\rho}-1)\bar{\phi}\right]}, \quad c_{n1} = \frac{\bar{\rho}}{1+(\bar{\rho}-1)\bar{\phi}}, \quad c_{n2} = \frac{\frac{\bar{\rho}}{\omega} + \frac{(\bar{\rho}-1)\bar{\phi}}{\bar{f}}}{1+(\bar{\rho}-1)\bar{\phi}}.$$

By inspecting these coefficients, we know that except $c_{i2} < 0$, others are strictly positive. For similar reasons, we can express r from (51) as

$$r = c_{r1} + \kappa c_{r2}, \tag{65}$$

where

$$c_{r1} = \frac{[1-\beta(1-\delta)(1-\chi)]\bar{\rho}}{1+(\bar{\rho}-1)\bar{\phi}} - \beta(1-\delta)\chi\bar{\rho}, \quad c_{r2} = \frac{[1-\beta(1-\delta)(1-\chi)](\frac{\bar{\rho}}{\omega}+\frac{(\bar{\rho}-1)\bar{\phi}}{\bar{f}})}{1+(\bar{\rho}-1)\bar{\phi}} > 0.$$

Since money exists, $\bar{L} \geq 0$ from (54) in equilibrium. The denominator of equation (54) has to be positive.

Then, $\bar{L} \geq 0$ iff $\left(\bar{\rho}\bar{\rho}^i + \bar{\rho}^n\right)\bar{d} + \bar{\rho}^i \left[\frac{(1-\alpha)\bar{r}}{\alpha} + \bar{q}^n(\beta^{-1} - 1)\right] \geq 0$. That is,

$$\delta(1 - \bar{\phi}\bar{q}^i) + \frac{\bar{\rho}^i \left[\frac{(1-\alpha)\bar{r}}{\alpha} + \bar{q}^n(\beta^{-1} - 1)\right]}{(\bar{\rho}\bar{\rho}^i + \bar{\rho}^n)} \geq \chi(\bar{r} + (1-\delta)\bar{\phi}\bar{q}^i),$$

or,

$$\left[\frac{\bar{\rho}^i}{(\bar{\rho}\bar{\rho}^i + \bar{\rho}^n)}\frac{(1-\alpha)}{\alpha} - \chi\right]\bar{r} + \frac{\bar{\rho}^i \left(\beta^{-1} - 1\right)\bar{q}^n}{(\bar{\rho}\bar{\rho}^i + \bar{\rho}^n)} - \bar{\phi}\left[\delta + \chi(1-\delta)\right]\bar{q}^i + \delta \geq 0.$$

Let $\zeta_r = \left[\frac{\bar{\rho}^i}{(\bar{\rho}\bar{\rho}^i + \bar{\rho}^n)}\frac{(1-\alpha)}{\alpha} - \chi\right]$, $\zeta_n = \frac{\bar{\rho}^i\left(\beta^{-1} - 1\right)}{(\bar{\rho}\bar{\rho}^i + \bar{\rho}^n)}$, and $\zeta_i = \bar{\phi}\left[\delta + \chi(1-\delta)\right]$, then we can use the expressions q^i, q^n, and r from (64) and (65) to derive:

$$\kappa\left(\zeta_r c_{r2} + \zeta_n c_{n2} - \zeta_i c_{i2}\right) \geq -\delta - \zeta_r c_{r1} - \zeta_n c_{n1} + \zeta_i c_{i1}.$$

Notice that $\zeta_r = \chi\left[\frac{1}{\chi\bar{\rho} + (1-\chi)/\bar{\rho}^\sigma}\frac{1-\alpha}{\alpha} - 1\right] > \chi\left[\beta^{-1}\frac{1-\alpha}{\alpha} - 1\right] > 0$, $\zeta_n > 0$, and $\zeta_i > 0$, and we must have $\zeta_r c_{r2} + \zeta_n c_{n2} - \zeta_i c_{i2} > 0$. This implies that $\tilde{\kappa}_1$ can be solved as

$$\kappa \geq \frac{-\delta - \zeta_r c_{r1} - \zeta_n c_{n1} + \zeta_i c_{i1}}{\zeta_r c_{r2} + \zeta_n c_{n2} - \zeta_i c_{i2}} = H(\chi, \beta, \delta, \alpha, \xi, \eta, \omega) \equiv \tilde{\kappa}_1.$$

Finally, by using the expression (54) and other equilibrium conditions, one can show that $\frac{d\bar{L}}{d\kappa} > 0$ and $\bar{L} < 0$ for a sufficiently small κ. That is, $\bar{L}(\kappa)$ is an increasing function of κ, or liquidity value increases with search costs κ. This fact implies that liquidity value $\bar{L}$ only cross zero once at $\kappa = \tilde{\kappa}_1$. When $\kappa < \tilde{\kappa}_1$, $L(\kappa) < 0$ and the assumption of co-existence of private claims and money is not verified.

Now, we have κ_1 and κ_2, and we are left to characterise the economy when $\kappa \in [\kappa_2, +\infty)$ and $\kappa \in [0, \kappa_1)$. When $\kappa \in [\kappa_2, +\infty)$, money is valued and $\bar{\rho} = \rho^*$, while $\bar{\phi} = 0$. When $\kappa \in [0, \kappa_1)$, we know that workers do not hold money and $B_{+1}^n = 0$. Therefore, the asset pricing formula needs to be modified to

$$\beta\mathbb{E}_\Gamma\left[\frac{u'(\rho_{+1}^n C_{+1})}{u'(\rho^n C)}\frac{(\chi\rho_{+1} + 1 - \chi)}{\Pi_{+1}}\right] + \nu = 0,$$

where $\nu > 0$ is the Lagrangian multiplier attached to $B_{+1}^n \geq 0$. In the steady state, one can still imagine that a government promises to pay return $\bar{\Pi} = 1$ on money, but no one is willing to hold such low-yielding asset. Then, $\bar{\rho}$ increases to ρ^* as ν decreases to zero (when κ increases to κ_1). As a result, $\bar{\phi} = \xi\left(\frac{(1-\omega)\bar{\rho}}{\omega}\right)^{1-\eta}$ is also an increasing function of κ. Finally, in order to solve $\bar{\rho}$ in this region, we can set $\bar{L} = 0$ in (54). That is, $\bar{\rho}$ solves the following equation

$$\left(\bar{\rho}\bar{\rho}^i + \bar{\rho}^n\right)\bar{d} + \bar{\rho}^i\left[\frac{(1-\alpha)\bar{r}}{\alpha} + \bar{q}^n(\beta^{-1} - 1)\right] = 0,$$

where all variables can be expressed as a function of $\bar{\rho}$ if we use equilibrium conditions (48)-(56). Finally, the rising κ can increase or decrease $\bar{q} = \frac{\bar{\rho} + \kappa(\bar{\rho}/\omega - 1/\bar{f})}{1 + (\bar{\rho} - 1)\bar{\phi}}$ since $\bar{\rho}$ increases with κ even if Assumption (A1) holds. This is different from the proof in Corollary B.3 in which $\bar{\rho}$ is fixed at ρ^* when both private claims and money exist. $\square$