Abstract

How can a small special interest group successfully get an inefficient transfer at the expense of a much larger group with many more resources available for lobbying? We consider a simple model of collusive organizations that provide a public good in the form of effort and have a fixed cost per member of acting collusively. Our key result is that the willingness of such a group to pay for a given prize depends on whether the prize is fungible - that is, whether the prize can be used to pay for itself. If the prize is fungible, as in the case of a transfer payment, a smaller group always has an advantage. If the prize is non-fungible - civil rights for example - willingness to pay first increases then decreases with the size of the group. We use the theory to study agenda setting both with and without blackmail by the politician showing that in general the small group is not too greedy: when it wins it optimally chooses to pre-empt the large group by choosing a prize small enough to equal the large group participation cost.

Keywords: Organization, Group, Collusion. Public Good

Jel codes: C72, D7, D72

† Department of Economics, WUSTL and WUI. Email: david@dklevine.com
‡ Università di Palermo. Email: salvatore.modica@www.unipa.it
Acknowledgments

We are especially grateful to Michele Boldrin, Michael Chwe, Drew Fudenberg, Andrea Galeotti, Zacharias Maniadis, Eric Maskin, Andrea Mattozzi, Andy Postelwaite, Debraj Ray, seminar participants at the St. Louis Federal Reserve Bank and Venice Conference on Economic Theory and several anonymous referees of an earlier version. We are grateful to the EIEF, to NSF Grant SES-08-51315 and to MIUR PRIN 20103S5RN3 for financial support. This project is related to the research agenda of the ADEMU project, “A Dynamic Economic and Monetary Union”. ADEMU is funded by the European Union's Horizon 2020 Program under grant agreement N° 649396 (ADEMU).

The ADEMU Working Paper Series is being supported by the European Commission Horizon 2020 European Union funding for Research & Innovation, grant agreement No 649396.

This is an Open Access article distributed under the terms of the Creative Commons Attribution License Creative Commons Attribution 4.0 International, which permits unrestricted use, distribution and reproduction in any medium provided that the original work is properly attributed.
1. Introduction

There is a basic puzzle about lobbying: how can a small special interest group successfully get an inefficient transfer at the expense of a much larger group with many more resources available for lobbying? Olson (1982) and others such as Becker (1983) have argued that this is because small groups are likely to be more effective than large groups but without providing much in the way of a theory about why this might be the case, and whether there are exceptions. By contrast empirical results on the relation between group size and strength in this case are mixed, see, for example, the survey by Potters and Sloof (1996). In this paper we examine a simple model where two groups of different size compete for a prize. The prize to a group takes the form of a transfer from the other group, and in trying to win the transfer the groups bribe a politician by offering him a payment. To solve the public good problem of contribution the groups must pay a fixed cost per member. A formal model of monitoring leading to this result can be found in Levine and Modica (2016). Here we explore conditions under which a small group is and is not more effective than their larger rival. Our main finding is that fungibility - whether the prize can be used to pay for itself - plays a key role. For example monetary subsidies such as farm subsidies are fungible since they can be used to pay the politicians who provide the subsidies, while benefits such as civil rights are not fungible as they do not increase the resources available for lobbying. In the case of a fungible prize we find that the idea that smaller groups are more effective is basically correct. In the case of a non-fungible prize it is true only up to a point: a group that is too small lacks the resources to submit a high bid, so that the effectiveness initially increases with group size; but then eventually decreases.\(^3\) In our conclusion we present some evidence that indeed small groups are much more effective at garnering fungible than non-fungible prizes.

Our model of lobbying is similar to those used in earlier work such as Dixit, Grossman and Helpman (1997) and Rama and Tabellini (1998) in that we include the possibility that lobbyists purchase influence in a menu auction. Those papers consider lobbies that compete with each other such as trade-unions and business groups and do not analyze the effect of group size. In this work the general public is represented only indirectly in the form of a preference by government officials for efficiency. Here we are instead interested specifically about why a small special interest group can “out lobby” a larger general interest group. Why do bankers and farmers “win” over taxpayers?

We apply the model also to agenda setting, considering that either group can choose the size of the prize. If the small group can set the agenda we find that it will generally choose a relatively small prize - some subsequent back of the envelope calculations concerning farm subsidies show that this is plausible. If the large group can set the agenda fungibility plays a key role. In the case of a fungible prize the large group cannot get the prize. In the case of a non-fungible prize the large group will choose a very large prize. We also consider the role of the politician. We focus on

---

\(^3\)Of course different models may deliver different predictions. Dixit (2004), Chapter 3 for example considers a two-period model of bilateral trading where misbehavior by a given individual in the first random match can be punished if her second partner knows that when the second match occurs. Assuming that this information is harder to come the larger the group yields the result that larger groups are less capable of enforcing “fair” trade.
a case where the politician must affiliate with one of the lobbies prior to agenda setting. When this is not the case the politician may “blackmail” the lobbies by threatening each with the agenda of the other if they do not pay up. In practice this is probably dangerous, and we show that in some circumstances it will result in the lobbies colluding to get rid of the politician.

The model potentially provides an explanation of the following paradox: Olson (1965) and others provide substantial evidence that small groups are effective at winning subsidies while larger groups are not. However we also observe frequently the suppression of minority rights by a majority; here the larger group trying to deny rights seems much more effective than the smaller group trying to keep their rights. We propose that the reason is due to the role of fungibility - small groups are effective in garnering small prizes regardless of fungibility, while large groups are effective only in garnering large non-fungible prizes - and civil rights seem to be in that category.

The literature on lobbying and other interest groups is large. Generally these models have fallen into four categories. Some treat the strength of the group as a black box and proceed with a working assumption, generally one in which strength decreases with size (Olson (1965), Becker (1983), Becker (1986)), or in the case of Acemoglu (2001) that strength increases with group size for a relatively small and a relatively large group. A second class of models treats collusive groups as individuals - effectively ignoring internal incentive constraints - and focuses instead on information differences between the groups: examples are Nti (1999), Persson and Tabellini (2002), Kroszner and Stratman (1998), Laffont and Tirole (1991), Austen Smith and Wright (1992), Banks and Weingast (1992), Damania, Frederiksson and Mani (2004), Green and Laffont (1979), Laffont (2000) and Di Porto, Persico and Sahuguet (2013). Dixit, Grossman and Helpman (1997) is similar, but allows the endogenous possibility that groups either act non-collusively, or collusively as a single individual. A few papers assume that leaders of the group can distribute benefits differentially (this may or may not be what Olson (1965) has in mind by “selective incentives”) so that there is no public goods problem: see for example Nitzan and Ueda (2011) and Uhlaner (1989). Finally Pecorino (2009), Lohmann (1998), Esteban and Ray (2001) and Esteban, J. and J. Sakovics (2003) treat the problem of individual contribution within a group as a voluntary public goods contribution problem. None of these papers addresses the issue of fungibility. We should also mention Mitra (1999), that goes in the direction opposite of ours: the paper assumes a fixed cost of forming a group - in contrast to our conclusion that there is a fixed cost per person in the group - so the more people there are the easier it is to overcome the fixed cost.

---

4 This is consistent with our results, since we show that strength increases with size for a small group, and for a relatively large group, the opposition is small, and therefore weak.

5 Olson’s concept is a bit slippery. He may have in mind people who are not in a group benefiting from the activity of the group - although this view of voluntary group participation runs somewhat counter to his notion of what constitutes a group. He argues that the group should devise auxiliary services (free lawyers, insurance) which “selectively” benefit only group members. It is not entirely clear why it would not be better to free ride on the group and pay directly for the auxiliary services, unless the group has some cost advantage in providing those services. In our setting members to not have the option of leaving the group - which is to say that they can not avoid being punished by group members. For example, farmers cannot avoid being shunned by neighboring farmers by refusing to join a farm association.
2. The Model

There are three agents, \( k \in \{S, L, P\} \). The first two agents are collusive lobbying groups where \( S \) means “small” and \( L \) means “large” and the number of members in group \( k \) is \( N_k \) where \( N_S < N_L \). The third agent is a politician.

2.1. The Economic Environment

Transfer payments between the three agents are possible. The status quo is that all agents get 0. Each group \( k \) can make a transfer \( V_k \) to the other group which receives \( \beta V_k \) where \( 1 > \beta > 0 \) is the efficiency of the transfer. Any group that is not making a transfer to the other group may make a payment \( p_k \geq 0 \) to the politician.\(^6\) In order to make a strictly positive payment to the politician the group must incur a cost \( cN_k \) where \( 1 > c > 0 \) is a per member cost of organizing and enforcing the payment from group members. That is, we follow Olson (1965) in recognizing that the groups face a public good problem and follow Levine and Modica (2016) in assuming that this can be overcome by a monitoring schemes that has a fixed cost per group member that must be monitored. Utility is linear in these payments and transfers.

Feasible transfer payments are subject to resource constraints. There are two types of resources: fungible resources are valued equally by all three groups - they represent money, goods or services. Non-fungible resources are valued only by the lobbying groups. The represent “rights,” for example, the right to bear arms, to have an abortion, to marry, to sit at the front of the bus and so forth. The politician must receive fungible resources. We consider two different economic environments: the case in which all resources are fungible and the case in which the transfer payments are made entirely from non-fungible resources. In both cases each group member is endowed with a unit of fungible resources which can be used to make payments to the politician and for organizing the group. In the non-fungible case these fungible resources are not used to make transfers and each group member is also endowed with \( \nu \) units of resources that can be used only to make transfers. We limit attention to the case where \( \nu > 1 \) so that more non-fungible resources are available than fungible resources.

Specifically the first resource constraint is that the transfer from group \( k \) must satisfy \( V_k \leq \nu N_k \) where we take \( \nu = 1 \) in the fungible case. Transfer payments to the politician must come from fungible resources, so in the non-fungible case the payment must satisfy \( p_k \leq (1 - c)N_k \). In the fungible case the transfers \( V_{-k} \) from the other group are fungible and may also used to pay the politician so the payment must satisfy \( p_k \leq (1 - c)N_k + \beta V_{-k} \). It is useful to use the dummy variable \( \psi \in \{0, 1\} \) to denote whether the environment is fungible or not, where \( 1 \) means fungible, so that for \( k \in \{S, L\} \) we may write the resource constraint for paying the politician as \( p_k \leq (1 - c)N_k + \psi \beta V_{-k} \).

\(^6\)Note that if the payment is split among a number of politicians as long as the particular politician in question receives a fixed share this does not change his incentives.
We first consider what happens when a particular agenda is set in the sense that a proposal is on the table to transfer a given amount $V_a$ from group $-a$ to the agenda setter group $a$ and the politician must decide whether or not to implement the proposal. We regard the politician as a seller who "sells" his decision (yes or no) to one of the groups - who we regard as buyers - in exchange for payment. Consider five standard mechanisms that the politician might use: an all-pay auction, a second price sealed bid auction, a first price sealed bid auction, a menu auction or a take-it-or-leave-it demand. In an all-pay auction, first analyzed by Hillman and Riley (1989), both groups submit bids, the highest bid wins - so if $a$ wins then the proposal is implemented and if $-a$ wins it is not - and both groups pay their bid. In a second price sealed bid auction - which is similar to a first price oral auction - both groups submit bids, the highest bid wins, and the winning group pays the bid of the losing group. In a first price sealed bid auction both groups submit bids, the highest bid wins and the winning group pays their own bid. In a menu auction each group places a bid for both winning and losing and pays the winning bid if they win and the losing bid if they lose. Menu auctions, also known as common agency, originally introduced in Bernheim and Whinston (1986b), are commonly studied mechanisms in the literature on buying influence such as Grossman and Helpman (1992) Dixit, Grossman and Helpman (1997) Grossman and Helpman (2001) or Ram and Tabellini (1998). With a take-it-or-leave-it demand the politician designates a group to whom the demand is made and sets a bid and if the group meets that bid they win and pay the bid, otherwise they lose and pay nothing. So if the politician makes a take-it-or-leave-it demand to $a$ and the group meets the demand the bid is paid and the proposal is implemented; if it does not meet the demand neither group pays anything and the proposal is not implemented. If the take-it-or-leave-it demand is addressed to $-a$ and the group meets the demand the bid is paid and the proposal is not implemented; if it does not meet the demand neither group pays anything and the proposal by $a$ is implemented.

It is useful here to contrast lobbying with voting, since lobbying, to a certain extent, is voting with money. In voting the mechanism is certainly that of the all-pay auction - that is the groups turn out their voters (their bids) and the highest bid wins. Yet the losing party also has to bear the cost of turning out their voters despite the fact they do not get the prize. Lobbying through campaign contributions may have a similar flavor, as campaign contributions may be made in advance of political favors being granted, and potentially both groups may contribute to the politicians campaign. However, many payments to politicians are made either ex post or contemporaneously - for example, jobs after the politician leaves office, jobs for relatives of the politician, donations to future campaigns, and of course outright bribes either in the form of cash or favors. Hence, unlike voting, it makes sense to think of mechanisms where payment is made only if the favor is delivered as well as the all-pay auction.

\footnote{Common agency introduced in Bernheim and Whinston (1986a) is conceptually similar to a menu auction but assume that bidders are not constrained to make non-negative bids. This model has not been widely used in the political economy literature and we do not examine it here.}
To analyze these five mechanisms it is useful to introduce the concept of willingness to pay, as measured, for example, by a Becker, DeGroot and Marschak (1964) elicitation mechanism. Let \( U_a = \beta V_a \) for the agenda setter and \( U_{-a} = V_{-a} \) for the other group denote the respective value of winning to each group. Let \( \psi_a = \psi \) for the agenda setter and let \( \psi_{-a} = 0 \). Then total willingness to pay of group \( k \) is given by

\[
W_k = \min\{(1 - c)N_k + \psi_k \beta V_{-a}, \max\{0, U_k - cN_k\}\}
\]

(WTP)

In the case of the agenda setter pursuing a fungible prize this reduces to \( W_k = \max\{0, U_k - cN_k\} \), which is decreasing in \( N_k \) - the basic Olsonian idea that larger groups are less effective because they face a stronger public goods problem. In the remaining cases, however, we have \( W_k = \min\{(1 - c)N_k, \max\{0, U_k - cN_k\}\} \) which for small \( N_k \) increases linearly with \( N_k \) so that very small groups are ineffective due to their lack of resources for bidding. For these cases there is an “optimal” group size neither too big nor too small that maximizes willingness to pay.

Remark. A natural question is why since a smaller group faces a smaller problem (here in terms of fixed cost) a larger group does not just “act like a smaller group” in order to increase its willingness to pay. But a subgroup of size \( M_k < N_k \) would only receive a share of the prize: \( (M_k/N_k)U_k \). Then the answer is straightforward: the willingness of the subgroup to pay is

\[
\min\{(1 - c)M_k + (M_k/N_k)\psi_k \beta V_{-a}, \max\{0, (M_k/N_k)U_k - M_k c\}\}
\]

so that the willingness of the subgroup to pay is always a fraction \( M_k/N_k \) of the willingness of the entire group to pay.

We can now characterize equilibrium for each of the five mechanisms, where we use standard refinements. Call the group \( d \) with the least willingness to pay the disadvantaged group and the group \(-d\) the advantaged group.

**Theorem 1.** Suppose that \( W_a > 0 \). In the all-pay auction there is a unique Nash equilibrium which is in mixed strategies. The advantaged group plays uniformly on \( (0, W_d) \), the disadvantaged group does not bid with probability \( (W_{-d} - W_d)/W_{-d} \) and places the remaining probability uniformly on \( (0, W_d) \). The expected payment to the politician is

\[
\frac{W_{-d} + W_d}{2W_{-d}} W_d.
\]

Group \(-d\) gets an expected utility of \( W_{-d} - W_d \) and group \( d \) gets nothing. In the second-price auction there is a unique equilibrium in which the groups use weakly undominated strategies: both groups bid their willingness to pay and the expected payment to the politician is \( W_d \) and the expected utility of the two groups is exactly the same as in the all-pay auction. In the first price auction and the menu auction there is a unique truthful equilibrium in which the two groups both bid \( W_d \), the
advantaged group wins and the expected payment to the politician and the expected utility of both
groups is identical to that in the second price auction. In the take-it-or-leave-it demand case the
politician charges group \(-d\) its willingness to pay \(W_{-d}\).

Remark. These are all known results. The all-pay auction is discussed in Hillman and Riley (1989)
and Levine and Mattozi (2016). The take-it-or-leave-it demand and second price auctions are
discussed in most textbooks. For the menu auction, “truthfulness” as introduced in Bernheim and
Whinston (1986b) requires that a bid of zero be placed for losing. Hence with two alternatives it
is the same as a first price auction. “Truthfulness” further requires that the loser bid their value.
Hence the advantaged group should bid just a bit more than the disadvantaged party and win, and
in the limit should win by placing the same bid.

We should emphasize first that the widespread equivalence of the different auctions is primarily
because values are commonly known, while most of the auction literature considers the far more
difficult case in which values are private information. Two summarize: the disadvantaged group
never gets anything. Otherwise there are three cases: the take-it-or-leave it demand, the all-pay
auction and the second price, first price and menu actions which are all the same. The take-it-or-
leave it demand is best for the politician and worst for the advantaged group. The advantaged group
is indifferent between all the different auctions, while the politician dislikes the all-pay auction.
Since everyone agrees or is indifferent to one of the first price, second price or menu auctions over
the all-pay auction, we assume that the all pay auction is not used. Since the first price, second
price and menu auctions are all the same, for concreteness and simplicity we focus on the second
price auction.

2.3. The Mixed Mechanism

Between the second-price auction and the take-it-or-leave-it demand the politician obviously
does better with the take-it-or-leave-it demand and the advantaged group does better with the
second price auction. How much rent can the politician in fact extract from the two groups? On
the one hand it seems that the politician should be able to extract at least what he can get in a
second-price auction by playing the groups against one another. On the other hand the groups
may resist a take-it-or-leave-it demand that leaves them with no possibility of surplus. In effect the
answer depends upon the bargaining power of the politician. One simple way to capture this idea
in a simple game form is to use a mechanism that randomizes between a second-price auction and
a take-it-or-leave-it demand. That is, we can think of all three agents submitting bids \(p_S, p_L, p_P\),
with the politician also designating one of the groups as a target \(\tau \in \{S, L\}\) for his bid. With
probability \(1 > \alpha > 0\) the game is determined by whether group \(\tau\) has bid enough to meet the
politician’s demand (bid) as with a take-it-or-leave-it demand, while with probability \(1 - \alpha\) the
game is determined by the bids of the two groups as in a second-price auction.

To understand how this mechanism works notice that the amount that either group pays for
winning is independent of its bid. The targeted group faces a randomly drawn price equal to \(p_{-\tau}\)
with probability \(1 - \alpha\) and equal to \(p_P\) with probability \(\alpha\), wins if its bid \(p_{\tau}\) is at least equal to the
randomly drawn price, but pays only the randomly drawn price. If it wins, the proposed transfer does or does not take place as the targeted group is the agenda setter or not. If \( \tau \) loses the opposite happens: the agenda is implemented iff \( \tau = -a \). In this case if \( \tau \) loses to the politician the agenda setter obtains the transfer for free. Note that here “losing” must mean “the opponent wins” which is why when the non-agenda setter is targeted and loses the take-it-or-leave it auction the agenda setter must get the transfer for free.

In the case of the non-targeted group with probability \( \alpha \), its bid does not matter, although it may get its preferred policy implemented for free if the targeted group falls short in the bidding against the politician. With probability \( 1 - \alpha \), it wins if and only if its own bid \( p_\tau \geq p_{-\tau} \), that is, it faces a second-price auction.

Since - regardless of whether a group is targeted or not - the amount that it pays for winning is independent of its bid, it is weakly dominant for both groups to bid their willingness-to-pay. Given that, the only possible equilibrium play of the politician is to target the advantaged group \( \tau = -d \) and to bid \( p_P = W_{-d} \).

3. Agenda Setting

Transfers are determined by bargaining between the three agents. Specifically we consider the following game-form:
1. The politician chooses a group \( a \in \{S, L\} \) to affiliate with. The group chosen is called the agenda setter.
2. The agenda setter may opt out and the status quo remains, or may propose an agenda for the amount of transfer \( 0 \leq V_{-a} \leq \nu N_{-a} \) to be paid by the other group.
3. All three agents submit bids \( p_k \). The politician designates a target group \( \tau \in \{a, -a\} \) for his bid. The bids should satisfy \( 0 \leq p_{-a} \leq (1 - c)N_{-a} \), \( 0 \leq p_a \leq (1 - c)N_a + \psi \beta V_{-a} \) and \( 0 \leq p_P \leq (1 - c)N_{\tau} + \psi \tau \beta V_{-\tau} \).
4. If the two groups bid zero the status quo remains. Otherwise, with probability \( \alpha \), the price to be paid to the politician is his bid \( p_P \) and with probability \( 1 - \alpha \), it is the lowest bid. When the price is the bid of the politician \( \tau \) wins and pays \( p_P \) if and only if his bid is at least that of the politician: \( p_\tau \geq p_P \); otherwise the politician is not paid and group \( -\tau \) wins. When the price is the lowest bid the highest bidder wins, and in case of a tie the agenda setter wins; in both cases the politician is paid the price by the winner.
5. If the non-agenda setter wins the status quo remains. If agenda setter wins the transfer is made.

The notion of equilibrium is subgame perfect equilibrium with three mild refinements: (1) no player plays a weakly dominated strategy, (2) if the agenda setter is indifferent to submitting a bid she does not do so, and (3) if the politician is indifferent between targeting the two groups she targets the agenda setter. The first assumption is self-explanatory and leads to the groups bidding their value. The second can be viewed as a lexicographic preference for not bidding that arises.
from a small cost of preparing a bid. The third can be viewed as a mild ability of the politician to commit to the group to which she affiliates.

3.1. Agenda Setting Equilibrium

We say that the agenda setter $a$ has a winning agenda if there is a feasible choice $V_{-a} \leq \nu N_{-a}$ for which the agenda setter bid/willingness to pay is greater than that of the non-agenda setter $W_a > W_{-a}$. The optimal agenda is a winning agenda for which the difference in willingness to pay is the greatest, since the net utility of the agenda setter is increasing in that difference and equal to $(1 - \alpha)(W_a - W_{-a})$. Notice that in case of equal willingness to pay the agenda setter earns zero, so will choose to opt out. In Appendix 1 we prove

Theorem 2. If the large group has a winning agenda the optimal agenda is $\nu N_S$; if the small group has a winning agenda its optimal agenda is $cN_L$.

In the fungible case: if $\beta \leq N_S/N_L$ both groups opt out; otherwise the politician affiliates with the small group.

In the non-fungible case: when $\beta\nu > (1 - c) + cN_L/N_S$ the politician affiliates with the large group; when $\nu N_S/N_L < \beta\nu \leq (1 - c) + cN_L/N_S$ the politician affiliates with the small group; and otherwise both groups opt out. The bids are given in the following table:

<table>
<thead>
<tr>
<th>Table 1: Equilibrium Bids/Willingness to Pay</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fungible case,</strong> small group sets agenda $cN_L$</td>
</tr>
<tr>
<td><strong>Non-fungible case</strong></td>
</tr>
<tr>
<td><strong>Bid by S</strong> $\min{(1 - c)N_S, \beta c N_L - cN_S} &gt; 0$</td>
</tr>
<tr>
<td><strong>Bid by L</strong> $0$</td>
</tr>
</tbody>
</table>

If the transfer is too inefficient (\beta small) the status quo is maintained. The overall message is that fungible issues or low stakes (\nu small) favor the small group while non-fungible issues with high stakes favor the large group. When it wins the small group is not too “greedy” in the sense that it asks only for $cN_L$ while it could ask for as much as $N_L$; by contrast the large group, unlike the small group, when it wins asks the most it can possibly get. Moreover, amount that the small group wins $cN_L$ is increasing in the fixed cost $c$. Notice too that only relative group size matters, the absolute size of groups is irrelevant.  

The reader may notice that the result also implies that holding all else fixed in the non-fungible case, if the small group is small enough relative to the large group it will win. This may seem to go against the main theme of the paper, but remember we consider only two groups and take group sizes as given. We cannot say, therefore, that it would not be advantageous for several groups to join forces. Moreover, if the cost of forming a group is non-null, as in Mitra (1999), the groups we actually observe cannot be too small. 

---

8The reader may notice that the result also implies that holding all else fixed in the non-fungible case, if the small group is small enough relative to the large group it will win. This may seem to go against the main theme of the paper, but remember we consider only two groups and take group sizes as given. We cannot say, therefore, that it would not be advantageous for several groups to join forces. Moreover, if the cost of forming a group is non-null, as in Mitra (1999), the groups we actually observe cannot be too small.
4. Blackmail

We have assumed that the politician affiliates with one group before bids are submitted. But since the affiliation is valuable to the groups, why do they not offer to pay the politician to affiliate with them? Or to put it differently - why does not the politician accept bids from both groups then decide with whom to affiliate. Intuition suggests that this may be lucrative for the politician: by telling each group “if you do not give me a good bid I will pass the other group’s agenda and you will be really sorry” each group will be willing to pay a great deal. On the other hand since groups may wind up paying more than the agenda is worth to them - the value to themselves plus the value to the other group - engaging in this type of political blackmail may be dangerous for the politician: the groups do not much like this and may collude to get rid of the politician. Here we consider a simple model that allows for the possibility both of blackmail and of removal of the politician.

We now elaborate the lobbying game form as follows:

1. The politician either chooses a group \( a \in \{S, L\} \) to affiliate with (called as before the agenda setter) or he does not - in which case we say he chooses to be “opportunistic”.

2. Each group chooses either to attempt removal of the politician, to block removal of the politician or to remain neutral. To attempt removal or block removal incurs a small cost which we model as a lexicographic preference for remaining neutral in case of indifference.

3. If one group attempts removal of the politician and the other group does not block it the politician is removed, and everyone gets 0. Otherwise the game continues.

In case the politician is not removed the game continues:

4. If the politician has affiliated with a group the agenda setting game of the previous section is played.

5. If the politician has chosen to be opportunistic each group \( k \in \{S, L\} \) proposes an agenda consisting of a transfer \( 0 \leq V_{-k} \leq \nu N_{-k} \) to be paid for by the other group and submits a bid \( 0 \leq p_k \leq (1 - c)N_k + \psi \beta V_{-k} \). The politician designates a target group \( \tau \) and submits a bid \( 0 \leq p_P \leq (1 - c)N_\tau + \psi \beta \nu N_{-\tau} \).

6. If both groups bid zero the status quo remains. Otherwise, with probability \( \alpha \) the price is the bid of the politician and with probability \( 1 - \alpha \) it is the lowest bid. When the price is the bid of the politician \( \tau \) wins if \( p_\tau \geq p_P \); if \( p_\tau < p_P \) politician is not paid and \( -\tau \) wins. When the price is the lowest bid the highest bidder wins and in case of a tie the target group wins.

7. The winning group has their agenda implemented and pays the price.

The notion of equilibrium is that in each subgame we must have Nash equilibrium in weakly undominated strategies. We use two additional refinements in addition to the lexicographic tie-breaking rule about removing the politician already mentioned. The first has to do with bids. In the blackmail subgame demands and bids are submitted simultaneously. This means that weak dominance has no bite. Recall that in a second-price auction there are many equilibria. For example: the loser might bid zero and the winner bid the loser’s willingness to pay - in which case the winner gets the item for free. This is ordinarily ruled out through weak dominance. We
cannot do so in the blackmail game, so we instead assume that the equilibrium must be robust to a small probability of the other bid being random which we model as assuming that conditional on the equilibrium demand of the other group and the own demand each group does in fact bid their willingness to pay and the politician targets the group with the highest willingness to pay and bids that amount. Second, the loser of the auction is indifferent to the demand. This raises an issue similar to that in bidding: there are many equilibria some in which the loser demands little and some in which the loser bids much. This is not reasonable if there is a small chance that your demand - perhaps being recognized as being just by the political system - will be accepted. In that case when indifferent you should also make the highest possible demand in case it should be accepted. So as an additional refinement we assume that when indifferent the highest demand must always be made. These refinements lead to a unique outcome.

Note incidentally that we assume that the politician does not submit his bid after the demands are known, which would put the winner in the position of a Stackelberg leader being able to shave his demand to pay less to the politician. The politician has incentive to commit to his bid simultaneously to avoid this.

One issue: why not assume that the game is sequential move? That is, first demands are submitted then observing the demand of the other group bids are submitted. However, from a descriptive point of view it seems to us most likely that given the politician is taking bids, the groups say “here is what I want and here is what I will pay” rather than “here is what I want, and we’ll argue later over what I’ll pay.” Second, as we will see, in the simultaneous move game the politician gets the most possible in any extensive form, hence has no reason to prefer a different mechanism. We should also acknowledge that with the refinements described above the simultaneous move game is much easier to analyze than the sequential move game.

4.1. Blackmail Equilibrium

In Appendix 2 we prove

**Theorem 3.** The only cases in which the politician chooses to be opportunistic are in the non-fungible case if

\[
\beta \nu > (1 - c) + c \frac{N_L}{N_S} + \alpha (1 - c) \left[ \frac{N_L}{N_S} - 1 \right]
\]

in which case the large group wins; and in the fungible case if \( \beta > 1 - c \) and

\[
(1 - \alpha) [\beta - (1 - c)] \left[ \frac{N_L}{N_S} - 1 \right] > 1
\]

in which case the small group wins. When the politician is opportunistic each group proposes the maximum possible \( V_{-k} = \nu N_{-k} \) and bids the maximum possible \( (1 - c) N_k + \psi \beta \nu N_{-k} \). In the remaining cases the politician affiliates with a group and the result is as in Theorem 2.

Overall the result is not terribly different than the main result - with non-fungible prizes favoring the large group and fungible prizes favoring the small group. It is interesting in the non-fungible...
case to contrast the condition for blackmail and the large group winning

$$\beta \nu > (1 - c) + c \frac{N_L}{N_S} + \alpha (1 - c) \left[ \frac{N_L}{N_S} - 1 \right]$$

with the condition for the large group winning when there is no blackmail

$$\beta \nu > (1 - c) + c \frac{N_L}{N_S}.$$ 

We see that the former condition always implies the latter, so that the possibility of blackmail does not additionally favor the large group, but rather when the stakes $\beta \nu$ are moderate the large group wins and is not blackmailed, but when the stakes are large enough the politician will turn to blackmail. The less effective is the politician at bargaining (the smaller is $\alpha$) the lower the stakes for which the politician will turn to blackmail. Put differently, blackmail by the politician enables him to attain a greater share if he is an ineffectual bargainer - but since he cannot commit to a modest demand, blackmail is only useful if he is unable to make a large demand. Basically the same circumstances which favor the large group are also likely to lead to blackmail.

By contrast blackmail is not so likely over fungible issues. If $\beta < 1 - c$ it will never occur. Otherwise it is large values of $N_L/N_S$ which both favor the small group without blackmail and are likely to lead to blackmail.

5. Discussion

The model has several implications. First, fungible prizes are more favorable to small groups than non-fungible prizes. Second, a small group should not be too greedy in agenda setting. Third, a higher fixed cost is more favorable to the small group. The world is a complicated place with many issues and in addition to lobbying where there are fixed costs that favor smaller groups, political decisions are also influenced by voting which as Levine and Mattozi (2016) show is more favorable to large groups. Moreover many political decisions are made by courts, and while these decisions are influenced by political calculations and lobbying the mechanism does not match that described in our model. Never-the-less it is useful to ask whether the complicated world reflects in a broad sense the general implications of the model. Some rough back-of-the-envelope calculations show that there is promise in this direction.

One place to look is to see how political decisions reflect public opinion. Do decisions favoring a group have substantial public support or limited public support? The model suggests that for fungible prizes widespread public support is not so important while for non-fungible prizes it is. Two significant non-fungible issues have been civil rights for blacks and civil rights for gays. In both cases significant advances have occurred when public support has become widespread.

Long term polling by Gallup\(^9\) asks about willingness to vote for a black person for President.

which may be taken as an indicator of general attitudes towards civil rights. In 1958 only 38% responded positively. By 1959 this rose to about 50% where it remained until about 1963 when it rose to 60%, dipped briefly in 1967 and then rose steadily to about 95% by the year 2000. Civil rights have been largely reflective of these public attitudes towards blacks. The “separate but equal” doctrine permitting racial discrimination in a variety of domains, but most significantly education was established in 1896 in Plessy v. Ferguson, and although it was repudiated in law in 1954 in Brown v. Board of Education, desegregation was not immediately implemented: George Wallace’s stand in the school house door taking place in 1963 - well after turn of public opinion, and the landmark legislation was the 1964 Civil Rights Act. Political action occurred only when the size of the group supporting civil rights became large. We find a similar story with respect to gay civil rights. The Pew Research center finds that in 2003 only 32% of Americans favored same-sex legal marriage - this increased steadily, reaching parity by 2011. From 1975 to 2000 various states and the Federal government passed a series of laws banning gay marriage. By 2009 only seven states had recognized gay marriage. This rose to thirteen by 2013 and to fifty with the Supreme court decision in 2015. Again the recognition of rights - non-fungible as it is - seems to have followed public opinion and indeed, majority public opinion.

By contrast if we look at an important fungible issue - farm subsidies - we see that support for large farms which receive the bulk of subsidies has only 15% popular support. While there are only about 2 million farms in the US it is not just farmers that benefit from farm subsidies. An upper bound should be the rural population of the US of about 60 million people or roughly 20 million households out of the 120 million U.S. households - which is also about 15%. So we see that a minority of roughly 15% is effective at getting a fungible prize from the remaining 85%. This number 15% is similar to the fraction of the population that is either black or gay - yet those groups have been ineffectual in realizing the non-fungible prize of civil rights until they achieved the support of roughly a majority.

Another way to get a handle on the effectiveness of small groups in competing for fungible prizes to look at how many of them there are. For example, the Italian yellow pages for example list 21,788 associations “sindacali e di categoria”. These groups - largely trade unions - have two main functions: they negotiate with firms over contracts and they lobby government for favors. If we look at the geographical distribution of these groups we can get an idea of the relative importance of these two functions. In Rome there are almost 1500 groups, in Milan around 1000 and in Bologna about 400. Looking at GDP, we see that Lombardia (the region of Milan) produces twice that of Lazio (where Rome is), and Emilia Romagna (containing Bologna) 20% less than Lazio. Why then does Lazio have 50% more groups than Lombardia despite having half the GDP and four times the number of groups as Emilia despite having similar GDP? It is natural to think that the reason is that Lazio contains Rome where Italian governmental functions are centralized. So it

---

10 pewforum.org/2015/07/29/graphics-slideshow-changing-attitudes-on-gay-marriage
11 www.worldpublicopinion.org/pipa/articles/brunitedstatescanadara/602.php
12 Source ISTAT. Figures for 2014 in millions of Euros are: Lazio 166, Lombardia 313, Emilia 130.
seems that perhaps as many as 1000 of these groups are primarily lobbying for fungible benefits from the Italian government and bureaucracy. Needless to say this is a large number of lobbying groups and all represent a relatively small number of people. In a similar vein we notice that in the U.S. there are around 10,000 registered lobbyists.\textsuperscript{13}

It is the presence of a fixed cost per member that prevents a large group from being effective. But is the level of fixed cost needed to explain the data plausible? Since data are readily available let us examine farm subsidies in the U.S.\textsuperscript{14} As we observed, $N_L$ is about 85\% and $N_S$ is about 15\% of households, so that $N_L$ is indeed much larger than $N_S$. From the U.S. budget farm subsidies run about $20$ billion per year, or, with 100 million non-farm households about $200$ per household. Now when the small group wins (taking the base model for reference) we have $V_L = cN_L$, whence $c = V_L/N_L = $200. To put this in perhaps more meaningful units, we observe that annual per capita income in the U.S. is about $50,000$ per year and the labor force is about half the population, so that income per worker is about $100,000$. Hours worked per worker per year are about 1700, meaning that the hourly income per worker is about $60$. So $200$ per household translates into an opportunity cost per person for participating in a group of roughly half a working day per year. This seems a plausible number.

The model also has a more refined implication that the amount of the benefit accruing to politicians in the form of bribes depends on $\alpha$. If politicians have little bargaining power then $\alpha$ is small and they get little. If they have a lot of bargaining power the should be able to get (in the case of farm subsidies) nearly $20$ billion per year. There are several ways of getting ballpark numbers about the size of bribes. Here is one piece of evidence

"The net worth of the 70 richest delegates in China’s National People’s Congress, which opens its annual session on March 5, rose to 565.8 billion yuan ($89.8 billion) in 2011, a gain of $11.5 billion from 2010, according to figures from the Hurun Report, which tracks the country’s wealthy. That compares to the $7.5 billion net worth of all 660 top officials in the three branches of the U.S. government.”\textsuperscript{15}

One estimate of the annual value of bribes received by top Chinese officials is the increase in their wealth - $11.5$ billion. China currently is of similar size in total real GDP as the U.S. Suppose that the portion of the economy subject to discretionary transfers in China is similar in size to the U.S. agricultural sector. Then $11.5$ billion in bribes is consistent with the idea that U.S. agricultural subsidies are commensurate with the overall size of favors paid by government officials - this would imply a substantial $\alpha$ although - since there are sectors other than agriculture - considerably less than 50\%. By contrast, if we assume that wealth among top U.S. officials increased as much as that of Chinese officials, after accounting for the fact that U.S. official are much less wealthy, we estimate the value of bribes by top U.S. government officials at about $1$ billion. If we look at

\begin{footnotesize}
\begin{itemize}
\item \textsuperscript{13}https://www.opensecrets.org/lobby/
\item \textsuperscript{14}This data can be conveniently found in the St. Louis Fed FRED.
\item \textsuperscript{15}Bloomberg News, February 26, 2012: “China’s Billionaire People’s Congress Makes Capitol Hill Look Like Pauper.
\end{itemize}
\end{footnotesize}
direct payments to U.S. politicians in the form of campaign contributions we find from Floating Path (2012) that about $1 billion is contributed to presidential campaigns. Since these take place every four years, but there are also congressional, state and local elections which are less costly but more frequent, we can take this also as a ballpark estimate of the value of bribes accruing to U.S. politicians. This suggests that in the U.S. $\alpha$ is quite small, less than 5%.

Turning to the bigger picture: the theory suggests that cost of inefficiency $(1 - \beta)eN_L$ depends on fundamentals and not on bargaining power $\alpha$ which simply determines how much politicians walk away with. Seen this way, while the evidence is in favor of a much higher $\alpha$ in China than in the U.S. - the theory says that from an allocational point of view - the amount of inefficient transfers - it may not make much difference. Notice, by the way, the fact that $\alpha$ is clearly much higher in China than the U.S. is suggestive that in societies that are more extractive in the sense of Acemoglu and Robinson (2012) politicians have more bargaining power.

Finally, the reason that lobbyists should not be too greedy is to avoid provoking the larger group into paying the fixed cost. We have seen this happen - for example, in the case of Stop Online Piracy Act (SOPA) in the U.S. This was an effort by copyright lobbyists to use Federal power to enforce their copyright claims. It had 31 sponsors in the U.S. House of Representatives. It was also an overreach. A large number of groups, including both technology firms and most notably Wikipedia launched a lobbying campaign against the bill. Despite the 31 sponsors the bill then died in committee and never came to the floor of the House.

Going back to Mancur Olson: his original idea that small groups are stronger does not take account of the fact that groups face budget constraints. These constraints vary considerably depending on whether the transfer groups seek is fungible or not - and this has a big impact on group behavior. When the prize is fungible small groups have a significant advantage over large ones as Olson suggests. When the prize is not fungible larger groups are advantaged provided they can extract enough value from the small group. This may explain the apparent paradox that when it comes to special financial favors small groups seem very effective, but when it comes to large non-financial issues - such as minority rights - large groups are more effective.

References


http://www.floatingpath.com/2012/02/20/buying-elections-newer-trend/


"Voter Participation with Collusive Parties" [11/01/15] (with A. Mattozzi)


Appendix 1: Proof of the Main Theorem

**Theorem.** [Theorem 2 in the text] If the large group has a winning agenda the optimal agenda is $\nu N_S$; if the small group has a winning agenda its optimal agenda is $c N_L$.

In the fungible case: if $\beta \leq N_S/N_L$ both groups opt out; otherwise the politician affiliates with the small group.

In the non-fungible case: when $\beta \nu > (1-c) + c N_L/N_S$ the politician affiliates with the large group; when $\nu N_S/N_L < \beta \nu \leq (1-c) + c N_L/N_S$ the politician affiliates with the small group; and otherwise both groups opt out. The bids are given in the following table:

<table>
<thead>
<tr>
<th>Fungible case, small group sets agenda $c N_L$</th>
<th>Bid by $S$</th>
<th>$c(\beta N_L - N_S) &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-fungible case</td>
<td>Bid by $L$</td>
<td>0</td>
</tr>
</tbody>
</table>

$\beta \nu > (1-c) + c N_L/N_S$  
$\nu N_S/N_L < \beta \nu \leq (1-c) + c N_L/N_S$  
$\beta \leq N_S/N_L$  
$\beta > N_S/N_L$

**Proof.** We study optimal agendas and the politician’s choice. Instead of willingness to pay which involves constraints based on the size of the group, whether the group is an agenda setter, and the fungibility of the prize, it is useful to ignore the constraints and consider the desire to pay. For the agenda setter $a$ this is $\beta V_a - c N_a$ and for the non-agenda setter this is $V_a - c N_a$. Both are increasing in $V_a$ but the desire of the non-agenda setter increases more rapidly. Define the crossover point $\hat{V}_a \equiv c(N_a - N_a)/(1 - \beta)$ as the point where the two desires are equal. To the right of this point the non-agenda setter has a higher desire. This means that if the constraints on his ability to pay do not bind he is at least as willing to pay as the agenda-setter. To the left of the crossover point the same is true of the agenda setter. We can also define the payoff point $\tilde{V}_a \equiv c N_a/\beta$ as the point where the desire of the agenda setter is zero. To the right of this point the agenda setter may possibly wish to set an agenda, to the left of this point never.
We first analyze the right of the crossover point, that is, $V_{-a} > \hat{V}_{-a}$. Here there is a winning agenda only if the constraint binds on the non-agenda setter, that is $W_{-a} = (1-c)N_{-a}$. Moreover since the bid of the non-agenda setter cannot increase once the constraint binds the agenda setter should propose the highest possible agenda, that is $V_{-a} = \nu N_{-a}$. For this to be a winning bid the willingness of the agenda setter to pay must be strictly greater than $(1-c)N_{-a}$ which is impossible for $a = S$ in the non-fungible case and otherwise true if and only if $\beta \nu N_{-a} - c N_{a} > (1-c)N_{-a}$ which is equivalent to $\beta \nu > (1-c) + c N_{a}/N_{-a}$.

In case $a = L$ the crossover point $\hat{V}_{-a} < 0$ and so the large group has a winning agenda if and only if $\beta \nu > (1-c) + c N_{L}/N_{S}$. Note that in the fungible case this is impossible.

In the case of the small group the crossover point is positive, so we must analyze the left of the crossover point. The small group will not propose any agenda below the payoff point $\hat{V}_{L} = c N_{S}/\beta$. There are two cases depending on which of $\hat{V}_{L}$ or $\hat{V}_{L}$ is larger. Notice that $\hat{V}_{L} \leq \hat{V}_{L}$ may be written as $\beta \leq N_{S}/N_{L}$.

If $\hat{V}_{L} \leq \hat{V}_{L}$ then there is no winning agenda for the small group below the crossover point, so the small group is in the same boat as the large group: it has a winning agenda if and only if the transfer is fungible $\beta > (1-c) + c N_{S}/N_{L}$. However this is inconsistent with $\beta \leq N_{S}/N_{L}$ so the small group has no winning agenda.

For $\beta \leq N_{S}/N_{L}$ we now have the complete picture. In the non-fungible case the small group has no winning agenda, and the large group has a winning agenda if and only if $\beta \nu > (1-c) + c N_{L}/N_{S}$. Hence either $\beta \nu \leq (1-c) + c N_{L}/N_{S}$ and no agenda is submitted, or $\beta \nu > (1-c) + c N_{L}/N_{S}$ in which case the large group submits the winning agenda $V_{S} = \nu N_{S}$. In the fungible case neither group has a winning agenda.

We now analyze the remaining case $a = S$ for $\beta > N_{S}/N_{L}$ that is $\hat{V}_{L} < \hat{V}_{L}$. If the highest feasible bid lies below $\hat{V}_{L}$ that $\nu N_{L} \leq \hat{V}_{L} = c N_{S}/\beta$, or equivalently, $\beta \nu \leq c N_{S}/N_{L}$ then there is no winning agenda to propose. Otherwise to small group is willing to propose an agenda to the right of the payoff point. Observe that the large group bids zero if and only if $V_{L} \leq c N_{L}$ and note that $c N_{L} > \hat{V}_{L} = \beta c N_{S}$. So there there is no point in proposing an agenda less than $c N_{L} = \min\{c N_{L}, \nu N_{L}\}$. Since $c < \nu$ so that larger agendas are feasible then the willingness to pay of the large group rises faster than the small group as long as the large group is not constrained. Hence either the small group should propose $c N_{L}$ or should propose enough that the constraint binds, in which case it is optimal to propose $\nu N_{L}$. However, in the non-fungible case if the constraint binds on the large group then the small group cannot win the bidding. In the fungible case proposing $c N_{L}$ gives $W_{S} - W_{L} = \beta c N_{S} - c N_{S}$ and proposing $\nu N_{L}$ gives $W_{S} - W_{L} \leq \beta N_{L} - c N_{S} - (1-c)N_{L}$. It can be checked the the former is always larger than the latter, so that in all cases the optimal winning agenda for the small group is $c N_{S}$.

That covers the fungible case as we already know that the large group has no winning agenda in that case. In the non-fungible case if $\beta \nu \leq (1-c) + c N_{L}/N_{S}$ the large group has no winning agenda so the politician affiliates with the small group provided $\beta \nu > c N_{S}/N_{L}$ so that the small group has a winning agenda. Otherwise the large group will propose the agenda $V_{S} = \nu N_{S}$ resulting in the
politician receiving \((1 - \alpha)(1 - c)N_S + \alpha \min\{\beta \nu N_S - cN_L, (1 - c)N_L\} \geq (1 - c)N_S\) while the agenda of the small group of \(V_L = cN_L\) gives the politician \(\alpha \min\{\beta cN_L - N_S, N_S(1 - c)\} \leq \alpha N_S(1 - c)\). This implies that the large group agenda is strictly preferred by the politician. \(\square\)

Appendix 2: Proof of the Blackmail Theorem

**Theorem.** [Theorem 3 in the text] The only cases in which the politician chooses to be opportunistic are in the non-fungible case if

\[
\beta \nu > (1 - c) + c \frac{N_L}{N_S} + \alpha (1 - c) \left[ \frac{N_L}{N_S} - 1 \right]
\]

in which case the large group wins; and in the fungible case if \(\beta > 1 - c\) and

\[
(1 - \alpha) [\beta - (1 - c)] \left[ \frac{N_L}{N_S} - 1 \right] > 1
\]

in which case the small group wins. When the politician is opportunistic each group proposes the maximum possible \(V_{-k} = \nu N_{-k}\) and bid the maximum possible \((1 - c)N_k + \psi \beta \nu N_{-k}\). In the remaining cases the politician affiliates with a group and the result is as in Theorem 2.

We prove this through a series of lemmas.

**Lemma 1.** If the politician is opportunistic both groups propose the maximum possible, that is group \(k\) demands \(V_{-k} = \nu N_{-k}\)

**Proof.** The game is simultaneous move. Given the bidding/demand strategy of the other group and the own bid utility is weakly increasing in the demand that is made. That is, because the game is simultaneous move a higher demand has no effect on the bids or demands of the other group, so you might as well ask for as much as you can. In case of indifference we have assumed you propose the maximum possible. \(\square\)

**Corollary 1.** The losing group strictly prefers the status quo to a opportunistic politician.

**Proof.** Since the losing group gets \(-\nu N_{-k}\) if the politician is opportunistic and zero at the status quo. \(\square\)

**Lemma 2.** If the politician is opportunistic both groups in equilibrium bid the maximum possible, that is, group \(k\) bids \((1 - c)N_k + \psi \beta \nu N_{-k}\) where recall that \(\psi\) reflects whether the prize is fungible or not.

**Proof.** Since we have assumed both groups bid their willingness to pay given the proposals, and we know the equilibrium proposals from Lemma 1, this is just a matter of showing that the maximum possible bid \((1 - c)N_k + \psi \beta \nu N_{-k}\) is less than or equal to the desire to pay \(\nu \beta N_{-k} - cN_k + \nu N_k\), or, rearranging that inequality \((1 - \nu)N_k \leq (1 - \psi) \nu \beta N_{-k}\). Since by assumption \(\nu \geq 1\) this inequality must hold. \(\square\)
Non-fungible case.

**Lemma 3.** In the non-fungible case the opportunistic politician remains in office if and only if

\[ \beta \nu \geq (1 - c) + c \frac{N_L}{N_S} + \alpha (1 - c) \left[ \frac{N_L}{N_S} - 1 \right] \]

in which case the large group wins. The politician chooses opportunism if he is able to remain in office.

**Proof.** The large group equilibrium bid is \((1 - c)N_L\) and the small group equilibrium bid is \((1 - c)N_S\) so the winner is the large group who pays \((1 - \alpha)(1 - c)N_S + \alpha (1 - c)N_L\) and gets \(\beta \nu N_S - cN_L\). Hence the net utility of winning for the large group is \(\beta \nu N_S - cN_L - (1 - c)\left[ (1 - \alpha)N_S + \alpha N_L \right]\). When this is strictly negative the politician is removed the small group being indifferent. The condition for the politician to remain in office is therefore that this be non-negative, which may be rewritten as

\[ \beta \nu \geq c \frac{N_L}{N_S} + (1 - c)(1 - \alpha + \alpha \frac{N_L}{N_S}) = (1 - c) + c \frac{N_L}{N_S} + \alpha (1 - c) \left[ \frac{N_L}{N_S} - 1 \right]. \]

Finally, if he is opportunistic and not removed the politician gets \((1 - c)[(1 - \alpha)N_S + \alpha N_L]\); if not, in the range of \(\beta \nu\) above he affiliates with the large group and gets (see last paragraph of the proof of Theorem 2) \((1 - \alpha)(1 - c)N_S + \alpha \min\{\beta \nu N_S - cN_L, (1 - c)N_L\}\) which is less or equal than what he gets if opportunistic. Hence he will choose to be opportunistic if he is not removed by doing so.

**Lemma 4.** In the fungible case the opportunistic politician remains in office if and only if \(\beta > 1 - c\) and

\[ (1 - \alpha)[\beta - (1 - c)](N_L - N_S) - N_S > 0 \]

in which case the small group wins. The politician chooses opportunism if he is able to remain in office.

**Proof.** In the fungible case recall that \(\nu = 1\) so that the large group equilibrium bid is \((1 - c)N_L + \beta N_S\) and the small group equilibrium bid is \((1 - c)N_S + \beta N_L\). The condition that the small group equilibrium bid is larger is \(1 - c - \beta < 0\) or \(\beta > 1 - c\). There are three cases.

Case \(\beta > 1 - c\): the small group wins, getting \(\beta N_L - cN_S\) and paying \(\alpha [(1 - c)N_S + \beta N_L] + \)
\[ (1 - \alpha) [(1 - c)N_L + \beta N_S]. \] This gives a net utility of

\[
\begin{align*}
\beta N_L - c N_S - \alpha \beta N_L + (1 - c)N_S - (1 - \alpha) \beta N_L + \beta N_S \\
= [\beta - \alpha \beta - (1 - \alpha)(1 - c)]N_L - [c + \alpha(1 - c) + (1 - \alpha)\beta]N_S \\
= (1 - \alpha)[\beta - (1 - c)]N_L - [1 - (1 - c) + \alpha(1 - c) + (1 - \alpha)\beta]N_S \\
= (1 - \alpha)[\beta - (1 - c)]N_L - [1 - (1 - \alpha)(1 - c) + (1 - \alpha)\beta]N_S \\
= (1 - \alpha)[\beta - (1 - c)](N_L - N_S) - N_S 
\end{align*}
\]

The leader is not removed when this expression is non-negative. As can be seen by inspection this happens if: \( \alpha \) is small, \( \beta \) is large and \( \frac{N_L}{N_S} \) is large.

Case \( \beta < 1 - c \): the large group wins. The net utility of the large group in this case is

\[
\begin{align*}
\beta N_S - c N_L - \alpha \beta N_S + (1 - c)N_L - (1 - \alpha) \beta N_S + \beta N_L \\
= (1 - \alpha)\beta N_S - c N_L - \alpha(1 - c)N_L - (1 - \alpha)(1 - c)N_S - (1 - \alpha)\beta N_L \\
= (1 - \alpha)[\beta - (1 - c)]N_S - [c + \alpha(1 - c) + (1 - \alpha)\beta]N_L \\
< (1 - \alpha)[\beta - (1 - c)]N_S < 0 
\end{align*}
\]

last inequality from \( \beta < 1 - c \). Consequently in this case the politician always prefers to affiliate with a group.

Case \( \beta = 1 - c \). In this case whoever wins (the targeted group) has negative utility. Indeed if the small group wins it gets

\[
\begin{align*}
\beta N_L - c N_S - (1 - c)(N_S + N_L) \\
= (1 - c)N_L - c N_S - (1 - c)(N_S + N_L) = -N_S 
\end{align*}
\]

analogously for the large group. Politician will therefore not choose to be opportunistic in this case.

Finally we observe that if the politician is opportunistic and not removed he gets \( \alpha [(1 - c)N_S + \beta N_L] + (1 - \alpha) [(1 - c)N_L + \beta N_S] \) while if he is not he affiliates with small group and gets \( \alpha c(\beta N_L - N_S) < \alpha (\beta N_L - N_S) < \alpha [(1 - c)N_S + \beta N_L] \) he prefers to be opportunistic if he can get away with it. \( \square \)