Abstract

This paper proposes a theory in which aggregate shocks also produce idiosyncratic risk which in turn introduces a demand channel that we argue is relevant for understanding the Great Recession. We study a model in which households are subject to uninsurable idiosyncratic employment shocks, firms set prices subject to nominal rigidities, and the labor market is characterized by matching frictions and by downward inflexible wages. Higher risk of job loss and worsening job finding prospects during unemployment depress goods demand because of a precautionary savings motive amongst employed households. Lower goods demand produces a decline in job vacancies and the ensuing drop in the job finding rate in turn triggers higher precautionary saving setting in motion an amplification mechanism. The amplification mechanism is absent from standard macroeconomic models and depends on the combination of incomplete financial markets and frictional goods and labor markets. The model can account for key features of the Great Recession in response to the observed changes in the job separation rate and an increase in search efficiency heterogeneity estimated from the matching function. We argue that the latter shock is required to reconcile the large increase in the incidence of longer term unemployment observed during the Great Recession.
Keywords: Job uncertainty, unemployment, incomplete markets.

Jel codes: E21, E24, E31, E32, E52

Acknowledgments

We are grateful for insightful comments from Marco Bassetto, Jeff Campbell, Mariacristina de Nardi, Marty Eichenbaum, Christian Hellwig, Per Krusell, Nicolas Petrosky-Nadeau, Jose-Victor Rios-Rull and seminar participants at numerous conferences and workshops. We acknowledge financial support from the ESRC through the Centre for Macroeconomics. This project is related to the research agenda of the ADEMU project, "A Dynamic Economic and Monetary Union". ADEMU is funded by the European Union's Horizon 2020 Program under grant agreement N° 649396 (ADEMU).
1 Introduction

The U.S. and many other Western economies have still not fully recovered from the Great Recession, the longest and deepest recession since the 1930’s. The U.S. labor market outcomes during the Great Recession have been particularly grave and have involved not only a persistent rise in the level of unemployment, but also a surge in the share of longer-term unemployed workers. This paper puts forward a macroeconomic theory of how such labor market weaknesses can motivate a decline in aggregate demand and how this may produce an amplification mechanism. In particular, we show that a combination of frictions in goods, labor and financial markets, fosters an environment in which changes in job uncertainty impact on goods demand due to precautionary savings against idiosyncratic employment risk and where such changes in goods demand are transmitted to the supply side resulting in changes in labor demand that feed back to the demand side. We apply this theory to U.S. data for the Great Recession and argue it may have been partly responsible for the severity of the Great Recession.

We consider a model in which workers face job loss risk during employment and uncertain job finding prospects during unemployment. Workers cannot purchase unemployment insurance contracts and therefore have to rely on government provided unemployment benefits and private savings for consumption smoothing. In such an incomplete markets setting, changes in the risk of job loss or in the probability of finding a new job during unemployment impact on aggregate demand through employed workers’ precautionary savings. As a result, the decline in aggregate demand that can derive from worsening labor market conditions may be far larger than the income loss of the workers that actually suffer a job loss.\(^1\) We embed this mechanism in a macro model with downward inflexible real wages and in which variations in aggregate demand are transmitted to the supply side because of nominal rigidities in price setting. The introduction of nominal rigidity is a simple way of allowing fluctuations in aggregate demand to impact on equilibrium allocations while the assumption of downward rigid wages is motivated by the lack of a decline in real wages in the U.S. during the Great Recession despite the large increase in the number of job seekers as indicated by unemployment statistics.

We adopt a Diamond-Mortensen-Pissarides style matching framework of the labor market. In order to address the surge in long-term unemployment observed in the U.S. during the Great Recession, we extend the standard matching model with heterogeneity across workers in their job search efficiency. Specifically, we assume that unemployed workers differ in their expected unemployment durations and

\(^1\)Carroll and Dunn (1997) and Carroll, Dynan and Krane (2004) also stress the impact of labor market uncertainties on demand due to precautionary savings.
that these differences emerge either upon job loss or during an unemployment spell which introduces negative duration dependence of job finding rates.\(^2\) The heterogeneity in search efficiency allows us to address the surge in longer term unemployment but is also important for the amplification mechanism because lower search efficiency during unemployment increases the precautionary savings motive amongst the currently employed workers.

We formalize these ideas in an Aiyagari-Huggett-type incomplete markets model with uninsurable idiosyncratic employment and job finding risks and with multiple unemployment states. Firms are monopolistically competitive and face quadratic costs of changing nominal prices. A fiscal authority provides unemployment benefits while the monetary authority sets the short-term nominal interest rate according to a Taylor rule. We allow for aggregate shocks to the job separation rate and to the probability that unemployed workers have low search efficiency. The model is computationally very challenging but we introduce assumptions that allow us readily to analyze its properties using a standard perturbation approach which may be of an independent interest.

We demonstrate that the model generates a very intuitive amplification mechanism in which the absence of insurance against idiosyncratic risk implies that an increase in individual job uncertainty - due to higher risk of job loss and/or longer expected unemployment duration - spurs a decline in goods demand because it motivates precautionary savings amongst the employed workers. The drop in goods demand in turn leads firms to post fewer vacancies which impacts negatively on the job finding rate. This interaction produces an amplification mechanism because employed workers increase their savings even further when perceiving higher job uncertainty due to the drop in probability of finding a job should they lose their current job. We simulate a calibrated version of the model in response to the short burst in the rate of inflow to unemployment observed in the U.S. at the onset of the Great Recession and to a shock to the risk of becoming a low search efficient unemployed worker. The latter shock is estimated imposing a Cobb-Douglas matching function and assuming that variations in the matching function residual derive from changes in the composition of the unemployment pool. Importantly, apart from the specification of the matching function, this estimate relies only upon our assumptions regarding the laws of motion of the stocks of employed and unemployed workers and is independent of all other features of the model. We find that, in response to these two shocks, the model produces a rise in the unemployment rate and a drop in vacancy postings that are very similar to the empirical counterparts observed during the Great Recession. Moreover, the model is also consistent with the movements along

\(^2\)Ahn and Hamilton (2014) and Hornstein (2011) investigate the importance of duration dependence and heterogeneity for the increase in unemployment during the Great Recession.
and the outward shift of the Beveridge curve.

A key insight of our analysis is that it is the combination of frictions in financial, goods and labor markets that generates the amplification mechanism. When workers can insure against idiosyncratic risk, labor market uncertainties have minor impact on aggregate demand because households have no incentive to engage in precautionary savings against idiosyncratic risk thus eliminating the demand channel. When prices are flexible, price adjustments eliminate the transmission of shocks from the demand side to the supply side. If wages are fully flexible, the wage adjustment is sufficient to neutralize the amplification mechanism unless workers have little bargaining power. We also demonstrate that monetary policy plays an important role. In standard New Keynesian models that rest on the representative agent assumption, aggressive changes in nominal interest rates in response to deviations of inflation from its target can eliminate the inefficiencies deriving from nominal rigidities. In the heterogeneous agent model that we examine the impact of nominal rigidities is amplified through the aggregate demand channel. This makes aggressive monetary policy extra effective because it not only addresses inefficient variations in mark-ups but also neutralizes the aggregate demand amplification. Moreover, we show that the local indeterminacy region of the parameter space is large and depends crucially on agents’ risk aversion. The more risk averse are agents the more the aggregate demand channel matters and the more aggressive does monetary policy need to be to rule out expectational equilibria.

We are not the first to study the impact of unemployment risks on the economy. Krusell and Smith (1999) and Krusell et al (2009) examine the impact of short-term and long-term unemployment risks with self-insurance. Challe and Ragot (2015) study like us the impact of precautionary savings in an incomplete markets setting. Guerrieri and Lorenzoni (2015) examine an incomplete markets setting with nominal rigidities focusing upon the impact of tightening borrowing constraints. Gornemann, Kuester and Nakajima (2012) and McKay and Reis (2014) both study incomplete markets models with labor and goods market frictions but focus upon very different questions from us. Gornemann, Kuester and Nakajima (2012) examine the distributional effects of monetary policy when agents face unemployment risk while McKay and Reis (2014) focus upon the impact of automatic fiscal stabilizers.

Our analysis is also related to the rapidly growing literature on ‘uncertainty shocks’ that has followed Bloom’s (2009) contribution. However, whilst much of the existing literature has emphasized the impact of changes in second moments of aggregate shocks, we stress the effects of changes in the first moments of job separation and job finding rates which trigger demand variations because of the their impact on idiosyncratic risk. In our model the absence of unemployment insurance contracts implies that

---

3 Basu and Bundick (2012) and Schaal (2012) also investigate the impact of uncertainty and news shocks in models with
variations in the probability of job loss and in the job finding rate impact on agents’ perception of idiosyncratic risk. An interesting aspect of this is that uncertainty is partially endogenous and rises in recessions when the job finding rate typically is low. We show that these uncertainty effects may be of first-order importance in a macro setting when combined with frictions in goods and labor markets.\textsuperscript{4}

The remainder of this paper is structured as follows. Section 2 reviews the labor market impact of the Great Recession. In Section 3 we present the model. Section 4 examines the quantitative properties of the model and provides an analysis of the Great Recession. Section 5 provides some robustness analysis. Section 6 summarizes and concludes.

2 The Great Recession and the Labor Market

The financial crisis produced one of the longest and deepest recessions in U.S. history. According to the NBER, the contraction lasted 18 months (December 2007 - June 2009), the longest since the Great Depression. The Great Recession also triggered a major deterioration of labor market conditions.\textsuperscript{5} The unemployment rate rose from 4.7 percent in July 2007 to 10 percent by October 2009, and has subsequently remained stubbornly high, see Figure 1. The increase in unemployment witnessed during this episode is large but not out of line with previous U.S. recessions. In the early 1980’s recession, for example, the unemployment rate peaked at 10.8 percent and the OPEC I recession saw it rising to 9 percent. However, compared to other recessions, the rise in unemployment has been very stubborn and 7 years after the onset of the recession, the unemployment rate is still well above its pre-recession level.

The flows in and out of unemployment provide a useful way to gain some further insight into the determinants of the change in unemployment. We measure the average instantaneous job finding rate, $p_f^t$, and the average job separation rate, $p_l^t$ as:

\[
p_f^t = \frac{m_t}{u_t-1}
\]

\[
p_l^t = \frac{e_t}{n_{t-1}}
\]

where $u_t$ is the level of unemployment, $n_t$ the stock of employment, $m_t$ the flow of workers from unemployment to employment, and $e_t$ the number of (permanent) job separations. All data were

\textsuperscript{4}Leduc and Liu (2013) provide time-series evidence that changes in ‘uncertainty’ impact on aggregate demand and argue that labor market risks are important determinants of risk.

\textsuperscript{5}See Daly et al (2011), Elsby, Hobijn and Sahin (2010), Hall (2010), Katz (2010), and Rothstein (2011) for excellent discussions of the labor market during the Great Recession.
obtained from the Current Population Study (CPS) apart from $e_t$ which we got from the Bureau of Labor Statistics.

Figure 2 illustrates $p_t^f$ and $p_t^l$. The initial rise in unemployment was triggered by a rapid increase in the unemployment inflow rate in the period from early 2008 to late 2009. However, the persistence of the rise in the level of unemployment derives from a large and stubborn decline in the unemployment outflow rate which drops dramatically during 2009 and since recovers only marginally. Thus, the main issue that needs explaining is how the recession produced such a large and persistent drop in the job finding rate and this is one of the main targets of the model that we construct in Section 3.

A key aspect of the labor market impact of the Great Recession is the surge in the incidence of longer term unemployment. Figure 3 illustrates the time-series for the share of unemployed workers who have been out of work for 6 months or more. In the postwar sample prior to the Great Recession, the share of longer term unemployed displays moderately countercyclical movements with its highest level of 26 percent occurring in the early 1980’s recession. During the Great Recession instead, this indicator of longer term unemployment surged from 17.5 percent in August 2007 to 45.3 percent in April 2010 and it still hovers well above 30 percent, see also Rothstein (2011) and Wiczer (2013).

The data suggest that the rise in longer term unemployment is related to increased heterogeneity of job finding prospects amongst the unemployed. Perhaps the simplest way of seeing this is to compare the average duration of unemployment with the inverse of the instantaneous job finding rate. Suppose that the job finding rate is constant within a month and that all unemployed face the same probability of finding a job. In this case the law of motion for average duration of unemployment, $d_t$, can be formulated as:

$$d_t = \left(1 - p_t^f\right) (d_{t-1} + 1) \frac{u_{t-1}}{u_t} + p_t^l \frac{n_{t-1}}{u_t}$$ (1)

In the non-stochastic steady-state $d = 1/p^f$ where $p^f$ is the long-run value of the job finding rate.\(^6\) Thus, under the assumption of homogenous search efficiency, the average duration of unemployment in the data should be close to the inverse of the instantaneous job finding rate unless there are large shocks to the flows in and out of unemployment.

Figure 4 illustrates the time-series of average unemployment duration in the United States together with the inverse of the average instantaneous job finding rate and the estimate of average unemployment duration derived from (1). The inverse job finding rate tracks the BLS estimate of the average unemployment duration very closely until the Great Recession. From the end of 2007, however, the

\(^6\)To see this note that $d = (1 - p^f) / p^f + (p^l / p^f) (n/u)$ and that $n/u = p^l / p^f$. 

5
two graphs deviate very significantly and the BLS estimate of unemployment duration rises approximately twice as much as the inverse of the job finding rate. This result indicates either measurement error, and/or the impact of large shocks and/or that there is heterogeneity in the search efficiency of the unemployed. We can check for the importance of large shocks by drawing a comparison with the estimated unemployment duration derived from (1). This estimate is essentially identical to the inverse of the job finding rate. Thus, we conclude that the increase in duration observed in the U.S. during the Great Recession requires one to allow for heterogeneous job finding prospects and/or consider sources of measurement error. We will concentrate on the first of these.

Another much discussed feature of the Great Recession is its impact on the Beveridge curve. Figure 5 illustrates the relationship between vacancies and unemployment using CPS estimates of unemployment and JOLTS estimates of the number of vacancies. We discriminate between the pre-Great Recession period and the period thereafter (from 2007:12). During the early parts of the recession, unemployment approximately doubled while the number of vacancies fell by around 50 percent which jointly produced a striking movement down the Beveridge curve. In the course of the initial part of the recession, the labor market conditions worsened considerably but the dynamics of unemployment and vacancies appear consistent with the pre-crisis Beveridge curve. From late 2009, however, there is instead evidence that the Beveridge curve shifted outwards, indicating a less efficient matching between workers looking for employment and firms looking for new hires, see also Barlevy (2011).

We take away from this that the persistent decline in the job finding rate is key for understanding the large and persistent decline in unemployment during the Great Recession, that the increase in average unemployment duration is related to an increase in heterogeneity amongst the unemployed, and that there have been substantial movements along the Beveridge curve followed by an outward shift in this relationship. We will construct a model and ask whether it is consistent with these observations and whether the labor market weaknesses may have been a key factor behind the severity of the recession.

3 Model

We consider a heterogenous agents economy with frictions in financial, labor and goods markets. The economy consists of workers, firms owned by entrepreneurs, and a government which is in charge of monetary and fiscal policies.

Workers. There is a continuum of mass 1 of workers indexed by $i \in (0,1)$. Workers are risk averse, infinitely lived, and maximize the expected present value of their utility streams.
A worker is either working or unemployed. Employed workers (indexed by \( r_{i,t} = n \)) earn a real wage \( w_t \) but lose their current job with probability \( \rho_t = [0,1] \). Unemployed workers search for jobs and receive unemployment benefits \( \xi < w_t \). There are two types of unemployed workers which differ in their search efficiency and therefore in their job finding probabilities, \( \eta_{t,t} \). Unemployed workers with high search efficiency \( (r_{i,t} = s) \) face a shorter expected unemployment spell than unemployed workers with low search efficiency \( (r_{i,t} = l), 0 \leq \eta_{t,t} \leq \eta_{s,t} \leq 1 \). Upon job loss, a newly unemployed worker enters the high search efficiency pool with probability \( \varphi_{s,t} \in [0,1] \), and the low search efficiency pool with the complement probability \( \varphi_{l,t} = 1 - \varphi_{s,t} \). Moreover, during an unemployment spell type \( s \) unemployed workers risk making a transition to type \( l \), an event that occurs with probability \( \omega_t \in [0,1] \). The model therefore includes two sources of heterogeneity in job finding rates, ‘unobserved heterogeneity’ and negative duration dependence both of which imply that workers who have been unemployed for longer periods on average have lower job finding rates than newly unemployed workers.\(^7\)

The timing is as follows. At the beginning of the period, the aggregate labor market shocks are realized. After this, unemployed workers and firms with job vacancies match and new employment relationships are established. This is followed by production and consumption. At the end of the period, job separations are effectuated. Thus, employed workers face idiosyncratic uncertainty about the identity of job losers and about their search efficiency should they lose their jobs. By the same token, unemployed workers face idiosyncratic risk about the identity of current and future job finders and high search efficiency unemployed workers also risk making a transition to low search efficiency.\(^8\)

Workers cannot purchase unemployment insurance contracts. Consumption smoothing is accomplished through government provided unemployment benefits and through self-insurance by saving in a riskless nominal bond, \( b_{i,t} \). Workers maximize utility subject to the following borrowing constraint and sequence of budget constraints:

\[
\begin{align*}
\text{b}_{i,t}^h & \geq b^\text{min}, \quad t \geq 0 \\
\text{c}_{i,t} + b_{i,t}^h & = n_{i,t}w_t + (1 - n_{i,t})\xi + \frac{R_{t-1}}{1 + \pi_t}b_{i,t-1}^h, \quad t \geq 0
\end{align*}
\]

where \( b^\text{min} \) is a borrowing limit, \( c_{i,t} \) denotes a consumption basket, \( R_{t-1} \) is the gross nominal interest

\(^7\)See also Barnichon and Figura (2012), Hornstein (2012), Krusell and Smith (1999) and Krusell et al (2009), for models with multiple unemployment states.

\(^8\)Of course, households also face idiosyncratic uncertainty about the identity of future job losers.
rate, $\pi_t$ denotes the net inflation rate in period $t$. $n_{i,t}$ indicates the household’s employment state:

$$n_{i,t} = \begin{cases} 
1 & \text{if worker } i \text{ is employed in period } t \\
0 & \text{if worker } i \text{ is unemployed in period } t 
\end{cases}$$

Let $V (b_i^h, r_t, S)$ be the expected present discounted utility of a household given its bond holdings, its labor market status, $r_t$, and the aggregate state vector, $S$. The Bellman equation for an employed household is given as:

$$V \left( b_i^h, n, S \right) = \max_{c_i, b_i^{ht}} \left\{ U \left( c_i \right) + \beta \mathbb{E} \left( 1 - \sum_{g=s,l} \rho \varphi_g (1 - \eta_g') V \left( b_i^{ht}, n, S' \right) \right) \right. \\
\left. + \beta \mathbb{E} \sum_{g=s,l} \rho \varphi_g (1 - \eta_g') V \left( b_i^{ht}, g, S' \right) \right\}$$

subject to the borrowing constraint and to the budget constraint in equation (3) setting $n_i = 1$. $U$ is an increasing and strictly concave utility function. $\beta \in (0, 1)$ is the subjective discount factor, and $\mathbb{E}$ is the conditional expectations operator. The terms on the right hand side of (4) are (i) the instantaneous utility flow $U (c_i)$, (ii) the probability of being employed next period $(1 - \sum_{g=s,l} \rho \varphi_g (1 - \eta_g'))$ times the continuation value of being employed, and (iii) the probability of being state $g$ unemployed next period $(\rho \varphi_g (1 - \eta_g'))$ times the respective continuation values.

The Bellman equation for a type $s$ unemployed worker is:

$$V \left( b_i^h, s, S \right) = \max_{c_i, b_i^{ht}} \left\{ U \left( c_i \right) + \beta \mathbb{E} \left( 1 - \omega \right) \left[ \eta_s' V \left( b_i^{ht}, n, S' \right) + (1 - \eta_s') V \left( b_i^{ht}, s, S' \right) \right] \right. \\
\left. + \beta \mathbb{E} \omega \left[ \eta_l' V \left( b_i^{ht}, n, S' \right) + (1 - \eta_l') V \left( b_i^{ht}, l, S' \right) \right] \right\}$$

subject to (2) and (3) setting $n_i = 0$. A type $s$ unemployed worker remains type $s$ with probability $(1 - \omega)$ and in that case finds a job with probability $\eta_s'$, and makes a transition to type $l$ with probability $\omega$ and then finds a job with probability $\eta_l'$. This last transition incorporates the possibility of negative duration dependence.

Finally, the Bellman equation for a type $l$ unemployed workers is given as:

$$V \left( b_i^h, l, S \right) = \max_{c_i, b_i^{ht}} \left\{ U \left( c_i \right) + \beta \mathbb{E} \left[ \eta_l' V \left( b_i^{ht}, n, S' \right) + (1 - \eta_l') V \left( b_i^{ht}, l, S' \right) \right] \right\}$$

subject to (2) and (3) setting $n_i = 0$.

The two Bellman equations (5) and (6) differ because the two types have different job finding prospects and because only type $s$ unemployed workers face the risk of making a transition to a less efficient search state. Since $w > \xi$, $V \left( b_i^h, n, S \right) \geq V \left( b_i^h, s, S \right)$ for all $b_i^h$ and $S$ so that no employed
household has an incentive to voluntarily leave their current job. Under the condition that \( \eta_s' \geq \eta_t' \),
\[ V(b^h, s, S) \geq V(b^h, l, S) \] for all \( b^h \) and \( S \).

The consumption index \( c_i \) is a basket of consumption goods varieties:
\[ c_i = \left( \int_j \left( c^j_i \right)^{1-1/\gamma} d_j \right)^{1/(1-1/\gamma)} \] (7)
where \( c^j_i \) is household \( i \)'s consumption of goods of variety \( j \) and \( \gamma > 1 \) is the elasticity of substitution between consumption goods. Household \( i \)'s demand for variety \( j \) is given as:
\[ c^j_i = \left( \frac{P_j}{P} \right)^{-\gamma} c_i \] (8)
where \( P_j \) is the nominal price of variety \( j \) and \( P \) is a price index:
\[ P = \left( \int_j P_j^{1-\gamma} d_j \right)^{1/(1-\gamma)} \] (9)

Entrepreneurs. Consumption goods are produced by a continuum of measure \( \Psi < 1 \) of monopsonistically competitive firms indexed by \( j \in (0, \Psi) \) which are owned by risk neutral entrepreneurs. Entrepreneurs discount utility at the rate \( \beta \) and make decisions on the pricing of their goods, on vacancy postings, and on their consumption and savings policies. In return for managing (and owning) the firm, they are the sole claimants to its profits. We assume that entrepreneurs can save but face a no-borrowing constraint. This no-borrowing constraint implies that the entrepreneur finances hiring costs through retained earnings.\(^9\)

Entrepreneurs make all their decisions simultaneously but we can separate them into the hiring and pricing decisions made when acting as firm owners and the consumption and savings choices made when acting as households. Output is produced according to a linear technology:
\[ y_{j,t} = n_{j,t} \] (10)
where \( n_{j,t} \) denotes entrepreneur \( j \)'s input of labor purchased from the workers. Firms hire labor in a frictional labor market. The law of motion for employment in firm \( j \) is given as:
\[ n_{j,t} = (1 - \rho_{t-1}) n_{j,t-1} + h_{j,t} \] (11)
\[ h_{j,t} = \psi_t v_{j,t} \] (12)

\(^9\)In the stationary equilibrium, \( \beta < 1/((R/((1 + \pi))) \) so entrepreneurs will be borrowing constrained. This derives from the assumption that households are risk averse while entrepreneurs are assumed risk neutral.
where \( h_{jt} \) denotes hires made by firm \( j \) in period \( t \), \( v_{jt} \) is vacancies posted, and \( \psi_t \) is the job filling probability. Firms are assumed to be sufficiently large that \( \psi_t \) can be interpreted as the fraction of vacancies that leads to a hire.\(^{10}\) The cost of posting a vacancy is given by \( \mu > 0 \). Real marginal costs are therefore:

\[
mc_{jt} = w_t + \frac{\mu}{\psi_t} - \beta \mathbb{E}_t \left[ (1 - \rho_t) \frac{\mu}{\psi_{t+1}} \right]
\]  

(13)

Following Rotemberg (1982), firms face quadratic costs of price adjustment. Given risk neutrality, entrepreneurs set prices to maximize the present discounted value of profits:

\[
\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left( \left( \frac{P_{jt+s}}{P_{t+s}} - mc_{jt+s} \right) y_{jt+s} - \frac{\phi}{2} \left( \frac{P_{jt+s} - P_{jt+s-1}}{P_{jt+s-1}} \right)^2 y_t \right)
\]

subject to:

\[
y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{\gamma} y_t
\]

Equation (15) is the demand for goods variety \( j \). \( y_t \), can be interpreted as aggregate real income. In equation (14) \( \phi \geq 0 \) indicates the severity of nominal rigidities in price setting with \( \phi = 0 \) corresponding to flexible prices. The first-order condition for this problem is given as:

\[
\left( 1 - \gamma + \gamma mc_{jt} \frac{P_t}{P_{jt}} \right) y_{jt} = \phi \frac{P_t}{P_{jt-1}} \left( \frac{P_{jt} - P_{jt-1}}{P_{jt-1}} \right) y_t - \phi \beta \mathbb{E}_t \left[ \left( \frac{P_{jt+1}}{P_{jt}} \right) \left( \frac{P_{jt+1} - P_{jt}}{P_{jt}} \right) P_t y_{t+1} \right]
\]

(16)

The entrepreneurs’ consumption and savings decisions are the solutions to the following dynamic programming problem:

\[
W(b_j', n_j, S) = \max_{d_j, b_j', n_j} \{ d_j + \beta \mathbb{E} W(b_j'', n_j', S') \}
\]

subject to the employment transition equation and the budget and borrowing constraints:

\[
d_j + b_j'' + wn_j + \frac{h_j}{\psi} = \frac{P_j}{P} n_j - T^e + \frac{R_{-1}}{1 + \pi} b_j'
\]

(18)

\[
b_j'' \geq 0
\]

(19)

where \( d_j \) denotes entrepreneur \( j \)'s consumption and \( b_j'' \) their bond purchases. Condition (19) imposes the no-borrowing constraint on entrepreneurs. \( T^e \) denotes a lump-sum tax imposed on employers to cover the government provided unemployment benefits.

---

\(^{10}\) This assumption - which is equivalent to assuming that \( \Psi \) is sufficiently smaller than 1, can be relaxed which would produce ex-post heterogeneity across firms.
Labor Market. We assume that \( w_t \geq \bar{w} \) as long as \( \bar{w} \) is consistent with the joint match surplus being non-negative and with workers preferring to work rather than being unemployed.\(^{11}\) The assumption of downward wage rigidity squares well with the U.S. experience during the Great Recession which witnessed wages failing to fall below their pre-recession level. We will later investigate the importance of this assumption.

The matching technology is given as:

\[
m_t = \varrho (u_{a,t})^\alpha (v_t)^{1-\alpha} \tag{20}
\]

where \( m_t \) denotes the measure of matches at date \( t \), and \( v_t \) is the aggregate measure of vacancies posted by the firms. \( \varrho > 0 \), and \( \alpha \in (0, 1) \) are constant parameters. \( u_{a,t} \) is a measure of the number of effective (‘active’) searchers at the beginning of the period:

\[
u_{a,t} = \left( (1 - \omega_{t-1}) u_{s,t-1} + \rho_{t-1} \phi_{s,t-1} n_{t-1} \right) + q \left( u_{l,t-1} + (\omega_{t-1} u_{s,t-1} + \rho_{t-1} \phi_{l,t-1} n_{t-1}) \right) \tag{21}
\]

where \( u_{r,t} \) is the measure of type \( r \) unemployed workers at date \( t \). \( q \in (0, 1] \) is the probability that a type \( l \) unemployed worker is searching for a job at date \( t \) and is an indicator of relative search efficiency. When \( q < 1 \), type \( l \) unemployed workers face longer expected unemployment spells than type \( s \) unemployed workers.

The job filling probability and the job finding probabilities are given as:

\[
\psi_t = \varrho \theta_t^{-\alpha} \tag{22}
\]
\[
\eta_{s,t} = \varrho \theta_t^{1-\alpha} \tag{23}
\]
\[
\eta_{l,t} = q \eta_{s,t} \tag{24}
\]

where \( \theta_t = v_t / u_{a,t} \) denotes labor market tightness. The laws of motion of the stocks of employed and unemployed workers are given as:

\[
n_t = (1 - \rho_{t-1}) n_{t-1} + m_t \tag{25}
\]
\[
u_{s,t} = (1 - \eta_{s,t}) \left( (1 - \omega_{t-1}) u_{s,t-1} + \rho_{t-1} \phi_{s,t-1} n_{t-1} \right) \tag{26}
\]
\[
u_{l,t} = (1 - \eta_{l,t}) \left( u_{l,t-1} + (\omega_{t-1} u_{s,t-1} + \rho_{t-1} \phi_{l,t-1} n_{t-1}) \right) \tag{27}
\]

Government. The government is in charge of monetary and fiscal policies. We assume that the government balances the budget period-by-period:

\[
u_t \xi = \Psi T_t^e \tag{28}
\]

\(^{11}\)We have checked that the match surplus is positive for all matches in all the results that we report.
where $u_t = u_{s,t} + u_{l,t}$.

Monetary policy is specified by a rule for the short-term nominal interest rate:

$$R_t = \bar{R} \left( \frac{1 + \pi_t}{1 + \bar{\pi}} \right)^{\delta}$$

(29)

where $\bar{R}$ is the long-run nominal interest rate target, $\bar{\pi}$ is the inflation target, and $\delta$ denotes the (semi-)elasticity of the nominal interest rate to deviations of inflation from its target.

**Stochastic Shocks.** We allow for shocks to the job separation rate\(^{12}\), $\rho_t$, to $\varphi_{l,t}$, the stochastic variable which determines the share of job losers who flow into the two types of unemployment, and to $\omega_t$ which determines the probability of search efficiency loss during an unemployment spell. We assume that:

$$\rho_t = \bar{\rho} + z_{\rho,t}$$

(30)

$$\varphi_{l,t} = \bar{\varphi}_l + z_{\varphi,t}$$

(31)

$$\omega_t = \bar{\omega} + z_{\omega,t}$$

(32)

$$z_{\rho,t} = \lambda_{\rho} z_{\rho,t-1} + \varepsilon_{\rho,t}$$

(33)

$$z_{\varphi,t} = \lambda_{\varphi} z_{\varphi,t-1} + \varepsilon_{\varphi,t}$$

(34)

$$z_{\omega,t} = \lambda_{\omega} z_{\omega,t-1} + \varepsilon_{\omega,t}$$

(35)

where $\bar{\rho}, \bar{\varphi}_l, \bar{\omega} \in (0,1)$ determine the steady-state values of the stochastic variables and $\lambda_{\rho}, \lambda_{\varphi}, \lambda_{\omega} \in (-1,1)$ their persistence. We assume that $\varepsilon_t \sim \mathcal{N}(0, \Sigma)$ where $\varepsilon_t = (\varepsilon_{\rho,t}, \varepsilon_{\varphi,t}, \varepsilon_{\omega,t})'$. $\varepsilon_{\rho,t}$ is assumed orthogonal to $\varepsilon_{\varphi,t}$ and $\varepsilon_{\omega,t}$ while these two latter shocks may be correlated.

We take no firm stand on the sources of the shocks to search efficiency, $\varepsilon_{\varphi,t}$ and $\varepsilon_{\omega,t}$. Hall (2014) argues that the composition of job losers during the Great Recession shifted towards those with on average smaller job finding rates which would be consistent with a positive innovation to $\varepsilon_{\varphi,t}$. Sahin et al (2014) document an increase in “mis-match” between vacancies and job seekers in the dimensions of occupations and industries during the Great Recession which would be another source of a positive innovation to $\varepsilon_{\varphi,t}$. Kroft et al (2014) instead document an increase in negative duration dependence after 2008 as would be consistent with innovations to either $\varepsilon_{\varphi,t}$ or $\varepsilon_{\omega,t}$. Sterk (2015) suggests that

\(^{12}\)The assumption that the job separation rate is exogenous is made only for simplicity and it would be feasible to allow for endogenous separations by e.g. introducing match specific shocks, see Mortensen and Pissarides (1994). However, we find it useful to assume exogenous job separations as this set-up can be thought of as capturing the impact of a variety of different shocks (such as productivity shocks, financial shocks, idiosyncratic demand shocks) through a single shock allowing us to focus on the amplification mechanism which is our main interest in this paper.
falling house prices may have limited labor mobility during the Great Recession adding another source of increase in $\varphi_{t,t}$ and $\omega_t$. In short, there are multiple reasons for why heterogeneity in job finding prospects may have gone up during the Great Recession. Our approach is to capture these through $\varphi_{t,t}$ and $\omega_t$ and analyze the extent to which their impact under some circumstances may be amplified.

**Equilibrium.** We focus upon a recursive equilibrium in which workers act competitively taking all prices for given while firms act as monopolistic competitors setting the price of their own variety taking all other prices for given. In equilibrium, firms will be symmetric because of the absence of idiosyncratic productivity shocks, state contingent pricing and because we assume that they are large enough that job separation and vacancy filling probabilities can be treated like fractions. We denote relative prices by $p_{j,t} = P_{j,t}/P_t$ and note that symmetry implies that the equilibrium relative price equals 1 for all goods. In the symmetric equilibrium, the optimal price setting condition therefore simplifies to:

$$1 - \gamma + \gamma mc_t = \phi \pi_t (1 + \pi_t) - \phi \beta E_t \left[ \pi_{t+1} (1 + \pi_{t+1}) \frac{y_{t+1}}{y_t} \right]$$

(36)

As is well-known, models with incomplete markets and aggregate shocks are cumbersome to solve numerically. In this paper we follow Krusell, Mukoyama and Smith (2011) and impose that the borrowing constraint $b^\text{min} = 0$. Under this assumption there is no aggregate savings vehicle available to workers and, in equilibrium, since unemployed workers cannot issue debt, all workers consume their income every period. Nonetheless, since employed workers face the risk of losing their job, they have an incentive to save and will therefore be on their Euler equations. For the same reason, although workers cannot save in equilibrium, the model still features a precautionary savings motive and this will impact on equilibrium quantities through the real interest rate which will have to adjust to induce employed workers not to save. The great advantage of imposing this borrowing constraint is that the aggregate wealth distribution is degenerate which simplifies the computational aspects of the model very considerably. Moreover, Ravn and Sterk (2012) find that allowing for individual savings - and therefore for a non-degenerate wealth distribution - has only limited impact on aggregate dynamics. The aggregate state vector under the $b^\text{min} = 0$ borrowing constraint is given as $S_t = (u_{i,t}, u_{s,t}, \rho_t, \varphi_{s,t}, \omega_t)$ which does not involve the wealth distribution.

We let $c(b^h, r, S)$ and $b^{ht'}(b^h, r, S)$ denote the decision rules that solve the workers’ problems (depending on their labor market status), and $h(b^e, n, S)$, $d(b^e, n, S)$ and $b^{et'}(b^e, n, S)$ the solutions to the entrepreneurs’ problem. We can now define the equilibrium formally:

---

13 The idea here is that geographical mobility (and therefore job finding rates) may have declined for households which experienced low or negative equity due to falling house prices.
Definition 1 A recursive monopolistic competition equilibrium is defined as a state vector S, pricing kernels \((w(S), \pi(S))\), decision rules \((c(b^h, r, S), b^{ht} (b^h, r, S))_{r=n,s,t}, (d(b^e, n, S), b^{et} (b^e, n, S), h(b^e, n, S))\) and \(p_j (b^e, n, S)\), value functions \((V^n (b^h, S), V^{u,s} (b^h, S), V^{u,l} (b^h, S))\) and \(W (b^e, n, S), \) and government policies \((T^e (S), R (S))\) such that

(i) given the pricing kernel, the government policies, and the aggregate and individual states, the household decision rules solve the workers problems;

(ii) given the pricing kernel, government policies, and the aggregate state, the entrepreneur decision rules solve the entrepreneurs’ problem and \(p_j (b^e, n, S) = 1\) for all \(j\) and all \((b^e, n, S)\);

(iii) asset, goods and labor markets clear:

\[
0 = \int b^{ht} (b^h, r, i, S) di + \Psi \int b^{et} (b^e, n_j, S) dj = 0
\]

\[
y - \mu v - \frac{\phi}{2} \pi^2 y = \int c^h (b^h, r, i, S) di + \Psi \int d_j (b^e, n_j, S) dj
\]

\[
y = \int n_i b^h di
\]

\[
\int h_j (b^e, n_j, S) dj = \varrho (u_{a,t})^\alpha (v_t)^{1-\alpha}
\]

\[
n_t = (1 - \rho_{t-1}) n_{t-1} + \varrho (u_{a,t})^\alpha (v_t)^{1-\alpha}
\]

\[
u_{a,t} = ((1 - \omega_{t-1}) u_{s,t-1} + \rho_{t-1} \varphi_{s,t-1} n_{t-1}) + q (u_{t-1} + (\omega_{t-1}s_{t-1} + \rho_{t-1} \varphi_{s,t-1} n_{t-1})),
\]

\[
u_{s,t} = (1 - \eta_{s,t}) ((1 - \omega_{t-1}) u_{s,t-1} + \rho_{t-1} \varphi_{s,t-1} n_{t-1})
\]

\[
u_{l,t} = (1 - \eta_{l,t}) (u_{l,t-1} + (\omega_{t-1} s_{t-1} + \rho_{t-1} \varphi_{l,t-1} n_{t-1}))
\]

(iv) the government budget constraint is satisfied and the nominal interest is given by the policy rule in equation (29).

4 Quantitative Results

4.1 Calibration

We solve the model numerically using a standard perturbation approach (see the Appendix for details). The calibration targets and parameter values are summarized in Tables 1 and 2.

One model period corresponds to a calendar month. The household utility function is assumed to be given as:

\[
U (c_{i,t}) = \frac{c_{i,t}^{1-\sigma} - 1}{1 - \sigma}, \sigma \geq 0
\]
σ is the degree of relative risk aversion which matters for the household savings response to uncertainty. We set \( \sigma = 1.5 \) which is in the mid-range of empirical estimates of Attanasio and Weber (1995), Eichenbaum, Hansen, and Singleton (1988), and many others who have examined either household data or aggregate time series. We assume an annual real interest rate of 5 percent in the steady state and set the subjective discount factor equal to 0.992 for both workers and entrepreneurs. This value is low relative to standard representative agent models but because of idiosyncratic risk and incomplete markets, agents have a strong incentive to engage in precautionary savings and a low real interest rate is required to induce zero savings in equilibrium.

When calibrating the parameters pertaining to the labor market we must confront the issue that the two types of unemployment are not directly observable. We address this issue by calibrating the relevant parameters on the basis of information on labor market statistics that are informative about the moments of interest. We target a steady-state unemployment rate of 5 percent. The parameters \((q, \varphi, \omega)\) and the steady-state job finding probability for type \(s\) unemployed workers, \(\eta_s\), are calibrated by targeting the following moments: First, according to CPS data for the post 1970 sample, on average 15 percent of job losers experience unemployment spells of 6 months or more. Secondly, the monthly hazard rate for the newly unemployed is 43 percent, see Rothstein (2011). Third, we introduce information on duration dependence from Kroft et al.’s (2013) estimate of the relationship between job finding probabilities and the length of an unemployment spell. These authors estimate the job finding probability of individuals out of work for \(d\) months at date \(t\) using a polynomial specification for the hazard. In particular, they assume that the job finding rate depends on the length of the unemployment spell and on labor market tightness as:

\[
\eta_t (\theta_t, d) = A (d) m_0 \theta_t^{1-\nu}
\]

\[
A (d) = (1 - a_1 - a_2) + a_1 e^{-b_1 d} + a_2 e^{-b_2 d}
\]

Using panel data from the CPS for the 2002-2007 sample (and controlling for demographic variables), Kroft et al (2013) estimate \(\hat{a}_1 = 0.314, \hat{a}_2 = 0.393, \hat{b}_1 = 1.085\) and \(\hat{b}_2 = 0.055\). We target the implied values of the relative hazard, \(A (d) / A (0)\), for integer values of \(d\) going up to 15 months.

We find that \(q = 0.468, \varphi = 0.229, \omega = 0.219\) and \(\eta_s = 0.586\). Thus, 77 percent of job losers are estimated to flow into the high search efficiency state upon job loss and thereafter face a moderate risk of 22 percent per month of loss of search efficiency during unemployment. Unemployed workers with

\[14\]Although type \(s\) unemployed workers have shorter expected unemployment spells than type \(l\) unemployed workers, some of the type \(s\) unemployed will be out of work for extended periods due to bad luck.
high search efficiency are more than twice as likely to find a job as low search efficiency unemployed workers implying that the unemployment state has a substantial impact on the expected unemployment duration. In the steady-state, these parameter values imply that the average duration of the stock of type $s$ unemployed workers is 1.48 months, that the corresponding number for type $l$ unemployed workers is 4.10 months while the share of unemployed workers out of work for 6 months or more is 15.9 percent. The calibration matches closely the hazard function estimated by Kroft et al’s (2013).\(^{15}\) Finally, to match the 5 percent unemployment rate, we set the steady-state job separation rate ($\bar{p}$) equal to 3.9 percent per month.

The benefit level, $\xi$, is calibrated by targeting a decline in consumption of 11.7 percent upon unemployment, a value which corresponds to Hurd and Rohwedder’s (2010) estimate of the average household spending impact of a job loss.\(^{16}\) We assume that the matching function elasticity to unemployment, $\alpha$, is equal to 65 percent and we normalize $\varrho = 1$. $\mu$, the vacancy cost parameter, is calibrated by targeting an average hiring cost of 4.5 percent of the quarterly wage bill per worker. Given other parameters, this implies that $\mu = 0.19$.

We set the average mark-up equal to 20 percent which implies that $\gamma$, the elasticity of substitution between goods, is equal to 6. $\phi$, the parameter that determines the importance of price adjustment costs, is calibrated to match a price adjustment frequency of 5 months. This value is conservative but consistent with the estimates of Bils and Klenow (2004).\(^{17}\) This implies that $\phi = 96.9$. We assume that the government’s inflation target $\pi = 0$ so that it pursues price stability and we set $\delta = 1.5$, a conventional value in the new Keynesian literature.

Finally, we estimate the parameters of the stochastic processes for $\rho_t$, $\varphi_{l,t}$ and $\omega_t$. The persistence of $\rho_t$ and the variance of its innovation are estimated using JOLTS data on layoffs and BLS estimates of the employment rate for a sample ranging from 2003 to 2014.\(^{18}\) We find $\hat{\lambda}_\rho = 0.91$ and $\hat{\sigma}_\rho/\bar{p} = 0.0067$. To estimate the persistence and volatility of $\varphi_{l,t}$ and $\omega_t$ we must again confront the issue that search efficiency is unobserved. However, we can use the model to ‘back out’ processes for these two stochastic

\(^{15}\)Figure A.1 in the Appendix illustrates the hazard function implied by our model evaluated in the steady-state together with Kroft et al’s (2013) estimates.


\(^{17}\)To be precise, we calibrate $\phi$ by exploiting the equivalence between the log-linearized Phillips curve implied by our model and the Phillips curve implied by the Calvo model.

\(^{18}\)Our analysis focuses on the Great Recession and its aftermath and we choose the sample period accordingly.
shocks given the above estimate of the relative search efficiency, \(q\), the matching function parameters, \(\varphi\) and \(\alpha\), and data on the unemployment outflow rate and labor market tightness. It follows from the matching function that:

\[
\nu_{a,t} = \nu_{t-1} \left( \frac{\eta_t}{\varphi} \right)^{1/\alpha} \left( \frac{v_t}{u_{t-1}} \right)^{1-1/\alpha}
\]

(37)

where \(\bar{\eta}_t\) is the average job finding rate amongst the stock of unemployed. We further assume proportionality between \(\varphi_{t,t}\) and \(\omega_t\) which implies that the disturbances to these two flows are perfectly correlated and proportional, i.e. \(\frac{\varphi_t}{\omega_t} = z_{\varphi, t}\). Under this assumption, we can use the estimates of \(\nu_{a,t}\) together with the transition equations (26) – (27) to derive estimates of \(\varphi_{s,t}\) and \(\omega_t\), see Appendix 2 for details, from which we can estimate the persistence and variance of the two shocks. Using data from JOLTS and the CPS (for the 2003-2014 sample) we find \(\hat{\beta} = 0.99\) and \(\hat{\sigma}_{\varphi}/\hat{\sigma}_{\omega} = \hat{\nu}_{\omega}/\hat{\omega} = 0.072\).\(^{19}\)

4.2 Results

The Impact of Labor Market Shocks. We start by examining the impact of job separation shocks and changes in the composition of unemployed workers. We compare the results of the benchmark economy with two alternative economies. In the first of these we assume that prices are flexible (\(\phi = 0\)) but retain the incomplete markets assumption. Comparing this economy with the benchmark model is informative about the extent to which nominal rigidities matter for the transmission mechanism. In the second alternative economy we assume that individuals can insure against idiosyncratic shocks within large families (but retain the presence of nominal rigidities). In this alternative economy there is risk sharing within the family so consumption is equalized across household members independently of their labor market status which allows us to understand the extent to which idiosyncratic risk is important for our results. In the risk sharing model the family maximizes utility subject to the single budget constraint:

\[
c_t + b_t^h = n_t w_t + (1 - n_t) \xi + \frac{R_{t-1}}{1 + \pi_t} b_{t-1}^h, \quad t \geq 0
\]

where \(n_t\) is the fraction of employed household members in period \(t\).

Figure 6 illustrates the responses of key aggregate variables to a one standard deviation increase in the probability of job loss. Variations in the job termination rate have only moderate effects on

\(^{19}\)Since the frequency of the data is monthly, measured job openings and layoffs are rather noisy. To avoid very erratic shocks, we pre-smooth the data using a 6 month moving average filter. The parameters of the shock processes are obtained by regressing the shock variables on their values lagged with one year. We then compute the monthly persistence parameters that are implied by the regressions.
equilibrium unemployment in standard matching models because rising unemployment implies declining job filling costs which triggers higher vacancy postings. In the benchmark model we instead find that an increase in job separations produces a large increase in unemployment. Relative to the increase in the job separation rate, unemployment rises a lot in the benchmark economy and in a very persistent manner which also produces a surge in the share of longer term unemployed workers.\textsuperscript{20} Thus, not only do workers see the job loss risk rising, but they also experience a worsening of job finding prospects during unemployment.

Figure 7 repeats the analysis for a joint one standard deviations increase in $\varphi_{s,t}$ and in $\omega_t$. This combination of shocks generates a decrease in average search efficiency because a larger proportion of job losers flow into type $l$ unemployment and because a larger share of the existing high search efficiency unemployed workers suffer a loss of search efficiency. We find that these shocks also produce a persistent increase in both the level of unemployment and in the share of longer term unemployed workers. Similarly to the job separation shock, the decline in search efficiency leads to a persistent decline in vacancy postings and in the job finding rate.

In order to understand better the results it is instructive to consider the Euler equation for employed workers and the first order condition for price setting:

$$\frac{\partial U(c^n)}{\partial c^n} = \beta \mathbb{E} \frac{R}{1 + \pi}(1 - \rho [\varphi_s (1 - \eta'_s) + (1 - \varphi_s) (1 - \eta'_l)]) \frac{\partial U(c^{uu})}{\partial c^{uu}}$$

$$+ \rho \varphi_s (1 - \eta'_s) \frac{\partial U(c^{u,ss})}{\partial c^{u,ss}} + \rho (1 - \varphi_s) (1 - \eta'_l) \frac{\partial U(c^{u,l})}{\partial c^{u,l}}$$

(38)

$$1 - \gamma + \gamma \left( w + \frac{\mu}{\omega} - \beta \mathbb{E} (1 - \rho_x) \frac{\mu}{\psi} \right) = \phi (1 + \pi) \pi - \beta \phi \mathbb{E} (1 + \pi') \pi' \frac{\psi'}{\psi}$$

(39)

where $c^n$ denotes the consumption level of an employed worker, $c^{u,s}$ is consumption of a high search efficiency unemployed worker and $c^{u,l}$ is the consumption level of an unemployed worker with low search efficiency.

Employed workers are on their Euler equation because they have an incentive to save. Increased risk of job loss (higher $\rho$) stimulate higher desired savings because it implies lower expected income and because of increased idiosyncratic employment risk. Declining search efficiency (lower $\varphi_s$) and worsening job finding prospects during unemployment (lower $\eta'_s$ or $\eta'_l$) imply longer expected unemployment spells and therefore also trigger a decrease in expected household income and an increase in idiosyncratic income risk both of which stimulate savings. Thus, when labor market conditions worsen, employed workers’ demand for consumption goods falls at the current real interest rate.

\textsuperscript{20} The increase in job separations produces an initial short-lived drop in the fraction of long-term unemployed workers because of the inflow of newly unemployed workers.
Equation (39) is the optimal price setting condition in the symmetric equilibrium and it relates inflation to real marginal costs and to growth in aggregate output. Due to nominal rigidities (as indexed by $\phi$), firms find it optimal to phase in changes in the optimal price level gradually over time. Adverse labor market conditions impact negatively on aggregate goods demand for the reasons just discussed. In response to the drop in goods demand, it therefore follows that inflation must fall which needs to be accompanied by lower marginal costs. Since the real wage is assumed downward inflexible, a fall in marginal costs has to come through a decline in the cost of hiring which requires the job filling rate, $\psi$, to increase. Thus, vacancy postings have to fall which explains the persistent drop in the job finding rate. Finally, in response to the decline in inflation, the central bank cuts nominal interest rates sufficiently strongly to generate a drop in the real interest rate.

Thus, the model produces a simple and intuitive amplification mechanism in which labor market shocks trigger declining goods demand which are transmitted to the supply side and produce a fall in labor demand. It is this transmission mechanism from the demand side to the supply side that produces amplification because fewer vacancies further depress labor market conditions and therefore stimulate a further fall in goods demand.\footnote{The no-borrowing constraint that we have imposed implies that the real interest rate does the full adjustment but it should be understood that the transmission mechanism is one in which demand contracts in response to worsening labor market conditions.}

The transmission mechanism depends crucially on the combination of nominal rigidities in price setting and on the lack of insurance against unemployment. To see this, Figures 5 and 6 also report the impact of the labor market shocks for the two alternative economies described above. When prices are flexible, job separation shocks have little impact on unemployment and lead only to a minor increase in incidence of longer term unemployment. In this economy, price flexibility neutralizes the need for a fall in marginal costs and firms take advantage of low hiring costs (due to the increase in unemployment) to post more vacancies. The job finding rate therefore falls only marginally which stops the amplification mechanism that operates in the benchmark economy. When the fraction of low search efficiency unemployed workers increases, it becomes more costly to fill vacancies because the job filling rate declines. For that reason, this shock leads to a decline in vacancy postings which depresses the job finding rate and spurs precautionary savings. However, price flexibility once again eliminates the need for a large decrease in vacancies and there is therefore no amplification of the shock. Thus, even if workers are exposed to idiosyncratic risk, price flexibility implies limited impact of deteriorating labor market conditions.
When workers can insure against idiosyncratic employment shocks through risk sharing within large families changing labor market conditions no longer impact on idiosyncratic risk and savings are therefore determined only by intertemporal considerations. An increase in the job separation rate impacts little on the household because its effect on expected family income is minor. For that reason, aggregate demand is rather unresponsive to changes in the job separation rate in the economy with insurance. The intertemporal savings motive is also small in the case of shocks to the share of low efficiency searchers and the presence of insurance within the family eliminates the precautionary savings effect. Thus, there is therefore little amplification of labor market shocks when households can insure against idiosyncratic risk.

In summary, the amplification mechanism that arises in the benchmark economy derives from the combination of nominal rigidities and the lack of insurance against idiosyncratic risk. Nominal rigidities produces a channel through which variations in goods demand impact on job creation while the lack of job insurance implies that changes in job prospects impact on goods demand.

**The Great Recession.** We now examine the extent to which the mechanisms of the model may be important for understanding key features of the Great Recession. We derive estimates of the sequences of the shocks, \((\varepsilon_{\rho,t}, \varepsilon_{\varphi,t}, \varepsilon_{\omega,t})_{t=2007:1}^{2014:8}\) and we feed them into the model to produce counterfactual experiments. \(\varepsilon_{\rho,t}\) is estimated by matching the observed U.S. time-series on the employment-to-unemployment transition rate while \(\varepsilon_{\varphi,t}\) and \(\varepsilon_{\omega,t}\) are estimated using the same “model free” approach as above on the basis of BLS and JOLTS data by matching the observed matching function residual. In order to avoid having too erratic shocks, we smooth both data series with a 6 months moving average filter. We then feed the resulting shock processes into the model economy and simulate it in response to this particular sequence of shocks.

The upper panels of Figure 8 illustrate the shocks that we estimate for the Great Recession. As discussed in Section 2, the Great Recession witnessed a spur of job separations which started in early 2008, peaked in early 2009, and lasted only until the end of that year. The shock to search efficiency is estimated to be much more persistent. We find a steady rise in the share of new job losers that flow into low search efficiency and in the share of high search efficiency workers that experience a drop in search efficiency. The drop in the average search efficiency starts in 2008 and continues throughout 2009/10 peaking in early 2011 and thereafter slowly diminishes. It is useful to compare this shock to search heterogeneity with other measures. For that purpose we also illustrate \(\varepsilon_{m,t}:\)

\[
\varepsilon_{m,t} = \log \left[ \frac{1}{\varrho} \left( \frac{m_t}{u_{t-1}} \right)^{1/\alpha} \left( \frac{v_t}{u_{t-1}} \right)^{1-1/\alpha} \right]
\]
\( \varepsilon_{m,t} \) is the matching function residual estimated by an econometrician who (wrongly) assumes homogeneous search efficiency amongst the unemployed. The implied matching function residuals are very similar to the estimates of Barlevy (2011) and correspond to a 40-45 percent adverse shock to the matching function over the 2007-2011 period and a 20 percent recovery thereafter. We also illustrate the fraction of newly unemployed workers who report to be “permanent” job losers. As argued by Hall and Schulhofer-Wohl (2013), permanent job losers have lower job finding rates than other types of job losers and variations in this fraction therefore reflect changes in average search efficiency. This fraction increases from 23 percent prior to the recession in 2007 to 45 percent by early 2010. Thereafter it gradually declines towards its pre-recession level. It therefore mirrors quite precisely the matching function residual implied by the shock to the heterogeneity in search efficiency that we estimate.

Figure 9 illustrates the impact of the shocks on the level of unemployment, the number unemployed workers out of work for 6 months or more (relative to the labor force), and on vacancies. In Section 2 we argued that the large drop in the job finding rate and its very slow recovery thereafter are key for understanding the severity of the Great Recession. A key focus of our analysis is therefore whether the model can account for this response of the job finding rate to the two labor market shocks just discussed. The answer to this is affirmative: Not only does the model reproduce the timing and the size of the fall in the job finding rate, but it is also consistent with the very persistent nature of the declining job finding prospects. Moreover, we find that the model accounts very closely for the adjustments in unemployment and vacancies. As in the U.S. data, the level of unemployment surges in 2008 while the number of vacancies implodes. Thereafter the increase in unemployment dissipates only very slowly over time while vacancies recover slightly faster. The results demonstrate that the amplification mechanism discussed in the previous sections is quantitatively important.

Figure 9 also reports the share of longer term unemployed (the number of unemployed workers out of employment for 6 months or more as a share of the total number of unemployed). This share soared during the recession increasing from 15 percent prior to the recession to above 40 percent during 2010-2012. The benchmark economy is consistent with the rise in the incidence of longer term unemployment in the early part of the recession and with the very stubborn nature of the rise in this labor market indicator. The model, however, is not fully able to account for the size of the rise in the incidence of longer term unemployment since the peak in the share of longer term unemployed workers in the model economy is just below 30 percent which is smaller and occurs earlier than in the U.S. data. Nevertheless, the model does generate a significant shift in the composition of the unemployed towards unemployment states with longer duration.
Finally, the bottom left panel of Figure 9 displays the conditional standard deviation of income one month ahead for currently employed workers, scaled by the current level of income.\textsuperscript{22} This is a measure of the income uncertainty in the model which partly is endogenous as it depends on the job finding rate. We find that income uncertainty surges during 2008 and remains at an elevated level until 2013, after which it decreases somewhat. Recall that there is little amplification of the shocks in the flexible price model. Therefore, by comparing with the corresponding measure in the economy without nominal rigidities we can evaluate the endogenous component of this uncertainty measure. We find that income uncertainty rises significantly less in the flexible price economy than in the benchmark model especially in the early part of the recession (the rise in income uncertainty by early 2009 is almost twice as large in the benchmark economy as in the flexible price version of the model). It follows that the model produces an important endogenous and countercyclical source of income uncertainty.

Combining the implications for the impact of the labor market shocks on unemployment and on job vacancies produces the Beveridge curve illustrated in Figure 10. In the U.S. economy, there was a rapid slide down the Beveridge curve in the early part of the recession followed by an outward shift of this relationship and a gradual recovery in both indicators. Such counter-clockwise Beveridge curve movements are not unusual during recessions but the current episode is more dramatic than what is observed during most other recessions. We find that the model accounts very accurately for both the movement down the Beveridge curve that occurred in 2008-2009 and the subsequent outward shift of the Beveridge curve.

One might wonder about the extent to which job separation shocks and shocks to the search efficiency matter individually for these results. To understand this, Figures 8 and 9 also display the paths of the relevant aggregates when we assume that the U.S. economy was hit only by job separation shocks. In the absence of shocks to the flows of workers to the two search efficiency states, the model can account for the initial rise in unemployment in late 2008 and for the initial drop in vacancies. However, the shocks to search efficiency are key for understanding both the size and the persistence of the rise in unemployment and for the very long and deep decline in job vacancies. In addition when we exclude the shocks to $\varphi_t$ and to $\omega_t$, the model produces only a very minor increase in the share of longer term unemployed workers and little change in income uncertainty post 2010. This demonstrates that an \textit{increase} in heterogeneity amongst the unemployed is important for accounting for both the severity of the Great Recession and for the surge in the incidence of longer term unemployment.

\textsuperscript{22}To compute the conditional standard deviations we use a Gauss-Hermite approximation with 36 nodes. We do not plot the uncertainty measure for the full insurance version of the model, as it is close to zero throughout the sample.
We can go further in understanding the mechanisms by examining the results when eliminating frictions in goods or financial markets. When we assume that prices are flexible, the labor market shocks leave vacancies almost unaffected. For that reason, the worsening labor market conditions have a much smaller impact on unemployment than in the data and lead to a very minor rise in the incidence of longer term unemployment. Perhaps most strikingly, the flexible price model implies an extremely counterfactual horizontal Beveridge curve. Interestingly, the model with insurance against idiosyncratic shocks generates very similar results to the flexible price model. The labor market shocks have minor impact on aggregate goods demand in this economy and for that reason firms have little incentive to post fewer vacancies in the wake of the worsening labor market conditions. The end result is a limited increase in unemployment, a minor increase in the incidence of longer term unemployment, and a very counterfactual horizontal Beveridge curve.

The key lesson we take away from this analysis is that the amplification mechanism derives from the combination of frictions in goods, labor and financial markets. Focusing on each of these frictions in isolation may not be informative about the joint effects and this potentially explains why past research has concluded against the importance of labor market shocks for macroeconomic fluctuations. It is important to stress that the shocks to search heterogeneity are not directly responsible for longer term unemployment incidence. When we assume either risk sharing within large households or fully flexible prices, these shocks are effectively neutralized because they either have little impact on goods demand (when assuming risk sharing) or are not transmitted to vacancy postings (when assuming flexible prices).

5 Extensions and Robustness Analysis

In this section we investigate four further issues. We first analyze further the sources of heterogeneity in search efficiency amongst the unemployed. Next, we examine the importance of the downward inflexible wage assumption and we look at how the monetary policy response matters for the amplification mechanism. Finally, we compare the results for the Great Recession with those for the early 1990’s recession.

Search Efficiency Heterogeneity: Amplification vs. Propagation. An important feature of our model is the heterogeneity in search efficiency amongst the unemployed. It is this aspect of the model that allows us to address the substantial increase in the incidence of longer term unemployment which has occurred during the Great Recession. Above we also argued that this feature is important for accounting for the magnitude of the increase in unemployment and for its persistence.
Our analysis allows for heterogeneity in search activity to materialize either upon job loss or during an unemployment spell (due to negative duration dependence). These two sources of heterogeneity in search efficiency play distinct roles. Heterogeneity in job search efficiency upon job loss impacts on employed workers’ consumption and savings decisions directly, cf. the Euler equation (38). When employed workers perceive higher risk of flowing directly into the low search efficiency state should they lose their job, they increase their desired precautionary savings. Increased risk of loss of search efficiency during an unemployment spell instead impacts on the average job finding rate but does not directly influence employed workers’ savings choices and is therefore not directly important for the amplification mechanism. Negative duration dependence instead may act as a propagation mechanism through which temporary increases in the number of job losers has persistent effects because the pool of newly unemployed workers risk experiencing a loss in their search efficiency during an unemployment spell.

We now wish to understand the extent to which the two flows separately have been important for the labor market dynamics during the Great Recession. Figure 11 repeats the Great Recession experiment from the previous section under two alternative scenarios. In the first of these we assume that the probability of search efficiency loss during an unemployment spell, $\omega_t$, remains constant during the Great Recession and equal to its steady-state value of $\bar{\omega} = 21.9$ percent per month. This experiment informs about the importance of changes in the two sources of heterogeneity for the labor market outcomes. In the second experiment we set $\omega = 0$ so that negative duration dependence is eliminated altogether so that heterogeneity in search efficiency derives solely from heterogeneous job finding prospects upon job loss.

We find that the path of the economy generated when assuming $\omega_t = \bar{\omega}$ is very similar to the one in which we allow for both sources of changes in the extent of heterogeneity. This similarity relates both to the share of longer term unemployed workers and to the level of unemployment. Thus, increased heterogeneity in job finding prospects upon unemployment is quantitatively much more important than increased negative duration dependence. This result derives from the impact on precautionary savings discussed above and it is consistent with the findings of Ahn and Hamilton (2014) who - studying CPS data - find that recessions are times when there is an increased inflow of workers with low job finding probabilities into unemployment. Our results go one step further and demonstrate that such a compositional change is important for the severity of the Great Recession because of its impact on aggregate demand.

We also find only a moderate effect of eliminating negative duration dependence altogether ($\omega = 0$).
which indicates that the propagation mechanism that derives from workers experiencing a loss of search
efficiency during an unemployment spell is rather limited. The reason for this is that our calibration
implies that type $s$ workers find jobs quite fast (within 1.5 months on average in the steady-state) and
face a quite small risk of a loss of search efficiency. Indeed, in the steady-state a type $s$ unemployed
worker only faces a 13 percent risk of experiencing a transition to state $l$ during an unemployment
spell. This risk is too small to matter much quantitatively. It follows that in the setting we examine,
heterogeneity in search efficiency upon job loss is much more important for macroeconomic outcomes
than negative duration dependence.

The Role of Wage Flexibility. We have assumed that real wages are downward inflexible. This
assumption appears consistent with Figure 12 which shows that while the average real compensation
per hour worked in the Business Sector grew by approximately 25 percent from the mid-1990’s to the
beginning of 2007, real wages have remained unchanged since the onset of the Great Recession. We
will now ask two questions: First, to which extent do our results depend on this rigidity? Secondly, are
there circumstances under which a lack of a fall in real wages may arise as an equilibrium outcome?

In order to investigate these issues we assume that wages are determined according to a non-
cooperative Nash bargaining game between firms and workers. We assume that in new matches, once
workers and firms have been matched (but before a wage has been agreed upon), a worker enters the
two unemployment pools with probability $\varphi_{s,t}$ and $1 - \varphi_{l,t}$, respectively, in exactly the same manner
as a worker who enters unemployment from an existing match. This assumption combined with the
borrowing constraint, makes the outcome under Nash bargaining particularly simple because the wage
offered to a new worker is independent of their unemployment state.

We report results for a wide range of values of the workers’ bargaining power which includes both
Hagedorn and Manovskii’s (2008) calibration of workers receiving 5 percent of the match surplus to
‘traditional’ values of this parameter of 50 percent. Figure 13 illustrates the impact of a job separation
shock on unemployment and on real wages. We report the maximum increase in unemployment relative
to the corresponding value in the benchmark model. Similarly, we show the maximum decline in the
real wage as a percentage of the steady-state real wage.

We find that higher bargaining power on the part of workers implies higher wage flexibility in

---

23 Alternatively, one can assume that workers retain their unemployment status but in this case the equilibrium would
entail one-period wage dispersion which seems unreasonable.

24 We assume that workers enjoy leisure when unemployed and calibrate the utility value of leisure so that the steady-state
equilibrium real wage implies a 5 percent unemployment rate.
equilibrium and a significantly smaller maximum response of unemployment. For example, when workers and firms have the same bargaining power, the maximum response of unemployment is less than 25 percent of the corresponding response in the benchmark economy. Low values of the workers’ bargaining power instead imply similar responses to labor market shocks to those we found when assuming inflexible real wages.

To understand these results, consider the impact of an increase in the job separation rate on the joint surplus. A higher job separation rate lowers the value of a filled job because it increases the probability that an existing match is terminated. It also lowers workers’ outside option because of its impact on the job finding rate. Hence, the joint match surplus declines and this puts a downward pressure on real wages which relieves the pressure on firms to cut vacancy postings. The higher the workers’ bargaining power, the larger is the fall in real wages and the smaller is the decline in vacancy postings. Whether the increase in job separations impact mostly on real wages or on vacancy postings matters for employed workers’ savings choices because the former of these have no impact on the precautionary savings motive since it has no idiosyncratic risk component. For that reason, the amplification mechanism is neutralized when workers have a large bargaining power but not for low values of this parameter.

As we have noted above, real wages did not decline much, if at all, during the Great Recession. The results presented here imply that this may either be consistent with workers have little bargaining power or with real wages are downward inflexible. These two alternative scenarios have very similar implications for equilibrium quantities and would therefore be difficult to disentangle empirically.

**The Role of Monetary Policy.** It is standard intuition in the monetary economics literature that aggressive responses of nominal interest rates to inflation can neutralize the inefficiencies that derive from nominal rigidities while too weak responses to inflation produce locally indeterminate equilibria. It is unclear whether similar results should be expected to hold in the heterogenous agents model considered in the current paper but the strength of the amplification mechanism implies that the monetary policy response may potentially be very important.

In order to investigate this issue, Figure 14 reports the impact of job separation rate shocks on unemployment as a function of two key parameters, \( \delta \) and \( \sigma \).\(^{25}\) \( \delta \) determines the response of the nominal interest rate to deviations of inflation from its target while \( \sigma \) determines the extent to which workers respond to employment risk. We indicate by different colors the amplification of the labor market shocks in the benchmark economy by normalizing the maximum impact on unemployment of

---

\(^{25}\)The results for mismatch shocks are nearly identical so we do not report them here.
the job separation shock with the equivalent response in a flexible price economy. A dark blue color means no amplification relative to the flexible price economy with lighter shades of blue and yellow and orange colors indicating ever increasing degrees of amplification. The white area corresponds to combinations of $\delta$ and $\sigma$ that are inconsistent with local determinacy of the equilibrium where inflation is on target.

We find that sufficiently aggressive monetary policy rules succeed in neutralizing the amplification mechanism while interest rate rules similar to those typically assumed in the New Keynesian literature instead produce a large amount of amplification. In our calibration ($\delta = \sigma = 1.5$), both shocks are significantly amplified but increasing $\delta$ to around 2 neutralizes much of the amplification relative to the flexible price allocation. More aggressive of monetary policy responses provide stabilization by moderating the agents’ expectations regarding the impact of the shocks on equilibrium inflation and vacancy postings and thus impact directly on the mechanism through which labor market shocks are amplified.

Our results also show that higher degrees of risk aversion demand more aggressive policy rules in order to provide stabilization and that, the higher is the degree of risk aversion, the more aggressive has the interest rate rule to be in order to ensure local indeterminacy of the intended equilibrium. These features derive from the impact of risk aversion on precautionary savings. When agents are more risk averse, there is a stronger impact of labor market uncertainty on precautionary savings which motivates the need for a more aggressive monetary policy stance in order to stabilize the economy. In the indeterminacy region, equilibria can exist in which agents’ expectations of worsening labor market outcomes and low inflation drives down aggregate demand thereby motivating firms to hire less labor and leading the economy to a high-unemployment-cum-low-inflation self-fulfilling equilibria. Thus, the design of the monetary reaction function is critical in the incomplete markets set-up analyzed in this paper.

**The Early 1990’s Recession.** An important check on the extent to which the mechanisms that we have emphasized are important can be conducted by applying the analysis to other recessions. For this reason we now examine the early 1990’s recession. This recession witnessed a more shallow contraction in output, an increase in the unemployment rate from 5 percent in 1989 to above 7 percent in 1992 (a smaller increase than during the Great Recession) and a moderate increase (minor relative to the

---

$^{26}$ $\sigma$ also impacts on the determinacy region in standard New Keynesian models, see e.g. Gali (2008), but only because it impacts on the response of consumption to real interest rates. In our model, a more important source of impact is through precautionary savings.
Great Recession) in the share of longer term unemployment (see Figures 2 and 3). Similarly to the Great Recession, the early 1990’s recession was associated with an increase in the job loss rate and a persistent decline in the job finding rate. Thus, there are similarities but also interesting differences between the Great Recession and the early 1990’s recession.

One issue that we must confront when applying the model to the early 1990’s recession is that we do not have access to the same data as those that we used to estimate the shocks for the Great Recession because we relied on JOLTS data for the layoff rate and for the number of job openings. We follow Shimer (2005) and estimate the job termination rate on the basis of CPS measures of the number of unemployed workers with duration below 5 months.\textsuperscript{27} We use Barnichon’s (2010) estimate of the “composite help wanted index” to derive a measure of vacancies.\textsuperscript{28}

Using these data we then repeat the exercise from Section 4.2 estimating the labor market shocks and computing counterfactual paths of the key variables. Figure 15 illustrates the estimates of the job separation shock and the search heterogeneity shocks for the early 1990’s recession. The increase in the estimated job separation rate is marginally smaller than what was observed in the Great Recession but very persistent. The heterogeneity shock instead is very small relative to the Great Recession and we actually find a small reduction in the fraction of workers that flow into the low search efficiency state from mid-1990 to mid-1991. This coincides with a small improvement in matching efficiency in the early part of the recession. From mid-1991 matching efficiency worsens and we find a moderate increase in the fraction of workers flowing into low search efficiency.\textsuperscript{29}

Figure 16 illustrates the counterfactual paths for unemployment, vacancies, the share of longer term unemployed, the job finding rate and the measure of income risk discussed earlier. The benchmark model matches the data very well in terms of the paths of unemployment and vacancies. It also accounts for the decline in the job finding rate in the early part of the 1990’s recession but not quite for the extent of the fall in the job finding rate from 1992 to 1994. By contrast, when we assume either flexible prices or lack of idiosyncratic income risk, the model implies little rise in unemployment and that vacancies and the job finding rate increase rather than fall. Thus, the amplification mechanism that we have discussed appears to be just as important for understanding the early 1990’s recession as for the Great Recession. However, as anticipated, an important difference between these two recessions is that the

\textsuperscript{27} We correct for the redesign of the CPS in 1994 using the adjustment proposed in Elsby, Michaels and Solon (2009).

\textsuperscript{28} We scale Barnichon’s composite index with the post 1970 index and we linearly detrend the data.

\textsuperscript{29} We do not show the fraction of permanent job losers for this period, as this data series is only available from 1994 onwards.
search heterogeneity shock appears of little importance for the 1990’s recession since the countercyclical paths of the economy are very similar with and without allowing for this shock.

Figure 17 illustrates the Beveridge curves. The data indicate that the US economy experienced a slide down followed by an outward shift of the Beveridge curve during the 1990’s recession much like what was witnessed during the Great Recession. However, both the movement along the Beveridge curve and the outward shift were much smaller in the earlier of these two recessions. The benchmark model again manages to account very well for these joint dynamics in unemployment and vacancies with the difference vis-à-vis the Great Recession being that the heterogeneity shock is not important for the extent to which the model provides a good fit to the Beveridge curve dynamics.

We conclude from this that the central amplification mechanism that we have stressed - the decline in aggregate demand due to labor market risks and its transmission to the supply side - is equally important for the early 1990’s recession as we claim it is for the Great Recession. The main contrast between these two recessions was that the search heterogeneity shock was not important for the early 1990 recession. For that reason, according to our results, this recession witnessed little rise in longer term unemployment and a smaller rise in idiosyncratic income uncertainty which meant that this recession produced less dramatic outcomes for the economy. Yet, in the absence of idiosyncratic income uncertainty or in the presence of flexible prices, the early 1990’s recession would barely have increased unemployment.

6 Conclusions and Summary

We have shown how frictions in labor markets that interact with goods and financial markets frictions can lead to a significant amplification of labor market shocks in a general equilibrium framework. At the heart of our theory is the idea that labor market shocks that produce job uncertainty can reduce aggregate goods demand because of precautionary savings. A calibrated version of the model can account not only for the increase in unemployment observed in the U.S. during the Great Recession but also for much of the movements in the Beveridge curve. It is the transmission of weak aggregate demand to aggregate supply that produces these results because of an endogenous amplification mechanism.

Our emphasis on job uncertainty deriving from idiosyncratic employment risk and uncertain outcomes of labor market search offers an additional route through which macroeconomic uncertainty can impact on the economy. Much recent literature has focused upon uncertainty shocks deriving from changes in the volatility of aggregate variables (such as TFP or policy related variables) and shown that such uncertainty can be important for understanding aggregate fluctuations, see e.g. Bloom (2009). As
Carroll and Dunn (1997) we have instead focused on the idiosyncratic uncertainty implications of first moment shocks to job loss and job finding probabilities. We plan in future work to further examine the importance of this channel using micro level data on consumption and labor supply.

Our theory has abstracted from aggregate savings and we imposed that workers cannot go into debt. These assumptions are appealing from a computational perspective but it would be interesting to relax them both so that one can also evaluate the impact on aggregate savings and investment. It would also be interesting to investigate the impact of unemployment insurance policies. Unemployment insurance duration is usually extended during U.S. recessions and the Great Recession is no exception to this. Our exercise does take this into account because we focus entirely upon recessions and the calibration of the level of benefits targets Hurd and Rohwedder’s (2010) estimates of the consumption loss due to unemployment shocks during the Great Recession. Yet, it would be of interest to investigate further how such cyclical variations in unemployment benefits impact on the aggregate outcomes. We plan to pursue each of these issues in future work.

7 References


**Daly, Mary, Bart Hobijn, Ayşegül Şahin and Rob Valletta.** 2011. “A Rising Natural Rate of Unemployment: Transitory or Permanent?” WP 2011-05, Federal Reserve Bank of San Francisco.


7.1 Appendix 1: Solving and simulating the model

From the no-borrowing constraint it follows that, in equilibrium, the individual workers’ consumption levels are given as $c^u = c^u_s = \xi$ and $c^n = \bar{w}$. We can then reduce the model down to a core dynamic subsystem of only three equations: the household’s Euler equation for bonds, Equation (38), the firms’ pricing condition, Equation (39) and the monetary policy rule, Equation (29), which can be written as:

$$
\bar{w}^{-\sigma} = \beta \mathbb{E} \frac{R}{1 + \pi} \left\{ (1 - \rho \varphi_s (1 - \eta'_s) + (1 - \varphi_s) (1 - \eta'_l)) \right\} \bar{w}^{-\sigma} \\
+ \rho \varphi_s (1 - \eta'_s) \xi^{-\sigma} + \rho (1 - \varphi_s) (1 - \eta'_l) \xi^{-\sigma} \\
1 - \gamma + \gamma \left( \bar{w} + \bar{y} \frac{1}{1+\sigma} \mu \eta^{\frac{1}{1+\sigma}} - \beta \mathbb{E} (1 - \rho) \bar{y} \frac{1}{1+\sigma} \mu \eta^{\frac{1}{1+\sigma}} \right) = \phi (1 + \pi) \pi + \beta \tilde{\phi} \mathbb{E} (1 + \pi') \pi \frac{\bar{y}'}{\bar{y}} \\
R_t = \mathbb{R} \left( \frac{1 + \pi_t}{1 + \bar{\pi}} \right)^{\delta}
$$

We use this dynamic system of three equations, together with the exogenous shock processes, to solve for the equilibrium laws of motions of $\eta_{s,t}$, $R_t$ and $\pi_t$ using a standard perturbation method.\textsuperscript{30} We can then simulate these variables for a given time path of the exogenous shocks.

Simulations of the remaining variables are obtained using the original nonlinear equations. Given a simulation for $\eta_{s,t}$, $R_t$ and $\pi_t$ and the shocks, we can use Equations (22)-(23) directly to obtain simulations for $\theta_t$ and $\psi_t$. Next we jointly use Equations (20)-(21) and (25)-27) to recursively find the simulation paths for $m_t$, $u_{a,t}$, $n_t$, $u_{s,t}$ and $u_{l,t}$, given initial conditions for $u_{s,t}$ and $u_{l,t}$. To find the time path for vacancies and the unemployment rate, we use the condition that $v_t = \theta_t / u_{a,t}$ and $u_t = 1 - n_t$. The average job finding rate is computed as $\frac{m_t}{u_{a,t} + n_t}$. Finally, a simulation for the share of unemployed for 6 months or less is obtained by recursively keeping track of the mass of unemployed in each of the two states by duration, ranging from 1 to 6 months.

7.2 Appendix 2: Retrieving shocks from the data

We retrieve the search efficiency shocks $z_{\varphi,t}$ and $z_{\omega,t}$ from the data using an iterative accounting procedure based on only the model’s labor market transition equations and the matching function. Importantly, the values of the shocks are found independently of the model solution for $\eta_{s,t}$, $R_t$ and $\pi_t$. Thus, key features of the model’s amplification mechanism such as the degrees of price stickiness, risk aversion and market incompleteness have no impact on the values of the shocks we retrieve from the data.

\textsuperscript{30}This is done after substituting out $\eta'_{l,t} = q \eta_{l,t}$ in the Euler equation and exploiting that the term $\frac{\phi'}{\phi}$ drops out under a first-order approximation of the model around a steady-state with $\pi = 0$. 

34
Our procedure is based on three data series: (i) the civilian unemployment rate from the BLS, (ii) the layoff rate from JOLTS, and (iii) the number of job openings from JOLTS. The first step is to retrieve a time series for the number of active searchers, \( u_{a,t} \), from the data. To this end, we re-write the matching function, Equation (20), as:

\[
m_t = \varrho \left( \frac{u_{a,t}}{u_{t-1}} \right)^{\alpha} (v_t)^{1-\alpha}.
\]

and note that \( \left( \frac{u_{a,t}}{u_{t-1}} \right)^{\alpha} \) is the time-varying residual of an aggregate matching function that abstracts from heterogeneity. Thus, a by-product we obtain a time series for the aggregate matching function residual and the shocks we back out map exactly into this residual.

To proceed, we rearrange the above equation to obtain an expression for \( u_{a,t} \):

\[
u_{a,t} = u_{t-1} \left( \frac{\tilde{\eta}_t}{\varrho} \right)^{1/\alpha} \left( \frac{v_t}{u_{t-1}} \right)^{1-1/\alpha}
\]

where \( \tilde{\eta}_t = \frac{m_t}{u_{t-1}} = \frac{1-u_t-(1-\rho_{t-1})(1-u_{t-1})}{u_{t-1}} \). Given time series for \( u_t, \rho_t \) and \( v_t \) we can evaluate the entire right-hand side of the above equation. For consistency with the model’s steady state, we take the log of both sides of the equation and subtract the mean, which gives:

\[
\hat{u}_{a,t} = \hat{u}_{t-1} + 1/\alpha \tilde{\eta}_t + (1 - 1/\alpha) (\hat{v}_t - \hat{u}_{t-1})
\]

where a hat denotes a log deviation from the mean.\(^{31}\) We then construct a time series for the fraction of active searchers as \( u_{a,t} = e^{\tilde{u}_a + \hat{u}_{a,t}} \), where \( \tilde{u}_a \) is the steady-state level of active searchers in the model.

Given \( u_{a,t} \) we can compute the job finding rate among searchers in each period:

\[
\eta_{s,t} = \frac{m_t}{u_{a,t}} = \frac{\tilde{\eta}_t}{u_{a,t}}
\]

which also gives a time series for \( \eta_{l,t} = q\eta_{s,t} \). Next, we back out the shocks using the following iterative procedure:

1. Start at \( t = 1 \), initializing \( u_{s,0} \) and \( u_{t,0} \) are at their steady-state levels.

2. Using our proportionality assumption \( \omega_t = (\overline{\omega}/\overline{\varphi}_t) \varphi_{l,t} \) and \( \varphi_{s,t,-1} = 1 - \varphi_{l,t-1} \), re-write equation (21) to obtain an expression for \( \varphi_{l,t-1} \):

\[
\varphi_{l,t-1} = \frac{u_{a,t} - u_{s,t-1} - \rho_{x,t-1} n_{t-1} - q u_{l,t-1}}{- (\overline{\omega}/\overline{\rho}_l) u_{s,t-1} - \rho_{x,t-1} n_{t-1} + q (\overline{\omega}/\overline{\rho}_l) u_{s,t-1} + q\rho_{x,t-1} n_{t-1}}
\]

\(^{31}\)The scaling’s parameter \( \varrho \) drops out of the equation. The matching parameter \( \alpha \) is set to 0.65, to be consistent with our model calibration.
Evaluate the right-hand side to obtain $\varphi_{l,t-1}$, and compute the implied values of $\omega_{t-1}$ and $\varphi_{s,t-1}$.

3. Use Equations (27) and (27) to compute $u_{s,t}$ and $u_{l,t}$.

4. Set $t = t + 1$ and go back to step 2 until $t = T$.

5. Once the time series for $\varphi_{l,t}$ and $\omega_{t}$ have been obtained, back out $z_{\varphi,t}$ and $z_{\omega,t}$ from Equations (33) and (34).

Our procedure uses data over the period January 2003 until August 2014.

### 7.3 Appendix 3: The Nash bargaining problem

Under the assumptions discussed in the main text, the Nash Bargaining solution satisfies:

$$
\kappa \frac{\mu}{\psi} = (1 - \kappa) \left[ V(b, e, S) - \varphi_l V(b, l, S) - (1 - \varphi_l) V(b, s, S) \right] / U'(w),
$$

where $\kappa$ is the bargaining power of the worker and $\frac{\mu}{\psi}$ is the surplus of a match to an entrepreneur, which is equal to the (expected) resource cost of hiring a new worker. The term between square brackets on the right-hand side is the surplus of a match to a household, which equals the difference between the value obtained in an employment relationship, $V(b, e, S)$, minus the expected value outside the relationship, $\varphi_l V(b, l, S) + (1 - \varphi_l) V(b, s, S)$. The multiplicative term $U'(w)$ on the right-hand side converts the utility surplus of the employed worker into units of resources using her marginal utility.

To solve the model, we add the Nash bargaining condition to the system of equations, as well as the worker value expressions, Equations (4)-(6), setting $c = w$ for the employed worker’s value and $c = \xi$ for the unemployed workers of each type. The latter requirement follows from the borrowing constraint $b \geq 0$ which implies that—as in the baseline model—workers consume their current-period income streams.

Relative to the baseline model, the system to be solved thus contains four additional equations (value functions for the three worker types and the Nash Bargaining condition) and four additional variables (worker values for the three types and the real wage). We solve this model using first-order perturbation on the entire system of model equations.

### 7.4 Appendix 4. The Hazard Function

We calibrate $\Delta = (\varphi, \omega, q, \eta_s)$ by targeting a 15 percent steady-state share of workers out of employment with a duration of unemployment of 6 months or more, a steady-state monthly job finding probability
of 43 percent, and the hazard function implied by Kroft et al’s (2013) estimates. These authors estimate the hiring probability of individuals out of work for \( d \) months at date \( t \) using a polynomial specification for the hazard:

\[
\eta_t(\theta_t,d) = A(d) m_0 \theta_t^{1-\nu}
\]

\[
A(d) = (1 - a_1 - a_2) + a_1 e^{-b_1 d} + a_2 e^{-b_2 d}
\]

Using panel data from the CPS for the 2002-2007 sample their estimates are \( \hat{a}_1 = 0.314, \hat{a}_2 = 0.393, \hat{b}_1 = 1.085 \) and \( \hat{b}_2 = 0.055 \). Third, we introduce information on duration dependence from Kroft et al’s (2013) estimate of the relationship between hiring probabilities and the length of an unemployment spell. These authors estimate the hiring probability of individuals out of work for \( d \) months at date \( t \) using a polynomial specification for the hazard. In particular, they assume that the job finding rate depends on the length of the unemployment spell and on labor market tightness as:

\[
\eta_t(\theta_t,d) = A(d) m_0 \theta_t^{1-\nu}
\]

\[
A(d) = (1 - a_1 - a_2) + a_1 e^{-b_1 d} + a_2 e^{-b_2 d}
\]

Using panel data from the CPS for the 2002-2007 sample for \( d \) going up to 15 months (and controlling for demographic variables), Kroft et al (2013) estimate \( \hat{a}_1 = 0.314, \hat{a}_2 = 0.393, \hat{b}_1 = 1.085 \) and \( \hat{b}_2 = 0.055 \). We calibrate \( \Delta \) by minimizing the quadratic form:

\[
W = \left[ \Sigma(\Delta) - \Sigma^{\text{target}} \right]' \left[ \Sigma(\Delta) - \Sigma^{\text{target}} \right]
\]

where \( \Sigma(\Delta) \) are the model implied moments and \( \Sigma^{\text{target}} \) are the targets just listed. The minimization problem delivers the estimates \( \Delta = (0.229, 0.219, 0.468, 0.586) \). The figure below illustrates the implied hazard function along with Kroft et al’s (2013) estimates.
Figure A.1: Hazard function
## 7.5 Tables and Figures

### Table 1: Targets and Parameter Values

<table>
<thead>
<tr>
<th>Targets</th>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05 unemployment rate (u)</td>
<td>96.9 price adjustment cost parameter</td>
</tr>
<tr>
<td>0.586 job finding rate among searchers (ηs)</td>
<td>6 elast. subst. goods varieties</td>
</tr>
<tr>
<td>0.045 s.s. hiring cost as a fraction of the quarterly wage (μ/ψw)</td>
<td>0.992 discount factor</td>
</tr>
<tr>
<td>0.15 fraction of long-term unemployed (&gt;6 months)</td>
<td>1.5 coefficient of relative risk aversion</td>
</tr>
<tr>
<td>0.117 consumption loss upon unemployment (ξ/ψ)</td>
<td>1.5 interest rate rule parameter on inflation</td>
</tr>
<tr>
<td>0 net inflation rate (π)</td>
<td>0.039 steady state job termination rate</td>
</tr>
<tr>
<td>0.05 annual net interest rate (R^12−1)</td>
<td>0.91 persistence parameter termination rate</td>
</tr>
<tr>
<td>5 average price duration (in months) in equivalent version with Calvo pricing</td>
<td>0.667 std. dev. ρ shock as a percentage of p</td>
</tr>
<tr>
<td>0.229 steady state fraction of job losers into s-pool</td>
<td>0.219 steady state fraction from s-pool to l-pool</td>
</tr>
<tr>
<td>0.99 persistence of shocks to fraction of job losers into l-pool</td>
<td>7.20 std. dev. ϕ_l shock as a percentage of ϕ_l</td>
</tr>
<tr>
<td>0.99 persistence of shocks to fraction from s-pool to l-pool</td>
<td>0.99 persistence of shocks to fraction from s-pool to l-pool</td>
</tr>
<tr>
<td>7.20 std. dev. ω shock as a percentage of ω</td>
<td>0.19 matching efficiency parameter</td>
</tr>
<tr>
<td>0.468 probability of search for unemployed in l-pool</td>
<td>0.830 real wage</td>
</tr>
<tr>
<td>0.65 matching function elasticity</td>
<td>0.733 unemployment benefit</td>
</tr>
</tbody>
</table>
| 0.004 steady-state net nominal interest rate | 39
### Table 2: Stationary State Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^n$</td>
<td>0.830</td>
<td>consumption employed</td>
</tr>
<tr>
<td>$c^{u,s}$</td>
<td>0.733</td>
<td>consumption unemployed in $s$-pool</td>
</tr>
<tr>
<td>$c^{u,l}$</td>
<td>0.733</td>
<td>consumption unemployed in $l$-pool</td>
</tr>
<tr>
<td>$\eta_s$</td>
<td>0.586</td>
<td>job finding rate unemployed in $s$-pool</td>
</tr>
<tr>
<td>$\eta_l$</td>
<td>0.274</td>
<td>job finding rate unemployed in $l$-pool</td>
</tr>
<tr>
<td>$u_s$</td>
<td>0.017</td>
<td>mass in $s$-pool</td>
</tr>
<tr>
<td>$u_l$</td>
<td>0.033</td>
<td>mass in $l$-pool</td>
</tr>
</tbody>
</table>
Figure 1: The US Civilian Unemployment Rate

Figure 2: Estimates of Job Separation and Job Finding Rates
Figure 3: Share of unemployed workers who have been out of work for 6 months or more

Figure 4: Average Unemployment Duration and Estimates from Job Finding Rates
Figure 5: The Beveridge Curve
Figure 6: The Impact of Job Separation Shocks
Figure 7: The Impact of Search Heterogeneity Shocks
Figure 8: The Great Recession: Shocks
Figure 9: The Great Recession
Figure 10: Actual and Counterfactual Beveridge Curves: 2007-2014
Figure 11: The Importance of Different Sources of Heterogeneity
Figure 12: Real hourly compensation
Figure 13: The Importance of Increased Duration Dependence
Figure 14: Monetary Policy and Amplification
Figure 15: Early 1990’s Recession: Shocks
Figure 16: Early 1990’s Recession
Figure 17: Actual and Counterfactual Beveridge Curves: 1990-1995