Downsizing, Job Insecurity, and Firm Reputation

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November 29, 2004

Abstract

This paper studies how firms make layoff decisions in the presence of adverse shocks. In this uncertain environment, workers’ expectations about their job security affect their on-the-job performance. This productivity effect of job insecurity forces firms to strike a balance between laying off redundant workers and maintaining survivors’ commitment when deciding on the amount and timing of downsizing. This framework offers an explanation of conservative employment practices (such as zero or reduced layoffs) based on firms having private information about their future profits. High retention rates and wages can signal that the firm has a bright future, boosting workers’ confidence. Moreover, the model provides clear predictions about when waves of downsizing will occur as opposed to one-time massive cuts.

JEL Classifications: J21, J23, D21, D82

Keywords: Downsizing, layoffs, employment policies

*The authors gratefully acknowledge financial support from DGES and FEDER under BEC2003-00412 and Barcelona Economics (CREA). Jeon also acknowledges the Ramon y Cajal grant from the Spanish ministry of Science and Technology. We thank Ken Ayotte, Heski Bar-Isaac, Patrick Bolton, Jan Boone, Lee Branstetter, Antonio Cabrales, Maria Guadalupe, Maia Guell, Ernesto Villanueva, and audiences at Columbia, NYU-Stern, Tilburg, and the UPF Applied Lunch for helpful comments. We also thank Sergi Lanau and Miquel Angel Serra for research assistance.

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1 Introduction

In 2002, companies in the United States announced layoffs of 1.96 million workers, with firms such as American Express, Lucent, Hewlett-Packard, and Dell Computer conducting multiple rounds in the same year.\textsuperscript{1} The tragedy of 9/11 had reverberated throughout the economy, leaving businesses scrambling to adjust. Workers had to face the consequences and those consequences were grim - Farber (2003) estimates that for displaced workers the average decline in weekly earnings was 10.6 percent and the average re-employment probability was 65 percent.\textsuperscript{2} Despite the potentially large impact on welfare, there is no clear picture about how downsizing is conducted. In this paper, we investigate factors that affect both the amount and timing of downsizing.

We present a simple model of firms’ downsizing decisions when they face adverse shocks. Firms must take into account that uncertainty about the possibility of being laid off tomorrow affects workers’ performance today. This creates a link between current and future employment decisions of the firm and implies that the firm will not automatically adjust its workforce to coincide with the current shock. Instead, the firm will try to strike a balance between laying off redundant workers and maintaining the survivors’ commitment to their work. This framework permits us to clearly identify conditions which lead to waves of downsizing, one-time massive cuts, and zero-layoff policies.

To be concrete, we look at a two period model in which firms face an unexpected negative shock (which is observed simultaneously by the firms and the workers) in period one. In period two, the profitability of a firm can either rebound or face a further negative shock; this information is known to firms ex-ante but may or may not be known to workers. This second shock may reflect fluctuations specific to the firm and/or the firm’s preparation or sensitivity to downturns. We model perceived job insecurity as a worker’s expected probability of being let go in the future. Increased job insecurity can reduce workers’ commitment to their work and make them more likely

\textsuperscript{1}See Cascio (2002) for details.
\textsuperscript{2}These numbers are for workers in 2001 who suffered a displacement between 1999 and 2001. The consequences may be even more severe: Jacobson, Lalonde, and Sullivan (1993) estimate that high tenure workers who had been displaced suffered a loss of 25 percent of their predisplacement earnings even five years after having separated from their former firms.
to look for other positions. Incentives for working are provided through the wage - higher job insecurity implies higher wages must be paid, forming the basis for our results.

First, we analyze the impact that private information on the part of the firm about its future prospects can have on the firm’s staffing decision. After a strong negative demand shock occurs industry-wide, firms may have private information about their prospects and how well they are prepared to deal with the shock, while workers face significant uncertainty.\(^3\) We find that conservative downsizing policies (i.e. zero or minimal layoffs) allow firms to signal that their future is bright. This increases workers’ confidence in the firm and hence their commitment to working. Examples of minimal layoff employment practices abound. In the aftermath of the 9/11 disaster, airlines reduced their staff by 20% on average in response to dramatically reduced business. Southwest Airlines, on the other hand, did not lay off or furlough anyone.\(^4\) And despite strong downturns in the financial markets, financial firms Lehman Brothers and Edward Jones insisted on keeping their staff intact.\(^5\) Informal zero layoff policies are not infrequent (47 of the 100 companies that made Fortune's 2002 list of the “100 Best Companies to work for” have them).\(^6\)

Second, we formalize the notion that the timing of downsizing can vary substantially. It is quite common to hear about massive layoffs and/or waves of downsizing. On average two-thirds of firms that lay off employees in a given year do so again the following year.\(^7\) Specifically, we call a one time sweeping cut in the workforce a “big-bang” and waves of cuts “gradualism” and provide an explanation based on job insecurity for why either may be chosen.\(^8\)

\(^3\)With regards to worker uncertainty, Greenhalgh (1982, p. 156) says, “When an organization is declining, however, rumors usually paint a much worse picture than the situation warrants. These rumors depress workers’ general perception of job security before any changes are introduced by management”.


\(^7\)Taken from U.S. Department of Labor. Moreover, although one may think that downsizing is ‘lumpy’ due to factory and office closings, Davis, Haltiwanger and Schuh (1996) find that among manufacturing firms, only “23% of job destruction takes place at plants that shut down”.

\(^8\)Dewatripont and Roland (1992a, 1992b, 1995) were the first to study the relative merits of gradualism versus a big-bang strategy in the context of reforms in transition
and Kreps (2000), in their textbook on human resources, discuss the basic costs and benefits of the approaches and state, “by moving boldly and rapidly, companies may minimize the long-term psychological damage” while “a one-time massacre runs the risk of cutting too much”. Within the model we are able to be very precise about what factors determine which policy is used. We find that a big-bang benefits the firm by increasing survivors’ commitment to their work (through the elimination of job insecurity) while imposing a cost on the firm of excessive (more than optimal) layoffs. The big-bang is more likely when (i) workers’ future job prospects (conditional on being fired) are better and (ii) the firm’s marginal profitability is lower due to either technology or demand shocks.

A fundamental assumption of the paper is that job insecurity demotivates workers. Formally, we model this through moral hazard; job insecurity makes workers want to dedicate their effort to looking for other jobs. Demotivation as a consequence of downsizing is well known among managers. In Bewley’s 1999 survey, 41% of businesses responded that layoffs hurt the morale of survivors for a long time. Greenhalgh (1982) discusses the negative impacts of job insecurity, and proposes that “decisions regarding change must optimize job security to minimize dysfunctional worker response”. Moreover, workers are very aware of the uncertainty that faces them. Schmidt (1999), looking at the General Social Survey, finds that workers’ beliefs about the probability of job loss track the unemployment rate and aggregate downsizing patterns quite well.

Our theory of conservative employment practices is based on reputational concerns. Both Holmstrom (1981) and Carmichael (1984) examine reputation in the labor market, but in a rational expectations framework. We include asymmetric information and thus reputation becomes defined by a (multidimensional) signaling game á la Milgrom and Roberts (1986). In their paper, price and amount spent on advertising are signals of product economies. They focus on the role of private information about workers’ types (1992a, 1992b) and on learning in the presence of aggregate uncertainty (1995) while, in our setting, neither of the two elements is present and the trade-off between both strategies is driven by job insecurity.


There exists another kind of model of reputation based on signaling (Kreps and Wilson (1982) and Milgrom and Roberts (1982)) where the player with private information faces long run (but finite) interactions and can choose among a limited number of actions (e.g. fight or accommodate).
quality, while in our model a firm signals its type through the wage and the amount of retained workers. Another paper along these lines is Bagwell and Ramey (1988).

The issues we analyze are related to two strands of the labor market contracting literature. The relational contracting literature (Bull (1987), Baker, Gibbons and Murphy (1994), MacLeod and Malcomson (1989), and Levin (2003)) models a firm’s reputation in the labor market as a zero-one variable in an infinitely repeated game, making the framework difficult to adapt for the analysis of how a firm should design its downsizing policy when faced with unexpected shocks. The implicit contract literature (Azariadis (1975), Bailey (1974), and Gordon (1974)) assumes the commitment of a firm to wages contingent on anticipated shocks as well as risk aversion. In our model we focus on short-term contracts signed after an unexpected shock occurs (in an extension, we look at long-term contracts) and risk neutrality.

Moral hazard forms the basis of our analysis. The main effect that drives our results, that workers must be compensated for job insecurity, also appears in efficiency wage models that extend Shapiro and Stiglitz (1984) to product market fluctuations - namely Rebitzer and Taylor (1991), Saint-Paul (1996), and Fella (2000). Rebitzer and Taylor (1991) and Fella (2000) restrict the probability of being laid off to being non-zero (we allow it to be zero, making marginal costs of the firm potentially discontinuous) and focus on different issues (dual labor markets and firing costs, respectively). Saint-Paul (1996) allows the probability of being laid off to be zero, but he assumes that firms can commit at period $t$ to employment and the wage level in period $t+1$, which is what provides workers employed at $t$ with incentives for effort. Lastly, Jeon and Laffont (1999) study downsizing in the public sector as a static mechanism design problem where workers have private information.

In section 2, we define the model. In section 3 we analyze the game under complete information. Section 4 examines the asymmetric information game. In section 5, we look at what would happened under both information structures if long-term contracting was possible. In section 6, we analyze social welfare and, in section 7, we discuss robustness to alternate specifications. In section 8 we conclude.
2 Environment

2.1 Workers

There is a mass 1 of homogenous workers. We consider a very simple model of moral hazard. An employed worker has two possible choices of unobservable effort, high \( e = 1 \) or low \( e = \alpha \) with \( 0 < \alpha < 1 \). There are two possible outputs, a high one equal to \( y_h \) and a low one equal to 0, where the probability of producing the output of \( y_h \) is equal to \( e \). We model two different benefits of shirking \( e = \alpha \) that a worker may gain utility from. First, as usual, his disutility of working increases in the level of effort. More precisely, let the disutility of effort associated with \( e \) be given by \( e^2 \). Second, shirking gives the worker more time to search for other jobs.\(^{11}\) Specifically, conditional on being laid off at the end of period \( t \), a worker who exerted effort \( e \) in period \( t \) has probability \( (1 - \gamma e) a_{t+1} \) (with \( \gamma e \in (0, 1) \)) of finding a job in period \( t + 1 \), where \( a_{t+1} \) represents the unconditional probability of finding a job in period \( t + 1 \).\(^{12}\)

To formalize the idea of job insecurity, let \( p_i^t \) be the expected probability of a worker employed by a type \( i \) firm (firm types will be defined later on) in period \( t \) to remain employed at the same firm in period \( t + 1 \). Given the total number \( n_i^t \) of workers employed by a type \( i \) firm in period \( t \), we define \( p_i^t = \min \left\{ \frac{E n_i^{t+1}}{n_i^t}, 1 \right\} \) where \( En_i^{t+1} \) is the expectation of workers in \( t \) about firm \( i \)’s employment level in \( t + 1 \). We assume for now that the firm cannot commit to long-term contracts (this is dropped in Section 5). Let \( \bar{w}_i^t \) and \( w_i^t \) be firm \( i \)’s wages associated with high and low output respectively in period \( t \). We assume that workers are protected by limited liability such that the wages must be larger than \( w_0 \); for example, \( w_0 \) could represent a minimum wage, utility from self-employment, or unemployment benefits.

A worker employed by firm \( i \) in period \( t \) thus has the following utility depending on his choice of effort:

\(^{11}\) There is a large literature about on-the-job search. A survey can be found in Pissarides (2000).

\(^{12}\) Hence \( a_{t+1} \) is a function of the number of unemployed workers \( u \) and the number of vacancies \( v \) and is between 0 and 1. For most of the paper, we fix \( a_{t+1} \) as exogenous. However, in Section 7.2, we endogenize it to understand the labor market more fully. We endogenize it simply by setting it equal to \( \frac{u}{v} \), although it seems clear from our results that a more general functional form would yield similar conclusions.
\[ U_t(e) = e\bar{w}_t^i + (1-e)\bar{w}_t^j - e^2 + \delta[p_t^i V_{t+1}^{e,\text{in}} + (1-p_t^i)(1-\gamma_e e)a_{t+1}V_{t+1}^{e,\text{out}} + (1-p_t^j)(1-(1-\gamma_e e)a_{t+1})V_{t+1}^{u}] \]

where \( V_{t+1}^s \) is the expected present discounted value in period \( t+1 \) of remaining employed within the firm (superscript \( s=\text{“e,in”} \)), working at a different firm (superscript \( s=\text{“e,out”} \)), and being unemployed (superscript \( s=\text{“u”} \)) and \( \delta \) is the discount rate common to firms and workers.

Assuming the firm wants to implement high effort, the incentive constraint takes the form of
\[ U_t(1) \geq U_t(\alpha), \]
which reduces to:
\[ (IC_t^i) \quad \bar{w}_t^i - \bar{w}_t^j \geq 1 + \alpha + \delta(1-p_t^i)\gamma_e a_{t+1}(V_{t+1}^{e,\text{out}} - V_{t+1}^{u}) \]

Since all that matters for giving incentives is the difference between the wages, and since wages are costly for the firm, this implies that the firm will set \( \bar{w}_t^j \) as low as possible (i.e. \( \bar{w}_t^j = \bar{w}_0 \)). Hence, we have:
\[ \bar{w}_t^i = \bar{w}_0 + 1 + \alpha + \delta \max \left\{ 1 - \frac{E n_{t+1}^i}{n_t^i}, 0 \right\} \gamma_e a_{t+1}(V_{t+1}^{e,\text{out}} - V_{t+1}^{u}) \equiv \bar{w}^i(n_t^i) \]

(1)

It is reasonable to assume that \( V_{t+1}^{e,\text{out}} > V_{t+1}^{u} \). The optimal wage thus takes into account both the possibility of job loss and its consequences. More job uncertainty or better outside offers make the worker less attached to her current job. A higher wage must then be paid to maintain worker effort.

Plugging in the optimal wage, the utility conditional on being employed in firm \( i \) at time \( t \) (given \( p_t^i \)) is given by:
\[ V_{t}^{e,\text{in}}(p_t^i) \equiv \bar{w}_0 + \alpha + \delta \left\{ p_t^i V_{t+1}^{e,\text{in}} + (1-p_t^i)(1-a_{t+1})V_{t+1}^{u} \right\} . \]

\(^{13}\)Here we don’t allow for the firm to fire the worker for low output (as in Shapiro and Stiglitz (1984)). This is done for simplicity, but the main idea that more job security (higher \( p_t^i \)) reduces the amount needed to compensate the worker holds in both cases. As mentioned in the introduction, this effect also can be found in pure shirking stories (without on-the-job search) a la Shapiro and Stiglitz (1984). Rebitzer and Taylor (1991) and Fella (2000) do not allow for discontinuity in \( p_t^i \) (they have no min operator), while Saint Paul (1996) does. Saint Paul, however, assumes that firms can commit at period \( t \) to employment and the wage level in period \( t+1 \), which is what provides workers employed at \( t \) with incentives for effort. This implies that the intertemporal decision is backward-looking.
Assuming that if a worker is unemployed for a period, she receives $w_0$ and her probability of finding a job in the next period is $(1 - \gamma_u)\alpha_{t+1}$, with $\gamma_u \in (0, 1)$, the utility of an unemployed person in period $t$ is equal to $V_t^u \equiv w_0 + \delta[(1 - \gamma_u)\alpha_{t+1}V_{t+1}^{e,\text{out}} + (1 - \alpha_{t+1}(1 - \gamma_u))V_{t+1}^u]$. Therefore, the participation constraint is satisfied if the following holds:

$$(PC^i_t) \quad V_t^{e,\text{in}}(p^i_t) \geq V_t^u$$

The participation constraint strictly holds for any $p_t^i$ if $V_{t+1}^{e,\text{in}} \geq V_{t+1}^{e,\text{out}}$. In the two period labor market model that we discuss, we assume $V_{t+1}^{e,\text{in}} = V_{t+1}^{e,\text{out}}$ since the second period has no job insecurity and wages therefore don’t differ between firms. Hence, employed workers earn rents from moral hazard. In a potentially richer model, the participation constraint may bind. The job insecurity effect would still be present, but the firm’s marginal costs would rise, further reducing employment.\(^\text{14}\)

2.2 Firms

There is a mass $M$ of firms in the industry we consider. The firms have two possible sources of labor supply, their workers from the previous period (whom we will call original workers) and workers from the general labor market (whom we will call new workers). We assume that original workers are more productive for the firm than new workers, i.e. there exists firm-specific human capital. Original workers thus produce $y^o_h = 1$ and new workers produce $y^n_h = \phi$ with $0 < \phi < 1$. Define the total output of the workers to be $N_t = n^o_t + \phi n^n_t$. In our formulation, wages are not connected to $y_h$; hence firms strictly prefer re-hiring original workers to replacing them with new workers.

We consider a two period model. In period one, the industry has an adverse shock and all the firms have the same profit function $f(N_t, \theta_1)$ gross of the wage payment, where $\theta_1$ is a parameter which represents the shock that is common to all the firms. Therefore all the firms downsize their labor force in period one (we will formalize this in an assumption later). However, firms are heterogeneous in terms of how well they adapt to the adverse shock. More

\(^{14}\)These effects would be exacerbated if we included labor market competition. In the current formulation we implicitly assume that upon rejecting an offer, workers become unemployed for the period.
precisely, firm $i$ either adapts well to the shock and has the profit function $f(N_2^G, \theta_2^G)$ in period 2 or does not adapt well and has the profit function $f(N_2^B, \theta_2^B)$ in period 2. We call a firm with $\theta_2 = \theta_2^G$ a good type and a firm with $\theta_2 = \theta_2^B$ a bad type. It is common knowledge that a proportion $\nu$ of the firms have type $\theta_2 = \theta_2^G$. Formally, the index $i \in \{G, B\}$ denotes a firm’s type.

We make the following assumptions about the profit function of the firms:

**Assumption 1:**

$$f(N, \theta^G_2) > f(N, \theta_1) > f(N, \theta^B_2)$$

$$f_1(N, \theta^G_2) > f_1(N, \theta_1) > f_1(N, \theta^B_2),$$

$$f_{11}(N, \theta) < 0 \text{ for all } \theta \in \{\theta_1, \theta^G_2, \theta^B_2\}.$$

Shocks are defined here as affecting the profit function - hence a shock could be related to either the demand side or the cost side.

We assume that firm $i$ has a better understanding of how well it can adapt to the adverse shock than workers. This is formalized by assuming that in period 1 firm $i$ has private information about $\theta_2$. After its realization, $\theta_2$ is known to everybody. We note that the profit function introduced above is conditional on inducing high effort.

### 2.3 Timing

Since we consider a two-period model, in period two there is no possibility of future production and therefore workers face no job insecurity. This is captured by setting $p_2^i = 1$ for $i = G, B$.

The timing within a period $t$ is given by:

1. A shock $\theta_t$ hits the firm and is observed by both the firm and its workers.
2. Each firm decides the amount of original workers to retain and their wage.
3. Original workers decide whether to accept or reject the firm’s offer.
4. Each firm decides the amount of new workers to hire and their wage.
5. New workers decide whether to accept or reject the firm’s offer.
6. Workers exert effort, production occurs, profits are realized, and payments are made.

There are two things to remark about the timing. First of all, the parameter \( \theta_2 \) is known by the firm at least one period ahead. In our complete information analysis, the workers will know in period one what type of shock the firm faces in period 2, while in the asymmetric information analysis the workers will be uncertain about which shock will hit the firm. Secondly, in the first period, by assumption, the firms are downsizing. Consequently there will be no hiring of new workers in period 1, although new workers may be hired in period 2.

We begin by examining the complete information solution.

3 Complete Information

We begin the analysis of short-term contracts under complete information by working backwards and looking first at period two. The second period analysis will be the same under both complete and asymmetric information, since there is no issue of how expectations about the firm’s future decisions affect current job security.

3.1 The Second Period

Since the second period is the last period and \( p_i^2 = 1 \) for \( i = G, B \), the wage for both types of firm is equal to \( w_2 = w_0 + 1 + \alpha \). This implies that the value of being retained by one’s firm in period 2, \( V_{i:in}^2 = w_0 + \alpha \), is greater than the value of being unemployed, \( V_{i:un}^2 = w_0 \). Firm \( i \)’s maximization problem in period two is defined as:

\[
\max_{n_{2i}^e, n_{2i}^n} f(n_{2i}^e + \phi n_{2i}^n, \theta_{2i}) - w_2(n_{2i}^e + n_{2i}^n) \\
\text{s.t.} \quad n_{2i}^e \leq n_i^1, \quad n_{2i}^n \geq 0
\]

Associating the multiplier \( \lambda \) with the first constraint on \( n_{2i}^e \) and \( \psi \) with the second constraint on \( n_{2i}^n \), we get the following two first order conditions:

\[
f_1(n_{2i}^e + \phi n_{2i}^n, \theta_{2i}) - w_2 - \lambda = 0 \\
(\phi f_1(n_{2i}^e + \phi n_{2i}^n, \theta_{2i}) - w_2 + \psi = 0
\]
By subtracting the second condition from the first and using the facts that the marginal product of labor is positive and \( \phi < 1 \), it is clear that at least one of the constraints binds. The solution depends on how many original workers are left from the previous period. When there are a large number of original workers (\( n_1^i \) large), the firm lays off original workers and does not hire any new workers. The optimal amount of original workers to retain in this case is given by \( \bar{n}_2^{oi} \), where:

\[
f_1(\bar{n}_2^{oi}, \theta_2^i) = w_2
\]

Therefore, for any \( n_1^i > \bar{n}_2^{oi} \), \( n_2^{oi} = \bar{n}_2^{oi} \) and profits are constant.

For \( n_1^i < \bar{n}_2^{oi} \), all original workers are kept (\( n_2^{oi} = n_1^i \)). The firm decides to hire new workers if the amount of original workers is very small. We define \( n_2^{ni} \) as the number of new workers hired and \( \tilde{N}_2^i \) as the total effective labor output from new and original workers, which both follow from the equation:

\[
f_1(\tilde{N}_2^i, \theta_2^i) = \frac{w_2}{\phi}
\]

The number of new workers hired is \( n_2^{ni} = \frac{\tilde{N}_2^i - n_1^i}{\phi} \), and new workers are hired only when \( n_1^i < \tilde{N}_2^i \).

Lastly, for the range \( \tilde{N}_2^i < n_1^i < \bar{n}_2^{oi} \), no new workers are hired and all the original workers are retained. To summarize, we define the profits in period two as:

\[
\pi_2^i(n_1^i) = \begin{cases} 
    f(\tilde{N}_2^i, \theta_2^i) - w_2\frac{\tilde{N}_2^i - n_1^i}{\phi} & \text{if } n_1^i < \tilde{N}_2^i \\
    f(n_1^i, \theta_2^i) - w_2n_1^i & \text{if } \tilde{N}_2^i < n_1^i < \bar{n}_2^{oi} \\
    f(\bar{n}_2^{oi}, \theta_2^i) - w_2\bar{n}_2^{oi} & \text{if } n_1^i > \bar{n}_2^{oi}
  \end{cases}
\]

### 3.2 The First Period

Suppose that all the firms’ types are common knowledge in period one. Workers are concerned about their probability of being retained in period 2. From the previous section, we saw that all original workers are retained when \( n_1^i < \bar{n}_2^{oi} \), so \( p_l^i = \min \{ \frac{\bar{n}_2^{oi}}{n_1^i}, 1 \} \). We assume that both types of firms are downsizing in period one (which is equivalent to assuming \( \bar{n}_2^{G} < 1 \)). This implies that no new workers will be hired in period one.
Firm $i$’s maximization problem in period one is defined as:

$$\max_{n_i^1} f(n_i^1, \theta_1) - w^i(n_i^1)n_i^1 + \delta \pi^2_i(n_i^1)$$

The first order condition sets marginal profitability $f_1(n_i^1, \theta_1)$ equal to the marginal cost of retaining an additional original worker $MC^i(n_i^1)$:

$$MC^i(n_i^1) = (w_0 + 1 + \alpha)(1 - \frac{1 - \phi}{\phi}) \quad \text{if } n_i^1 < \tilde{N}_2^i$$
$$= (w_0 + 1 + \alpha)(1 + \delta) - \delta f_1(n_i^1, \theta_2^i) \quad \text{if } \tilde{N}_2^i < n_i^1 < \tilde{n}_2^i$$
$$= (w_0 + 1 + \alpha) + \delta \gamma \alpha \quad \text{if } n_i^1 > \tilde{n}_2^i$$

(4)

Let $n_i^*_{\text{eq}}$ denote the solution satisfying the above condition, which is unique. Furthermore, let $n_i^*$ denote the optimal static level of employment. This is the optimal level of employment in $t=1$ when $\delta = 0$ and is defined by:

$$f_1(n_i^*, \theta_1) = w_0 + 1 + \alpha.$$  

We are now ready to describe the equilibrium employment levels.

**Proposition 1** Under complete information on $\theta_2$ and assumption 1,

1. The good type chooses $n_i^{G*} \in (n_i^1, \tilde{n}_2^G]$ in period 1. In period 2, it doesn’t fire anyone and hires either zero or a positive amount of new workers.

2. The bad type either chooses a big-bang strategy ($n_i^{B*} = \tilde{n}_2^B$) or a gradual downsizing strategy ($n_i^{B*} \in (\tilde{n}_2^B, n_i^1)$). In the first case, there is no further downsizing in period 2 while in the second case, downsizing occurs in both periods and $n_i^{B*} - \tilde{n}_2^B$ workers are laid off in period two.

The proof is in the appendix.

The actions of the good firm are very intuitive: since demand rebounds in period two, a good type firm retains more original workers than the static optimal level in period one and has no reason to fire any of them in period two. Nevertheless, as $\theta_1$ becomes worse, a good type firm is likely to lay off more original workers in period one and to hire new workers in period two.

For the bad firm, there are two possible cases, one where $n_i^{B*} = \tilde{n}_2^B$ and one where $n_i^{B*} > \tilde{n}_2^B$. In the first case, the bad firm, which faces adverse

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15Note that we can simplify $w^i(n_i^1)$ (previously defined in equation 1) notationally; since all firms offer the same wage in period 2 $V_{t+1}^{\text{out}} - V_{t+1}^{\text{in}} = \alpha$. 

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shocks in both period one and period two, lays off workers only once - in period one. In period two the firm makes no further labor force adjustments. We call this strategy “big bang”, since the firm drops the axe on its employees in one blow. This is depicted in Figure 1 as point B. When the firm lays off workers in both periods (i.e. when $n_{1B}^* > \bar{n}_2$), we say that the firm resorts to a policy of “gradualism”, where the firm adjusts its labor supply every time there is an adverse shock. Gradualism can be seen in Figure 1 as point C. It is clear from the figure that the discontinuity in the marginal cost of retaining a worker due to job insecurity gives rise to the two possible solutions.

It is important to point out that in either big-bang or gradualism, the amount of workers retained by a bad type at the end of period two is the same ($\bar{n}_2$). If job insecurity didn’t affect the survivors’ effort levels, a bad type would keep $n_1^*$ amount of workers in period one and lay off $n_1^* - \bar{n}_2$ of them in period two. However, job insecurity reduces survivors’ commitment to their job, forcing firms to pay higher wages to induce high effort. Therefore, when choosing $n_1^B$, a bad type faces a tradeoff between increasing the amount of workers retained in period one and reducing their job insecurity. This tradeoff can make it optimal to completely remove job insecurity of the survivors by
choosing a big-bang strategy \( (n_1^B = \tilde{n}_2^B) \).

We can now analyze what determines whether a firm engages in a big-bang or gradual downsizing strategy. In general, given a level of job insecurity, the larger the expected outside option, the higher a premium the workers command, making the big-bang more likely. The value of the outside option in turn depends on employment opportunities, job search, and labor market tightness. In addition, lower marginal productivity for the firm can make it more likely to make sweeping cuts. This may be due to its fundamental production process, or the shocks which hit the firm. A larger negative shock in period one reduces the marginal productivity of all workers, making a high wage more costly and big-bang more likely. A smaller negative shock in period two increases \( \tilde{n}_2^B \) and implies that the number of people to be downsized is smaller in both periods. With more workers retained, the marginal productivity of the last worker in period one is lower, making it too costly to pay a high wage and big-bang more likely. We summarize these determinants in the following corollary:

**Corollary 1**  *Big-bang is more likely if*

1. *Workers’ future job prospects (conditional on being fired) are better,* i.e. if
   
   (i) *job search is effective* \( (\gamma_e \text{ high}) \)
   
   (ii) *the labor market is very slack* \( (a \text{ high}) \)
   
   (iii) *the value of finding employment in the following period is large* \( (V_{e,\text{out}} - V_u \text{ high}) \)

2. *The firm’s marginal productivity is low*
   
   (i) *in absolute terms: due to technology or product market competition*
   
   (ii) *relative to wages: when the first period shock is worse* \( (\theta_1 \text{ smaller}) \) *or*
   
   *the second period shock is not as bad* \( (\theta_2^B \text{ larger}) \)

It is natural to wonder about how gradualism takes place - does the majority of downsizing take place in period one or period two? That answer is also given to us by the corollary. Conditional on being in a regime of gradualism, the factors which made the big-bang more likely also make the amount of downsizing larger in period one relative to period two.

Although an empirical analysis is outside the scope of this paper, it is worth examining which directions the corollary points us towards. Waves of layoffs create job insecurity for survivors, increasing the firm’s marginal cost of retaining a worker in period one. Greenhalgh, Lawrence, and Sutton
(1988) find similar results when reviewing the management literature: “The negative effects of waves of layoffs have been reported in case studies of the Atari Corporation (Sutton, Eisenhardt, and Jucker (1986)), Amex (Reibstein (1985)), and American Telephone and Telegraph (Guyon (1986)), and they have been noted in the decline of the hospital industry (White (1985)). To avoid this stress, managers make cuts that exceed the expected oversupply”.

Wages may be different between firms in period 1 if the bad firm has a policy of gradualism. The bad firm must pay higher wages to compensate for job insecurity. In some sense, this is a compensating differential, although the worker is not directly choosing between jobs at a good and a bad firm. Examples of a wage premium for job insecurity are plentiful:

- At United Airlines, most of the “75,000 employees... had bought a majority stake in the airline, taking huge pay cuts in return for a commitment that none of United’s employee-owners would be laid off for five years”. Moreover, “the list of pilots seeking jobs at United has swelled to more than 10,000, even though the airline now pays less than some of its biggest rivals”.

- Moretti (2000) examines the compensating differential in the agricultural sector for temporary work over permanent work and find that it is between 9.36 and 11.9 percent of the average worker’s hourly wage.

- Dial and Murphy (1995), in their study of General Dynamics, observe that the wage premium for working in “the competitive defense industry” reflected specialized skills and a “compensating differential for risky employment in an industry with historically variable demand”.

One might argue, on the other hand, that general job insecurity should decrease wages since it decreases outside opportunities and bargaining power (this thesis was originally put forth by Alan Greenspan (1998)). This does not conflict with our model. Firstly, if outside potential job opportunities are worse the wage demanded by workers will decrease. Secondly, we assume

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18 He also provides a literature review of compensating differentials related to unemployment risk. The results are mixed, but most previous estimations suffered from sample selection problems and unobserved individual heterogeneity.
that some firms rebound from the initial negative shock. If the economy is in a persisting recession, then this may not be true and a wage differential between firms may disappear. We have little to say about wage dynamics, since the wages in the second period are essentially fixed.

The productivity results in the corollary suggest that industries may differ substantially in their layoff policies. High productivity or profitability industries should be more stable. Davis and Haltiwanger (1999) state that for manufacturing “the relative volatility of destruction [to job creation] falls with trend growth and rises with firm size, plant age and the inventory-sales ratio” while Farber notes that professional services have low and consistent rates of job loss. On the negative side, Bewley (1999, section 13.3) provides evidence that managers don’t consider labor market conditions when making layoff decisions, although they are aware of on-the-job search for other jobs.

4 Asymmetric information

We now assume that firms have private information about the shock they will face in period two: $\theta^G_2$ or $\theta^B_2$. Workers at a firm have an ex-ante belief that with probability $\nu$, $\theta_2 = \theta^G_2$ and with probability $1 - \nu$, $\theta_2 = \theta^B_2$. The private information may reflect a firm’s superior knowledge of how well prepared it is for demand shocks or of overall market conditions and trends. As we have seen in the section on complete information, the second period labor demand has a serious effect on the firm’s decisions in both periods. The difference between the second period labor demand for a good firm and a bad firm creates a possible role for adverse selection when types become private information. The good firm has no incentives to masquerade as the bad firm (i.e. choosing the bad firm’s wage and employment levels). The reason is that it could easily have chosen them in complete information, but found it optimal not to do so. The bad type, on the other hand, was restricted in its choices. It could not choose the wage-employment level pair that the good type chose, because it had to offer higher wages to compensate workers for a higher probability of being laid off in the second period. The minimum wage that the bad firm could offer was $w^B(n_1) = w_0 + 1 + \alpha + \delta \max\{(1 - \bar{n}_o^B n_1), 0\} \gamma e^{\alpha a}$. 

15
The bad firm has incentives to pretend it is the good firm when:

\[ f(n^*_G, \theta_1) - (w_0 + 1 + \alpha)n^*_G + \delta \pi^G_2(n^*_G) > f(n^*_B, \theta_1) - w^B(n^*_B)n^*_B + \delta \pi^B_2(n^*_B) \]

Since \( n^*_G \geq n^*_B \) and the bad firm always downsizes to \( \bar{n}^B_2 \) in period 2, it must be that \( \pi^B_2(n^*_G) = \pi^B_2(n^*_B) \). Hence the following assumption is sufficient for establishing the existence of an adverse selection problem when there is asymmetric information.

**Assumption 2:** \[ f(n^*_G, \theta_1) - (w_0 + 1 + \alpha)n^*_G > f(n^*_B, \theta_1) - w^B(n^*_B)n^*_B \]

We first study the fully separating equilibrium and then the pooling equilibrium\(^{20}\). The equilibrium concept employed is Perfect Bayesian Equilibrium and we refine the set of equilibria using the Cho-Kreps (1987) intuitive criterion. The model presents a two-dimensional signaling problem: firms may use both the period 1 employment level and wages of original workers to signal. This problem is similar to that of Milgrom and Roberts (1986) in two ways. First, Milgrom and Roberts also have two dimensions of signaling, prices and advertising. Second, advertising is dissipative in their model, meaning that it does not affect demand and hence does not interact with the type of the firm. Wages in our model function in a similar way. Under complete information, any wage above \( w^i(n^*_i) \) (with \( i = G, B \)) involves extra cost for a firm at no gain. Nevertheless, we will see that excess wages may be used in equilibrium.

4.1 Separating Equilibrium

In the separating equilibrium, the good firm chooses in period 1 an employment level \( n_S \) and a wage \( w_S \) for workers such that the bad firm does not have any incentives to masquerade as the good firm. Specifically, we define the belief structure of workers, \( \mu(n_1, w_1) \), as the probability that the firm is good given its first period employment and wage decisions. This then implies that in the separating equilibrium \( \mu(n_S, w_S) = 1 \). Moreover, if the separating equilibrium exists, the bad firm is recognized as bad. It will then choose its employment and wage optimally, opting for the solution to the complete information case \( (n^*_B, w^B(n^*_B)) \). This implies a belief of \( \mu(n^*_B, w^B(n^*_B)) = 0 \).

\(^{20}\)We ignore semi-separating equilibria, where a firm may have mixed strategies.
Lastly, notice that some choices \((n_1, w_1)\) are not feasible due to the need to provide incentives, namely any \(w_1 < w^G(n_1)\).

Two incentive constraints define the set of separating equilibria \((n_S, w_S)\). First, the bad firm must prefer being recognized to masquerading:

\[
f(n_1^*, \theta_1) - w^B(n_1^*)n_1^* + \delta \pi^B_2(n_1^*) \geq f(n_s, \theta_1) - w_sn_s + \delta \pi^B_2(n_s) \quad (IC_B)
\]

Second, the good firm must prefer separating to being perceived as the bad firm. When the good firm is perceived to be the bad firm, any employment level above \(\bar{n}_o^B\) in the first period will make it necessary to compensate workers with higher wages, as workers will believe the layoff probability in the second period will be positive. The wage level necessary to prevent quits is the same that the bad firm must use to prevent quits: \(w^B(n_1)\). Since beliefs are not pinned down off the equilibrium path\(^{21}\) we assume that the beliefs of workers are such that any feasible choice of the bad firm (i.e. an \(n_1\) and \(w_1 \geq w^B(n_1)\)) is believed to have come from the bad firm, or \(\mu(n_1, w_1) = 0\).

We denote \((n_{GB}, w^B(n_{GB}))\) as the optimal choice of the good type when workers believe that it is the bad type. The incentive constraint for the good firm is thus:

\[
f(n_s, \theta_1) - w_sn_s + \delta \pi^G_2(n_s) \geq f(n_{GB}, \theta_1) - w^B(n_{GB})n_{GB} + \delta \pi^G_2(n_{GB}) \quad (IC_G)
\]

We will establish the result using a graphical argument. The curves and solution are depicted in Figures 2 and 3. For now, we assume that the optimal full information choice for the bad firm was that of gradualism, where the solution was \(n_1^*\) and \(w^B(n_1^*)\). The results for a big-bang solution are qualitatively the same.

We begin the analysis by defining isoprofit curves in \((n_1, w_1)\) space. The curve \(ISO_B\) represents all of the employment-wage pairs for the bad firm which yield the same profits as its full information choice. The curve \(ISO_G\) depicts the employment-wage pairs for the good firm which yield the same profits as its choice when it is believed to be the bad firm, \((n_{GB}, w^B(n_{GB}))\). By the definition of the incentive constraints, these curves are the minimum

\(^{21}\)Cho-Kreps is not of any use here, since both firms’ equilibrium choices will dominate the payoffs of choices where \(w > w^B(n_1)\).
level of profits that the firms can achieve in a separating equilibrium \((n_S, w_S)\). The curves are both tangent to the \(w^B(n_1)\) curve at points \((n_1^B, w^B(n_1^B))\) and \((n_{GB}, w^B(n_{GB}))\). These are depicted in Figure 2 as points B and C, respectively. From the isoprofit curve of the bad firm, we see that, 1) Gradualism is preferred to big-bang (point B is preferred to point A) and 2) the bad firm prefers the good firm’s full information choice to its own (point D is preferred to point B).

The isoprofit curves intersect only once since they satisfy a weak single crossing property: 
\[
\left(\frac{dw_1}{dn_1}\right|_{\theta_2=\theta^G} - \frac{dw_1}{dn_1}\right|_{\theta_2=\theta^B} \geq 0.
\]
This implies that the slope of the isoprofit curve for the good firm is always greater than or equal to the slope of the curve for the bad firm. This is straightforward to show, and follows from the fact that keeping an extra worker in the first period is (weakly) more profitable for the good firm. The inequality is strict everywhere except when \(n_1 < \bar{N}_2^B\) and \(n_1 \geq \bar{\bar{n}}_2^G\). The former inequality will never be relevant, since a bad firm must always get at least as much profits as in the full information solution, and the region indicated by the former inequality gives less profits than in full information. In the case of the latter inequality, both firms lay off workers in period two and their second period profits do not change with \(n_1\), implying that the slope of their isoprofit line does not change with their type. This will be relevant for both the separating and the pooling equilibrium.

The area below the isoprofit curve for the good firm and above the isoprofit curve for the bad firm satisfies both incentive constraints. All choices in this area are thus equilibrium dominated for the bad firm, hence Cho-Kreps assigns \(\mu(n_1, w_1) = 1\) to these choices. The signaling problem then amounts to the good firm maximizing its profits subject to \(IC_B\) holding with equality.\(^{22}\)

The solution \((n_S, w_S)\) is characterized by:

**Proposition 2** With asymmetric information and assumptions 1 and 2, a separating equilibrium always exists and will take one of two possible forms:

1. The good firm chooses \(n_S \in (n_1^G, \bar{n}_2^G]\) and \(w_S = w_0 + 1 + \alpha\) in period 1 and retains all workers (possibly hiring new ones) and pays the same wage in period 2.

\(^{22}\)We did not discuss beliefs for the area below the curve \(w^B(n_1)\) and above the upper envelope of the two isoprofit curves because for any beliefs these choices would yield lower profits for the firms.
2. The good firm chooses $n_S > \bar{n}_2^{G}$ and $w_S > w_0 + 1 + \alpha$ in period 1 and chooses $\bar{n}_2^{G}$ and $w_0 + 1 + \alpha$ in period 2.

In both solutions, the bad firm chooses its symmetric information levels.

The result of the proposition is depicted in figures 2 and 3. There are two possible cases. The first is where the asymmetric information problem is ‘small’ in the sense that point D (the good type’s symmetric information solution) would increase the bad firm’s profits only a small amount. In this case, the good firm can use only increased employment levels to signal, holding the wage fixed at $w_0 + 1 + \alpha$. This is evident in Figure 2; since the good firm’s isoprofit curve always has greater slope than the bad firm’s, the tangency can only occur at the kinked part (denoted as point S1 in the graph). In the second case, depicted in Figure 3, the asymmetric information problem is ‘large’; the bad firm has large incentives to masquerade as the good one. In this case, there are a range of tangencies, since both firms’ isoprofit lines have the same slope in the area where $n_1 \geq \bar{n}_2^{G}$. This is represented by a hollow oval in Figure 3. All of these solutions involve the good firm increasing its level of employment above $n_1^{G}$ and its wage strictly above $w_0 + 1 + \alpha$. Hence, when the asymmetric information problem is more difficult, the good firm
must resort to using both employment level and wages to signal its type.

Under asymmetric information, a good type can reduce job insecurity of survivors only by retaining more workers than necessary in period one. Therefore, it faces the tradeoff between reducing personnel and decreasing the job insecurity of the survivors. The effectiveness of signaling comes from the fact that it is less costly for the good firm to reduce its downsizing in period 1 in the interval $\bar{n}_2^{GB} < n_1 < \bar{n}_2^{G}$. An increase in period 1 employment increases the good firm’s second period profits while not affecting the bad firm’s second period profits. A wage increase will be a part of the signal when employment increases so much that the good firm creates some job insecurity in period 2.

When we compare this to the full information solution, we see how a conservative employment policy works and its associated costs. Good firms will reduce their downsizing in the first period to signal, reducing their profits and potentially forcing them to not hire and maybe even fire people in the second period. This lies in contrast to the symmetric information solution, where job insecurity is diminished by downsizing more workers. The reduction in downsizing may be so large as to imply zero layoffs in period one for the good
firm despite the optimality of positive layoffs in complete information. This is evident from case 2 of the proposition if $n_S > 1 > \bar{n}_2^G$.

4.2 Pooling Equilibria

In a pooling equilibrium, both types of firm choose the same employment and wage levels $(n_P, w_P)$. When a worker observes these, she is unable to update her information set and believes that the firm is good with probability $\nu$. In order to prevent quits, the firm must offer a wage high enough to compensate workers for the probability of being laid off; taking into account the uncertainty of the firm’s type, $w_P$ must then be greater than or equal to:

$$w^P(n_1) = w_0 + 1 + \alpha + (1 - \nu)\delta \max\{(1 - \bar{n}_2^B n_1), 0\} \gamma_e \alpha a + \nu \delta \max\{(1 - \bar{n}_2^G n_1), 0\} \gamma_e \alpha a$$

Despite this restriction on the choice of wage, isoprofit curves take the same shape as in the previous section. They are determined by incentive constraints for type $i$ (i=B,G) where type $i$ prefers $(n_P, w_P)$ to the optimal deviation of type $i$. We assume that off the equilibrium path a deviation is believed to come from the bad type, or equivalently, $\mu(n_1, w_1) = 0$. Therefore the best deviations are $(n^*_B, w^B(n^*_B))$ for the bad type and $(n_{GB}, w^B(n_{GB}))$ for the good type.

A pooling equilibrium can only occur in the area where the single crossing property does not hold, i.e. where both the good and the bad firm would be in a firing regime in period two and their second period profits are fixed with respect to first period employment. When the single crossing property does hold, it is always possible to find a $(n_1, w_1)$ between the isoprofit curves that the bad firm would never choose and that the good firm prefers. Since the bad firm would never choose this point irrespective of being recognized as the good or bad firm, the intuitive criterion assigns $\mu(n_1, w_1) = 1$ to the beliefs at this point, leaving the good firm able to select it and be recognized as the good firm.

In figure 4, we depict pooling equilibria as the shaded region. Any point in the region satisfies the criteria that the single crossing property does not hold, both firms earn at least as much as they would if recognized as the bad firm (points B and C are the respective points of maximum profits when the bad type and the good type, respectively, are recognized as bad firms), and there are no other profitable deviations. Notice that since the slope of the
isoprofit curve of the good firm is always larger than that of the bad firm outside of this region, the good firm’s isoprofit curve lies below that of the bad firm. Thus both the good firm and the bad firm are making profits in the pooling equilibrium that are not feasible (and at least as large as) when recognized as a bad firm. Feasibility for a bad firm, of course, is defined as the wage being above the line $w_B(\cdot)$.

If the pooling equilibria exist, there are a continuum of them. Existence depends on whether the isoprofit curve for the bad firm, defined by the bad firm’s incentive constraint, extends into the area where the single crossing property doesn’t hold, namely $n_1 \geq \bar{n}_2^{\alpha_G}$ and $w_1 \geq w_P(n_1)$. The following proposition characterizes the equilibria.

**Proposition 3** Assuming existence, any pooling equilibria $(n_P, w_P)$ has $n_P \geq \bar{n}_2^{\alpha_G}$ and $w_P \geq w_P(n_1)$. In a pooling equilibrium, a bad type firm never adopts the big-bang strategy.

In the pooling equilibria, both firms raise their employment levels above their full information levels. They both pay a wage above $w_0 + 1 + \alpha$ in the first period. This wage may actually be dissipative in the sense that it can be
more than the minimum wage demanded by workers to compensate them for expected job insecurity. Both firms downsize in the second period as well, due to keeping too many employees in the first period. Here asymmetric information can make it more likely to observe waves of downsizing, as both the bad firm and the good firm choose policies of gradualism.

5 Long-Term Contracts

In the model, we have restricted firms to one period contracts with no long term commitments. Although this may seem more realistic when thinking about downsizing in an environment with adverse shocks, making commitments to workers can reduce job insecurity, making a study of long-term contracts particularly relevant. We find that long-term contracts increase employment and eliminate the asymmetric information problem.

5.1 Complete Information

When a firm may downsize in period two, long-term contracts can assuage the fears of workers and reduce job insecurity. The tradeoff is that by reducing some workers’ insecurity, the probability that other workers face of getting fired will rise. Controlling job insecurity may play an important role in the real world; contingent contracts of the type “I’m hiring you for this period, but will retain you for the next period at wage \( w \) with probability \( z \)” resemble union contracts, where downsizing is possible, but only if there are some compelling reasons for doing it (such as negative shocks to the firm’s economic environment).

Consider a bad firm. With commitment power to retain original workers, the firm can use two contracts which differ mainly in terms of the commitment level to job security: one contract specifies that workers will be retained for sure in period two while the other contract specifies that workers will be retained in period two only with probability \( z \). Let us call the first contract a long-term one and the second a short-term one for convenience. Let \( n_L \) \( (n_S) \) denote the number of original workers retained in period one through the long-term contract (the short-term contract). Let \( w_{tL} \) \( (w_{tS}) \) represent the period \( t \) wage specified by the long-term contract (the short-term contract). In addition, the short-term contract specifies \( z \), the probability of being retained in period two. Hence, in period two, the firm retains \( n_{2S} = zn_S \)
workers of the short-term group. Choosing $z$ is formally equivalent to choosing $n_{2S}$ such that $n_{2S} \leq n_S$ and we proceed with this substitution. We assume that the firm can commit at $t = 1$ to the level of job security and the wages $(w_{1L}, w_{2S})$ after observing $\theta_1$. Therefore, the maximization problem of the firm is given by:

$$\max_{n_L, n_S, n_{2S}, w_{1L}, w_{1S}, w_{2L}, w_{2S}} f(n_L + n_S, \theta_1) - w_{1L} n_L - w_{1S} n_S + \delta(f(n_L + n_{2S}, \theta_2^B) - w_{2L} n_L - w_{2S} n_{2S})$$

s.t. $n_{2S} \leq n_S$,  
$w_{1L} \geq w_0 + 1 + \alpha$,  
$w_{2L} \geq w_0 + 1 + \alpha$,  
$w_{1S} \geq w_0 + 1 + \alpha + \delta \gamma_e \alpha (1 - \frac{n_{2S}}{n_S})$,  
$w_{2S} \geq w_0 + 1 + \alpha$

where the constraint on $w_{1L}$ ($w_{1S}$) represents the incentive constraint to induce the workers to choose high effort in period $t$ under the long-term contract (the short-term contract). Since the second period is the last period, the second period incentive constraints are the same as the first period one for the workers with complete job security: therefore it is optimal to choose $w_{1L} = w_{2L} = w_{2S} = w_0 + 1 + \alpha$. In contrast, job insecurity affects the first period incentive constraint for the workers under the short-term contract. It is important to notice is that if we define $n_1 = n_L + n_S$ and $n_2 = n_L + n_{2S}$, the composition of $n_1$ and $n_2$ among $n_L$, $n_S$ and $n_{2S}$ doesn’t matter. This comes from looking at the first period wage bill; re-arranging, it becomes $(w_0 + 1 + \alpha) n_2 + (w_0 + 1 + \alpha + \delta \gamma_e \alpha a)(n_1 - n_2)$. Then, we can define the following reduced program:

$$\max_{n_1, n_2 (\leq n_1)} f(n_1, \theta_1) - (w_0 + 1 + \alpha)n_2 - (w_0 + 1 + \alpha + \delta \gamma_e \alpha a)(n_1 - n_2) + \delta \left[ f(n_2, \theta_2^B) - (w_0 + 1 + \alpha)n_2 \right].$$

From the program, it is straightforward to see that for any $n_2 < n_1$, increasing job security (i.e., increasing $n_2$) has the benefit of reducing the wage bill of period one by $\delta \gamma_e \alpha a$. This defines a period two cutoff $\hat{n}_2^B$ by $f(\hat{n}_2^B, \theta_2^B) = w_0 + 1 + \alpha - \gamma_e \alpha a$, where $\hat{n}_2^B > \tilde{n}_2^B$. If $n_1 > \hat{n}_2^B$, the firm finds it optimal to reduce its period two workforce to $\hat{n}_2^B$ and otherwise the firm chooses

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23We only look at the range of employment $n_L \geq \tilde{N}_2^B$, for which workers may be laid off in the second period (since this is where long-term commitments will have bite) and hence don’t include hiring new workers in the optimization problem.
$n_2 = n_1$. Since for $n_1 < \tilde{N}_2^B$, the marginal cost of increasing $n_1$ is the same as the one without commitment, we have the following first order condition with respect to $n_1$:

$$f_1(n_1, \theta_1) = \begin{cases} 
(w_0 + 1 + \alpha)(1 - \frac{\delta}{\phi}) & \text{if } n_1 < \tilde{N}_2^B \\
(w_0 + 1 + \alpha)(1 + \delta) - \delta f_1(n_1, \theta_2^B) & \text{if } \tilde{N}_2^B \leq n_1 < \hat{n}_2^B \\
(w_0 + 1 + \alpha) + \delta \gamma \alpha a & \text{if } n_1 > \hat{n}_2^B
\end{cases} \quad (5)$$

Comparing (5) with (4) reveals that commitment allows the firm to reduce the marginal cost of retaining workers for the $\overline{n}_2^B \leq n_1 < \hat{n}_2^B$ range. Since we have shown that choosing $n_1 < \overline{n}_2^B$ is not optimal without commitment, it is not optimal with commitment either. A big-bang arises when the bad firm chooses $n_1 \in [\overline{n}_2^B, \hat{n}_2^B]$ and is depicted in Figure 5 as point B. Gradualism occurs when $n_1 > \hat{n}_2^B$ and is depicted in Figure 5 as point C.

The existence of long-term contracts increases employment for the bad firm. Long-term contracts make workers cheaper by allowing firms to control job insecurity and reduce the marginal cost of retaining workers in period one. If the optimal choice without commitment $n_1^B$ belongs to $[\overline{n}_2^B, \hat{n}_2^B)$, this effect induces the firm to retain more workers in period one. Moreover,
when the optimal $n_1$ under long-term contracting is larger than $n_2^B$, long-term contracts increase second period employment from $\bar{n}_2^B$ to $\min\{n_1, \hat{n}_2^B\}$. Finally, we observe that long-term contracts don’t affect the good firm’s choices, since it never downsizes in period two (see Lemma 1, part 1) and does not need the commitment power.

5.2 Asymmetric Information

We have seen that long-term contracts, under complete information, can substantially improve the tools of the firm to manage job insecurity. When we consider long-term contracts in the situation where the second period shock to the firm is unknown to workers, we find that the signaling problem of the good firm disappears. The intuition is the following: since firms can commit to different levels of job security through binding long-term contracts, a worker’s belief about a firm’s type has no impact on his perception of job security - only the job security specified by his employment contract matters. Therefore, the signaling problem ceases to exist and each type of firm simply chooses its optimal decision under complete information. Hence all the workers retained by the good firm in period one will be hired under a long-term contract specifying full job security. The bad firm strictly prefers maintaining its optimal choices under complete information to mimicking the good firm’s choices; otherwise, it would have chosen the latter instead of the former under complete information, which is a contradiction.

6 Social welfare

Social welfare is defined as the sum of firms’ profits and workers’ payoffs. We study the social optimum when a benevolent social planner chooses wages, the number of original workers to be retained and the number of new workers to be hired taking into account the incentive and participation constraints of workers. In order to conduct the comparison between the market outcome and the social optimum on a similar ground, we first look at the case where the social planner cannot commit to a long-term contract. As we explain later, it turns out that short-term and long-term contracts are equivalent for

\[24\text{Including consumer surplus, where consumer surplus is increasing and concave in production (production is essentially equal to employment in the model), yields the same qualitative results and would make quantitative conclusions even stronger.}\]
the planner. Lastly, we assume that the planner can’t affect labor market matching frictions, so \(a\) is exogenously given.\(^{25}\)

Let us first study the social optimum in period two. The period two social welfare related to a type-\(i\) firm, denoted by \(SW^i_2\), is given as follows up to a constant\(^{26}\):

\[
SW^i_2(n^{oi}_2, n^{ni}_2; n^i_1) \equiv f(n^{oi}_2 + \phi n^{ni}_2, \theta^i_2) - (w_0 + 1) (n^{oi}_2 + n^{ni}_2)
\]

where \(n^{oi}_2 \leq n^i_1\). The wage \(w_2\) does not appear in \(SW^i_2\) since it only affects the distribution of profits between firms and workers, but \(w_2\) must be at least as large as \(w_0 + 1 + \alpha\) in order to satisfy the incentive constraint. The social marginal cost of retaining a worker is the worker’s outside option \(w_0\) plus the disutility of exerting effort \(1\). Let \(\left(\tilde{N}^{si}_2, \tilde{n}^{osi}_2\right)\) be defined as follows:

\[
\phi f_1(\tilde{N}^{si}_2, \theta^i_2) \equiv w_0 + 1 \equiv f_1(\tilde{n}^{osi}_2, \theta^i_2)
\]

Then, from the first-order conditions, the social optimum \((n^{stoi}_2, n^{stni}_2)\) is characterized as follows:

\[
\begin{align*}
    n^{stoi}_2 &= n^i_1, \\
    n^{stni}_2 &= \frac{\tilde{N}^{si}_2 - n^i_1}{\phi} & \text{if } n^i_1 < \tilde{N}^{si}_2 \\
    n^{stoi}_2 &= \tilde{n}^{osi}_2, \\
    n^{stni}_2 &= 0 & \text{if } \tilde{N}^{si}_2 < n^i_1 \leq \tilde{n}^{osi}_2
\end{align*}
\]

The period one social welfare related to a type-\(i\) firm, denoted by \(SW^i_1\), is given as follows up to a constant:

\[
SW^i_1(n^i_1) \equiv f(n^i_1, \theta^i_1) - (w_0 + 1) n^i_1
\]

where \(w_1\) does not appear in \(SW^i_1\) for the same reason that \(w_2\) does not appear in \(SW^i_2\) but it must be larger than or equal to \(w^i(n^i_1)\) in order to satisfy the incentive constraint. The government chooses \(n^i_1\) to maximize \(SW^i_1(n^i_1) + \delta SW^i_2(n^{stoi}_2(n^i_1), n^{stni}_2(n^i_1))\). The first order condition is given as follows:

\[^{25}\text{To keep the accounting simple, however, we assume that } a \text{ is the number of total people hired in period two over the number of total people looking for work (those fired in periods one and at the beginning of period two). This avoids having to give weight to jobs or workers from outside of the labor market that we describe.}\]

\[^{26}\text{The total period two social welfare is equal to } \left[ \nu SW^G_2 + (1 - \nu) SW^B_2 \right] M + w_0.\]
\[ f_1(n_1', \theta_1) = \begin{cases} (w_0 + 1) (1 - \delta \frac{1 - \varphi}{\bar{\varphi}}) & \text{if } n_1' < n_1^s \\
_0 + 1 & \text{if } n_1' > n_1^s \end{cases} \]

Let \( n_1^{**} \) denote the optimal \( n_1' \) satisfying the above first order condition.

In the next proposition, we make some comparisons regarding social welfare.

**Proposition 4**

1. When we compare the social optimum with the market outcome under complete information,
   (i) Each period, each type of firm employs less than the socially optimal number of workers.
   (ii). Big-bang is never socially optimal.

2. When we compare the market outcome under complete information with the one under asymmetric information, asymmetric information increases social welfare both in a separating equilibrium and in a pooling equilibrium as long as the increase in employment in either is not excessive.

**Proof.** Since the other results are straightforward, we prove only 1(ii).

We will prove that in the case of a bad type firm, the government finds it optimal to lay off some original workers in period two (i.e. gradualism is always optimal and \( n_1^{**B} > \tilde{n}_2^{osB} \)). Suppose \( n_1^{**B} \leq \tilde{n}_2^{osB} \). On the one hand, \( n_1^{**B} \leq \tilde{n}_2^{osB} \) implies \( f_1(n_1^{**B}, \theta_1) \geq f_1(\tilde{n}_2^{osB}, \theta_1) \). On the other hand, from the first order condition with respect to \( n_1 \), we know \( f_1(n_1^{**B}, \theta_1) \leq w_0 + 1 \) for \( n_1^{**B} \leq \tilde{n}_2^{osB} \) and we have \( f_1(\tilde{n}_2^{osB}, \theta_1) > f_1(\tilde{n}_2^{osB}, \hat{\theta}_2) = w_0 + 1 \), which is a contradiction.

The under-employment result is straightforward since a firm does not internalize workers’ utilities. Big-bang is never socially optimal since it is socially optimal for a bad type firm to lay off some workers in period two. This is because the social planner does not face any fundamental trade-off between laying off redundant workers and maintaining the commitment of

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27 The under-employment result makes sense only if there is involuntary unemployment in each period. This obviously holds in period one since we assume downsizing and also holds in period two as long as the good firms do not hire any new workers. If they hire new workers, a sufficient condition to have involuntary unemployment in period two is \((1 - \nu)\phi(1 - \tilde{n}_2^{osB}) > \nu(1 - \phi)(1 - \tilde{N}_1^G)\), where \( \tilde{N}_1^G \) is defined by \( f_1(\tilde{N}_1^G, \theta_1) \equiv (w_0 + 1 + \alpha) (1 - \delta \frac{1 - \varphi}{\bar{\varphi}}) \).
‘survivors’ to their work. The social planner internalizes workers’ utilities and any wage increase due to job insecurity has no impact on her objective function. Asymmetric information unambiguously increases period one social welfare since it increases employment levels in period one. Furthermore, an increase in period one employment can never induce a decrease in period two production. Therefore, asymmetric information improves social welfare.

Finally, the social optimum under long-term contracts is the same as the one under short-term contracts. Although long-term contracts are useful for firms when job insecurity forces them to pay higher wages, the social planner does not gain anything from a long-term contract since job insecurity does not affect the social marginal cost of retaining an additional worker. Without this intertemporal link, it can be shown that a series of short-term contracts can perfectly replicate a set of long-term and short-term contracts (and vice versa).

In a complete information environment, this implies that long-term contracts are welfare improving relative to short-term contracts (note that there is not overhiring since the marginal cost of long-term contracts is still larger than the social marginal cost). In a world with asymmetric information, however, social welfare under short-term contracts can be higher than the one under long-term contracts, since the need to signal with the short-term contracts can possibly increase employment beyond the long-term contract level.

7 Robustness

7.1 General Types

In the model, economic shocks are deterministic and firms have perfect foresight about what shock they will face. In this section, we relax that strict assumption to allow for uncertainty about what type of shock will hit. To be more specific, we assume that in the second period, the firm may face either the good shock $\theta^G_2$ or the bad shock $\theta^B_2$. We redefine a firm’s type to be the probability that it will face a good shock in period two. A “good” firm is more likely to rebound in period two and has a probability $x_g$ that a good shock will occur. A “bad” firm is more likely to fall further and has a probability $x_b$ that it will avoid that fate. We assume $x_g > x_b$. Firms don’t know what their future will be, but do know the probability distribu-
tion over shocks. Finally, we keep all other elements of the basic model with short-term contracts the same, and we will use previous notation to define the solution.

Let \( n^*_1(x) \) denote the optimal level of employment in period one given probability \( x \) and taking into account the effect on second period profits (hence, in terms of our previous analysis, \( n^*_1(0) = n^*_1B \) and \( n^*_1(1) = n^*_1G \)). The probability \( x \) affects the incentive constraint in period one in a simple manner; if we define the optimal first period wage as \( w(n_1, x) \), the incentive constraint yields:

\[
w(n_1, x) = wx^G(n_1) + (1 - x)w^B(n_1)
\]

In the solution to the basic model (section 3.2) the first order conditions take the form

\[
f_1(n_1, \theta_1) = MC_i(n_1) \quad \text{for} \quad i = G, B,
\]

where the right hand side is the marginal cost associated with an increase in retentions. The first order conditions which define the solution here take a similarly simple form:

\[
f_1(n_1, \theta_1) = xMC^G(n_1) + (1 - x)MC^B(n_1)
\]

We know that \( MC^G(n_1) = MC^B(n_1) \) for \( n_1 < \tilde{N}_2^B \) and \( n_1 > \tilde{n}_2^G \). For \( n_1 \in (\tilde{N}_2^B, \tilde{n}_2^G) \), marginal costs are such that \( MC^G(n_1) < MC^B(n_1) \). This leads us to the following proposition:

**Proposition 5** The number of workers retained in period one, \( n^*_1(x) \), weakly increases with \( x \).

(i) If a big-bang is optimal when \( x = 0 \), there is a threshold \( \hat{x}_{BB} \) such that for all \( x \leq \hat{x}_{BB} \), the big-bang is optimal (meaning that \( n^*_1(x) = \tilde{n}_2^B \)) and for \( x > \hat{x}_{BB} \), a gradual downsizing is optimal. The cutoff \( \hat{x}_{BB} \) is defined by

\[
\hat{x}_{BB} = \frac{f_1(\tilde{n}_2^B, \theta_1) - MC^B(\tilde{n}_2^B)}{MC^G(\tilde{n}_2^G) - MC^B(\tilde{n}_2^G)} \quad \text{(where the (-) superscript means “from the right”)}.
\]

Thus, when \( x \leq \hat{x}_{BB} \), \( n^*_1(x) \) does not change with \( x \) while when \( x > \hat{x}_{BB} \), \( n^*_1(x) \) strictly increases with \( x \).

(ii) If gradualism is optimal when \( x = 0 \), then it is optimal for all \( x \). There can be a threshold \( \hat{x}_{GR} \) such that for all \( x > \hat{x}_{GR} \), \( n^*_1(x) = \tilde{n}_2^G \) while for \( x \leq \hat{x}_{GR} \), \( n^*_1(x) \) strictly increases with \( x \). The cutoff \( \hat{x}_{GR} \) is defined by

\[
\hat{x}_{GR} = \frac{f_1(\tilde{n}_2^G, \theta_1) - MC^B(\tilde{n}_2^G)}{MC^G(\tilde{n}_2^G) - MC^B(\tilde{n}_2^G)} \quad \text{(where the (+) superscript means “from the left”)}.
\]
In order for the notion of a “good” firm versus a “bad” firm to be interesting, we define the probabilities $x_g$ and $x_b$ such that $n_1^*(x_g) > n_1^*(x_b)$. From the lemma, this implies that it cannot be the case that both $x_g$ and $x_b$ are less than $\hat{x}_{BB}$ when a big-bang was optimal in the basic model and it cannot be the case that both $x_g$ and $x_b$ are greater than $\hat{x}_{GR}$ when gradualism was optimal in the basic model.

The new environment implies that we must slightly redefine the concepts of big-bang and gradualism. A big-bang occurs when a massive downsizing takes place in period one and conditional on a further negative shock no future downsizing would occur. Now, of course, we may observe hiring in period two after a big-bang layoff, since it is possible that a good shock will be realized in period two. Similarly, a policy of gradualism is also defined conditional on a future negative shock- in that case the firm will decide to conduct further downsizing. A bad firm may therefore either conduct a big-bang or a gradual layoff. The good firm may conduct a gradual layoff should it face a negative shock in period two, but never will undertake a big-bang downsizing.

When the firm has private information about its probability of facing a good shock in period two, the results are very similar to our simple model. An adverse selection problem can exist, since the bad type is restricted to paying higher wages than the good type in period one, meaning that the bad type may increase its profits by masquerading as the good type. The single crossing property holds weakly once again, since the slopes of the isoprofits curves are the same for the intervals $n_1 < \hat{n}_2^{oB}$ and $n_1 > \hat{n}_2^{oG}$. Hence, in a separating equilibrium the choice of the good firm $(n_s, w_s)$ will involve increasing retention and possibly the wage in period one ($n_s > n_1^*(x_g)$ and $w_s = w(n_s, x_g) \geq w(n_1^*(x_g), x_g)$). A pooling equilibrium may also exist due to the failure of the single crossing property. The pooling equilibria would involve increased retention by both firms and a wage that represents an increase for the good firm, but may be a possible decrease for the bad firm.

### 7.2 Endogenizing Labor Market Slackness

Here we consider the complete information outcome and endogenize $a$, the unconditional probability of a worker finding a job, in order to study interactions between firms in the labor market. Firms’ downsizing policies affect $a$, which in turn affect the amount of layoffs chosen by each firm through the incentive constraint. More precisely, assume $a$ takes the functional form of
where \( l_I \) represents exogenous labor inflow and \( l_H \) exogenous labor hiring in other labor markets.

Note first that \( a \) has no impact on a good firm’s downsizing policy in period one. This is because the employees in a good firm do not face any job insecurity in period two and therefore \( a \) disappears in the incentive constraint that a good firm faces. We consequently represent \( a \) as \( a(n_{B1}^B) \) and find the following result:

\[
\frac{da}{dn_{1B}^B} \geq 0 \text{ if and only if } \gamma_e \geq \gamma_u; \quad \frac{d^2a}{(dn_{1B}^B)^2} > 0.
\]

Consider first the case in which those unemployed in period \( t \) can find jobs more easily in period \( t+1 \) than those who were employed in period \( t \) but laid off in period \( t+1 \) (i.e. \( \gamma_e > \gamma_u \)). Since \( \frac{da}{dn_{1B}^B} > 0 \) in this case, as bad firms increase the number of retained workers \( n_{1B}^B \) above \( \bar{n}_2^{oB} \), the marginal cost of retaining one more worker increases (due to the fact that the aggregate labor supply curve for the bad type firms has a strictly positive slope for \( n_{1B}^B \geq \bar{n}_2^{oB} \)). Since the marginal revenue from retaining one more worker is strictly decreasing (and the aggregate labor demand curve for the bad type firms has a strictly negative slope), there is a unique equilibrium. In contrast, when the unemployed can find jobs less easily than the employed (i.e. \( \gamma_e < \gamma_u \)), we have \( \frac{da}{dn_{1B}^B} < 0 \). Hence, the aggregate labor supply curve for the bad type firms has a strictly negative slope for \( n_{1B}^B \geq \bar{n}_2^{oB} \) and we can have multiple equilibria (either \( n_{1B}^B = \bar{n}_2^{oB} \) or \( n_{1B}^B > \bar{n}_2^{oB} \)); if a bad firm expects that all the other bad firms will choose a high (low) \( n_{1B}^B \), he thinks that the incentive constraint is easy (difficult) to satisfy because of a low (high) \( a \) and therefore finds it optimal to choose a high (low) \( n_{1B}^B \).

Last, note that \( a \) increases with \( \nu \). Therefore, as the proportion of good firms increases in the industry, bad firms lay off more workers and hence a big-bang scenario is more likely.
8 Conclusion

Managing job insecurity ranks as one of the central human resource tasks of a firm when faced with a shaky economic climate. The balance between laying off redundant workers and maintaining some level of job security forms the basis for a broad set of layoff patterns. These patterns, which differ in their amount of layoffs and timing, can have substantial effects on the welfare of workers and the economy in general.

This paper has offered as simple a model as possible to characterize the layoff practices of firms. We found that downsizing patterns (one time massive cuts versus waves of downsizing) can be distinguished and we isolated the contributions of firm productivity and labor market conditions to the firm’s decision. Moreover, we were able to explain implicit commitments of job security and zero-layoff policies as firms signaling that their future prospects are bright. Both asymmetric information and long-term contracts increase employment and can therefore be welfare enhancing relative to complete information short-term contracts.

Our paper represents a call for further empirical research into the specific causes of downsizing. As Butcher and Hallock (2004) state, “there is little academic work in economics that investigates how, when, and why firms make layoff decisions”. Underlying trends have become much clearer in the past 10 years thanks to Davis, Haltiwanger and Schuh (1996), Davis and Haltiwanger (1999), Farber (2003), and Baumol, Blinder, and Wolff (2003), while managerial intentions have been captured in Hallock (2003) and Bewley (1999). Although difficult because of data concerns, our analysis suggests that firm level analysis across sectors could yield rich insights.

Theoretically, it would be interesting to explore the consequences of risk aversion and insurance behavior (as in the implicit contracts literature) further. In addition, extending the model with more dynamic reputation concerns or adding heterogeneity in worker productivity might yield extra testable predictions.

References


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[27] Hallock, Kevin F. “A Descriptive Analysis of Layoffs in Large U.S. Firms Using Archival and Interview Data”, mimeo, University of Illinois at Urbana-Champaign, 2003.


We offer a proof of Proposition 1 in two parts. First we consider the good firm:

Lemma 1 Under assumption 1, a type G firm
1. Never fires workers in period two;
2. Retains in period one strictly more original workers than the static optimal level: $n^G_1 > n^*_1$;
3. Hires new workers in period two if and only if $\hat{N}^G_1 < \tilde{N}^G_2$, where $\hat{N}^G_1$ is defined by $f_1(\hat{N}^G_1, \theta_1) \equiv (w_0 + 1 + \alpha)(1 - \delta \frac{1 - \phi}{\phi})$.

Proof. 1. Suppose that a good firm lays off some original workers at period two, in which case $n^G_1 > \tilde{n}^G_2$. This implies from assumption 1 that $f_1(n^G_1, \theta_1) < f_1(\tilde{n}^G_2, \theta_1) < f_1(\tilde{n}^G_2, \theta^G_2)$. On the other hand, from the first order condition with respect to $n^G_1$, we have $f_1(n^G_1, \theta_1) = (w_0 + 1 + \alpha) + \delta \gamma \alpha a$ and, from the definition of $\tilde{n}^G_2$, we have $f_1(\tilde{n}^G_2, \theta^G_2) = w_0 + 1 + \alpha$. Hence, we have $f_1(n^G_1, \theta_1) > f_1(\tilde{n}^G_2, \theta^G_2)$, which is a contradiction.
Lemma 2 Under assumption 1, a type B firm

1. Never chooses $n_1^* < \tilde{n}_2^G$, which implies that it never hires in period two;

2. Retains strictly less original workers than the static optimal level in period one: $n_1^* B < n_1^*$;

Proof. 1. Suppose that a type B firm chose $n_1^* B < \tilde{n}_2^G$. This implies that $f_1(n_1^* B, \theta_1) < w_0 + 1 + \alpha = f_1(\tilde{n}_2^G, \theta_2)$. However, by assumption 1, $f_1(\tilde{n}_2^G, \theta_2) < f_1(n_1^* B, \theta_1)$ and by concavity, $n_1^* B < \tilde{n}_2^G$ also implies that $f_1(\tilde{n}_2^G, \theta_1) < f_1(n_1^* B, \theta_1)$, which gives us a clear contradiction.

2. From part 1, we know $n_1^* B \geq \tilde{n}_2^G$. Consider first the case $n_1^* B > \tilde{n}_2^G$ and suppose $n_1^* B \geq n_1^*$. One the one hand, $n_1^* B \geq n_1^*$ implies $f_1(n_1^* B, \theta_1) \leq f_1(n_1^*, \theta_1).$ On the other hand, from the first order condition with respect to $n_1^* B$, we know $f_1(n_1^* B, \theta_1) = (w_0 + 1 + \alpha) + \delta \gamma \alpha \alpha$ for $n_1^* B > \tilde{n}_2^G$, which is strictly larger than $f_1(n_1^*, \theta_1) = w_0 + 1 + \alpha$. Hence, there is a contradiction. Consider now the case $n_1^* B = \tilde{n}_2^G$. Since $f_1(n_1^*, \theta_1) = w_0 + 1 + \alpha = f_1(\tilde{n}_2^G, \theta_2)$ holds, from assumption 1, we must have $n_1^* > \tilde{n}_2^G$. •