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## Leverage Bounds with Default and Asymmetric Information

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## Abstract

This paper analyzes productivity and welfare losses from capital misallocation in a general equilibrium model of occupational choice and financial intermediation. It studies the effects of risk sharing with default and imperfect monitoring on the optimal allocation of resources and derives endogenous leverage bounds. Information frictions have large impact on entrepreneurs' entry and firm-size decisions due to endogenous collateral requirements derived from incentive compatible allocations. Leverage bounds derived from default and asymmetric information constraints are then used to simulate the tradeoff from a macroprudential policy aimed at mitigating the effects of unanticipated changes in information regime.

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JEL codes: E44; D82; J24

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## Acknowledgments

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## Abstract

This paper analyzes productivity and welfare losses from capital misallocation in a general equilibrium model of occupational choice and financial intermediation. It studies the effects of risk sharing with default and imperfect monitoring on the optimal allocation of resources and derives endogenous leverage bounds. Information frictions have large impact on entrepreneurs' entry and firm-size decisions due to endogenous collateral requirements derived from incentive compatible allocations. Leverage bounds derived from default and asymmetric information constraints are then used to simulate the tradeoff from a macroprudential policy aimed at mitigating the effects of unanticipated changes in information regime.

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## 1 Introduction

Geanakoplos (2006) and Geanakoplos and Fostel (2012) show that collateral rates or leverage can be more important to economic activity and prices than interest rates. The change in loan-to-value ratio on new loans is an important source of economic crises, more important that the debt-to-equity ratio. The collateral problem, rather than insufficient demand or irrational behavior, seems to be one of the main causes of the recent financial crisis. During a leverage cycle there is too much leverage in normal times and therefore too high asset prices and vice versa in bad times.

The maximum leverage ratio is now perceived as potentially effective countercyclical metric that helps to avoid excessive build-up during booms and the rapid deleveraging in times of stress, a process that can destabilize the whole financial system. Importantly, one of the main reasons for introducing leverage ratio restrictions was the observation that financial institutions that were

severely affected by the financial crisis showed strong risk-based capital ratios before the crisis. The maximum leverage ratio is supposed to complement the risk-based capital requirements as a simple, transparent, politically neutral, non-risk based "backstop" to mitigate cyclical fluctuations, being a tighter constraint in booms and a looser constraint in recessions. Countries that have used a maximum leverage for mortgages (Canada, Switzerland, Hong Kong) have fared much better during the Great Recession and have experienced much lower price volatility in housing and financing markets.

Overall, regulation of the leverage ratio is expected to protect the borrowers/lenders against the consequences of imprudent borrowing/lending, to constrain excessive credit growth and price increases, and to reduce the amplitude of economic booms and busts. Leverage regulation can be implemented within the financial system at the institutional level (bank capital to asset ratio) or embedded for borrowers in collateral downpayment margins and 'haircuts'.

This paper builds a general equilibrium model with financial intermediation and occupational choice. It derives endogenous leverage bounds arising from an imperfect monitoring problem and/or default. These leverage bounds are then used to simulate the tradeoff from the macro-prudential regulation: in good times, restricting leverage is costly as it limits the efficient allocation of resources to their most productive use. On the other hand, minimum downpayment requirements provide incentives for accumulation of assets that prevent excessive deleveraging during after a bad technology or information shock or after a change in the regulatory regime.

Using the evidence on financial frictions and entrepreneurial activity, I build a dynamic general equilibrium model with heterogeneous agents and occupational choice, financial constraints and endogenous financial markets. Each agent compares the expected value he or she would obtain from being a worker to the expected value of becoming an entrepreneur. A worker receives a wage while an entrepreneur establishes a firm with capital investment, employs other agents as workers, and realizes profit from a decreasing-returns-to-scale production technology. The occupational heterogeneity is important as workers (together with the less productive or unconstrained entrepreneurs) lend their assets to entrepreneurs who can use them more productively. Financial intermediation thus allocates resources to the most productive use, reduces financing constraints and increases efficiency.

A well functioning financial intermediation makes personal wealth less relevant when allocating productive capital to managerial skills: the most talented entrepreneurs can enter and operate firms closer to the optimal size. This in turn increases demand for labor and capital. General equilibrium effects are crucial for entry/exit decisions. Higher equilibrium wages and interest rates increase the opportunity cost of becoming an entrepreneur for the less talented agents. This further improves the skill composition of entrepreneurs and increases the amount of resources available to the most talented ones.

Under perfect risk-sharing and full information, financial intermediation allocates resources to the most productive use regardless of the collateral position of a borrower: the most talented entrepreneurs operate firms at the optimal size. As competition among entrepreneurs reduces their profits, welfare gains mostly apply to workers who benefit from higher equilibrium wages and the possibility to receive higher return on their savings (similar to Geanakoplos and Fostel 2012). Numerical simulations show that higher quality of financial markets increases efficiency, improves average welfare as well as reduces inequality.

With asymmetric information and/or default, financial markets cannot provide full insurance against risk. Instead, banks restrict lending by demanding collateral, that is, financial markets impose an endogenous maximum leverage bound on each borrower. Consequently, high productivity but poor entrepreneurs (firms) are not able to borrow the optimal amount of capital while
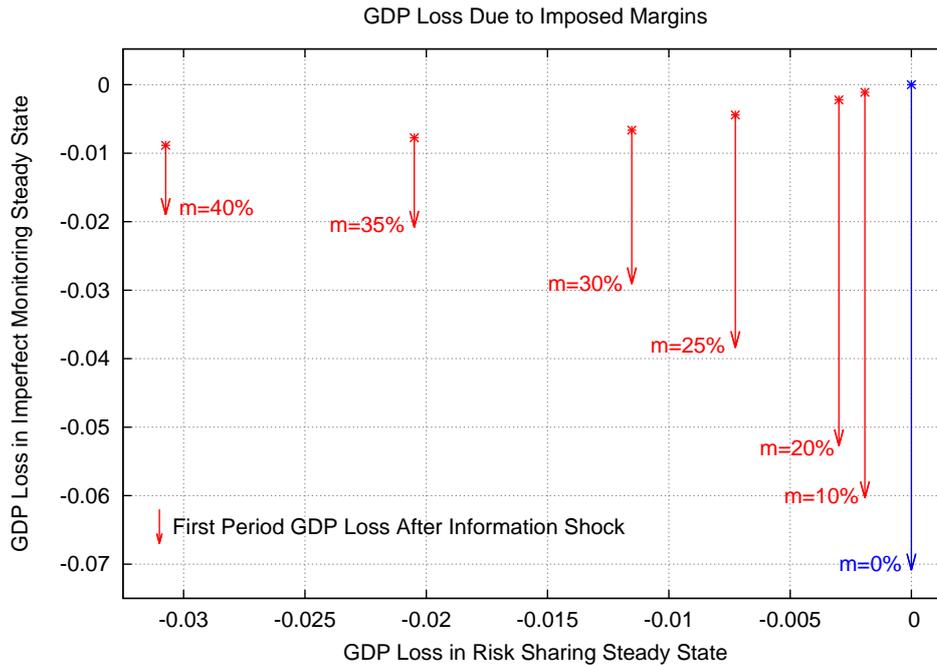
Figure 1: Efficiency Tradeoffs from Leverage Regulation

low productivity entrepreneurs with assets (collateral) become relatively more important.[1] The misallocation is especially severe with respect to the marginal entrepreneur who is indifferent between being a worker or an entrepreneur, and whose skills are higher if leverage is possible under efficient financial markets.

Incentive compatible allocations from these information and financial frictions can be mapped into leverage bounds, i.e., required downpayments or margins (the inverse of the leverage ratio). The average downpayment margin increases to 22% in the case of default, and up to 50% in the imperfect monitoring economy. When default is present together with imperfect monitoring, the margin is even higher, depending on the recovery rate. Naturally, these incentives lead to higher accumulation of assets in the steady state needed as a collateral for obtaining loans. Allocation inefficiencies have large adverse effects on aggregate productivity, welfare and inequality. Default in combination with information asymmetries lowers welfare by 7-14% (frictions increase the rent to entrepreneurs while welfare of workers falls by 25-40%).

These leverage bounds are then used to simulate policy regimes with imposed collateral margins on steady states with full risk sharing that are subsequently exposed to default and/or information frictions (imperfect monitoring). The macroprudential policies are implemented by imposing the same maximum leverage ratio (or, equivalently, minimum collateral margins) in both the initial risk-sharing steady state and the imperfect monitoring/default steady states. The goal is to evaluate the effects of macroprudential policies on the steady-state allocations and their behavior during the transition to new steady-state, i.e., the tradeoff between lower efficiency in the steady state and lower efficiency losses during a crisis.

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[1]Resource misallocation across individual production units may account for a substantial share of the income differences we observe across countries. For evidence see Hsieh and Klenow (2009), Banerjee et al. (2003), Banerjee and Munshi (2004), or Alfaro et al. (2008). For other explanations of resource misallocation see the references in Restuccia and Rogerson (2008). In the quantitative macro models, resource reallocation derived from removing inefficiencies in credit markets has large effects on output and measured TFP (see, among others, Buera et al. (2010), Erosa and Hidalgo-Cabrillana (2010), or Moll (2010)).

The goal is to provide a simple regulation framework for mitigating the adverse effects of information frictions. Figure 1 summarizes the efficiency tradeoffs from imposing leverage bounds. The stars show GDP losses in the risk-sharing (x-axis) and imperfect-monitoring (y-axis) steady states from imposing collateral margins on loans at 0%, 10%, ..., 40%. For example, imposing a 30% downpayment margin leads to 0.012% of GDP loss in the steady state with full risk sharing and to 0.07% loss in the imperfect monitoring steady state, both relative to the respective zero-margin steady states. When the change in information regime impacts the risk-sharing steady state, the fall of GDP in the first period of transition is 2.9% (arrow to $m = 30\%$). If there is no collateral margin requirement and the unanticipated change of the information regime occurs, the initial loss in transition is 7.1% (arrow to $m = 0\%$). In the long-run steady states, general equilibrium effects provide incentives to accumulate assets such that serve as buffer stock against risk and collateral for potential entrepreneurial projects. Consequently, the resulting steady state losses from leverage regulation can be contained around 1% of GDP for a 30% minimal downpayment regulation. In general, the higher the regulation, the greater are the GDP losses in the steady state and the smaller they are during a crisis.

The contribution of this papers is twofold. First, it develops a novel approach of modelling asymmetric information. In particular, the adverse selection problem for entrepreneurial or firm decisions can be applied to a wide variety of economic problems. Second, the paper simulates macroprudential policy in an environment with imperfect monitoring and default, using the derived leverage bounds. The incentive-compatible leverage bounds and optimal allocation of resources are analyzed in a single unified framework in a dynamic, general equilibrium economy.[2]

The appendix contains important proofs and additional results. The online appendix shows sensitivity results.

## 2   Entrepreneurial Activity and Credit Markets

This Section briefly describes the relationship between financial intermediation, entrepreneurship, and wealth in the U.S. data. Following Gentry and Hubbard (2000), define an entrepreneur as someone who combines upfront business investment with entrepreneurial skill to obtain the chance of earning economic profits. According to the Survey of Consumer Finances (SCF 1989), 8.7% households report active business assets greater than $5,000 (9.5% report business assets greater than $1,000). Similarly, in the Panel Study of Income Dynamics (PSID 1994), 10.4% of families own a business or have a financial interest in some business enterprise.

De Nardi et al. (2007) document that U.S. entrepreneurs are characterized by their high propensity to accumulate capital, risk taking, and committing skills and resources to their businesses. The Gini coefficient for family wealth is between 0.78 and 0.84, depending on the year and survey (PSID and SCF, respectively). The Gini coefficient for family income is 0.45 in the PSID and 0.54 in the SCF. In the PSID, the top 1 percent of families owns around 29% of the total household wealth and around 8% of the total income. The top 5 percent owns already 50% of the wealth and receives 20% of the income. Finally, the top decile owns more than 60% of the wealth and receives more than 32% of the income.

The percentage of business families increases in higher wealth classes: Quadrini (1999a) documents that about half of all families in the top 5% are business families. At the same time, the concentration of wealth among business families is not purely explained by the concentra-

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[2]For an exogenously imposed risk sharing see Heathcote et al. (2009). Karaivanov et al. (2006) estimate credit market imperfections due to limited liability and moral hazard in Thailand. Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997).

tion of income. Quadrini (1999b) and Gentry and Hubbard (2000) report that entrepreneurs are wealthy because they not only earn more income but also save relatively more than workers. Entrepreneurs, being such a small fraction of the population, receive 22% of the total income and own 40% of the total wealth. The ratio of wealth to income is about twice as large for business families (6.77 versus 2.94).

Available evidence suggests that entrepreneurs are constrained by their wealth. Based on the National Longitudinal Survey, Evans and Leighton (1989) find that men with greater assets are more likely to become self-employed all else being equal. They estimate in their model that entrepreneurs can borrow up to 50% of their current assets.

Personal wealth and housing are important sources of collateral. Lustig and Van Nieuwerburgh (2010) study borrowing-constrained households and find that when the value of housing relative to human wealth falls, loan collateral shrinks, borrowing (risk-sharing) declines, and the sensitivity of consumption to income increases. Adrian and Shin (2010) find a negative relationship between households' total assets and leverage.[3] Important contributions to the literature on leverage and credit markets include Geanakoplos and Zame (2013), Dubey et al. (2005), Adrian and Shin (2010), Adrian and Shin (2013), Geanakoplos and Fostel (2012), Geanakoplos and Zame (2013), Gromb and Vayanos (2002), Brunnermeier and Pedersen (2009), Lustig and Van Nieuwerburgh (2010), or Brunnermeier et al. (2011).

The Federal Reserve Survey of Terms of Business Lending reveals that small loans are more often secured by collateral. In 2000, of all commercial and industrial loans in the United States, 83% required collateral for loans smaller than $99,000, 74% for loans smaller than $1 million, 46.9% for loans smaller than $10 million, and only 31.7% for loans greater than $10 million. In Europe, the 2010 ECB survey of small and medium size enterprises (SME) shows that around 60% of small and medium size enterprises (SME) use at least one source of debt financing. The most prevalent source of debt financing has been the bank: 30% of companies have used bank overdraft facilities or a credit line and 26% have received a bank loan.

Lack of collateral is the most significant obstacle for establishing a firm, with about 15% of loans were fully rejected. See De Nardi et al. (2007) for a similar evidence for the United States. Overall, the three largest sources of funding are the principal owner, commercial banks, and trade creditors, which together account for over 70% of total small business finance.

Entrepreneurial income is more volatile than the labor income of workers. Heaton and Lucas (2000) find that the median standard deviation of the growth rate of nonfarm proprietary income is 64% annually while the median standard deviation of the growth rate of real wage income is only 35% annually. Entrepreneurial portfolios are very undiversified. Gentry and Hubbard (2000) find that active businesses account for 42% of entrepreneurs' assets (even in the top wealth classes). In the survey of Characteristics of Business Owners (2002), seventy percent of the owners of employer respondent firms reported that their business was their primary source of income. Not only face entrepreneurs high risk in their occupation, it also has a future value compared to initial income: Hamilton (2000) and Bohacek (2006) find evidence that most entrepreneurs enter and persist in business despite the fact that they have lower initial earnings in paid employment, with a median earnings differential of 35 percent. The turnover of business families is substantial.

Small firms pay fewer dividends, take on more debt, and invest more. In terms of the aggregate value of small firm debt, almost 90% of credit comes from traditional sources, mostly lines of credit and loans (Berger and Udell (1998)). Between 65 percent and 79 percent of entrepreneurs started

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[3]This evidence is well depicted by the National Financing Conditions Index (NFCI) or the Shiller-Case Home Price Index. For commercial and industrial loans see the Senior Loan Officer Opinion Survey on Bank Lending Practices by the Federal Reserve Board.

their own business and almost half of entrepreneurs use their own or their family's savings. Fazzari and Petersen (1988) report that internal finance in the form of retained earnings generates the majority of net funds for firms of all size categories: the average retention ratio is largest for small firms (80%) and lowest for the largest firms (50%).

At the same time, entrepreneurial activity is a very important feature of the U.S. economy. Small firms play an important role in innovation, technological change and productivity growth. Davis et al. (1996) show that the rates of job creation and job destruction in U.S. manufacturing firms decrease in firm size and that, conditional on the initial size, small firms grow faster than large firms. According to the U.S. Small Business Administration, small businesses account for half of all U.S. private-sector employment and produced 64 percent of net job growth in the United States between 1993 and 2008.

The models in this paper attempt to replicate this list of data on entrepreneurial activity. Motivated by the above empirical regularities, agents will be identified by their accumulated level of assets and entrepreneurial ability. In the presence of financial constraints, occupational choice and entrepreneurial decisions will be functions of this individual state and equilibrium prices.

# 3   The Economy

There is a large number of agents who differ in the amount of accumulated assets and their talent (productivity). Each agent decides whether to allocate his talent to be an entrepreneur and establish a firm or to be a worker and work for an entrepreneur. There exists a financial intermediation sector that provides credit services. All shocks are idiosyncratic and there is no aggregate uncertainty.

Each agent is endowed with a unit of time and evaluates streams of consumption $c$ with a utility function

$$E\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t\right)\right],$$

where $\beta \in (0,1)$ and $u : \Re_+ \to \Re$ is a bounded, strictly increasing, strictly concave, and twice differentiable continuous function that satisfies the Inada conditions.

The timing of events is as follows. At the beginning of every period, agents are identified by a level of accumulated assets $a \in A = [0,\infty)$ and by an idiosyncratic productivity shock $z \in Z = [\underline{z}, \overline{z}]$. This productivity level is carried from the previous period and represents a *signal* for the effective productivity the agent will have later in the period when production takes place, $z' \in Z$. Given $a$ and the signal $z$, first, each agents makes the occupational choice and decides whether to become a worker or an entrepreneur. Workers deposit their assets at the financial intermediaries and offer their labor services in the market. Entrepreneurs decide how much capital and labor to use in production. Importantly, entrepreneurs have to make their business plan and commit to capital and labor *before* their effective productivity shock $z'$ is realized. This feature of the model reflects the riskiness of entrepreneurial occupation. Furthermore, this setup allows for risk sharing and associated problems of imperfect monitoring. In the literature, entrepreneurs usually do not face any risk from running their businesses as their occupational and input decisions are made *after* the productivity shock is observed (for example Cagetti and De Nardi (2006), Meh (2008), or Restuccia and Rogerson (2008)). Each agent draws his effective productivity level $z'$ from a first-order Markov process with a monotone transition function $Q$ that satisfies the Feller property and the mixing condition. Denote the transition process of each occupation as $Q^W$ and $Q^E$.

At the end of the period, the effective productivity shock of each agent $z'$ is realized, production at firms takes place, workers are paid their wages and entrepreneurs realize profits or losses. Finally, each agent decides how much to consume $c$ and the amount of savings $a'$. The effective productivity shock $z'$ is carried to the next period as the signal for future productivity shocks. All assets depreciate at the rate $\delta \in (0, 1)$.

Banks are in the business of intermediating credit. The supply of credit comes from depositors, i.e. all workers as well as entrepreneurs with assets in excess of their capital needs. The demand for credit is from entrepreneurs whose efficient size of firm in terms of capital is larger than their accumulated assets. The intermediation technology is costless at an equilibrium interest rate $r$ described below.

## 3.1 A Worker's Budget Constraint

If an agent with assets $a$ and an idiosyncratic signal ability shock $z$ decides to be a worker, his budget constraint is

$$c + a' \leq (1 + r)a + wz', \tag{1}$$

where $r$ is the equilibrium interest rate, $w$ is the equilibrium wage common to all workers, and $z'$ is a worker's idiosyncratic productivity level.

## 3.2 An Entrepreneur's Budget Constraint

Entrepreneurs hire labor from a pool of perfectly diversified workers whose average productivity is $\tilde{z}$. An entrepreneur $(a, z)$ who commits $k$ units of capital, hires $n$ workers, and draws an idiosyncratic effective productivity shock $z'$, produces according to a production function

$$y(z'|a, z) = z' f\left(k(a, z), \tilde{z}n(a, z)\right) = z' \left(k(a, z)^\alpha (\tilde{z}n(a, z))^{1-\alpha}\right)^\theta, \tag{2}$$

where $\alpha \in (0, 1)$ and $\theta < 1$. The production function exhibits decreasing returns to scale which, as in Lucas (1978), can be thought of as capturing the presence of decreasing returns to managerial control.

The capital input $k$ can be smaller or greater than assets $a$. Denoting the amount borrowed $b = k - a$, an entrepreneur's budget constraint for each $z'$ is

$$c + a' \leq (1 - \delta)k + z'f(k, \tilde{z}n) - w\tilde{z}n - (1 + r)b \qquad \text{for each} \quad z' \in Z,$$

where $b > 0$ if the entrepreneur borrows and $b < 0$ if he is a depositor. Denote the transfer from the entrepreneur to the bank as the repayment amount when a productivity shock $z'$ is realized as $\rho(z') = (1 + r)b$ so that the budget constraint can be written consistently through all information environments as

$$c + a' \leq (1 - \delta)k + z'f(k, \tilde{z}n) - w\tilde{z}n - \rho(z') \qquad \text{for each} \quad z' \in Z. \tag{3}$$

I abstract from fixed costs associated with operating a business modeled in Hopenhayn and Rogerson (1993), among others, and think of the endogenous opportunity cost of forgone equilibrium wages/profits as the main determinant of entry/exit outcomes that arise from comparing the expected present discounted value of each occupation.[4]

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[4]Define the total income of an entrepreneur as $TI = ra + z'f(k, \tilde{z}n) - w\tilde{z}n - (r + \delta)k$, and operating profits

## 3.3 Recursive Formulation

At the stationary equilibrium, the problem of an agent who enters the period with the pair $(a, z)$ can be summarized by the value function

$$
\begin{aligned}
v(a, z) &= \max \left\{ v^W(a, z), v^E(a, z) \right\} \\
&= \max \left\{ \sum_{z'} v^W(a, z')\, Q^W(z, z'), \max_{k,n} \sum_{z'} v^E(a, z')\, Q^E(z, z') \right\},
\end{aligned} \tag{4}
$$

where the value for a worker and an entrepreneur, $I \in \{W, E\}$, respectively, equals

$$
v^I(a, z') = \max_{c,a'} \{ u(c) + \beta v(a', z') \}, \tag{5}
$$

subject to the occupation-specific budget constraints (1) or (3). Finally,

$$
a \in A \text{ with } \underline{a} = 0, \text{ and } k, n \geq 0, \tag{6}
$$

Notice again that entrepreneurs must commit capital and labor inputs before the effective productivity shock $z'$ is known. Note that it is optimal for an agent who decides to be a worker not to take any loan.

## 3.4 Endogenous Financing Constraint

The specification of the Inada-type utility function together with the uncertainty in entrepreneurial profits imply that agents with a low level of accumulated assets may be constrained with respect to the size of the entrepreneurial project. In particular, the total income must guarantee a nonnegative consumption for all possible realizations of profit,

$$
(1 - \delta)k + z' f(k, \tilde{z}n) - w\tilde{z}n - (1 + r)b \geq 0 \qquad \text{for all} \quad z' \in Z. \tag{7}
$$

Assume that in each period $Q^E(z, \underline{z}) > 0$ for all $z \in Z$ and that $\underline{z} = 0$. That is, the financing constraint must be satisfied for the lowest effective ability shock $\underline{z}' = 0$,

$$
(1 - \delta)k - w\tilde{z}n - (1 + r)b \geq 0 \qquad \text{for} \quad z' = 0. \tag{8}
$$

Given these assumptions on the transition process $Q^E$, poor agents with good entrepreneurial ideas may not be able to establish a firm or the firm size may be smaller than it would have been without the financing constraint.[5] For entrepreneurs with a high signal $z$, that is for those who would like to borrow and hire many workers, the potential failure of the project might prevent those with low savings to run a project at its efficient size. Furthermore, running a small firm might decrease the expected profits below the opportunity cost of running the project in the first place.[6] Therefore, the financing constraint may have important allocation effects on entry and especially on the firm size decisions.

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from running the firm as $OP = z' f(k, \tilde{z}n) - w\tilde{z}n - (r + \delta)k$.

[5] I set $Q^E(z, \underline{z}) > 0$ for all $z \in Z$ in each period only for a more intuitive interpretation of the financing constraint and the analysis of leverage and private risk premia in Section 7. Similar results will hold and a stationary equilibrium with an endogenous distribution of agents would also exist for $Q^N(z, \{\underline{z}\}) > 0$ for all $z \in Z$ in a finite number of periods $N > 0$.

[6] The main opportunity cost is the forgone equilibrium wage from being a worker. The future value of a project will be discussed in Section 7.

8

Note that this constraint applies not only to borrowing but also to depositing entrepreneurs. The financing constraint can also be motivated by limited contract enforceability.

## 3.5   Stationary equilibrium

At the aggregate level, the equilibrium outcome of these decisions is a probability measure $\lambda$ that determines the density over agents' individual states $(a, z)$, with a law of motion

$$\lambda'(a', z') = \sum_{(a,z'):a'=a'(a,z')} Q(z, z')\, \lambda(a, z),$$

where a transition function $Q(z, z') \equiv Q^W(z, z')|_W + Q^E(z, z')|_E$ determines the end of period productivity from the beginning of period productivity for each occupation. The measure of agents with next period's state $(a', z')$ consists of agents whose skills evolve to $z'$ and whose savings are $a'$.

## 3.6   Financial Intermediation

Financial intermediation consists of competitive banks that accept deposits and provide loans to entrepreneurs. Total deposits from workers and depositing entrepreneurs are

$$D \equiv \sum_{E \times W} \max\{0, a - k\}\, \lambda(a, z),$$

and the aggregate loans taken by entrepreneurs are

$$L \equiv \sum_{E \times W} \max\{0, k - a\}\, \lambda(a, z) = L.$$

I assume banks lend to a perfectly diversified pool of entrepreneurs and that banks do not face any risk so that their participation constraint is satisfied at zero profit. Banks have the capacity to process and share information on agents over time.

## 3.7   Stationary Recursive Competitive Equilibrium

The concept of stationary equilibria requires that assets supplied by all agents equal the amount of capital demanded by the entrepreneurs, that labor supply by workers equal the labor hired by entrepreneurs, and that all allocations be feasible for a time invariant probability measure.

**Definition 1** *A stationary recursive equilibrium with borrowing and lending is constant prices* $(r, w)$, *value functions* $(v, v^W, v^E)$, *policy functions* $(k, n, c, a')$, *a probability measure* $\lambda$, *and aggregate levels* $(A, K, D, L, Y, \tilde{z})$, *such that*

1. *at given prices the policy functions solve the optimization problem of each agent* $(a, z)$;
2. *the probability measure* $\lambda$ *is time invariant;*
3. *prices are such that markets clear: in particular, the financial market,* $D = L$, *and the labor market,*

$$\sum_W z' Q^W(z, z')\, \lambda(a, z) = \tilde{z} \sum_E n(a, z)\, \lambda(a, z),$$

9

*with*

$$\sum_W \lambda(a, z) + \sum_E \lambda(a, z) = 1;$$

*4. the aggregate feasibility constraint holds at equality: for goods*

$$\sum \{c(a, z') + \delta a'(a, z')\} Q(z, z') \lambda(a, z) = \sum z' f(k(a, z), \tilde{z} n(a, z)) Q^E(z, z') \lambda(a, z) = Y,$$

*and assets,*

$$A = \sum a \, \lambda(a, z) = \sum_E k(a, z) \, \lambda(a, z) = K$$

# 4   Risk Sharing

Contracts that could reduce the riskiness of production and alleviate the financing constraint might have important efficiency effects if they allow the more talented entrepreneurs to enter and/or run their firms at a more efficient size. In this Section I describe a risk sharing contracts for entrepreneurs under the assumption that all allocations are fully observable and contracts fully enforceable.[7] As the risk sharing contracts are actuarially fair, they are also provided by the representative bank. Similarly to borrowing and lending, the risk sharing transfer technology bears no cost.

An risk sharing contract allows entrepreneurs to insure against the risk represented by $z'$ by exchanging transfers from the bank that collects pooled profits from all entrepreneurs. Entrepreneurs will be able to choose the degree of output uncertainty they want to bear. In particular, assume that the entrepreneur can write a risk sharing contract that will insure a fraction of $x \in [0, 1]$ of the difference between his realized output and the expected output. That is, the entrepreneur $(a, z)$ who uses inputs $(k, n)$ and receives a productivity shock $z'$, sends to the bank a risk-sharing transfer equal to

$$x f(k, \tilde{z} n) (z' - E[z'|z]).$$

A negative transfer represents an insurance transfer from the bank in the case of lower than expected output, and vice versa. In the extreme cases, $x = 0$ represents no risk sharing as in the previous Section while $x = 1$ represents full risk sharing where the entrepreneur receives transfers that deliver the expected output in all states $z'$.

Denote the total transfer from the entrepreneur to the bank at the end of period after a shock $z'$ is realized as

$$\rho(z') = (1 + r)b + x f(k, \tilde{z} n) (z' - E[z'|z]), \tag{9}$$

where $b$ is the amount of loan taken at the beginning of period.

The value function with risk sharing is

$$v(a, z) = \max \left\{ \sum_{z'} v^W(a, z') Q^W(z, z'), \max_{\substack{x \in [0,1] \\ k, n}} \sum_{z'} v^E(a, z') Q^E(z, z') \right\}, \tag{10}$$

subject to the occupational budget constraints (1) or (3) with the risk-sharing repayment $\rho(z')$ for each $z'$ defined in equation (9). The endogenous financing constraint now becomes

$$(1 - \delta)k + (1 - x)z' f(k, \tilde{z} n) - w \tilde{z} n - (1 + r)b + x E[z' f(k, \tilde{z} n)|z] \geq 0 \quad \text{for all} \quad z' \in Z, \tag{11}$$

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[7]Default and imperfect monitoring will be analyzed in Sections 5 and 6.

which is for the lowest shock,

$$(1 - \delta)k - w\tilde{z}n - (1 + r)b + xE[z'f(k, \tilde{z}n)|z] \geq 0 \qquad \text{for} \quad z' = \underline{z}' = 0. \tag{12}$$

Compared to equation (8), the endogenous financing constraint is relaxed by the last term, the insured fraction of the expected output. For choices $x > 0$ the financing constraint is less binding than in the economy without risk sharing. In the case of full risk sharing, the entrepreneur faces no output uncertainty and operates at the efficient scale. Since risk sharing is costless, $x = 1$ is chosen in efficient allocations under full information. Detailed characterization of these allocation will be provided in Section 7.

The definition of a stationary recursive equilibrium with risk sharing is the same except for the added insurance decisions of entrepreneurs. With risk sharing, the representative bank now intermediates credit between agents as well as provides insurance services.

# 5  Default

In this Section, the borrowing contract between an entrepreneur and a bank can be subject to default on the loan repayment. I assume that a defaulting entrepreneur will have no access to financial markets and risk sharing from the next period on. Also, it is assumed that wages of workers are always honoured and that an entrepreneur defaults on the loan and subsequently does not undertake any risk sharing transfer with the bank. Naturally, defaulted entrepreneurs do not have access to risk-sharing transfers.

The value of being (already) forever in default is

$$v^D(a, z) = \max \left\{ \sum_{z'} v^{DW}(a, z') \, Q^W(z, z'), \max_{k,n} \sum_{z'} v^{DE}(a, z') \, Q^E(z, z') \right\}, \tag{D}$$

where the value for a previously defaulted worker and an entrepreneur, $I \in \{W, E\}$, respectively, equals

$$v^{DI}(a, z') = \max_{c,a'}\{u(c) + \beta v^D(a', z')\}, \tag{13}$$

subject to the budget constraints for a worker in default

$$c + a' \leq (1 - \delta)a + wz', \tag{14}$$

or a budget constraint for an entrepreneur in default,

$$c + a' \leq (1 - \delta)a + z'f(k, \tilde{z}n) - w\tilde{z}n, \tag{15}$$

and an exclusion from financial markets constraint

$$k \leq a \text{ and } x = 0. \tag{16}$$

In equilibrium, there will be no default. In preventing default, the contract must satisfy an incentive constraint where the present discounted value of honoring the contract is greater or equal than the present discounted value of defaulting. The gain from default is the current gain from a lower loan repayment; the cost is the present discounted value of being excluded from the financial market from the next period on.

11

Note that default applies only to borrowers with $b > 0$. Let $\gamma \in [0, 1)$ denote the fraction of the loan the bank recovers. If $\gamma = 0$, there is a full default.

For all $(a, z)$ and each $x(a, z) \in [0, 1]$, the repudiation constraint is an allocation $(k(a, z), n(a, z))$ such that for each realization of $z' \in Z$,

$$v^E(a, z') \geq \max_{\hat{c}, \hat{a}'} \left\{ u(\hat{c}) + \beta v^D(\hat{a}', z') \right\},$$ (17)

subject to a budget constraint

$$\hat{c} + \hat{a}' \leq (1 - \delta)k + z'f(k, \tilde{z}n) - w\tilde{z}n - \rho^D,$$ (18)

with $\rho^D = \gamma(1 + r)b$.

**Definition 2 (One-Period Gain from Default)** *For a borrowing entrepreneur characterized by $(a, z)$, the one-period gain from defaulting is*

$$\Delta^D = (1 - \gamma)(1 + r)b(a, z) > 0.$$

For derivation see Appendix. The current period gain from defaulting is positive as the entrepreneur repays only a part of the loan that the bank can recover. Note there is no gain from risk-sharing transfers that are zero in expectation when shocks $(z, z')$ are reported truthfully. As the gain from default is decreasing in the size of a loan, the bank will tend to provide smaller loans in order to provide default-free contracts in equilibrium. Again, the default constraint would be likely more binding on poor agents and high-skill agents, both with high leverage.[8]

### 5.0.1 Ex-Ante Default

An entrepreneur can also default ex-ante, that is keeping the borrowed capital only for consumption, not producing, and repaying only a fraction of the loan $\rho^D = \gamma(1 + r)b$. For all $(a, z)$ and $x \in [0, 1]$, the repudiation constraint is an allocation $(k, n)$ such that

$$v^E(a, z) = \max_{k, n} \sum_{z'} v^E(a, z') \, Q^E(z, z') \geq \max_{\hat{c}, \hat{a}'} \left\{ u(\hat{c}) + v^D(\hat{a}', \underline{z}) \right\},$$ (ex ante-D)

subject to a budget constraint

$$\hat{c} + \hat{a}' \leq (1 - \delta)k - \rho^D.$$ (19)

Note that there is no uncertainty. It is assumed that no production is followed by $z' = \underline{z}$ in the next period.

**Definition 3 (One-Period Gain from Ex-Ante Default)** *For a borrowing entrepreneur characterized by $(a, z)$, the one-period gain from defaulting ex-ante is*

$$E[\Delta^{DEA}|z] = (1 - \gamma)(1 + r)b(a, z) - \left( E[z'|z]f(k(a, z), \tilde{z}n(a, z)) - w\tilde{z}n(a, z) \right).$$

---

[8]In a different setup it might be Pareto superior to allow for optimal default in bad circumstances to default as in Dubey et al. (2005)).

For derivation see Appendix. The one-period gain from defaulting ex-ante is $\Delta^D$ minus the loss of expected output less wages paid to workers.

# 6    Imperfect Monitoring

Private information on entrepreneurial productivity constrains efficient allocation of resources available under full risk sharing. When $z$ is unobservable at all times, in order to assure truthful reporting, the bank specifies a contract consisting of a threat-keeping constraint needed for a true report of the signal shock $z$ and an incentive constraint required for a truthful report of the realized shock $z'$. In equilibrium, there will only be truthful reports at the cost of imperfect insurance and sub-optimal allocation of capital.

## 6.1    Ex-Post Incentive Compatibility Constraint

The ex-post, incentive constraint for a truthful report of $z'$ is specified first. For an entrepreneur with assets $a$, true initial signal shock $z$, who was assigned inputs $(k(a,z), n(a,z))$ and risk-sharing $x(a,z)$, and who later realized a true realization of output based on $z'$ but contemplates reporting a shock realization $\hat{z}'(z')$, the incentive compatibility constraint requires that for all $(a,z) \in A \times Z$, and all true $z' \in Z$,

$$u(c) + \beta v(a', z'; z) \geq \max_{\hat{c}, \hat{a}'} \left\{ u(\hat{c}) + \beta \hat{v}(\hat{a}', z'; \hat{z}') \right\} \qquad \text{for each} \quad \hat{z}' \in Z, \qquad \text{(IC)}$$

where the right-hand side maximization is subject to the budget constraint

$$\hat{c} + \hat{a}' \leq (1 - \delta)k + z'f(k, \tilde{z}n) - w\tilde{z}n - \rho(\hat{z}'),$$

where

$$\rho(\hat{z}') = (1 + r)b + x\left(\hat{z}'f(k, \tilde{z}n) - E\left[z'f(k, \tilde{z}n)|z\right]\right) = (1 + r)b + xf(k, \tilde{z}n)\left(\hat{z}' - E\left[z'|z\right]\right).$$

Note that the endogenous financing constraint remains the same as in equation (12) for the same choice of risk sharing and inputs.

Denote the choice of $\hat{z}'$ that maximizes the value of ex-post reporting as

$$\hat{z}'(z') \equiv \arg\max_{\hat{z}' \in Z} \max_{\hat{c}, \hat{a}'} \left\{ u(\hat{c}) + \beta \hat{v}(\hat{a}', z'; \hat{z}') \right\} \qquad \text{for all} \quad z' \in Z.$$

Truthful reporting requires that $\hat{z}'(z') = z'$ for all $z' \in Z$.

## 6.2    Ex-Ante Threat-Keeping Constraint (Adverse Selection)

The ex-ante, threat-keeping constraint represents an adverse selection problem. The constraint requires that the allocations are based on a reported shock $\hat{z}$ when the true shock is $z$. For an entrepreneur with assets $a$, true shock $z$, and misreported shock $\hat{z}$ carried over from the last period, let $\hat{v}^E(a, z; \hat{z})$ denote the present discounted value of being assigned inputs $(\hat{k} = k(a, z; \hat{z}), \hat{n} = n(a, z; \hat{z}))$ with risk-sharing $\hat{x} = x(a, z; \hat{z})$,

$$\hat{v}^E(a, z; \hat{z}) \equiv \max_{\substack{\hat{x} \in [0,1] \\ \hat{k}, \hat{n}}} \sum_{z'} \max_{\hat{c}, \hat{a}'} \left\{ u(\hat{c}) + \beta v(\hat{a}', z'; z') \right\} Q^E(z, z'), \qquad \text{(TK)}$$

13

subject to

$$\hat{c} + \hat{a}' \leq (1 - \delta)\hat{k} + z' f(\hat{k}, \tilde{z}\hat{n}) - w\tilde{z}\hat{n} - \hat{\rho}(z'),$$

where

$$\hat{\rho}(z') = (1 + r)\hat{b} + \hat{x} \left( z' f(\hat{k}, \tilde{z}\hat{n}) - E\left[ z' f(\hat{k}, \tilde{z}\hat{n}) | \hat{z} \right] \right) = (1 + r)\hat{b} + \hat{x} f(\hat{k}, \tilde{z}\hat{n}) \left( z' - E\left[ z' | \hat{z} \right] \right).$$

Note that the true value $v(\hat{a}', z'; z')$ on the right-hand side of (TK) is due to the incentive compatibility constraint (IC) applied to the one-period deviation from the current period onwards in the recursive formulation above, i.e., when $z' = \hat{z}'(z')$.

The endogenous financing constraint for the lowest realization of output $z' = 0$ is

$$(1 - \delta)\hat{k} - w\tilde{z}\hat{n} - (1 + r)\hat{b} + \hat{x} f(\hat{k}, \tilde{z}\hat{n}) E[z'|\hat{z}] \geq 0.$$

## 6.3   Truth-Telling Incentive Compatibility Contract

The incentive compatible contract must jointly include the threat-keeping constraint (TK, adverse selection) and the incentive compatibility constraint (IC), otherwise the agent would have incentives to deviate in either subperiod.

Finally, the adverse selection constraint must also guarantee that the contract is incentive compatible for those entrepreneurs who would choose to become workers for whom $\hat{v}^W(a, z; \hat{z}) = v^W(a, z)$ as wages are paid for each realization of $z'$ without any risk sharing.

Together, truth-telling requires for all $(a, z) \in A \times Z$ and all $\hat{z} \in Z$,

$$v(a, z) \geq \hat{v}(a, z; \hat{z}) = \max\left\{ \hat{v}^E(a, z; \hat{z}), v^W(a, z) \right\},$$

subject to the incentive compatibility constraint (IC) and threat-keeping (TK) constraints.

## 6.4   One-Period Gain from Imperfect Monitoring

For an $(a, z)$-entrepreneur contemplating deviations from truthful reporting it is possible to derive one-period gains from reporting any combinations $(\hat{z}, \hat{z}') \in Z \times Z$. Note that these are out-of-equilibrium allocations that can be nevertheless evaluated at equilibrium prices.

These gains can be decomposed into three parts. First is the loss in expected profits $E[\Delta\pi|z]$ from reporting a different signal $\hat{z}$ than the true signal shock $z$. This loss arises from suboptimal inputs $(k(a, \hat{z}), n(a, \hat{z}))$ assigned to to correct expectations based on the true signal shock $z$ in the decreasing-returns production function. A suboptimal inputs lead to a lower expected profit

$$E[\Delta\pi|z] \equiv E[\pi(k(a, \hat{z}), n(a, \hat{z}))|z] - E[\pi(k(a, z), n(a, z))|z] < 0.$$

In an example depicted in Figure 2, the agent has a low signal shock $z_L$ and contemplates reporting a higher signal shock $z_H$. For the true signal $z_L$, a capital stock $k(z_L)$ maximizes expected profits $E[\pi(k(z_L))|z_L]$, and similarly for the high shock. The expected loss in profits from misreporting $z_H > z_L$ is $E[\Delta\pi]$.

Positive gains from imperfect monitoring arise from positive risk-sharing transfers in expectation: from misreporting the ex-ante signal $z$ and/or the ex-post effective shock $z'$. Reporting a higher signal shock $\hat{z} > z$ delivers higher ex-ante, expected risk-sharing transfers. This gain arises from the adverse selection problem where the increasing, monotone $Q^E$-transition process implies
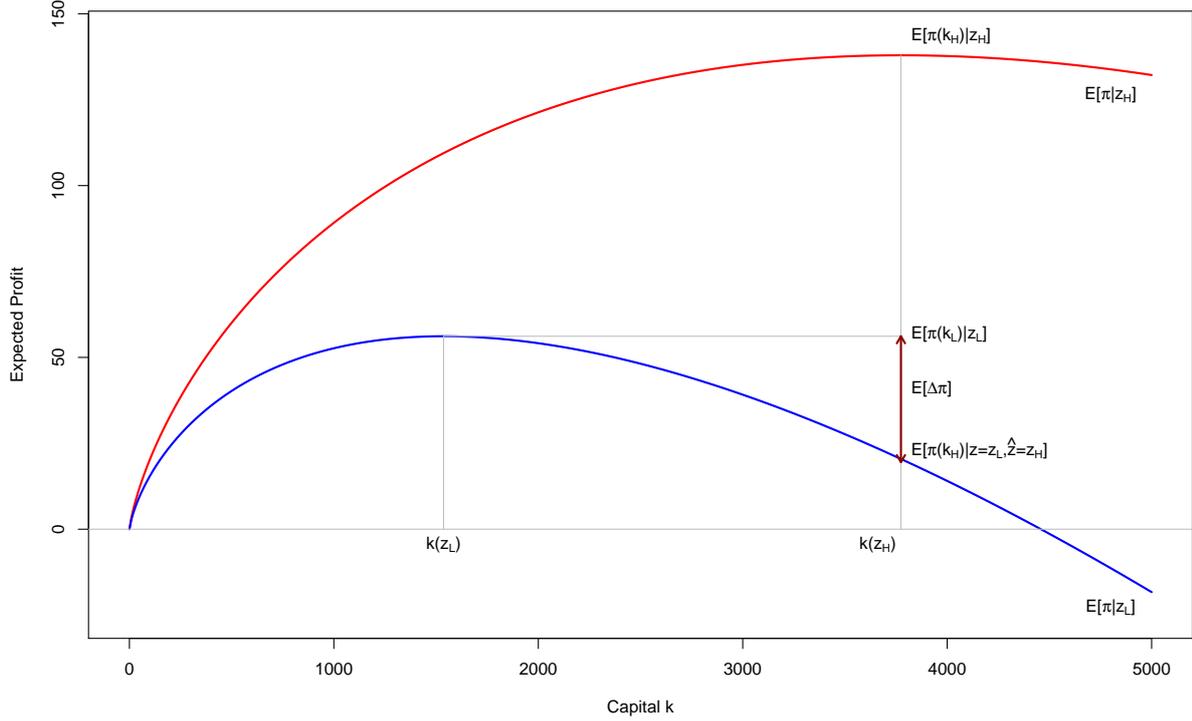
Figure 2: Expected Profit and Imperfect Monitoring

$E[z'|\hat{z}] \geq E[z'|z]$, that is a higher probability of realized output for which the entrepreneurs expects risk-sharing transfers from the bank,

$$x(a,\hat{z})f(k(a,\hat{z}),n(a,\hat{z}))\left(E\left[z'|\hat{z}\right] - E\left[z'|z\right]\right) > 0.$$

The ex-post gains arise from reporting $\hat{z}'(z') < z'$, that is directly manipulating the risk-sharing transfers ex-post, and in expectation,

$$x(a,\hat{z})f(k(a,\hat{z}),n(a,\hat{z}))E\left[z' - \hat{z}'(z')|z\right] > 0.$$

Note that for all three cases the agent forms the correct expectation based on the true signal shock $z$.

**Definition 4 (One-Period Gain from Imperfect Monitoring)** *For an entrepreneur characterized by* $(a,z)$, *the one-period gain from reporting any* $(\hat{z},\hat{z}') \in Z \times Z$ *is*

$$E[\Delta|z] = E[\Delta\pi|z] + \underbrace{\hat{x}f(\hat{k},\tilde{z}\hat{n})\left(E\left[z'|\hat{z}\right] - E\left[z'|z\right]\right)}_{Ex-Ante} + \underbrace{\hat{x}f(\hat{k},\tilde{z}\hat{n})E\left[z' - \hat{z}'(z')|z\right]}_{Ex-Post},$$

*where*

$$E[\Delta\pi|z] = E[z'|z](f(\hat{k},\tilde{z}\hat{n}) - f(k,\tilde{z}n)) - w\tilde{z}(\hat{n}-n) - (r+\delta)(\hat{k}-k) < 0.$$

For derivation see Appendix. Note that the gain applies to both borrowing and depositing entrepreneurs, i.e. all entrepreneurs with positive risk-sharing.

15

In equilibrium with truthful reporting, it must be that the one-period gains from imperfect monitoring are dominated by the value of long-term losses.

## 6.5 Default together with Imperfect Monitoring

When defaulting is an option together with the imperfect monitoring problem the incentive compatibility constraint (IC) for each ex-post realization shock $z'$ must also hold for the possibility of default, for all $(a, z) \in A \times Z$, and all $z' \in Z$,

$$u(c) + \beta v(a', z') \geq \max_{\hat{c}, \hat{a}'} \left\{ u(\hat{c}) + \beta v^D(\hat{a}', z') \right\}, \tag{D-IC}$$

where the right-hand side maximization is subject to the same budget constraint as in equation (18). As risk-sharing transfers are also provided by the bank, it is assumed that default must occur before risk-sharing transfers take place.

**Definition 5 (One-Period Gain from Imperfect Monitoring and Default)** *For a borrowing entrepreneur characterized by $(a, z)$, the one-period gain from imperfect monitoring and default for any $(\hat{z}, \hat{z}') \in Z \times Z$ is*

$$E[\Delta^D|z] = (1 - \gamma)(1 + r)\hat{b} - E[\Delta\pi|z],$$

*where*

$$E[\Delta\pi|z] = E[z'|z](f(\hat{k}, \tilde{z}\hat{n}) - f(k, \tilde{z}n)) - w\tilde{z}(\hat{n} - n) - (r + \delta)(\hat{k} - k) < 0.$$

For derivation see Appendix. The loss comes again from the lower profits in expectation $E[\Delta\pi|z]$ while the gain arises from the defaulted part of the loan $b(a, \hat{z})$. Note that there is no loss from risk-sharing transfers that are not provided to defaulting entrepreneurs.

## 6.6 Ex-Ante Default together with Imperfect Monitoring

When imperfect monitoring is combined with the possibility of ex-ante default, the threat-keeping constraint (TK) must hold also for the value of reporting a different signal $\hat{z}$ and defaulting immediately without using the inputs $(k(a, \hat{z}), n(a, \hat{z}))$ in production.

For all $(a, z, \hat{z})$, the threat-keeping (TK) and repudiation (ex ante-D) constraints together are

$$\hat{v}^E(a, z; \hat{z}) \geq \max_{\hat{c}, \hat{a}'} \left\{ u(\hat{c}) + v^D(\hat{a}', \underline{z}) \right\}, \tag{D-TK}$$

subject to a budget constraint

$$\hat{c} + \hat{a}' \leq (1 - \delta)\hat{k} - \hat{\rho}^D,$$

with $\hat{\rho}^D = \gamma(1 + r)\hat{b}$.

As risk-sharing is also provided by the bank, it is assumed that there are no risk-sharing transfers if there is a default.

**Definition 6 (One-Period Gain from Imperfect Monitoring and Ex-Ante Default)**
*For a borrowing entrepreneur characterized by $(a, z)$, the one-period gain from one imperfect monitoring and ex-ante default for any $\hat{z} \in ZZ$ is*

$$E[\Delta^{D-EA}|z] = (1 - \gamma)(1 + r)\hat{b} - (r + \delta)(\hat{b} - b) - \left( E[z'|z]f(k, \tilde{z}n) - w\tilde{z}n \right).$$

For derivation see Appendix. The gain from ex-ante default and adverse selection of reporting $\hat{z}$ is the gain from not repaying the assigned loan less the loan repayment on the difference with respect to the loan based on true signal, minus the loss from unrealized production. Note that there is no loss from risk-sharing transfers that are not provided to defaulting entrepreneurs.

## 6.7   The Economy without Financial Intermediation

Finally, I include a description of an economy without financial intermediation. In financial autarky, there is no financial intermediation (interest rate does not exist) and for all agents $b = 0$. Without financial sector, there is no possibility of risk sharing (and therefore, there is no space for imperfect monitoring). Therefore, each entrepreneur must finance his or her project from accumulated assets and faces full risk from volatile production. Otherwise, the structure of the this economy is identical. In particular, there still exists a labor market where workers can be hired at an equilibrium wage $w$ and entrepreneurs have access to the same production technology.

A worker now faces a budget constraint

$$c + a' \leq (1 - \delta)a + wz'.$$

Without financial intermediation, entrepreneurs must satisfy $k \leq a$. It is easy to show that it is always optimal to use all assets in production and adjust the number of workers. An entrepreneur has a budget constraint
$$c + a' \leq (1 - \delta)a + z'f(a, \tilde{z}n) - w\tilde{z}n.$$
The financing constraint can be written, for $z' = \underline{z} = 0$,

$$(1 - \delta)a - w\tilde{z}n \geq 0.$$

The definition of the stationary recursive competitive equilibrium is similar to that of the economy with financial intermediation except for the market clearing condition in the asset market. If the equilibrium exists, i.e., if there is a positive fraction of workers (entrepreneurs), the total amount of capital used in production is strictly smaller than the total amount of assets in the economy, $K < A$.

# 7   Characterization of Entrepreneurial Decisions

This Section characterizes the optimal allocations by entrepreneurs. Because productivity shocks multiply the production function and it is the only source of uncertainty, there is an optimal capital-labor ratio for depositors and borrowers independent of assets or productivity shock $z$, provided that the exogenous collateral does not bind.

**Proposition 1 (Capital-Labor Ratio)** *If the collateral constraint does not bind, the optimal capital-labor ratio equals*
$$\chi \equiv \frac{k}{n} = \left(\frac{\alpha}{1 - \alpha}\right)\left(\frac{w\tilde{z}}{r + \delta}\right),$$
*where $r$ and $w$ are the equilibrium prices.*

The proof follows directly from the first order conditions.

**Proposition 2 (Risk Sharing with Full Information)** *In an economy with full information, the optimal risk sharing is $x(a, z) = 1$ for all $(a, z)$.*

This results follows directly from the first order condition for the choice of risk sharing derived in the Appendix.

Intuitively, as the degree of risk sharing at an individual entrepreneur's level is costless, the optimal decision for risk averse entrepreneurs is to fully insure the profit of the firm. Therefore, in the risk sharing economy with full information, the entrepreneurs do not face any risk and the financing constraint is never binding.

## 7.1   Endogenous Bounds on Leverage

**Definition 7 (Leverage Ratio)** *Leverage is the the ratio of assets to equity, that is, the ratio of an entrepreneur's own assets plus debt over assets,*

$$\phi(a, z) \equiv \frac{a + b(a, z)}{a} = \frac{k(a, z)}{a}.$$

The bank for provides incentive compatible contracts by restricting risk-sharing and limiting the amount of the loan. The incentive compatible truth-telling contract can be expressed in terms of a maximum leverage ratio.

**Definition 8 (Collateral Constraint)** *For an entrepreneur characterized by $(a, z)$, and for each choice of risk sharing $x(a, z) \in [0, 1]$,*

$$\phi(a, z) \leq \kappa(a, z) \qquad with \; \kappa > 1. \tag{20}$$

For depositing entrepreneurs, the collateral constraint does not bind as $k < a$. For depositing entrepreneurs subject to imperfect monitoring, the only instrument the bank has is to limit the amount of risk sharing. For each $(a, z)$, the Lagrange multiplier $\eta$ on the collateral constraint can be used for the analysis of risk premia and allocation of capital across firms.

The endogenous financing constraint also imposes an upper bound on the leverage. Using the optimal capital-labor ratio we get the following Proposition for the upper bound on leverage derived from the maximum level of capital satisfying the financing constraint.

**Proposition 3 (Leverage Bound)** *For a borrowing entrepreneur $(a, z)$ with risk sharing $x(a, z) \in [0, 1]$, the upper bound on leverage is*

$$\overline{\phi}(a, z) = \min \left\{ \kappa(a, z), \frac{\alpha(1 + r)}{r + \delta} \left( 1 + x(a, z) \frac{E[z'|z] f(k(a, z), \tilde{z} n(a, z))}{(1 + r)a} \right) \right\}. \tag{21}$$

The upper bound on leverage comes either from the exogenous collateral constraint or from the endogenous financing constraint. The latter is decreasing in the equilibrium interest rate as a higher interest rate makes borrowing more costly which in turn might lead to a binding financing constraint.

Risk sharing $x(a, z)$ increases the right-hand term by guaranteeing income that can be used as collateral in the worst state when production is zero. This term represents the expected output over the gross return on assets. The upper bound on leverage is a function of wealth, expected output, parameters and equilibrium interest rate on loans. Without risk sharing, that

is when $x = 0$, the upper bound on leverage is independent of assets and is non-increasing in the equilibrium interest rate, $\frac{\partial \overline{\phi}}{\partial r} < 0$. For the usual values of the parameters $(\alpha, \delta)$ and the equilibrium loan rate $r$, $\alpha(1 + r)/(r + \delta) \gg 1$, so that the maximum level of capital as a function of assets lies above the 45-degree line.

The leverage bound also implies the minimal amount of down-payment on a loan, the margin, defined as $m = 1 - b/k$, i.e. one minus the loan-to-value ratio.

**Corrolary 1 (Collateral Margin Bound)** *For a borrowing entrepreneur $(a, z)$, the lower bound on collateral margin is*

$$\underline{m}(a, z) = \overline{\phi}(a, z)^{-1}. \tag{22}$$

## 7.2 Private Equity Premium

Financing and collateral constraints also imply private equity premia. For a simpler exposition, the marginal capital product is denoted $f_k \equiv f_k(k, \tilde{z}n)$ and the expected marginal utility of consumption $E[u_c(c)|z] = E[u_c(c(a, z'))|z]$.

**Proposition 4 (Private Equity Premium)** *For a borrowing entrepreneur characterized by $(a, z)$, the private equity premium is*

$$E[z'|z]f_k - r - \delta = \frac{\eta}{E[u_c(c)|z]} - (1 - x)\frac{\text{cov}\,(u_c(c), z'f_k)}{E[u_c(c)|z]}, \tag{23}$$

*where $\eta$ is the Lagrange multiplier associated with the collateral constraint.*

The private equity premium defines the extra return earned on the marginal investment into the business project instead of depositing it in the bank. The right-hand side consists of two non-negative terms: the first term represents the wedge related to the collateral constraint (representing the incentive constraints), the second term is the premium compensating the entrepreneur for the idiosyncratic risk of the business project (note that the covariance term is negative). The premium increases in the severity of the collateral constraint and in riskiness of the project. The private equity premium is derived in the Appendix. With risk sharing, the idiosyncratic risk of the project is lower by the insured part of the risk through risk sharing.

With full information and full risk sharing, both the financing and the collateral constraints are never binding and the private equity premium equals zero, $E[z'|z]f_k - r - \delta = 0$.

## 7.3 Abreu, Pearce, and Stacchetti

Convergence in sets as in Abreu et al. (1986) and Abreu et al. (1990).

Continuation values $(v', v^D) \in V \times V$ into $v \in V$. Continuation value functions must be admissible with respect to the set of value functions, $(v', v^D) \in W$, Admissible allocations map candidate continuation values tomorrow into new candidate values today, $B(W)$. The operator $B$ is monotone, if $W \subseteq W' \subseteq R$ then $B(W) \subseteq B(W')$. $B(\cdot)$ maps compact sets $W$ into compact sets $B(W)$. $W$ is self-generating if $W \subseteq B(W)$. If $W \subset R$ is bounded and self-generating, then $B(W) \subseteq V$. With $V \subseteq B(V)$, self-generation implies $V = B(V)$, the set of equilibrium values is the largest fixed point of $B$. Monotonicity and compactness allow iteration on $B$ to convergence from a large initial set $V \subseteq B(W_0) \subseteq W_0$.

## 7.4 Existence of Equilibrium

The occupational choice of an agent is based on the comparison of the expected present discounted value of each career. The following two assumptions guarantee the existence of a stationary recursive equilibrium with a positive fraction of the population in each occupation.

**Assumption 1** *The signal ability shock $\bar{z}$ is such that there exists an asset level $a^s$ for which*

$$\sum_{z'} v^W(a, z') \, Q^W(\bar{z}, z') \leq \sum_{z'} v^E(a, z') \, Q^E(\bar{z}, z') \text{ for all } a \geq a^s.$$

**Assumption 2** *The signal ability shock $\underline{z}$ is such that*

$$\sum_{z'} v^W(a, z') \, Q^W(\underline{z}, z') \geq \sum_{z'} v^E(a, z') \, Q^E(\underline{z}, z') \text{ for all } a \in A.$$

Both assumptions are related to the opportunity cost of each occupation. The first assumption requires that there be a shock sufficiently high so that agents with assets greater than a switching level $a^s$ become entrepreneurs: the expected value of entrepreneurship is greater than the expected value of choosing to work for a wage. Vice versa, the second assumption requires a shock sufficiently low so that agents with such a signal prefer to be workers.

The properties of value functions for each occupation follow the analysis in Bohacek (2006) and Stokey et al. (1989). The value function of each occupation, $v^I(a, z')$, is strictly increasing in each argument since the utility function is strictly increasing and strictly concave and a the constraint set is strictly increasing in assets and the effective ability shock. Due to the monotonicity of the transition matrix $Q$, the expected value functions of each occupations and the value function $v(a, z)$ are all increasing and continuous functions of both $a$ and $z$.[9]

## 7.5 Future Value of Entrepreneurship

The experience aspect contained in the monotone Markov process has important implications for the investment decisions of entering entrepreneurs. Contrary to the static model in Lucas (1978), where agents only consider the current expected incomes, it is the expected discounted present value of each career that determines an agent's occupational decision.

For a given level of signal ability shock $z \in Z$, an agent with assets at the switching level $a^s(z)$ is indifferent between working and undertaking an entrepreneurial project. Therefore, it must be the case that

$$\sum_{z'} v^W(a^s(z), z') \, Q^W(z, z') = \sum_{z'} v^E(a^s(z), z') \, Q^E(z, z'). \tag{24}$$

The first order intertemporal condition for any asset level $a$ and any realized effective ability shock $z'$ is just $u_c(c(a, z')) = \beta v_a(a'(a, z'), z')$ as there is no uncertainty about the agent's next period state. Using the usual envelope conditions and assuming interior solutions, the condition (24) can be rewritten, dropping the term $(1 + r)\beta$ on both sides, as

$$\sum_{z'} v_a(a'(a^s(z), z'), z') \, Q^W(z, z') = \sum_{z'} v_a(a'(a^s(z), z'), z') \, Q^E(z, z').$$

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[9]The value function $v(a, z)$—the outer envelope for the value functions at each shock level—may not be a concave function even if the value functions of workers and entrepreneurs are. Gomes et al. (2001) analyze a model of unemployment with a similar property. The operator on the value function satisfies the Blackwell's sufficient conditions for a contraction mapping. In this paper, we do not explore possible gains from randomization.

Entrepreneurship has a future value if the transition process $Q^E$ is sufficiently persistent, $\sum_{z'} z' Q^W(z,z') < \sum_{z'} z' Q^E(z,z')$. In other words, the marginal entrepreneurs are willing to sacrifice current consumption for having the opportunity to begin their business career that brings high returns only in the future. They invest a large share of their income and wealth in order to relax the credit constraint and to run their firm at the optimal size. For such agents the expected current income from business might be lower than the current expected wage. Without full risk sharing, the financing constraint might prevent the entrepreneur from running the firm at the optimal size and the above inequality might hold for several initial periods of entrepreneurship.[10]

## 7.6 TFP and Wedges

**Definition 9 (TFP Measure)** *The total factor productivity is measured as the ratio of output in the entrepreneurial economy to a corresponding output of a representative agent, constant returns to scale economy, using the same aggregate inputs in both economies,*

$$\Psi = \frac{Y}{K^s (\tilde{z}N)^{1-s}} \quad where \ s = w\tilde{z}N/Y. \tag{25}$$

The input shares in the CRS production function are defined as the labor income share in output in the entrepreneurial economy.

Wedges represent the credit market frictions as distortions in first-order conditions and resource constraints (see Chari et al. (2007)). For an economy with frictions (default or asymmetric information), the collateral constraint becomes binding for some agents. The positive Lagrange multiplier can be represented by a tax on the distorted allocations. The tax is such that the first-order conditions hold at zero Lagrange multiplier again. All wedges are evaluated at corresponding equilibrium prices. A zero wedge applies to entrepreneurs who do not face a binding collateral constraint.

**Definition 10 (Intertemporal Wedge $\tau_I$)** *The intertemporal wedge $\tau_I \in [0,1)$ is a subsidy on savings such that the first order intertemporal condition for an economy with frictions is for each $(a,z')$,*

$$u_c(c(a,z'))(1 - \tau_I(a,z')) = (1+r)\beta E[u_c(c'(a',z''))|z']. \tag{26}$$

In the budget constraint, the subsidy on savings $(1 - \tau_I)$ represents the incentives needed to save sufficient assets such that the collateral constraint is not binding in the next period. The subsidy equals

$$\tau_I(a,z') = 1 - \frac{(1+r)\beta E[u_c(c'(a',z''))|z']}{u_c(c(a,z'))}. \tag{27}$$

The aggregate investment wedge is

$$\bar{\tau}_I = \sum_{a,z} \tau_I(a,z')\, Q^E(z,z')\lambda(a,z).$$

**Definition 11 (Capital Wedge $\tau_K$)** *The capital wedge $\tau_K \in [0,1)$ is a tax rate on the cost of capital input that makes the collateral constraint not binding for each $(a,z)$,*

$$E\left[ u_c(c(a,z')) \left( z' f_k(k(a,z), \tilde{z}n(a,z)) - (r+\delta)(1 + \tau_K(a,z)) \right) | z \right] = 0. \tag{28}$$

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[10]In the search model with occupational choice by Gomes et al. (2001), consumption of searchers similarly decreases compared to workers who keep their jobs.

Table 1: Parameter Values

| | | |
|---|---|---|
| Discount factor | $\beta$ | 0.95 |
| Risk aversion | $\sigma$ | 1.00 |
| Span of control | $\theta$ | 0.88 |
| Depreciation rate | $\delta$ | 0.043 |
| Capital share | $\alpha$ | 0.32 |
| Loan recovery rate | $\gamma$ | 0.75 |

| | | |
|---|---|---|
| Productivity shocks | | |
| Entrepreneurs $Q^E$ | $\rho_E = 0.90$ | $\sigma_E = 0.50$ |
| Workers $Q^W$ | $\rho_W = 0.94$ | $\sigma_W = 0.20$ |
| Levels* | $z = \{ 0.00, \quad 1.00, \quad 1.25, \quad 1.50, \quad 1.75, \quad 2.00, \quad 2.25, \quad 2.50 \}$ | |

Notes: $\rho$ = serial correlation, $\sigma$ = standard deviation. *Workers' lowest ability shock $\underline{z} = 0.5$.

As $\eta$ is reduced to zero, the tax on capital input must make the capital more costly so that the input is not increased. The aggregate capital wedge is

$$\bar{\tau}_K = \sum_{a,z} \tau_K(a, z)\, \lambda(a, z).$$

**Definition 12 (Efficiency Wedge $\tau_Y$)** *The efficiency wedge $\tau_Y \in [0, 1)$ is a tax rate*

$$E\left[z' f(k^*(a, z), \tilde{z} n^*(a, z))|z\right](1 - \tau_Y(a, z)) = E\left[z' f(k(a, z), \tilde{z} n(a, z))|z\right], \qquad (29)$$

*where $k^*(a, z)$ and $n^*(a, z)$ are the efficient capital and labor inputs that satisfy for each $(a, z)$,*

$$E\left[z' f_k(k^*(a, z), \tilde{z} n^*(a, z))|z\right] - (r + \delta) = 0,$$
$$E\left[z' f_n(k^*(a, z), \tilde{z} n^*(a, z))|z\right] - \tilde{z} w = 0.$$

The efficiency wedge relates the chosen inputs $(k(a, z), n(a, z))$ to the efficient capital and labor inputs $(k^*(a, z), n^*(a, z))$ that are constrained neither by the financing nor the collateral constraint. In other words, the inputs $(k^*(a, z), n^*(a, z))$ represent the optimal size of the firm at equilibrium prices.[11] The aggregate efficiency wedge is

$$\bar{\tau}_Y = \sum_{a,z} \tau_Y(a, z)\, \lambda(a, z).$$

# 8 Results

This Section presents the results of numerical simulations of four stationary equilibria of the economy with risk sharing, default and asymmetric information, and in autarky.

## 8.1 Parameters

Parameters of the model are shown in Table 1 are standard for the U. S. economy as in Cooley (1995). The span of managerial control $\theta$ set at 0.88, a level close to the one estimated by Burnside (1996). The utility has the logarithmic form.

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[11]Because of the decreasing returns to scale, the wedge on the optimal size of the firm can be found from $k^*(a, z)(1 - \tau^F(a, z)) = k(a, z)$.

Table 2: Aggregate Allocations: Benchmark Economies with Zero Margins

| | Risk Sharing | | | | Autarky |
|---|---|---|---|---|---|
| | Full Information | | Imperfect Monitoring | | |
| | — | Default | — | Default | — |
| Entrep. (%) | 8.43 | 8.64 | 8.81 | 9.12 | 9.71 |
| Assets | 3.72 | 3.73 | 4.11 | 4.03 | 5.15 |
| Capital | | | | | 4.18 |
| Output | 1.17 | 1.16 | 1.11 | 1.04 | 0.94 |
| Consumption | 1.01 | 1.01 | 0.95 | 0.87 | 0.78 |
| $r$ (%) | 4.58 | 4.24 | 2.52 | 2.12 | — |
| $w$ | 1.43 | 1.41 | 1.23 | 1.09 | 0.79 |
| $K/Y$ | 3.17 | 3.20 | 3.69 | 3.87 | 4.43 |
| $L/GDP$ | 2.62 | 2.36 | 1.58 | 1.22 | — |
| TFP | 1.40 | 1.38 | 1.17 | 1.07 | 0.81 |
| Leverage | 11.32 | 4.81 | 2.08 | 1.51 | — |
| Margin (%) | 18.80 | 24.98 | 50.77 | 67.30 | — |
| PEP (%) | 8.52 | 0.09 | 0.27 | 1.20 | — |
| Wedge (%) | | | | | |
| Intertemporal | 0.00 | 2.09 | 6.10 | 5.77 | 8.60 |
| Efficiency | 0.00 | 19.51 | 60.20 | 62.38 | 40.58 |
| Firm Size | 0.00 | 21.52 | 63.65 | 65.49 | 41.64 |
| Welfare | 1.00 | 0.98 | 0.93 | 0.86 | 0.77 |
| Entrepreneurs | 1.51 | 1.75 | 2.78 | 3.25 | 3.41 |
| Workers | 0.94 | 0.92 | 0.75 | 0.65 | 0.48 |

Notes: Assets and capital are equal in all economies except autarky. $L/GDP$ equals total credit (loans) over output. Welfare measured in terms of consumption equivalent units.

The transition matrix for entrepreneurial skills has important implications for the degree of business persistence and accumulation of wealth by business families. I set the values of $Q^E$ and the levels of shocks $Z$ so that the model is able to replicate the first and second moments of the U.S. distribution of wealth. I set the serial correlation parameter $\rho_E$ at 0.90 in the benchmark calibration as in Covas (2006). In the following section of the paper I will study different persistence processes of entrepreneurial skills. The unconditional standard deviation $\sigma_E$ is set to match the first two moments of the U.S. distribution of wealth and income discussed in Section 2. I find that $\sigma_E = 0.5$, which implies a much riskier process that of workers (see Covas (2006), Hamilton (2000) and Moskowitz and Vissing-Jorgensen (2002)). Similarly to Veracierto (2001), I choose the effective ability shocks for the entrepreneurs $Z = [0, \bar{z}]$ with $Q^E(0,0) = 1$ so that an entrepreneur who fails with the lowest effective ability shock will prefer to be a worker in the following period. Also, $Q^E(z,0) > 0$ for all $z \in Z$ implies that all entrepreneurs terminate their businesses in finite time. The zero output probabilities entries in the transition matrix are calculated using annualized data from Table 1 in Evans (1987) on growth rates and exit rates of firms in the Small Business Data Base constructed by the Office of Advocacy of the U.S. Small Business Administration (SBA).

Workers draw $z'$ from a first-order Markov process $Q^W(z, z')$, approximated by the method of Tauchen and Hussey (1991), with the AR(1) coefficient $\rho_W = 0.94$ and the unconditional standard deviation $\sigma_W = 0.20$, which is in the range of estimates in the literature (see Hubbard et al. (1994), Storesletten et al. (2004), and Covas (2006)). The lowest possible value of their productivity shock is set to 0.5. This specification of shocks and their laws of motion imposes the

financing constraint in each period and satisfies the assumptions on the existence of a stationary equilibrium. I use the standard method of Tauchen and Hussey (1991) to approximate both processes by a Markov process with eight states.

Productivity parameters are specified so that the outcomes in the financial intermediation economy match the data for the U.S. economy, with entrepreneurs constituting 8-10% of the population and the average exit rate is around 5% (see Evans (1987)). The discount factor and depreciation rate lead to capital-output ratios equal to 3.17 in the risk sharing steady state.

## 8.2 Default Recovery Rates

The important parameter for bank loan recovery is calibrated to $\gamma = 0.75$. This level was used by Carlstrom and Fuerst (1997) and corresponds to the following U.S. statistics: the median recovery rate of 72% for senior bank loans in Moody's 1970-2000; discounted average recovery rates for senior bank loans 88.3% in S&P 1981-2000; nominal average recovery rates for senior bank loans of 77.1% in S&P 1981-2000; Fitch 2001 senior secured bank loans recovery of 73%; or Fitch 1991-1997 bank loans of 82%. Altman 1996-2001 bank loans: 60-86%. Altman (1984) estimates the sum of direct and indirect costs of bankruptcy at 20%.

## 8.3 Numerical Simulation

The algorithm for finding the steady state of each regime is relatively simple. To solve for the occupational decision, expected values of both options are computed first. I iterate on the wage and the interest rates (and insurance payments) until markets are cleared, banks have zero profit and the conditions of the stationary recursive competitive equilibrium are satisfied. Finally, I set the maximal level of assets high enough so that the upper bound of the stationary distribution of resources is endogenous. Convergence in sets is accommodated by starting with much higher initial guess of the value function than that in default (see Abreu et al. (1986) and Abreu et al. (1990)).

## 8.4 Steady State Aggregate Allocations

Table 2 shows the aggregate steady-state results for all simulated economies. Results are compared to the first column describing the efficient allocation with risk sharing and full information.

Frictions in the financial market increase the need for accumulating assets that are required to be held as collateral. All assets are used in production except for autarky, where almost 20% of assets lie idle. Output and consumption fall significantly (by around 8-15%) in economies with imperfect monitoring. Fraction of entrepreneurs is 8.43% and generally increases with frictions. It falls bellow this number only in autarky and the economy without risk sharing and with default.

General equilibrium effects are extremely important. In the full risk sharing economy, the optimal allocation of capital to skills increases demand for capital, which increases the interest rate and especially wages. Prices fall to much lower levels with frictions, up to 2.5% for interest rate and by more than 25% for wages (in autarky, wages fall by more than 50%). Correspondingly, the capital-output ratio increases and the ratio of credit to GDP falls.

Efficiency measures are presented at the bottom. TFP measures based on the constant return to scale equivalent show large losses related to imperfect monitoring, contrary to Hopenhayn (2011) and Midrigan and Xu (2010) who argue that financial market frictions are unlikely to

Table 3: Median Entrepreneurs: Benchmark Economies with Zero Margins

| | Risk Sharing | | | | Autarky |
| | Full Information | | Imperfect Monitoring | | |
| | — | Default | — | Default | — |
|---|---|---|---|---|---|
| Assets | 5.25 | 8.75 | 20.50 | 24.00 | 23.54 |
| Capital | 41.75 | 40.89 | 40.27 | 36.51 | 23.51 |
| Risk Sharing | 1.00 | 1.00 | 0.26 | 0.14 | — |
| Output | 13.19 | 12.91 | 10.97 | 9.73 | 5.80 |
| Income | 1.83 | 2.39 | 2.98 | 3.01 | 2.71 |
| Profit | 1.58 | 2.01 | 2.46 | 2.49 | 2.71 |
| Return (%) | 3.79 | 4.79 | 5.87 | 6.64 | 11.54 |
| MPK | 8.52 | 8.49 | 7.33 | 7.18 | 6.71 |
| Leverage | 7.26 | 4.59 | 2.03 | 1.55 | — |
| Margin (%) | 13.76 | 21.74 | 49.21 | 64.53 | — |
| PEP (%) | 0.00 | 0.10 | 0.02 | 0.87 | — |
| Wedge (%) | | | | | |
|   Intertemporal | 0.00 | 1.88 | 2.11 | 2.38 | 7.08 |
|   Efficiency | 0.00 | 21.98 | 53.64 | 59.69 | 49.76 |
|   Firm Size | 0.00 | 24.34 | 57.83 | 63.36 | 50.85 |
| Assets/Income | 2.87 | 3.67 | 6.87 | 7.98 | 8.68 |

imply large efficiency losses in an economy with relatively efficient capital markets. Also wedges required to eliminate market frictions are sizeable, at around 60%.

These effects have large effect on welfare, opposite for each occupation (computed as consumption equivalents). Default in combination with information asymmetries lowers welfare by 7-14%. As frictions increase the rent to entrepreneurs, their welfare increase dramatically, while the welfare of workers falls by 25-40%. In autarky, these changes are even more dramatic. As in Geanakoplos and Fostel (2012), workers suffer welfare losses from low wages inefficient production.

## 8.5 Entrepreneurs

Table 3 shows allocations of the median entrepreneur (that is, the median firm). The risk-sharing economy with full information allocates capital to entrepreneurs with highest skills regardless of their assets. Entrepreneurs do not need to have a buffer stock of assets, output is higher, and return lower than in other economies. Efficient allocation of resources to talent increases equilibrium prices and reduces income and profit of entrepreneurs.

In the risk-sharing steady-state, leverage is 7.26, corresponding to a 14% collateral margin, and it decreases to 2 in the imperfect monitoring steady state. Kalemli-Ozcan et al. (2013) find mean values of leverage for large non-financial listed U.S. firms to be very stable at around 2.3-2.4 and slightly larger for non-listed firms. In the 2003 Surveys of Small Business Finances the average leverage 2.85.

Absence of full risk sharing in economies with default and information frictions require all agents to accumulate more assets for self-insurance and collateral requirements that enable them to borrow if they become entrepreneurs. Leverage is much smaller, collateral margins increase to 22% with default and to 49% with imperfect monitoring. If combined with default, margins are at 64%. Median private equity premium is between 0 and 1%. Covas (2006) generates an average

Table 4: Distribution of Assets and Income

| | Risk Sharing | | | | Autarky |
|---|---|---|---|---|---|
| | Full Information | | Imperfect Monitoring | | |
| | — | Default | — | Default | — |
| Inequality (Gini) | | | | | |
| Income | 0.24 | 0.26 | 0.36 | 0.39 | 0.51 |
| Wealth | 0.62 | 0.63 | 0.88 | 0.93 | 0.94 |
| Share (E) | | | | | |
| Assets | 0.17 | 0.26 | 0.59 | 0.74 | 0.81 |
| Income | 0.17 | 0.21 | 0.33 | 0.35 | 0.51 |
| Asset Share (E) | | | | | |
| Top 1% | 0.07 | 0.07 | 0.21 | 0.30 | 0.34 |
| Top 5% | 0.26 | 0.26 | 0.59 | 0.73 | 0.77 |
| Top 10% | 0.41 | 0.42 | 0.81 | 0.92 | 0.95 |
| Top 20% | 0.62 | 0.64 | 0.96 | 1.00 | 1.00 |
| Income Share (E) | | | | | |
| Top 1% | 0.04 | 0.04 | 0.08 | 0.12 | 0.18 |
| Top 5% | 0.13 | 0.13 | 0.25 | 0.30 | 0.42 |
| Top 10% | 0.21 | 0.22 | 0.35 | 0.39 | 0.51 |
| Top 20% | 0.35 | 0.36 | 0.46 | 0.57 | 0.66 |

Notes: Share of assets and income of entrepreneurs (E).

equity premium of 2.8% in the steady state from a borrowing constraint especially among the small and financially constrained entrepreneurs (equity premium as much as 20%).

## 8.6 Distribution of Assets and Income

Table 4 shows the distribution of assets and income in the economy. Gini coefficients of inequality are lower than in the data for full risk sharing but very close in economies with frictions.

The more severe financial frictions the more unequal the economies become. Note that the large share of assets held by entrepreneurs is not due to a similarly large share of income: entrepreneurs save more in order to insure and accumulate collateral. With information frictions, top 5% of the population holds more than 50% of wealth. In default with low recovery rates entrepreneurs hold most of the assets. As in developing countries, in autarky they control 81% of assets and receive 51% of income.

## 8.7 Leverage Bounds

Figure 3 shows leverage bounds (dashed line) and the allocation of capital to entrepreneurs with the highest signal shock $z_H$ (solid line) as a function of assets at equilibrium prices in each steady state. The top left panel represents the optimal allocation in the economy with full risk sharing, where the allocation of capital is a horizontal line independent of assets. In all other steady states, the allocation of capital is sub-optimal, that is increasing in assets, i.e., in collateral available to an entrepreneur. Each alleviation of the financing constraint increases efficiency as entrepreneurs with a high skill signal $z$ can enter or expand their firms, while general equilibrium effects provide incentives for low skill entrepreneurs to reduce their firms or exit and become workers.

The leverage upper bounds increase as assets in the definition of the leverage bounds dominate over expected output. Leverage decreases in entrepreneurial wealth in all economies. The poorer
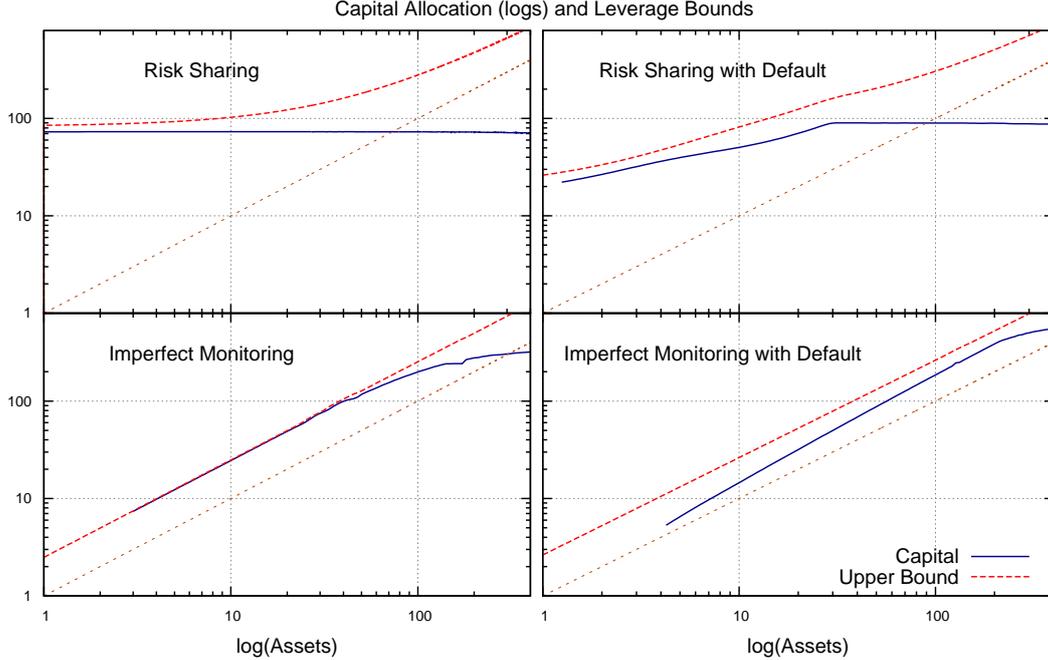
Figure 3: Capital Allocation and Leverage Bounds

Leverage bounds (dashed line) and the allocation of capital by entrepreneurs with the highest signal shock $z_H$ (solid line) as functions of assets.

entrepreneurs need to borrow while for wealthy entrepreneurs who are net depositors the ratio is less than one. As in Geanakoplos (2006) and Geanakoplos (2011), leverage cycle creates an inefficient mix of skills, especially with respect to the marginal entrepreneur who is indifferent between being a worker or an entrepreneur, and whose skills are higher if leverage is possible and financial markets work efficiently.

## 8.8   Optimal Capital

Figure 4 displays the ratio of capital to the optimal capital (unconstrained allocation) at the equilibrium prices in each steady state. These ratios are shown for three highest signal shocks $(z_H, z_M, z_L)$ as functions of assets (the marks represent deciles in asset distribution of entrepreneurs).

The ratio is one for all shocks in the full risk sharing economy. In all other panels, the ratio is less than one and lower for higher signal productivity shocks. This is important as market frictions constrain more the most productive entrepreneurs. The case of imperfect monitoring in the left-bottom panel is revealing as entrepreneurs with highest skill (+ lines) are constrained by potential adverse selection and misreporting to low levels of capital in order to not provide incentives for agents with lower skills to pretend they have higher skill.

Without the imperfect monitoring problem,the economies are more efficient as only entrepreneurs with the two highest shocks enter. Imperfect monitoring allow also the least efficient agents to enter (□ lines) as collaterized assets become relatively more important than skills. Note that the most severe frictions require more assets to enter.

With default, poor but highly skilled entrepreneurs cannot borrow as much and the ratio is

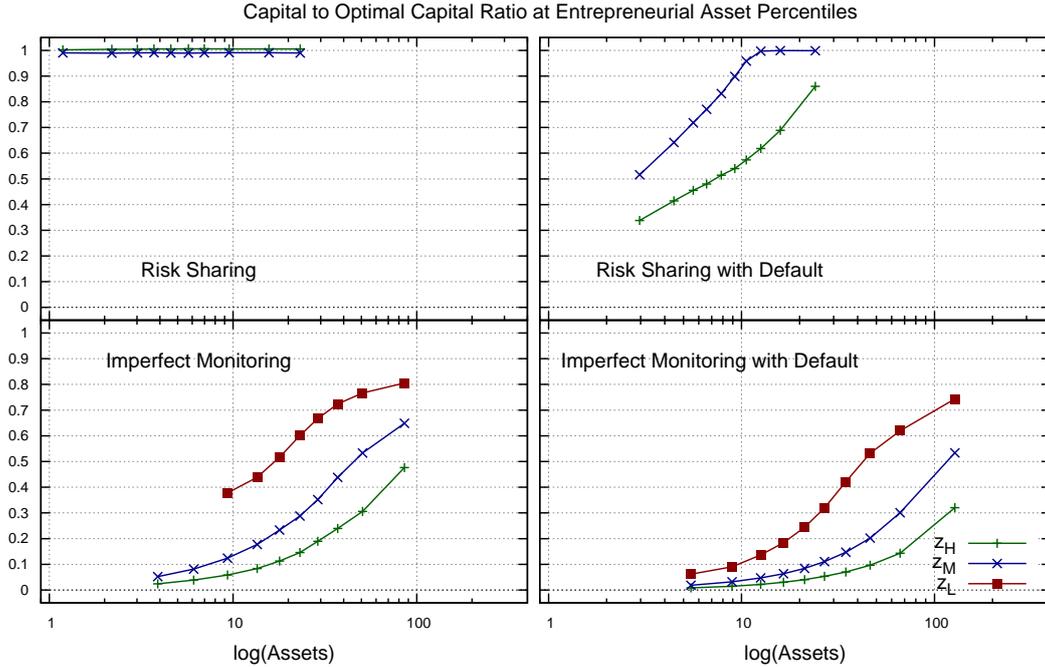Capital to Optimal Capital Ratio at Entrepreneurial Asset Percentiles

Figure 4: Optimal Capital Allocation

The ratio of capital to the optimal capital (unconstrained allocation) at the equilibrium prices in steady states. Ratios are shown for three highest signal shocks $(z_H(+), z_M(\times), z_L(\square))$ as functions of assets.

0.3 for the first wealth decile. Under imperfect monitoring, the capital allocation is extremely inefficient at low wealth levels: for the most skilled entrepreneurs, the ratio is below around 10% even for the median agent.

### 8.9 Macro-Prudential Policy: Maximum Leverage Regulation

During a leverage cycle there is too much leverage in normal times and therefore too high asset prices. In bad times, there is too little leverage and therefore too low asset prices. During the most recent leverage cycle after 2006, leverage gradually rose because of low volatility and technological innovation in financial markets had stretched the available collateral. After a bad and scary news (increase in mortgage delinquencies) increased volatility, leverage as well as prices fell dramatically. The average downpayment (margin) for mortgages fell from 14% in 2000 to 2.7% in the second quarter of 2006 (home prices reached their peak at the same time). After the crisis the margin increased to 25-30%. [12] Importantly, during this time period the interest rates have not changed.

Leverage bounds derived in this model are used to simulate the tradeoff from the leverage ratio macro-prudential regulation: in good times, restricting leverage is costly as it limits the efficient allocation of resources to their most productive use. On the other hand, the regulation provides incentives for accumulation of assets that prevent excessive deleveraging during a recession or after a change of regulatory framework. In this model, good times are modelled as steady states without information frictions with full risk sharing, with each steady state characterized by a different values of the collateral margin that can be mapped back to asymmetric information or

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[12]Hedge fund margins on AAA securities increased from 5% to 70% during the same time.
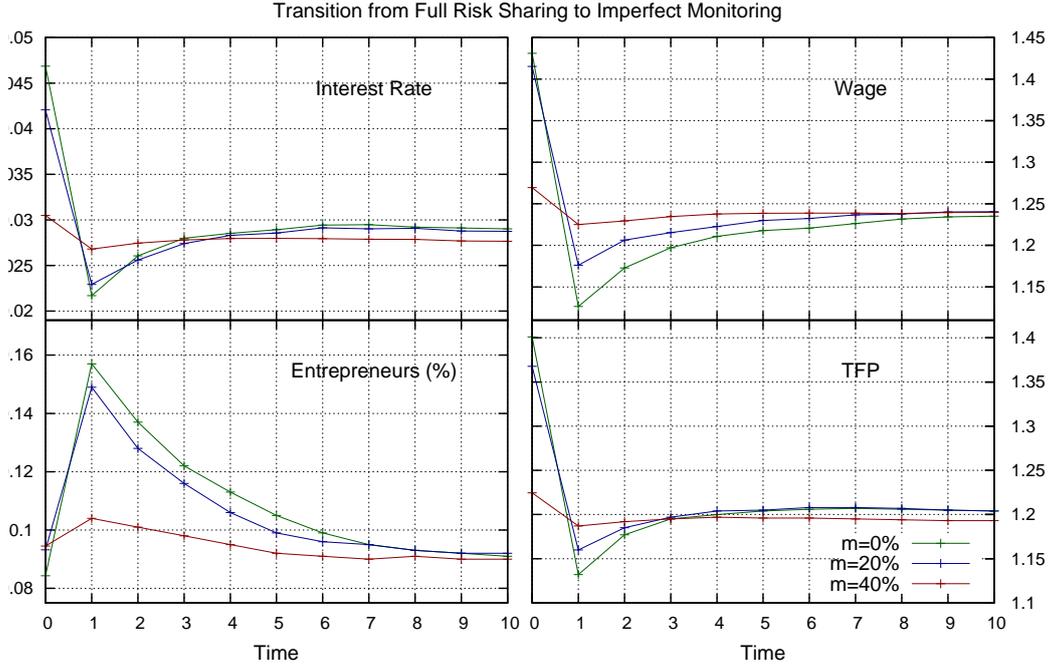
Figure 5: Transition After Information Regime Change

Transition from the steady state of the economy with full risk sharing to steady states of economies with imperfect monitoring. Leverage margins at 0%, 20% and 40%.

financial frictions: from 0% (no downpayment), to 10%, 20%, 30%, and 40% collateral margin required for each loan, respectively.

These steady states with full information, full risk-sharing are exposed to an unanticipated, permanent change in informational regime in which financial institutions start requiring truth-telling incentive compatible contracts for imperfect monitoring and/or default. In other words, the initial risk-sharing steady states represent the long moderation period when all actors believe they are in the good state of the economy when default or asymmetric frictions are not binding (for example, due to government policies promoting borrowing or home ownership). Then a scary bad news occurs and the markets realize the presence of asymmetric information problems and/or default and provide contracts with incentive-compatible allocations.

The question is then by how much and at which cost in terms of efficiency and welfare in both steady states the leverage ratio regulation alleviates the effects of the information regime change during the transition. The macroprudential policies are implemented by imposing the same maximum leverage ratio (or, equivalently, minimum collateral margins $m$) in both the initial risk-sharing steady state and the imperfect monitoring/default steady states. These policies are kept the same in the initial and terminal steady states as well as during the transition. The goal is to evaluate the effects of macroprudential policies on the steady-state allocations and their behavior during the transition to new steady-state. In other words, the numerical simulations analyze the maximum leverage regulation tradeoff between lower efficiency in the steady state and lower efficiency losses during a crisis.

Figure 5 shows transition paths for interest rate, wage, occupational choice, and TFP paths after the change of the information regime from full information to imperfect monitoring. (the unanticipated transition starts in period 1). The green line shows allocation for a zero margin ($m = 0\%$) requirement, the blue line for a $m = 20\%$, and the red line for a $m = 40\%$ margin
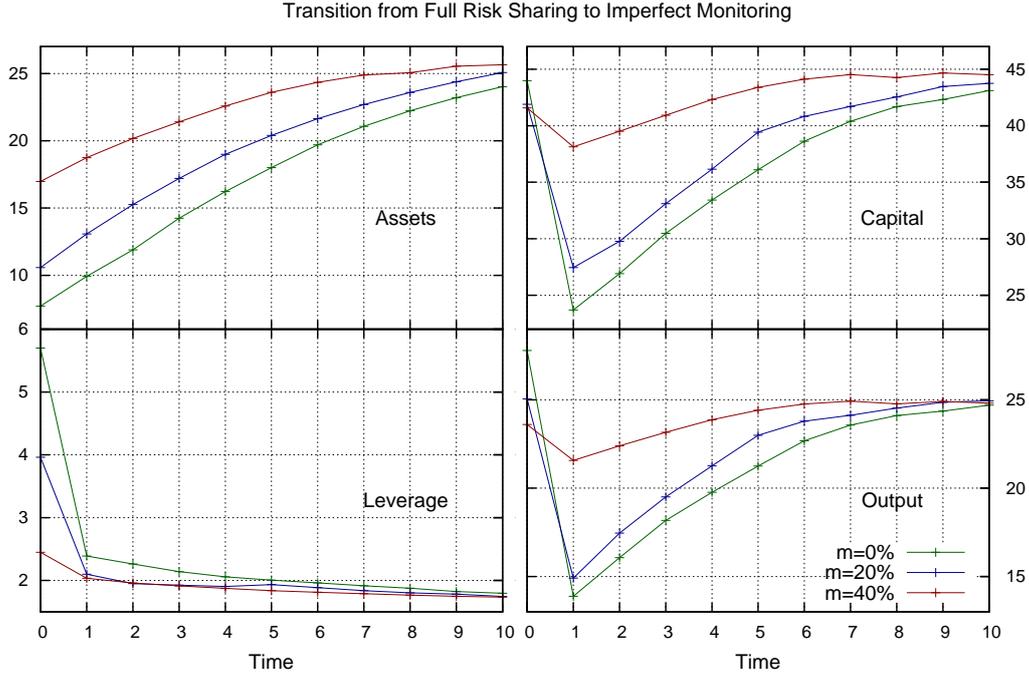
Figure 6: Transition After Information Regime Change

Transition from the steady state of the economy with full risk sharing to steady states of economies with imperfect monitoring. Leverage margins at 0%, 20% and 40%.

requirement.

If no minimal margins are imposed ($m = 0\%$), the suddenly imposed incentive compatible contracts in the first period reallocate resources to less skilled but more wealthy agents with collateral. As there is now lower demand for productive capital, interest rates fall from 4.5% to 2.2%, productivity falls by 25% and so do wages (by 15-20%). As the more efficient but poor entrepreneurs exit, more inefficient agents enter and the fraction of entrepreneurs almost doubles. When collateral margins are required, the reallocation is less severe, especially in the case of 40% downpayment.

Figure 6 shows transition paths for assets, capital, leverage and output per average entrepreneur. As imperfect monitoring polices require more capital as collateral, average entrepreneurs accumulate more assets but use much lower leverage. Capital and output recover after their initial falls.

Figure 1 summarizes the efficiency tradeoffs from imposing leverage bounds. The stars show GDP losses in the risk sharing (x-axis) and imperfect monitoring (y-axis) steady states from imposing collateral margins on loans at 0%, 10%, ..., 40%. For example, imposing a 30% downpayment margin leads to 0.012% of GDP loss in the risk sharing steady state and to 0.07% loss in the imperfect monitoring steady state, both relative to zero margin steady states. When the change in information regime impacts the risk-sharing steady state, the fall of GDP in the first period of transition is 2.9% (the arrow to $m = 30\%$). When there is no collateral margin requirement ($m = 0\%$), the initial loss in transition is 7.1%.

The initial and terminal steady states for different downpayment margin requirements are similar in aggregate levels of productive capital, output and consumption. This is due to general equilibrium effects that provide incentives to accumulate assets that serve as buffer stock and

collateral for potential entrepreneurial projects. The steady state losses from leverage regulation do not exceed 1% of GDP.

# 9    Conclusions

Possibly the best policy to prevent a future financial crisis is to act before it occurs. Macroprudential policies have been formulated in order to alleviate macroeconomic and financial markets imbalances. At the EU level, the Macroeconomic Imbalances Procedure (MIP) has been designed to detect, prevent and correct problems at their early stages by constructing a set of macroeconomic indicators with subsequent excessive imbalance procedures. Regulating maximal leverage in good times might be a policy that can achieve this end.

This model generates endogenous leverage bounds when a possibility of default and imperfect monitoring are present, individually or jointly, in an environment that does not display irrational behavior or expectations. These bounds provide a guidance for leverage management as a prevention against a future crisis.

The contribution of this papers is twofold. First, it develops a novel approach of modelling asymmetric information. In particular, the adverse selection problem for entrepreneurial or firm decisions can be applied to a wide variety of economic problems. Second, the paper simulates macroprudential policy in an environment with imperfect monitoring and default, using the derived leverage bounds. Many features important for macroprudential policies are left for future research: a countercyclical variation of the leverage ratio over the business cycle, alternative definition of accounting variables in the leverage ratio, differential regulation of different assets and contracts, the regulation of related indicators (loan-to-value, loan-to-income, among others), and finally, the degree of complementarity to risk-weighted capital requirements.

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# 10 Appendix

## 10.1 Optimal Allocations

### 10.1.1 Worker's Problem

A worker characterized by the state variables $(a, z)$ solves

$$v^W(a, z) = \left\{ E \left[ \max_{c, a'} \left\{ u(c) + \beta v(a', z') \right\} \right] \right\},$$

subject to the budget constraint (with a Lagrange multiplier $\mu$ for each $z'$)

$$c + a' \leq (1 + r)a + z'wn,$$

The envelope condition is

$$v_a^W(a, z) = (1 + r)E[\mu],$$

and the first order condition with respect to consumption for each $z'$ is

$$u_c(c) = \mu,$$

the first order condition with respect to savings for each $z'$

$$\beta v_a(a', z') = \mu.$$

These conditions imply that the usual first-order intertemporal condition

$$u_c(c) = (1 + r)\beta E[u_c(c')].$$

### 10.1.2 Entrepreneur's Problem

In the benchmark case without risk-sharing, an entrepreneur characterized by the state variables $(a, z)$ solves

$$v^E(a, z) = \max_{k, n} \left\{ E \left[ \max_{c, a'} \left\{ u(c) + \beta v(a', z') \right\} \right] \right\},$$

subject to the budget constraint (with a Lagrange multiplier $\mu$ for each $z'$)

$$c + a' \leq (1 + r)a + z'f(k, \tilde{z}n) - \tilde{z}wn - (r + \delta)k,$$

and the exogenous collateral constraint (with a Lagrange multiplier $\eta$)

$$0 \leq \kappa a - k.$$

The envelope condition is

$$v_a^E(a, z) = (1 + r)E[\mu] + \eta\kappa,$$

the first order condition with respect to capital input is

$$E\left[ \mu \left( z'f_k - (r + \delta) \right) \right] - \eta = 0,$$

the first order condition with respect to labor input is

$$E\left[\mu\left(z'f_n - \tilde{z}w\right)\right] = 0,$$

the first order condition with respect to consumption for each $z'$ is

$$u_c(c) = \mu,$$

the first order condition with respect to savings for each $z'$

$$\beta v_a(a', z') = \mu.$$

These conditions imply that, after the uncertainty is realized,

$$\beta v_a(a', z') = u_c(c),$$

and that

$$u_c(c) = (1 + r)\beta E[u_c(c')] + \eta\kappa.$$

If the collateral constraint does not bind and $\eta = 0$, this is the usual intertemporal condition. If it binds with respect to the next period's business project, the optimal decision is to decrease consumption today and save more.

With risk sharing, an entrepreneur $(a, z)$ solves

$$v^E(a, z) = \max_{\substack{x \in [0,1] \\ k,n}} \left\{ E\left[\max_{c,a'} \left\{u(c) + \beta v(a', z')\right\} | z\right]\right\},$$

subject to the budget constraint (again with a Lagrange multiplier $\mu$ for each $z'$)

$$c + a' \leq (1 + r)a + xE\left[z'|z\right]f(k, \tilde{z}n) + (1 - x)z'f(k, \tilde{z}n) - \tilde{z}wn - (r + \delta)k,$$

and the exogenous collateral constraint as above with a Lagrange multiplier $\eta$.

The first order conditions that differ from the benchmark case are with respect to capital input,

$$E\left[\mu\left(xE\left[z'|z\right]f_k + (1 - x)z'f_k - (r + \delta)\right)\right] + \eta xE\left[z'|z\right]f_k - \eta = 0,$$

with respect to labor input,

$$E\left[\mu\left(xE\left[z'|z\right]f_n + (1 - x)z'f_n - \tilde{z}w\right)\right] + \eta xE\left[z'|z\right]f_n = 0,$$

and the first order condition with respect to risk sharing

$$E\left[\mu\left(E\left[z'|z\right]f - z'f\right)\right] = 0.$$

### 10.1.3 Capital-Labor Ratio

From the first order condition with respect to capital and labor inputs,

$$f_k E\left[z'u_c(c)\right]\left(1 - \frac{1 - \alpha}{\alpha}\frac{r + \delta}{\tilde{z}w}\right) = \eta. \tag{30}$$

At asset levels where the collateral constraint does not bind $a > a_\kappa$ the unconstrained bor-

rowing entrepreneurs with $\eta = 0$ and

$$\chi \equiv \frac{k}{n} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w\tilde{z}}{r+\delta}\right).$$

For the constrained borrowing entrepreneurs $\eta > 0$, the left-hand side of equation (30) is positive only if

$$\chi^{\kappa} \equiv \frac{k}{n} < \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w\tilde{z}}{r+\delta}\right) = \kappa.$$

### 10.1.4 Expected Profit

Using the capital-labor ratio,

$$E[\pi|z] = E[z'|z]f(k,\tilde{z}n) - w\tilde{z}n - (r+\delta)k = E[z'|z]\chi^{(\alpha-1)\theta}k^{\theta} - \frac{r+\delta}{\alpha}k.$$

### 10.1.5 Optimal Allocation of Capital

From the first-order conditions and the capital-labor ratio,

$$k(z) = \left(\frac{\theta\alpha E[z'|z]\chi^{(\alpha-1)\theta}}{r+\delta}\right)^{\frac{1}{1-\theta}}.$$

## 10.2 Leverage Bounds

Using the capital-labor ratios $\chi$ in the endogenous financing constraint above, there is an upper bound for the collateral constraint above which it will not bind because the endogenous financing constraint would be violated,

$$k \leq \alpha\frac{1+r}{r+\delta}a + \frac{\alpha}{r+\delta}xE[z'|z]f(k,\tilde{z}n).$$

The collateral constraint will not bind for the depositing entrepreneurs.

## 10.3 Private Equity Premium

Using the fact that $\text{cov}(X,Y) = E[XY] - E[X]E[Y]$, from the first order condition for capital

$$\text{cov}\left(u_c(c), xE[z'|z]f_k + (1-x)z'f_k - r - \delta\right)$$
$$+ E[u_c(c)|z]E\left[xE[z'|z]f_k + (1-x)z'f_k - r - \delta\right] = \eta,$$

which can be simplified as

$$\text{cov}\left(u_c(c), (1-x)z'f_k\right) + E[u_c(c)|z]E\left[xE[z'|z]f_k + (1-x)z'f_k - r - \delta\right] = \eta.$$

Dividing by $E[u_c(c)]$ and rearranging,

$$(1-x)\frac{\text{cov}\left(u_c(c), z'f_k\right)}{E[u_c(c)|z]} + (1-x)E[z'|z]f_k - r - \delta + xE[z'|z]f_k = \frac{\eta}{E[u_c(c)|z]}.$$

37

$$E[z'|z]f_k - r - \delta = \frac{\eta}{E[u_c(c)|z]} - (1-x)\frac{\mathrm{cov}\,(u_c(c), z'f_k)}{E[u_c(c)|z]},$$

Applying the analysis from the capital-labor ratio above, the private equity premium is higher for the constrained borrowing entrepreneurs (with $\eta > 0$).

## 10.4    Degree of Risk Sharing

Using a notation $f \equiv f(k, \tilde{z}n)$, from the first order condition with respect to risk sharing,

$$E\left[u_c(c)\left(E\left[z'|z\right]f - z'f\right)|z\right] = 0,$$

$$\mathrm{cov}\,\left(u_c(c), E[z'|z]f - z'f\right) + E[u_c(c)|z]E\left[E[z'|z]f - z'f|z\right] = 0,$$

which implies

$$\mathrm{cov}\,\left(u_c(c), z'f\right) = 0.$$

As marginal utility from consumption is always positive, this can only hold only under full risk sharing $x = 1$ for all $(a, z)$.

## 10.5    One-Period Gains

### 10.5.1    One-Period Gains from Default

Using the notation $\hat{f} = f(k(a, \hat{z}), n(a, \hat{z}))$, the one-period expected gain from defaulting ex-post for each pair of $(z, \hat{z})$ is

$$
\begin{aligned}
E[\Delta|z] &= (1-\delta)k + E[z'|z]f - w\tilde{z}n - \rho^D - \left((1-\delta)k + E[z'|z]f - w\tilde{z}n - E[\rho(z')|z]\right) \\
&= (1+r)b + \underbrace{xfE[z' - E[z'|z]|z]}_{=0} - \gamma(1+r)b \\
&= (1-\gamma)(1+r)b.
\end{aligned}
$$

### 10.5.2    One-Period Gains from Ex-Ante Default

Using the notation $\hat{f} = f(k(a, \hat{z}), n(a, \hat{z}))$, the one-period expected gain from defaulting ex-post for each pair of $(z, \hat{z})$ is

$$
\begin{aligned}
E[\Delta|z] &= (1-\delta)k - \rho^D - \left((1-\delta)k + E[z'|z]f - w\tilde{z}n - E[\rho(z')|z]\right) \\
&= \underbrace{xfE[z' - E[z'|z]|z]}_{=0} - \gamma(1+r)b - \left(E[z'|z]f - w\tilde{z}n - (1+r)b\right) \\
&= (1-\gamma)(1+r)b - \left(E[z'|z]f - w\tilde{z}n\right).
\end{aligned}
$$

### 10.5.3 One-Period Gains from Imperfect Monitoring

Using the notation $\hat{f} = f(k(a, \hat{z}), n(a, \hat{z}))$, the one-period expected gain from imperfect monitoring for each pair of $(z, \hat{z})$ is

$$
\begin{aligned}
E[\Delta|z] &= (1 - \delta)\hat{k} + E[z'|z]\hat{f} - w\tilde{z}\hat{n} - E[\hat{\rho}(\hat{z}'(z'))|\hat{z}]|z] - \left((1 - \delta)k + E[z'|z]f - w\tilde{z}n - E[\rho(z')|z]\right) \\
&= (1 - \delta)(\hat{k} - k) + E[z'|z](\hat{f} - f) - w\tilde{z}(\hat{n} - n) - (1 + r)(\hat{b} - b) \\
&\quad + \underbrace{xfE[z' - E[z'|z]|z]}_{=0} - \hat{x}\hat{f}E[\hat{z}'(z') - E[z'|\hat{z}]|z] \\
&= \underbrace{E[z'|z](\hat{f} - f) - w\tilde{z}(\hat{n} - n) - (r + \delta)(\hat{k} - k)}_{E[\Delta\pi|z] \quad \text{Profit}} - \hat{x}\hat{f}E[\hat{z}'(z') - E[z'|\hat{z}]|z] \\
&= E[\Delta\pi|z] - \hat{x}\hat{f}E[\hat{z}'(z') - E[z'|\hat{z}]|z] \\
&= E[\Delta\pi|z] + \hat{x}\hat{f}E[E[z'|\hat{z}] - \hat{z}'(z')|z] \\
&= E[\Delta\pi|z] + \hat{x}f(\hat{k}, \tilde{z}\hat{n})\left(E\left[E\left[z'|\hat{z}\right] - z'|z\right]\right) + \hat{x}f(\hat{k}, \tilde{z}\hat{n})E\left[E\left[z'|z\right] - \hat{z}'(z')|z\right] \\
&= E[\Delta\pi|z] + \underbrace{\hat{x}f(\hat{k}, \tilde{z}\hat{n})\left(E\left[z'|\hat{z}\right] - E\left[z'|z\right]\right)}_{Ex-Ante} + \underbrace{\hat{x}f(\hat{k}, \tilde{z}\hat{n})E\left[z' - \hat{z}'(z')|z\right]}_{Ex-Post}.
\end{aligned}
$$

### 10.5.4 One-Period Gains from Imperfect Monitoring and Default

Using the notation $\hat{f} = f(k(a, \hat{z}), n(a, \hat{z}))$, the one-period expected gain from imperfect monitoring and default for each pair of $(z, \hat{z})$ is

$$
\begin{aligned}
E[\Delta|z] &= (1 - \delta)\hat{k} + E[z'|z]\hat{f} - w\tilde{z}\hat{n} - \hat{\rho}^D - \left((1 - \delta)k + E[z'|z]f - w\tilde{z}n - E[\rho(z')|z]\right) \\
&= (1 - \delta)(\hat{k} - k) + E[z'|z](\hat{f} - f) - w\tilde{z}(\hat{n} - n) + (1 + r)b - \gamma(1 + r)\hat{b} \\
&\quad + \underbrace{xfE[z' - E[z'|z]|z]}_{=0} \\
&= \underbrace{E[z'|z](\hat{f} - f) - w\tilde{z}(\hat{n} - n) - (r + \delta)(\hat{k} - k)}_{E[\Delta\pi|z] \quad \text{Profit}} + (1 - \gamma)(1 + r)\hat{b}.
\end{aligned}
$$

### 10.5.5 One-Period Gains from Imperfect Monitoring and Ex-Ante Default

Using the notation $\hat{f} = f(k(a, \hat{z}), n(a, \hat{z}))$, the one-period expected gain from imperfect monitoring and ex-ante default for each pair of $(z, \hat{z})$ is

$$
\begin{aligned}
E[\Delta|z] &= (1 - \delta)\hat{k} - \hat{\rho}^D - \left((1 - \delta)k + E[z'|z]f - w\tilde{z}n - E[\rho(z')|z]\right) \\
&= (1 - \delta)(\hat{k} - k) - \left(E[z'|z]f - w\tilde{z}n\right) - (1 + r)b) + \underbrace{xfE[z' - E[z'|z]|z]}_{=0} - \gamma(1 + r)\hat{b} \\
&= (1 - \gamma)(1 + r)\hat{b} - \left(E[z'|z]f - w\tilde{z}n - (r + \delta)(\hat{b} - b)\right).
\end{aligned}
$$

## 10.6 The Economy Without Financial Intermediation

For the economy without financial intermediation, the financing constraint implies

$$a \geq \left( \frac{w\tilde{z}n}{\delta k} + 1 \right) \delta k.$$

For some agents with lots of assets it may be optimal not to use all assets as capital. In that case, $k < a$ and $\eta = 0$. Plugging these values into first order conditions produces the capital labor ratio

$$\kappa^A \equiv \frac{k}{n} = \frac{\alpha}{1-\alpha} \frac{w\tilde{z}}{\delta}.$$

Notice for these agents the financing constraint is

$$k \leq \frac{\alpha}{\delta} a.$$

Because, in general, $\alpha > \delta$, these agents are not financially constrained. On the other hand, for some agents $k = a$ so that $\eta > 0$. In this case, the financing constraint reads

$$n \leq \left( \frac{1-\delta}{w\tilde{z}} \right) a.$$

Agents with relatively low levels of assets, $a$, and high skills may be constrained in their choices of inputs. For a given level of assets, these agents will show higher capital-labor ratios than unconstrained agents and, therefore, larger marginal productivity of capital.

With these expressions in mind, it is easy to see that opening up the credit market improves the aggregate efficiency. Constrained agents with high skills will tend to borrow as their marginal product of capital is high. On the contrary, low skill unconstrained entrepreneurs with low marginal productivity of capital will be inclined to deposit. Eventually, for some low skill entrepreneurs, it will be more convenient to deposit all their assets, exit entrepreneurship and be workers.