Policy Spillovers and Synergies in a Monetary Union

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Abstract
We provide a general equilibrium framework for analyzing the effects of supply and demand side policies, and the potential synergies between them, in an asymmetric monetary union that faces a liquidity trap and a slow deleveraging process in its ‘periphery’. We find that the joint implementation of pro-competition structural reforms in the periphery, a fiscal expansion in the ‘core’, and forward guidance about the three policies: forward guidance re-inforces the expansionary effects of country-specific policies, and the latter in turn improve the effectiveness of forward guidance.

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1 Introduction

The global financial crisis initiated in 2008 triggered a deep and prolonged setback for aggregate demand in all major industrialized economies, paving the way for persistently low inflation rates. In the European Monetary Union (EMU), the crisis has also revealed forcefully the imbalances and efficiency gaps across its member states. In most of the so-called ‘periphery’, the combination of high indebtedness, deleveraging and widespread dysfunctionalities in labor and product markets is feeding fears of a long-lasting scenario of weak and fragile growth, with adverse consequences for the entire union.

Against this context, many voices are calling for a simultaneous implementation of supply and demand side policies within the EMU. On the supply side, the removal of inefficiencies and the enhancement of market competition are invoked as the only lever available to the periphery so as to regain its competitiveness vis-à-vis the rest of the EMU. On the demand side, given that the periphery lacks sufficient fiscal space to stimulate domestic demand and that the European Central Bank is already constrained by the zero lower bound (ZLB), the attention has shifted towards the role of non-standard monetary policies and the possibility of expansionary fiscal measures in the ‘core’. Moreover, the policy debate has progressively moved towards two closely interlinked areas: first, the potential for policy-induced spillovers across countries within EMU and, second, the likely complementarities or synergies between non-standard monetary policy, structural reforms, and fiscal policy.\(^1\)

The issue of fiscal spillovers in a monetary union has recently been the subject of formal quantitative analysis, e.g. by Erceg and Lindé (2013) and Blanchard, Erceg and Lindé (2014). However, relatively less is known about cross-country spillovers induced by the adoption of structural reforms in one part of the union. Critically, even less is known about the potential synergies between such structural reforms, fiscal expansion in the rest of the union, and non-conventional monetary policies at the union level.

In this paper, we address these issues in the context of a model of an asymmetric

\(^1\)See, for example, Draghi (2014, 2015), European Commission (2014) and IMF (2014).
two-country (‘core’ and ‘periphery’) monetary union. In both countries, households and firms borrow long-term subject to collateral constraints. We construct a baseline scenario aimed at capturing key features of the current macroeconomic landscape in the EMU. First, the periphery is assumed to be hit by an adverse financial shock that tightens the collateral requirements on the loans to households and firms. This shock, combined with collateral constraints and long-run debt, gives rise to a protracted and costly process of deleveraging in the periphery with implications for the monetary union as a whole. Second, a union-wide demand shock causes a reduction in union-wide inflation that is large enough to drive the monetary authority’s nominal interest rate towards its ZLB. Both shocks combine to produce a long-lasting recession and persistently low inflation in the currency union as a whole.

Against this background, we analyze the effects of two types of country-specific macroeconomic policies: structural reforms in the periphery (consisting of reductions in price and wage-setters’ monopolistic rents), and a temporary increase in government spending in the core. We show that the cross-country spillovers of such policies depend critically on the incidence of the ZLB. Outside of the ZLB, structural reforms have a positive output effect on the periphery already on impact, but produce a slight positive impact on the core too, thanks to the monetary accommodation of the ensuing disinflationary pressures. A government expenditure expansion in the core, on the contrary, aggravates the recession in the periphery, as the central bank tightens its policy rate in response to the inflationary pressures coming from the core. By contrast, in a liquidity trap the sign of the previous cross-country spillovers are reversed. First, reforms in the periphery, which remain expansionary for the latter although less so, produce a negative (though relatively small) effect on the core. Similarly, absent the previously discussed monetary tightening, a fiscal expansion in the core produces sizeable positive spillovers for the periphery.

We next consider the possibility that the monetary authority follows a ‘forward guidance’ policy with the aim of raising area-wide GDP and inflation while in the liquidity trap. In particular, we analyze the case in which the central bank can credibly commit to keeping the interest rate at zero for two more quarters than what its standard rule would dictate. Such a policy is found to have positive effects, of a
similar magnitude, in the output of both regions. This last result is remarkable because, during the deleveraging phase (which lasts longer than the liquidity trap in our simulations), the credit flow in the periphery is frozen, such that credit-constrained agents are not exposed to the usual intertemporal consumption substitution channel of forward guidance. Thus, in our setup the effectiveness of forward guidance on the periphery seems to be more related to other transmission channels, such as the core-periphery trade channel and net worth effects on the balance-sheets of deleveraging agents.

We then quantify the synergies between the three policies, an exercise for which our nonlinear model (and our fully nonlinear solution method) is well suited. We find that the short-run expansionary effects of national stimulus measures (reforms in the Periphery and fiscal expansion in the Core) increases by a sizable amount when in parallel the monetary authority implements a policy of forward guidance. Conversely, the expansionary impact of forward guidance is largely enhanced when at the same time each country implements its respective policy package. Importantly, these positive synergies take place both for the monetary union as a whole and for each individual country.

We identify two prominent channels for these synergies. On the one hand, country-specific policies produce expansionary effects that run beyond the short term, especially in the case of structural reforms that may deploy permanent effects on output. Thus, the forward-guidance-driven reduction in long-run real interest rates raises the present-discounted value of such gains, via income and net worth effects, with the resulting positive effect on current consumption and investment (discounting effect). On the other hand, different country-specific policies have different effects on the endogenous path of the nominal interest rate when the latter follows a standard ZLB-constrained Taylor rule, both in terms of the date at which the nominal rate exits the ZLB (the lift-off date) and the intensity of its rise once outside the ZLB; we may refer to this channel as the lift-off effect. In particular, inflationary demand-side policies such as a fiscal expansion in the Core enjoy a positive lift-off effect, whereas the opposite is true for deflationary supply-side policies such as product market reforms in the Periphery. Our analysis stresses the importance of jointly implementing
forward guidance and both supply- and demand-side country-specific policies, as it is this package that brings together the discounting and (positive) lift-off effects and thus maximizes the positive synergies.

**Related literature.** By analyzing the joint implementation of demand and supply side policies, we contribute to a long-standing tradition in macroeconomics, with early contributions by Blanchard et al. (1985), Buiter (1987) and Bean (1994), among others. Our paper revisits this topic in the context of a quantitative modern dynamic general equilibrium (DGE) framework.

More specifically, our paper contributes to the literature on the evaluation of macroeconomic policies in a currency union in the context of quantitative DGE models. Our analysis shares several themes with previous contributions, such as the effects of national policies (fiscal expansion/consolidation, structural reforms, etc.), cross-country spillovers of such policies, the role of the ZLB in shaping the impact of such policies, and the effects of forward guidance by the monetary authorities in the face of a binding ZLB. Relative to this literature, which we summarize next, one important contribution is that we analyze quantitatively the synergies between national policies and (non-standard) union-wide monetary measures, in sync with recent policy debates in the EMU.

A recent literature studies the effects of country-specific fiscal policies, and the associated cross-country spillovers, in two-country monetary union models. Erceg and Lindé (2013) analyze different strategies of fiscal consolidation by one country, with a particular attention to the constraints imposed by currency union membership, including the possibility of a binding ZLB. In a similar framework, Blanchard, Erceg and Lindé (2014) study the spillovers of fiscal expansion in one country to the other under different assumptions about the incidence of the ZLB or the degree of home bias in government purchases, as well as the welfare implications of such an expansion. In addition to the analysis of synergies discussed above, we also build on this literature by studying the cross-country spillovers of structural reforms in one part of the currency union and how such spillovers depend on whether the ZLB binds or not.

The role of forward guidance about future interest rates as a means of alleviating the restrictions imposed by the ZLB is the subject of a recent and growing
literature, after the seminal theoretical analysis of Eggertsson and Woodford (2003). Levin et al. (2010), Campbell, Fisher and Justiniano (2012), Del Negro and Giannoni (2013), Benigno, Eggertsson and Romei (2014) and McKay, Nakamura and Steinsson (2015) are some notable recent examples of DSGE model-based analyses of forward guidance. We complement this literature by studying, in the context of a multi-country monetary union model, the interaction between forward guidance and different country/region-specific macroeconomic policies, both supply-side (in the periphery) and demand-side ones (in the core). Our analysis reveals an important role of forward-guidance in strengthening the expansionary effects of national supply and demand side policies.

Our paper is also related to a recent literature that studies the effects of structural reforms, via reductions in price and/or wage markups, in a currency union where the monetary authority is either constrained by the ZLB (Fernández-Villaverde, Guerrón-Quintana and Rubio-Ramírez, 2012; Eggertsson, Ferrero and Raffo, 2014; Gerali, Notarpietro and Pisani, 2015) or by its concern for nominal exchange rate stabilization (Galí and Monacelli, 2014). One contribution of our analysis to this line of research is to add fiscal expansion by the core countries in the union and forward guidance by the monetary authority, and to study the resulting complementarities across these policies.

Finally, Andrés, Arce and Thomas (2015) study the effects of structural reforms in a small open economy that belongs to a monetary union (with the resulting lack of monetary accommodation) and undergoes a prolonged process of private-sector deleveraging due to the coexistence of long-term debt, collateral constraints and a negative financial shock. As argued by those authors, the assumption of long-term debt brings model debt dynamics closer to those observed in actual deleveraging episodes, both historical ones and those currently ongoing in the EMU periphery. We build on their analyses by considering a two-country monetary union structure, which allows us to analyze the cross-country spillovers created by country-specific policies, as well as the synergies between such country-specific policies and the common monetary policy.

The rest of the paper is organized as follows. Section 2 lays out the model and
presents the calibration and solution method. Section 3 constructs our main baseline scenario, which includes a binding ZLB and deleveraging in the periphery. Section 4 analyzes the effects of country-specific macroeconomic policies (structural reforms in the periphery, fiscal expansion in the core) and forward guidance by the common monetary authority, both with and without ZLB. It then quantifies the synergies between these policies. Section 5 concludes.

2 Model

We now present a general equilibrium model of a monetary union with two countries or regions: the ‘Periphery’ (denoted by $H$) and the ‘Core’ (denoted by $F$). The union-wide population is normalized to 1, where a fraction $s$ live in the Periphery and the remaining $1 - s$ in the Core.

The real side of the economy is fairly standard. In each country, households obtain utility from consumption goods and from housing units. Consumption goods are produced using a combination of household labor, commercial real estate and equipment capital goods. Construction firms build real estate (both for residential and commercial purposes) using labor and consumption goods; the latter are also used as inputs by equipment capital goods producers. Consumption-goods and labor markets are both characterized by monopolistic competition and nominal rigidities.

On the financial side, the structure is as follows. In each country, there are three types of consumers: patient households, impatient households, and (impatient) entrepreneurs. In equilibrium, the latter two borrow from the former and from lenders in the other country. Debt contracts are long-term. In periods in which borrowers are able to receive new credit flows, they do so subject to collateral constraints. If the value of their collateral is too low for them to receive new credit flows, they just repay their outstanding debts at a fixed contractual rate. Real estate is the only collateralizable asset. We will henceforth refer to impatient and patient households as ‘constrained’ and ‘unconstrained’ households, respectively.

Finally, a common monetary authority sets the nominal policy interest rate using a standard Taylor rule and subject to the ZLB constraints.
All variables are in real terms and in per capita unless otherwise specified, with the consumption goods basket of each country acting as the numeraire in that country. From now onwards, we focus on the model structure in the Periphery country.\(^2\) The Core country is modelled analogously. All equilibrium conditions, including first-order conditions of agents’ optimization problems, are listed in Appendix A.

## 2.1 Households

There is a representative constrained household and a representative unconstrained household, denoted respectively by superscripts \(c\) and \(u\).

### 2.1.1 Cost minimization

Before analyzing dynamic household optimization, we first derive the static cost minimization problem, which is common to both households types (and to entrepreneurs). Households consume a basket of home and foreign goods, denoted respectively by subscripts \(H\) and \(F\),

\[
\begin{align*}
c^x_t &= \left( \omega_H^{1/\varepsilon_H} \left( c^x_{H,t} \right)^{(\varepsilon_H-1)/\varepsilon_H} + (1 - \omega_H)^{1/\varepsilon_H} \left( c^x_{F,t} \right)^{(\varepsilon_H-1)/\varepsilon_H} \right)^{\varepsilon_H/(\varepsilon_H-1)},
\end{align*}
\]

for \(x = c, u\); \(c^x_{H,t}\) and \(c^x_{F,t}\) are baskets of Home and Foreign good varieties, respectively,

\[
\begin{align*}
c^x_{H,t} &= \left( \int_0^1 c^x_{H,t}(z)^{(\varepsilon_p-1)/\varepsilon_p} d\varepsilon \right)^{\varepsilon_p/(\varepsilon_p-1)},
\end{align*}
\]

\[
\begin{align*}
c^x_{F,t} &= \left( \int_0^1 c^x_{F,t}(z')^{(\varepsilon'_p-1)/\varepsilon'_p} d\varepsilon' \right)^{\varepsilon'_p/(\varepsilon'_p-1)},
\end{align*}
\]

where \(\varepsilon_p, \varepsilon'_p > 1\) are the elasticities of substitution across Home and Foreign good varieties, respectively. Let \(P_{H,t}(z)\) and \(P_{F,t}(z')\) denote the prices of Home good variety

\(^2\)Given our focus on the decision problems from the point of view of the agents in the Periphery, we will also refer to them as Home agents, and to agents in the Core as Foreign agents. Likewise, goods produced in the Periphery and the Core will also be referred to as Home and Foreign goods, respectively.
z and Foreign good variety \( z' \) respectively. Household \( x = c, u \) minimizes nominal consumption expenditure, 
\[
\int_0^1 P_{H,t}(z) c_{H,t}^x(z) \, dz + \int_0^1 P_{F,t}(z') c_{F,t}^x(z') \, dz',
\]
subject to (1), (2) and (3). The first order conditions can be expressed as

\[
c_{H,t}^x = \frac{1}{H} P_{H,t} P_t^{-\varepsilon_H} c_t^x, \quad c_{F,t}^x = (1 - \omega_H) \left( \frac{P_{F,t}}{P_t} \right)^{-\varepsilon_H} c_t^x, \quad (4)
\]

for \( z, z' \in [0, 1] \), where

\[
P_t = (\omega_H P_{H,t}^{1-\varepsilon_H} + (1 - \omega_H) P_{F,t}^{1-\varepsilon_H})^{1/(1-\varepsilon_H)}, \quad P_{H,t} = \left( \int_0^1 P_{H,t}(z)^{1-\varepsilon_p} \, dz \right)^{1/(1-\varepsilon_p)}
\]

are the Periphery’s consumer price index (CPI) and producer price index (PPI), respectively, and where

\[
P_{F,t} = \left( \int_0^1 P_{F,t}(z')^{1-\varepsilon_p} \, dz' \right)^{1/(1-\varepsilon_p)}
\]

is a price index of Foreign goods. Nominal spending in Home and Foreign goods equal
\[
\int_0^1 P_{H,t}(z) c_{H,t}^x(z) \, dz = P_{H,t} c_{H,t}^x \quad \text{and} \quad \int_0^1 P_{F,t}(z') c_{F,t}^x(z') \, dz' = P_{F,t} c_{F,t}^x,
\]
respectively, whereas total nominal consumption spending equals \( P_{H,t} c_{H,t}^x + P_{F,t} c_{F,t}^x = P_t c_t^x \).

As noted before, consumption goods are also used as inputs by construction firms and equipment capital producers. The latter are assumed to combine home and foreign goods analogously to households. This gives rise to investment demand functions analogous to (4).

### 2.1.2 Unconstrained households

The unconstrained household maximizes

\[
E_0 \sum_{t=0}^{\infty} (\beta^u)^t \zeta_t \left\{ \log (c_t^u) + v \log (h_t^u) - \chi \int_0^1 \frac{n_t^u (i)^{1+\varphi}}{1+\varphi} \, di \right\},
\]
where $\zeta_t$ is a union-wide shock to the discount factor of all consumers, $n_t^u(i)$ are labor services of type $i \in [0, 1]$ and $h_t^u$ are housing units, subject to the following budget constraint (expressed in units of the consumption goods basket),

$$c_t^u + d_t + p_t^h [h_t^u - (1 - \delta_h) h_{t-1}^u] = \frac{R_{t-1}}{\pi_t} d_{t-1} + (1 - \tau_w) \int_0^1 \frac{W_t(i)}{P_t} n_t^u(i) di - T_t,$$

where $d_t$ is the real value of net holdings of riskless nominal debt, $R_t$ is the gross nominal interest rate at which Home agents lend and borrow, $\delta_h$ is the depreciation rate of real estate, $p_t^h$ is the real price of real estate, $\pi_t \equiv P_t/P_{t-1}$ is gross CPI inflation, $W_t(i)$ is the nominal wage for labor services of type $i$, $\tau_w$ is a tax rate on labor income and $T_t$ are lump-sum taxes.

### 2.1.3 Constrained households

The constrained household’s preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \left\{ \log (c_t^e) + \theta \log (h_t) - \chi \int_0^1 \frac{n_t^e(i)^{1+\varphi}}{1 + \varphi} di \right\},$$

where $\beta < \beta^u$, i.e. the constrained household is relatively impatient. The household faces the following budget constraint,

$$c_t^e + p_t^h [h_t - (1 - \delta_h) h_{t-1}] = b_t - \frac{R_{t-1}}{\pi_t} b_{t-1} + (1 - \tau_w) \int_0^1 \frac{W_t(i)}{P_t} n_t^e(i) di - T_t,$$

where $b_t$ is the real value of household debt outstanding at the end of period $t$.

Unlike in most of the literature, which typically assumes short-term (one-period) debt, we assume that debt contracts are long-term. In the interest of tractability, we assume that at the beginning of time $t$ the household repays a fraction $1 - \gamma$ of all nominal debt outstanding at the end of period $t - 1$, regardless of when that debt was issued.\(^3\) This type of perpetual debt is similar to the one proposed by Woodford.

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\(^3\)Total (gross) debt payments in each period are then $(1 - \gamma) + (R_{t-1} - 1)$ times nominal debt outstanding, i.e. the sum of amortization and interest payments.
(2001) as a tractable way of modelling long-term debt. In real terms, the outstanding principal of household debt then evolves as follows,

\[ b_t = \frac{b_{t-1}}{\pi_t} + b_t^{new} - (1 - \gamma) \frac{b_{t-1}}{\pi_t} = b_t^{new} + \gamma \frac{b_{t-1}}{\pi_t}, \]

where \( b_t^{new} \) is gross new credit net of voluntary amortizations, i.e. amortizations beyond the contractual debt repayment \((1 - \gamma) b_{t-1}/\pi_t\).

We assume that, in 'normal times' (in a sense to be specified below), household borrowing is subject to collateral constraints, as in Kiyotaki and Moore (1997). Following Iacoviello (2005), outstanding debt \( b_t \) cannot exceed a fraction \( m_t \) (the 'loan-to-value ratio', which we assume to be exogenously time-varying) of the expected discounted value of the household's residential stock: \( b_t \leq m_t R_t^{-1} E_t \pi_{t+1} p_{t+1}^h h_t \). For brevity, we will refer to such pledgeable value of collateral as collateral value. This debt limit, however, is only effective as long as it exceeds \( \gamma b_{t-1}/\pi_t \), which we will henceforth refer to as the contractual amortization path. Indeed, if the collateral value falls below such path, lowering \( b_t \) to the value of collateral would require lenders not only to reduce gross new credit to zero (its lower bound), but also to impose additional amortizations beyond those agreed in the contract (i.e. \( b_t^{new} < 0 \)). Since lenders cannot force borrowers to pay back faster than the contractual amortization rate, the contractual amortization path becomes the effective debt limit. Therefore, long run debt implies the following asymmetric borrowing constraint,

\[ b_t \leq R_t^{-1} m_t E_t \pi_{t+1} p_{t+1}^h h_t, \quad \text{if } \frac{m_t}{R_t} E_t \pi_{t+1} p_{t+1}^h h_t \geq \gamma \frac{b_{t-1}}{\pi_t}, \]

\[ b_t \leq \gamma \frac{b_{t-1}}{\pi_t}, \quad \text{if } \frac{m_t}{R_t} E_t \pi_{t+1} p_{t+1}^h h_t < \gamma \frac{b_{t-1}}{\pi_t}. \]

This asymmetry gives rise to a double debt regime. In 'normal times' in which collateral values exceed the contractual amortization path, debt is restricted by the former. In this baseline regime, households can receive new credit against their housing collateral, with the constraint that such new credit does not exceed the gap between
collateral values and the amortization path. However, in the face of shocks that reduce collateral values sufficiently, the economy switches to an alternative regime, in which new credit disappears and debt is restricted instead by the contractual amortization path. Notice that changes from one regime to the other take place endogenously, and may thus be affected by policy or by other shocks.

For future reference, we obtain here the optimal choice of housing,

\[ \lambda^c_t p^h_t = \frac{\zeta_t \beta}{h_t} + \beta E_t \lambda^c_{t+1} (1 - \delta_h) p^h_{t+1} + \xi_t \frac{m_t}{R_t} E_t \pi_{t+1} p^h_{t+1}, \tag{8} \]

where \( \lambda^x_t = \zeta_t / c^x_t \) and \( \xi_t \) are the Lagrange multipliers associated to the budget constraint of consumer type \( x = c, u, e \) and to the collateral constraint (eq. 6), respectively. Equation (8) illustrates that, when the collateral constraint is binding \((\xi_t > 0)\), the marginal value of housing is higher due to the possibility of borrowing against it. This possibility disappears once the economy enters into the alternative debt regime, in which the collateral constraint ceases to be effective.

### 2.2 Production

Entrepreneurs produce an intermediate good and sell it to retailers, who transform it into consumption good varieties. Entrepreneurs and retailers conform the consumption goods sector. In addition, construction firms produce real estate, both for residential and commercial use, whereas equipment capital is produced by capital goods producers. All sectors operate under perfect competition, except retailers who enjoy monopolistic power.

#### 2.2.1 Entrepreneurs

A representative entrepreneur produces an intermediate product and sells it to retailers at a perfectly competitive real (CPI-deflated) price \( m_t \). The entrepreneur

\[ b^{\text{new}}_t \leq m_t R_t^{-1} E_t \pi_{t+1} p^h_{t+1} h_t - \gamma b_{t-1} / \pi_t. \]
maximizes
\[ E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \log c^e_t, \]
with the consumption basket \( c^e_t \) defined analogously to (1), subject to
\[ c^e_t + p^h_t [h^e_t - (1 - \delta_h) h^e_{t-1}] + q_t [k_t - (1 - \delta_k) k_{t-1}] = mc_t y^e_t - \frac{W_t}{P_t} n^e_t + b^e_t - \frac{R_{t-1} b^e_{t-1}}{\pi_t} + \sum_{s=r,h,k} \Pi^e_s, \]
where \( y^e_t \) is output of the intermediate good, \( k_{t-1} \) is equipment capital with unit price \( q_t \), \( \delta_k \) is the depreciation rate of equipment capital, \( h^e_{t-1} \) is commercial real estate, \( n^e_t \) is a basket of labor services, \( W_t \) is a nominal wage index, \( b^e_t \) is the real value of entrepreneurial debt outstanding at the end of period \( t \), and \( \{\Pi^e_s\}_{s=r,h,k} \) are real profits from the retail, construction and equipment goods-producing sectors.\(^5\)

Entrepreneurs’ maximization is also subject to an asymmetric borrowing constraint analogous to the one on constrained households,
\[ b^e_t \leq R_t^{-1} m^e_t E_t \pi_{t+1} p^h_{t+1} h^e_t, \quad \text{if} \quad \frac{m^e_t}{R_t} E_t \pi_{t+1} p^h_{t+1} h^e_t \geq \gamma^e b^e_{t-1}, \quad (9) \]
\[ b^e_t \leq \gamma^e \frac{b^e_{t-1}}{\pi_t}, \quad \text{if} \quad \frac{m^e_t}{R_t} E_t \pi_{t+1} p^h_{t+1} h^e_t < \gamma^e \frac{b^e_{t-1}}{\pi_t}, \quad (10) \]
where we allow for a different loan-to-value ratio \( (m^e_t) \) and contractual amortization rate \((1 - \gamma^e)\) for entrepreneurs. Again, it is instructive to analyze here the optimal demand for commercial real estate,
\[ \lambda^h_t p^h_t = \beta E_t \lambda^e_{t+1} \left\{ mc_{t+1} \alpha_h \frac{y^e_{t+1}}{h^e_t} + (1 - \delta_h) p^h_{t+1} \right\} + \xi^e_t \frac{m^e_t}{R_t} E_t \pi_{t+1} p^h_{t+1}, \quad (11) \]
where \( \xi^e_t \) is the Lagrange multipliers associated to constraint (9). Analogously to the case of constrained households, in periods in which the collateral constraint
\(^5\)Notice that entrepreneurs are assumed to own the firms in the latter sectors. We adopt this specification because we are interested in analyzing how profit accumulation affects productive investment decisions, which in our model are made by the entrepreneurs.
binds ($\xi_t^e > 0$) the marginal value of commercial real estate is higher thanks to the possibility of borrowing against it.

### 2.2.2 Retailers

A continuum of monopolistically competitive retailers indexed by $z \in [0, 1]$ purchase the intermediate input from entrepreneurs at the real price $mc_t$, and transform it one for one into final good varieties. Retailers’ real marginal cost is thus $mc_t$. Each retailer $z$ faces a demand curve

$$y_t(z) = \left( \frac{P_{H,t}(z)}{P_{H,t}} \right)^{-\varepsilon_p} y_t \equiv y_t^d(P_{H,t}(z)),$$

where $y_t$ is aggregate demand of the basket of Home goods (to be derived below). Assuming Calvo (1983) price-setting, a retailer that has the chance of setting its nominal price at time $t$ solves

$$\max_{P_{H,t}(z)} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \theta_p)^s \lambda_t^e \frac{\chi_t^e}{\lambda_t^e} \left[ (1 - \tau_p) \frac{P_{H,t}(z)}{P_{t+s}} - mc_{t+s} \right] y_{t+s}^d(P_{H,t}(z)),$$

where $\theta_p$ is the probability of not adjusting the price and $\tau_p$ is a tax rate on retailers’ revenue. The first-order condition is standard (see Appendix), with all time-$t$ price setters choosing a common optimal price $\tilde{P}_{H,t}$. If retailers were able to reset prices in every period ($\theta_p = 0$), they would set

$$\tilde{P}_{H,t} = \frac{1}{1 - \tau_p \varepsilon_p - 1} P_t mc_t.$$

Therefore, the term $\frac{\varepsilon_p}{1 - \tau_p \varepsilon_p - 1}$ represents the desired price markup over nominal marginal cost, and thus measures the degree of monopolistic distortions in product markets.
2.2.3 Construction firms

A representative construction firm maximizes its expected discounted stream of profits, 
\[ E_0 \sum_{t=0}^{\infty} \beta^t \frac{X_t}{X_0} \Pi_t^h, \]
where \( \Pi_t^h = p_t^h I_t^h - \frac{W_t}{P_t} n_t^h - i_t^h \), subject to the production technology
\[ I_t^h = (n_t^h)^{\omega} \left\{ i_t^h \left[ 1 - \frac{\Phi_h}{2} \left( \frac{i_t^h}{i_{t-1}^h} - 1 \right)^2 \right]\right\}^{1-\omega}, \]
where \( n_t^h \) are labor services, \( i_t^h \) are consumption goods, and \( I_t^h \) are new real estate units.\(^6\)

2.2.4 Equipment capital producers

A representative equipment capital producer maximizes its expected discounted stream of profits, 
\[ E_0 \sum_{t=0}^{\infty} \beta^t \frac{X_t}{X_0} \Pi_t^k, \]
where \( \Pi_t^k = q_t I_t - i_t \), subject to the technology
\[ I_t = i_t \left[ 1 - \frac{\Phi_k}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \right], \]
where \( i_t \) are consumption goods, and \( I_t \) are new equipment capital goods.

2.3 Wage setting

Both entrepreneurs and construction firms use a basket of labor services by constrained and unconstrained households,
\[ n_t^s = (n_t^{s,c})^{\mu_s} (n_t^{s,u})^{1-\mu_s}, \]
where \( n_t^{s,x} \) are labor services provided by type-\( x \) household, \( x = c, u \), to each sector \( s = e, h \). We assume that both worker types (constrained and unconstrained) earn the same wage. Cost minimization then implies \( (1 - \mu_s) n_t^{s,c} = \mu_s n_t^{s,u} \), for \( s = e, h \).

\(^6\)We include labor services in the production function of construction firms so as to allow for long-run changes in real estate prices. Without labor in construction (\( \omega = 0 \)), real estate prices are always unity in the long run. More generally, it can be shown that \( p_t^{h,s} = (w_{ss}^s)^{\omega} \omega^{-\omega} (1 - \omega)^{-1-\omega}. \)
From each household type, each sector demands in turn a basket of labor service varieties,

\[ n_{t}^{s,x} = \left( \int_{0}^{1} n_{t}^{s,x}(i) \left( \varepsilon_{w}-1 \right) / \varepsilon_{w} \, di \right)^{\varepsilon_{w} / (\varepsilon_{w} - 1)}, \]

for \( x = c, u \) and \( s = e, h \), where \( \varepsilon_{w} > 1 \) is the elasticity of substitution across labor varieties \( i \in [0, 1] \). Cost minimization implies

\[ n_{t}^{s,x}(i) = (W_{t}(i) / W_{t})^{-\varepsilon_{w}} n_{t}^{s,x}, \]

for \( x = c, u \) and \( s = e, h \), where \( W_{t} \equiv \left( \int_{0}^{1} W_{t}(i) \left( 1 - \varepsilon_{w} \right) / \varepsilon_{w} \, di \right)^{1 / (1 - \varepsilon_{w})} \) is the nominal wage index. Total demand for each variety of labor services is thus

\[ n_{t}^{x}(i) \equiv n_{t}^{e,x}(i) + n_{t}^{h,x}(i) = \left( W_{t}(i) / W_{t} \right)^{-\varepsilon_{w}} \left( n_{t}^{e,x} + n_{t}^{h,x} \right) \equiv n_{t}^{d,x} \left( W_{t}(i) \right), \]

for \( x = c, u \). Total nominal wage income earned by each type-\( x \) household equals \( \int_{0}^{1} W_{t}^{x}(i) \, di = W_{t} n_{t}^{x} \), where \( n_{t}^{x} \equiv n_{t}^{e,x} + n_{t}^{h,x} \).

As in Erceg, Henderson and Levin (2000; EHL), nominal wages are set à la Calvo (1983). In particular, a union representing all type-\( i \) workers maximizes the utility of the households to which such workers belong. Then a union that has the chance to reset the nominal wage at time \( t \) chooses \( W_{t}(i) \) to maximize

\[ \sum_{x=c,u} \sum_{s=0}^{\infty} \left( \beta^{x} \theta_{w} \right)^{s} \left[ \lambda_{t+s}^{x} (1 - \tau_{w}) W_{t+s} \left( d_{t+s}^{d,x} (W_{t+s}(i)) \right)^{1 + \varphi} \right], \]

where \( \theta_{w} \) is the probability of not adjusting the wage and \( \beta^{c} = \beta \). All time-\( t \) wage-setters choose a common optimal wage \( \tilde{W}_{t} \); see the first-order condition in the Appendix. If workers were able to reset wages in every period (\( \theta_{w} = 0 \)), then they would charge a markup

\[ \frac{1}{1 - \tau_{w} \varepsilon_{w} - 1} \]

over a weighted average of constrained and unconstrained households’ marginal rates of substitution between consumption and labor. Therefore, the term \( \frac{1}{1 - \tau_{w} \varepsilon_{w} - 1} \) represents the desired wage markup, and thus measures the degree of monopolistic distortions in the labor market.
2.4 Fiscal authority

The fiscal authority demands a basket of Home good varieties analogous to (2), which we denote by $g_t$ and is exogenously determined. Thus, government demand for each Home variety $z$ is $g_t(z) = (P_{H,t}(z)/P_{H,t})^{-\varepsilon_p} g_t$. Assuming full home bias in government purchases, the total nominal value of government purchases is $\int_0^1 P_{H,t}(z) g_t(z) \, dz = P_{H,t} g_t$. For simplicity, we assume that the fiscal authority balances its budget period-by-period by adjusting lump-sum taxes $T_t$,

$$\tau_w \frac{W_t}{P_t} (n_t^c + n_t^u) + \tau_p \frac{P_{H,t}}{P_t} y_t + 2 T_t = \frac{P_{H,t}}{P_t} g_t.$$  

2.5 Common monetary authority

The common monetary authority sets the gross nominal policy interest rate $R_t^{MU}$ according to a simple inflation-based Taylor rule and subject to the zero bound on net interest rates,

$$R_t^{MU} = \max \left\{ 1, R_t^{MU} (\pi_t^{MU})^{\rho_{\pi}} \right\},$$  

where $\rho_{\pi} > 1$, $\bar{R}^{MU}$ is the long-run target for the policy rate, and

$$\pi_t^{MU} = s \pi_t + (1 - s) \pi_t^*$$

is a measure of the union-wide gross CPI inflation rate, where $\pi_t^* \equiv P_t^*/P_{t-1}^*$ is Foreign CPI inflation.

2.6 International linkages

In section 2.1.1 we derived Home agents’ optimal demand for imported (Foreign) goods. As regards the exports side of international trade, we assume that Foreign agents demand baskets of Home good varieties analogous to (2), denoted by $c_{H,t}^F$, $c_{H,t}^u$, etc. The law of one price is assumed to hold for each Home good variety, such
that \( P_{H,t}^*(z) = P_{H,t}(z) \) for all \( z \in [0,1] \), implying \( P_{H,t}^* = P_{H,t} \).\(^7\) Thus, export demand for each Home good variety \( z \) is \( x_t(z) = (P_{H,t}(z)/P_{H,t})^{-\varphi} x_t \), where real per capita exports equal

\[
x_t = \frac{1 - s}{s} \left( c_{H,t}^* + c_{H,t}^u + c_{H,t}^e + i_{H,t}^* + i_{H,t}^h \right)
= \frac{1 - s}{s} \left( 1 - \omega_F^* \right) \left( \frac{P_{H,t}}{P_t^*} \right)^{-\varphi_F} \left( c_{t}^* + c_{t}^u + c_{t}^e + i_{t}^* + i_{t}^h \right).
\]

In equation (14), \( \omega_F^* \) and \( \varphi_F \) are the relative weight on Foreign goods and the elasticity of substitution between Home and Foreign goods, respectively, in Foreign agents’ consumption and investment baskets, \( P_t^* \) is the Core’s CPI, and \( z_t^*, z = c^e, c^u, c^e, i, i^h \), are per capita demand for Home goods by the different Foreign agents.

As mentioned before, Home agents can lend to and borrow from foreigners and other domestic agents at a riskless nominal rate \( R_t \). We denote by

\[
nfa_t \equiv d_t - b_t - b_t^c
\]

the Periphery’s real (CPI-deflated) per capita net foreign asset position. Following standard practice in the literature, in order to guarantee stationarity of the net foreign asset position, we assume that \( R_t \) is given by

\[
R_t = R_t^{MU} \exp \left( -\psi \frac{P_t nfa_t}{P_{H,t} gdp_t} \right),
\]

where \( \psi > 0 \) and \( gdp_t \) is the real (PPI-deflated) per capita GDP, to be derived later.

### 2.7 Aggregation and market clearing

Each retailer \( z \) demands \( y_{t}^d(P_{H,t}(z)) \) units of the intermediate input, as given by (12). Total demand for the latter equals \( \int_0^1 y_{t}^d(P_{H,t}(z)) \, dz = y_t \Delta_t \), where \( \Delta_t \equiv \int_0^1 (P_{H,t}(z)/P_{H,t})^{-\varphi} \, dz \) denotes relative price dispersion. Market clearing in the

\(^7\)The same holds for Foreign good varieties: \( P_{F,t}(z') = P_{F,t}^*(z') \) for all \( z' \in [0,1] \), such that \( P_{F,t} = P_{F,t}^* \).
intermediate good market thus requires

$$k_{t-1}^{\alpha_k} (h_{t-1}^e)^{\alpha_h} (n_t^c)^{1-\alpha_h-\alpha_k} = y_t \Delta_t.$$  

Aggregate demand for the basket of Home good varieties is given by,

$$y_t = c_{c;H,t} + c_{u;H,t} + c_{e;H,t} + i_{H,t} + i_{H,1} + g_t + x_t.$$  

Total demand for real estate must equal total supply,

$$h_t + h_t^u + h_t^e = I_t^h + (1 - \delta_h) \left( h_{t-1} + h_{t-1}^u + h_{t-1}^e \right).$$  

Total demand for equipment capital must equal total supply: $k_t = I_t + (1 - \delta_k) k_{t-1}$. Labor market clearing requires $n_t^c + n_t^u = n_t^e + n_t^h$. We define real (PPI-deflated) per capita GDP as

$$\text{gdp}_t = y_t + \frac{P_t}{P_{H,t}} (q_t I_t - i_t) + \frac{P_t}{P_{H,t}} (p_t^h I_t^h - i_t^h)$$

$$= \frac{P_t}{P_{H,t}} c_t^{\text{tot}} + \frac{P_t}{P_{H,t}} (q_t I_t + p_t^h I_t^h) + \left[ x_t - \frac{P_{F,t}}{P_{H,t}} \left( c_{F,t}^{\text{tot}} + i_{F,t} + i_{F,1}^h \right) \right],$$

where in the second equality we have used (16) and $z_{H,t} = \frac{P_t}{P_{H,t}} z_t - \frac{P_{F,t}}{P_{H,t}} z_{F,t}$ for $z = c_t, c_t^u, c_t^e, i_t, i_t^h$, and where $c_t^{\text{tot}} = c_t^c + c_t^u + c_t^e$ is total consumption (total consumption imports $c_{F,t}^{\text{tot}}$ are defined analogously).

Zero net supply of nominal international bonds requires

$$sP_t nfa_t + (1 - s) P_t^s nfa_t^s = 0,$$

where the Core’s real per capita net foreign asset position, $nfa_t^*$, is defined analogously to (15). We may combine all domestic market-clearing conditions and budget constraints to obtain the Periphery’s current account identity,

$$nfa_t = \frac{R_{t-1}}{\pi_t} nfa_{t-1} + \frac{P_{H,t}}{P_t} x_t - \frac{P_{F,t}}{P_t} \left( c_{F,t}^{\text{tot}} + i_{F,t} + i_{F,1}^h \right).$$
2.8 Calibration and solution method

We calibrate our two-country monetary union model to the European Monetary Union, where the country labelled Periphery broadly represents the member states in the so-called EMU ‘Periphery’. As explained in the introduction, we are motivated by the recent experience of the peripheral EMU economies, where the private sector is still embarked in a gradual deleveraging process, and for which structural reforms in product and labor markets have been advocated as a means of fostering economic recovery.

The share of the total population that lives in the Periphery is set to $s = 1/3$, following Blanchard et al. (2014). The rest of the calibration closely follows Andrés, Arce and Thomas (2014), who calibrate a similar model to the Spanish economy.\(^8\)

The time period is a quarter. Some parameters will be calibrated by matching the model’s steady state to a number of empirical targets in 2007, the year prior to the start of the international financial crisis.\(^9\)

The discount factor of the impatient agents is set to $\beta = 0.98$, following Iacoviello (2005). For patient households, we choose $\beta^u = 1.025^{1/4}$, which is consistent with a steady state nominal interest rate of $R_{ss} = 1.025^{1/4} \pi_{ss} = \bar{R}^{MU} e^{-\psi(nfo^y_{ss})}$. We set the long-run inflation target $\bar{\pi}^{MU}$ to 1, which implies $\pi_{ss} = \pi^*_s = 1$ in a stationary equilibrium. Choosing $\bar{R}^{MU} = 1.021^{1/4}$ for the nominal policy interest rate, we then set $\psi$ to replicate net foreign assets over GDP in 2007, $nfo^y_{ss} = -79.3\%$. The inverse labor supply elasticity is set to $\varphi = 4$, consistently with a large body of micro evidence. The weight parameter in the consumption basket, $\omega_H$, is set to match gross exports over GDP in 2007 (26.9\%). Based on evidence for Spain in García et al. (2009), the price elasticity of exports and imports is set to $\varepsilon_F = \varepsilon_H = 1$.

The elasticities of substitution across varieties of consumption goods and labor services, $\varepsilon_p$ and $\varepsilon_w$, and the tax rates on retailers’ revenue and labor income, $\tau_p$ and $\tau_w$, determine the desired markups in product and labor markets, respectively. We set

---

\(^8\)We thus opt for calibrating the Home country to Spain, rather than building consolidated aggregates for the peripheral EMU economies.

\(^9\)We do not claim, however, that the Spanish economy was in (or close to) a steady state in 2007. Instead, our model’s steady state should be interpreted as the economy’s initial condition for the purpose of our simulation exercises.
\( \varepsilon_p = 7 \) and \( \tau_p = 0 \), implying an initial price markup of \( (1 - \tau_p)^{-1} \varepsilon_p/(\varepsilon_p - 1) = 1.17 \), which is broadly consistent with estimates by Montero and Urtasun (2013) based on Spanish firm-level data. Wage markups are hard to estimate empirically, so we adopt an alternative calibration strategy. We follow Galí (2011) in reinterpreting the EHL model of wage-setting in a way that delivers equilibrium unemployment (see Appendix B for details). Targeting an unemployment rate of 8.6\% in 2007, we obtain an initial wage markup of \( (1 - \tau_w)^{-1} \varepsilon_w/(\varepsilon_w - 1) = 1.43 \), which we achieve by setting \( \tau_w = 0 \) and \( \varepsilon_w = 3.31 \).

The elasticity of entrepreneurial output with respect to equipment capital and commercial real estate are set to \( \alpha_k = 0.11 \) and \( \alpha_h = 0.21 \), which are chosen to replicate the labor share of GDP in 2007 (61.6\%) and the share of equipment capital in the total stock of productive capital.\(^{11}\) As in Iacoviello and Neri (2010) we set \( \delta_h = 0.01 \), whereas \( \delta_k \) is set to a standard value of 0.025. The elasticity of construction output with respect to labor \( \omega \) is set to match the construction share of total employment in 2007 (13.4\%). The weight of utility from housing services, \( \vartheta \), is chosen to replicate gross household debt over annual GDP (80.2\%). The share of constrained and unconstrained workers in the labor baskets are set to \( \mu_h = \mu_u = \mu = 1/2 \). The scale parameters of convex investment adjustment costs, \( \Phi_h \) and \( \Phi_k \), are chosen such that the fall in construction and equipment capital investment in our baseline deleveraging scenario resembles their behavior during the crisis.\(^{12}\)

The Calvo parameters are set to \( \theta_p = 2/3 \) and \( \theta_w = 3/4 \), such that prices and wages are adjusted every 3 and 4 quarters on average, respectively. This is consistent with survey evidence for the Spanish economy (see e.g. Druant et al., 2009).

\(^{10}\)Our choice of \( \tau_p \) and \( \tau_w \) is motivated as follows. In this paper, we implement structural reforms by changing the elasticity parameters \( \varepsilon_p \) and \( \varepsilon_w \). Setting \( \tau_p = \tau_w = 0 \) allows us to isolate the effects of structural reforms from additional fiscal effects operating through the budget constraint of constrained households (in particular, through changes in lump-sum taxes \( T_t \)). See Andrés, Arce and Thomas (2015) for a discussion of the effects of reforms implemented via reductions in \( \tau_p \) and \( \tau_w \).

\(^{11}\)Using data from BBVA Research, we obtain that the value of equipment capital was 21.4\% of the total value of productive capital in 2007.

\(^{12}\)In particular, we set \( \Phi_h \) and \( \Phi_k \) such that the accumulated fall in construction and equipment capital investment 8 quarters after the financial shock replicate their accumulated fall 8 quarters after their peak in 2007:Q4 (24.5\% and 28\% respectively).
Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td>1/3</td>
<td>relative size of Home country</td>
</tr>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta^u, \beta^{us}$</td>
<td>0.994</td>
<td>unconstrained household discount factor</td>
</tr>
<tr>
<td>$\beta, \beta^*$</td>
<td>0.98</td>
<td>constrained household discount factor</td>
</tr>
<tr>
<td>$\varphi, \varphi^*$</td>
<td>4</td>
<td>(inverse) labor supply elasticity</td>
</tr>
<tr>
<td>$\vartheta, \vartheta^*$</td>
<td>0.38</td>
<td>weight on housing utility</td>
</tr>
<tr>
<td>$\varepsilon_p, \varepsilon_p^*$</td>
<td>7</td>
<td>elasticity of subst. across consumption varieties</td>
</tr>
<tr>
<td>$\varepsilon_w, \varepsilon_w^*$</td>
<td>3.31</td>
<td>elasticity of substitution across labor varieties</td>
</tr>
<tr>
<td>$\omega_H, \omega_F^*$</td>
<td>0.72, 0.86</td>
<td>weight on domestic goods in consumption basket</td>
</tr>
<tr>
<td>$\varepsilon_H, \varepsilon_F^*$</td>
<td>1</td>
<td>elast. of subst. between domestic and imported goods</td>
</tr>
<tr>
<td>Technology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_h, \alpha_h^*$</td>
<td>0.21</td>
<td>elasticity output wrt real estate</td>
</tr>
<tr>
<td>$\alpha_k, \alpha_k^*$</td>
<td>0.11</td>
<td>elasticity output wrt equipment</td>
</tr>
<tr>
<td>$\omega, \omega^*$</td>
<td>0.43</td>
<td>elasticity construction wrt labor</td>
</tr>
<tr>
<td>$\delta_h, \delta_h^*$</td>
<td>0.01</td>
<td>depreciation real estate</td>
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<tr>
<td>$\delta_k, \delta_k^*$</td>
<td>0.025</td>
<td>depreciation equipment</td>
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<td>$\mu, \mu^*$</td>
<td>0.5</td>
<td>share of constr. households in labor baskets</td>
</tr>
<tr>
<td>$\Phi_h, \Phi_h^*$</td>
<td>6.1</td>
<td>investment adjustment costs construction</td>
</tr>
<tr>
<td>$\Phi_k, \Phi_k^*$</td>
<td>2.4</td>
<td>investment adjustment costs equipment</td>
</tr>
<tr>
<td>Price/wage setting</td>
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<td></td>
</tr>
<tr>
<td>$\theta_p, \theta_p^*$</td>
<td>0.67</td>
<td>fraction of non-adjusting prices</td>
</tr>
<tr>
<td>$\theta_w, \theta_w^*$</td>
<td>0.75</td>
<td>fraction of non-adjusting wages</td>
</tr>
<tr>
<td>Debt constraints</td>
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<tr>
<td>$\tilde{m}, \tilde{m}^*$</td>
<td>0.70</td>
<td>household LTV ratio</td>
</tr>
<tr>
<td>$\tilde{m}^e, \tilde{m}^e*$</td>
<td>0.64</td>
<td>entrepreneur LTV ratio</td>
</tr>
<tr>
<td>$\gamma, \gamma^*$</td>
<td>0.98</td>
<td>amortization rate household debt</td>
</tr>
<tr>
<td>$\gamma^e, \gamma^{e*}$</td>
<td>0.97</td>
<td>amortization rate entrepreneurial debt</td>
</tr>
<tr>
<td>Monetary policy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi, \tilde{R}^{MU}$</td>
<td>1.5, $1.02^{1/4}$</td>
<td>Taylor rule coefficient, long-run policy rate</td>
</tr>
</tbody>
</table>
The parameters that regulate the debt constraints are calibrated as follows. According to data from the Spanish Land Registry office, loan-to-value ratios (LTV) for new mortgages prior to the crisis were slightly below 70 percent. We thus set $\bar{m} = 0.70$ for the household’s initial loan-to-value ratio. The entrepreneurial initial loan-to-value ratio is chosen to match the ratio of gross non-financial corporate debt to annual GDP (125.4% in 2007), which yields $\bar{m}_e = 0.64$. Finally, we calibrate the contractual amortization rates, $1 - \gamma$ and $1 - \gamma^e$, in order to replicate the average age of the stock of outstanding mortgage debt prior to the crisis. This yields $1 - \gamma = 0.02$ and $1 - \gamma^e = 0.03$ per quarter.\(^{13}\)

For the Core, for simplicity we assume a fully symmetric calibration, with two exceptions. First, the weight on Periphery goods in the consumption basket of Core consumers, $\omega^*_F$, is set in order to normalize the terms-of-trade in the initial steady state to $1$.\(^{14}\) Second, we allow for an additional parameter in the interest rate premium of the Core and set it such that interest rates are the same in both countries in the initial steady state.\(^{15}\)

Finally, we assume a standard value of $1.5$ for the Taylor rule coefficient $\phi$, which together with the long-run target for the policy rate chosen above ($\bar{R}^{MU} = 1.02^{1/4}$) completes the specification of the monetary policy rule. Table 1 summarizes the calibration.

---

\(^{13}\)Under our debt contracts (with a constant fraction of outstanding debt amortized each period), the average age of the debt stock converges in the steady state to $\gamma/(1 - \gamma)$ and $\gamma^e/(1 - \gamma^e)$ for households and entrepreneurs, respectively. According to calculations by Banco de España, based on data from the Land Registry office and large financial institutions, the average age of outstanding mortgage debt prior to the crisis was close to 12.5 years for households and 8 years for nonfinancial corporations and entrepreneurs. This yields $\gamma = 12.5 \times 4/(12.5 \times 4 + 1) = 0.98$ and $\gamma^e = 8 \times 4/(8 \times 4 + 1) = 0.97$.

\(^{14}\)Unlike in the case of $\omega_H$, which was calibrated to match an exports target for the Home country (equivalently, an imports target, given the target for the NFA-to-GDP ratio), $\omega^*_F$ cannot be targeted to the Foreign country’s exports because these must equal the Home country’s imports in the model.

\(^{15}\)In particular, we assume $R^*_t = R^{MU}_t \exp \left[ -\psi^* \left( P^*_t a^*_t / P^*_t / gdp^*_t \right) + \psi^*_0 \right]$, with $\psi^* = \psi$, and set $\psi^*_0$ such that $R_{ss}^* = R_{ss}$.\(^{*}\)
2.8.1 Solution method

We assume perfect foresight in all our simulations. We solve for the fully nonlinear equilibrium path, using a variant of the Newton-Raphson algorithm developed by Laffargue (1990), Boucekkine (1995) and Juillard (1996) (LBJ). As discussed in the previous section, our assumption of long-run debt contracts gives rise to two debt regimes for households and entrepreneurs. If collateral values are above the contractual debt amortization paths, then debt levels are restricted by the former, according to equations (6) and (9). If the opposite holds, then new credit flows collapse to zero and debt is restricted by the contractual amortization path (equations 7 and 10). Moreover, the presence of the ZLB on nominal interest rates (see equation 13) implies that the economy may also switch endogenously between two monetary policy regimes, depending on whether the ZLB binds or not. We have therefore extended the LBJ algorithm to allow for endogenous changes of both debt and monetary policy regimes. In particular, the dates at which these regime changes take place are solved as equilibrium objects.

3 Baseline scenario: deleveraging and the ZLB

In this section we construct a baseline scenario that is meant to capture some important features of the current economic situation in the EMU and, particularly, in its peripheral economies. On the one hand, the latter economies are experiencing a protracted process of private-sector deleveraging. With this aim, we will first simulate the effects of a deleveraging shock in the Periphery, assuming the common monetary authority is able to reduce nominal interest rates so as to partially counteract the resulting fall in union-wide inflation.

On the other hand, the European Central Bank is currently restricted in its ability to further reduce nominal interest rates, as the latter are already very close to the zero bound. Thus, we will consider a second scenario in which, simultaneously to the deleveraging shock, a negative union-wide demand shock occurs that pushes the monetary authority's nominal interest rate against its ZLB. The latter scenario, with
both private sector deleveraging in the Periphery and a binding ZLB, will constitute the main baseline scenario with respect to which we will evaluate the effects of, and synergies between, alternative macroeconomic policies.

### 3.1 Adjustment to deleveraging out of the ZLB

In order to better understand the effects of a deleveraging shock in our model of collateral constraints and long-run debt, we first subject the model economy to a negative financial shock in the Periphery that reduces the availability of credit for borrowers. Our ‘credit crunch’ consists of an unexpected, gradual, permanent drop in the LTV ratios of both households and entrepreneurs, $m_t$ and $m^e_t$ respectively. In particular, we assume an autoregressive process for both LTV ratios: $x_t = (1 - \rho^x) \bar{x} + \rho^x x_{t-1}$, $x = m, m^e$, where we set $\rho^m = \rho^{m^e} = 0.75$. We then simulate an unanticipated fall in the long-run LTV ratios ($\bar{m}, \bar{m}^e$) of 7.5 percentage points from their baseline values in Table 1, which accords well with recent experience in Spain.\footnote{Data from the Spanish Land Registry office shows that average LTV ratios for new mortgages declined by 7.7 percentage points in the 6 years between 2007:Q3 and 2013:Q3.}

Figure 1 displays the response to the credit crunch of collateral values and contractual amortization paths, together with the actual equilibrium path of outstanding debt, both for entrepreneurs and households in the Periphery. Before the shock ($t = 0$), the economy rests in the steady state of the baseline regime, where debt levels equal pledgeable collateral values.\footnote{Indeed, the fact that constrained households and entrepreneurs are both more impatient than unconstrained households, $\beta < \beta^u$, guarantees that the collateral constraint binds for both agents in the steady state.} The credit crunch shock drives collateral values below the contractual amortization paths already on impact ($t = 1$). Therefore, the economy switches on impact to the alternative regime in which entrepreneurial and household debt stocks decay at the contractual amortization rates. In this phase, the economy undergoes a gradual and prolonged deleveraging process.

Eventually, collateral values rise again above the contractual amortization path, at which point borrowers are able to regain access to fresh funds. We denote by $T^*$ and $T^{**}$ the time at which the endogenous regime change takes place for entrepreneurs.
Figure 1: Debt dynamics after a deleveraging shock in the Periphery and households, respectively. Notice that collateral values and debt both experience a surge at the time of the regime change. This is because real estate becomes again valuable as collateral (see equations 8 and 11), which pushes up borrowers’ demand for real estate, and hence its price. Thus, $T^*$ and $T^{**}$ also represent the duration of the deleveraging phase for entrepreneurs and households. In the scenario analyzed here, the equilibrium duration of the deleveraging phase is $T^* = 10$ quarters for entrepreneurs and $T^{**} = 17$ quarters for households, the latter being longer due mainly to the lower amortization rate of household debt ($1 - \gamma < 1 - \gamma^e$).\footnote{Figure 1 shows that the debt constraints (7) and (10) are binding during $t = 1, \ldots, T^{**} - 1$ and $t = 1, \ldots, T^* - 1$, respectively, whereas the collateral constraints (6 and 9) are binding for $t \geq T^{**}$ and $t \geq T^*$, respectively. We have verified that the corresponding Lagrange multipliers are indeed strictly positive in the relevant periods, both in the baseline scenario and in all subsequent simulations. Results are available upon request.}

Figure 2 shows the response of both countries to the deleveraging shock in the Periphery.\footnote{In all figures, all variables are in %, except interest rates (real and nominal), which are in annualized percentage points.} In the latter, the shock produces a deep and protracted recession, which ends around the period in which entrepreneurs regain access to new loans ($t = 10$).
Such recession is due to the fall in domestic demand (consumption and investment); the latter is only partially counteracted by an improvement in net exports, thanks to the Periphery’s improvement in competitiveness vis-à-vis the Core in the first few years and the contraction in domestic demand.\textsuperscript{20} The resulting union-wide deflation leads the monetary authority to reduce nominal interest rates according to the Taylor rule, which produces a mild economic expansion in the Core.

### 3.2 Adjustment to deleveraging at the ZLB

We move next to our main baseline scenario, where, contemporaneously to the deleveraging shock in the Periphery, a common negative demand shock affects both countries. In particular, we assume an unanticipated temporary increase in consumers’ discount factors. Assuming \( \zeta_t = \zeta_{t-1} e^{\gamma_t} \), we set \( u^c_1 = 0.005 \), i.e. discount factors increase on impact by 2 annualized percentage points, and \( \rho_\zeta = 0.90 \); we choose this calibration such that the short-run fall in union-wide GDP replicates approximately that of EMU GDP during the last recession.\textsuperscript{21}

As shown in Figure 3, the fall in union-wide inflation in this scenario is large enough to make the monetary authority’s nominal interest rate hit the ZLB constraint on impact. After 4 quarters, the latter constraint ceases to bind, and nominal interest rates increase gradually in sync with union-wide inflation. Not surprisingly, this scenario is more severe for both countries than that displayed in Figure 2: peripheral GDP falls more on impact, whereas the core now enters in recession for a few quarters. Overall, our baseline scenario draws a picture of prolonged economic downturn and persistently low inflation at the union level.

\textsuperscript{20}The response of variables such as consumption, investment, terms-of-trade and net exports are not shown in the figures for brevity, but are available upon request.

\textsuperscript{21}In particular, union-wide GDP falls by about 0.85\% in the first two quarters of the simulation. This is close to the accumulated fall in EMU GDP in the first two quarters of the last recession, which amounted to 0.96\%.
Figure 2: The effects of a deleveraging shock in the Periphery
Figure 3: Baseline scenario: deleveraging in the Periphery and a binding ZLB
4 Macroeconomic policies at the ZLB

The baseline scenario constructed in the previous section is meant to broadly capture some of the main macroeconomic difficulties that the EMU currently faces: sluggish aggregate demand (aggravated in the Periphery by an ongoing deleveraging process), persistently low inflation, and nominal interest rates at their zero bound. Such a scenario poses significant challenges for economic authorities in the EMU. Among the measures considered in order to foster recovery in the euro area, three have attracted particular attention from the economic authorities: (i) structural reforms in product and factor markets in countries with weaker public finances (mainly countries in the ‘periphery’); (ii) countercyclical fiscal policies in those economies with fiscal room to implement them (all of them in the ‘core’), and (iii) non-standard monetary policy measures by the ECB aimed at pushing down the interest rate curve beyond the zero-constrained short-end, such as forward guidance about the future path of policy interest rates.\textsuperscript{22}

We now use our model to analyze the effects of these economic policy measures. We start by looking at the effects of country-specific policies: structural reforms and countercyclical fiscal policies.

4.1 Country-specific policies: structural reforms and fiscal expansion

*Structural reforms.* We implement structural reforms by means of an unanticipated, permanent reduction in desired price and wage markups in the Periphery, $\varepsilon_p / (\varepsilon_p - 1)$ and $\varepsilon_w / (\varepsilon_w - 1)$ respectively. Both are assumed to fall by 1%, following Eggertsson et al. (2014).\textsuperscript{23} Figure 4 displays the *marginal* effects of these reforms (i.e. with respect to the baseline scenario without such reforms), depending on whether the baseline scenario features the union-wide negative demand shock, i.e. a binding

\textsuperscript{22}See, for example, Draghi (2014), Coeuré (2014), European Commission (2014), and IMF (2014).

\textsuperscript{23}In particular, $\varepsilon_p$ increases from 7 to 7.45, and $\varepsilon_w$ increases from 3.31 to 3.39.
As a natural outcome of the greater degree of competition and efficiency in product and labour markets, structural reforms give rise to transitory lower inflation rates. This deflationary pressure tends to depress ceteris paribus the aggregate demand in the Periphery, via the increase in the real value of debt (‘debt deflation’ channel). The latter effect is amplified when nominal interest rates cannot be reduced further (dashed lines in Figure 4), thereby prompting an increase in real short-term interest rates that adversely impacts on consumption and investment.

These contractionary effects are however dominated, even in the short term, by a combination of expansionary channels. First, reforms have permanent positive effects on income and consumption, the anticipation of which leads to higher consumption and investment in the short-run. Second, the previous effect also benefits demand for real estate, pushing up its price and the value of borrowers’ collateral. This ‘collateral channel’ fosters spending by borrowers once they regain access to new loans, thus reinforcing the medium and long-run gains in activity. Third, the improvement in the Periphery’s competitiveness vis-à-vis the core gives rise to a significant and lasting increase in its exports and in domestic demand for its own goods. All these effects give rise to a strong positive effect on peripheral GDP.

In the Core, the spillover effect from these reforms depends critically on the incidence of the ZLB. Outside of it, the reduction in the nominal policy interest rate produces a (small) increase in GDP. At the ZLB, however, monetary policy cannot counteract the Core’s loss of competitiveness, giving rise to a temporary contraction.

**Fiscal expansion.** We now consider the effects of a fiscal expansion in the Core, implemented through an exogenous temporary increase in government expenditure. Assuming \( g_t^* = \rho g_{t-1}^* + u_{it}^g \), we set \( u_{1t}^g \) such that \( g_t^* \) increases on impact by 1% of (ex-ante) Core GDP, or 0.67% of (ex-ante) union-wide GDP, which closely resembles the size of the initial public contribution to the so-called ‘Juncker plan’ for the financing of public infrastructures;\(^{25}\) we also set \( \rho_g = 0.75 \), such that the plan has a half-life

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\(^{24}\)In figures 4 through 9, which display the marginal effects of alternative macroeconomic policies, spikes typically reflect policy-induced changes in the endogenous duration of households’ and entrepreneurs’ deleveraging processes \((T^*, T^{***})\).

\(^{25}\)The ‘Juncker plan’ (technically, European Fund for Strategic Investments) aims for an initial push to direct public investment of 63 bn euros, i.e. 0.66% of 2014 EMU GDP. In broader terms,
Figure 4: Marginal effects of structural reforms in the Periphery

Marginal effect of structural reforms in product and labor markets in Periphery, without zero lower bound.
Marginal effect of structural reforms in product and labor markets in Periphery, at the zero lower bound.
Baseline

Deviations from the respective baseline scenario, except nominal interest rates which are in levels.
of about a year. As reflected in Figure 5, the fiscal stimulus deploys clearly positive effects on the Core’s economic activity, but has opposing effects on the Periphery’s GDP depending on whether the economy is in a liquidity trap.

When monetary policy is not restricted by the ZLB, the positive effects of stronger activity in the Core on the Periphery through the exports channel is neutralized by the monetary tightening in response to higher union-wide inflation. The net spillover effect on the Periphery is actually negative in the short term, although it disappears quickly. By contrast, at the ZLB, the inflationary pressure stemming from the Core causes a reduction in real interest rates in both countries. This favors the Periphery both through higher exports (thanks to higher spending in the Core) and higher domestic demand, the result being a relatively sizable and persistent positive spillover effect.

4.2 Forward guidance about monetary policy

The previous subsection has considered the effects of country-specific policies, and how such effects depend on the incidence of the ZLB. We now turn our attention to the effects of ‘forward guidance’ by the common monetary authority when the latter is constrained by the ZLB.

Figure 6 shows the effects (relative to our baseline scenario with a binding ZLB) that would follow from a commitment by the central bank to keep interest rates at zero for two more quarters than what its Taylor rule would dictate in the baseline scenario, i.e. until period $t = 5$ included. This non-standard monetary policy measure allows to boost GDP in both regions in the short run. The main channel, common to both regions, is the reduction in long-run real interest rates relative to the baseline scenario. The subsequent expansion in activity prompts an increase in inflation in both regions which, coupled with the fact that the nominal policy rate is stuck at zero for a number of periods, amplifies the decline in long-run real rates.

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26 As explained in Section 3.2, in the baseline scenario the policy rate exits the liquidity trap at $t = 4$. 

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the plan aims for an increase in total (public and private) investment of about 315 bn euros, or 3.3% of euro-area GDP, over a period of three years (2015-2017). 

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Figure 5: Marginal effects of a fiscal expansion in the Core
Figure 6: Marginal effects of forward guidance
and hence the positive impact on economic activity.

It is worth emphasizing that forward guidance produces significant expansionary effects on both countries, which are of similar magnitude on impact, in spite of the presence of binding collateral constraints. This non-Ricardian feature is particularly acute in the Periphery since, while deleveraging, no new credit flows to existing debtors. Absent this last feature of the model, forward guidance would produce an irrealistically high expansionary effect on impact,\(^{27}\) an issue that has received some attention in the recent literature on forward guidance in DSGE models (the forward guidance puzzle).\(^{28}\)

## 5 Policy synergies

The previous exercises show that the three types of policies considered have the potential to alleviate the costs associated with negative real and financial shocks. The three policies however are implemented by different authorities: the two national governments, and the common monetary authority. As discussed before, in policy circles increasing attention is being devoted to the potential gains that could be achieved if the different authorities within the EMU were to jointly implement their respective policy/reform packages. Thus, a key question to ask in the context of our model is whether some complementarities or synergies exist between the policies considered thus far. In particular, we now investigate to what extent each policy reinforces the effects of the other. The non-linear nature of our model (together with our reliance on a fully non-linear solution method) makes it well suited for analyzing this issue.

We start by quantifying how forward guidance modifies the effectiveness of country-specific policies. Figure 7 compares the marginal effects of jointly implementing structural reforms in the Periphery and fiscal expansion in the Core \textit{vis-à-vis} two

\(^{27}\)Specifically, in the model version in which the Periphery does not enter the slow deleveraging regime, the impact of forward guidance would be of around two and a half time bigger than the one in our baseline .

different reference scenarios that differ in the monetary policy stance: one in which the monetary authority passively follows its ZLB-constrained Taylor rule (which corresponds to our baseline scenario in section 3.2), and one in which the monetary authority implements a forward guidance policy as formulated in the previous subsection. Clearly, country-specific policies are more effective, both in the Periphery and the Core, when in parallel to such policies the central bank commits to a lower future path for its policy rate.

We now analyze to what extent the implementation of national policies favors or hinders the effectiveness of forward guidance by the common monetary authority. Figure 8 compares the marginal effects of forward guidance relative to two different scenarios: one in which national authorities implement their respective policy pack (reforms in the Periphery, fiscal expansion in the Core), and one in which they do not. Again, we find sizable synergies between both groups of policies: forward guidance is more effective in fostering economic activity, both in the Core and in the Periphery, when governments in the latter countries carry out their respective measures.

To summarize, our analysis suggests that, in a scenario characterized by a liquidity trap and a prolonged deleveraging process in a sizable part of the monetary union, the joint implementation of country-specific policy stimuli and forward guidance by the common monetary authority may give rise to first-order gains in short-run economic activity, not just in the union as a whole but also in each individual country.

5.1 Inspecting the synergy channels

So far we have analyzed the synergies between forward guidance, on the one hand, and a combination of country-specific policies, on the other. In order to gain further insights on the sources of these synergies, here we analyze the interaction between forward guidance and individual country-specific policies, i.e. we consider separately structural reforms in the Periphery and fiscal expansion in the Core. Moreover, we also distinguish between reforms in product markets and labor markets, as both types of reforms may differ in their potential for synergies. Figure 9 displays the marginal effects of (i) a labor market reform, (ii) a product market reform, both in
Figure 7: Marginal effects of national policies, with and without forward guidance in the baseline
Figure 8: Marginal effects of forward guidance, with and without national policies in the baseline
the Periphery, and (iii) fiscal expansion in the Core, relative to two different reference scenarios: one where the monetary authority follows the ZLB-constrained Taylor rule (the baseline scenario described in section 3.2), and one where it announces and applies forward guidance. Again, the latter is defined as a commitment to keeping nominal interest rates at zero until 2 quarters after the lift-off date in the baseline, no-policy-change scenario. We find that, unlike the labor market reform, the product market reform displays negative synergies with forward guidance, whereas the fiscal expansion in the Core has clearly positive synergies.

To understand these results, we focus on two different mechanisms through which synergies arise in our framework. On the one hand, country-specific policies unleash positive effects on domestic GDP that go beyond the short term. This is particularly the case for structural reforms (both in product and labor markets), as these have permanent expansionary effects on the Periphery’s GDP, as shown by the solid lines in Figure 9. Thus, the fall in long-run real interest rates induced by forward guidance amplifies the present-discounted value of such future expansionary effects. This induces an additional stimulus in current consumption and investment decisions, giving rise to an increase in area-wide economic activity and inflation in the short term. We may refer to this channel as the discounting effect.

On the other hand, country-specific policies imply different endogenous effects on the nominal interest rate path when monetary policy follows the standard ZLB-constrained Taylor rule. As shown in Figure 10, a demand-side stimulus such as a fiscal expansion in the Core brings forward the lift-off date for the policy rate by 1 quarter, which tends to buffer the expansionary impact of this measure. However, if the central bank commits to keeping interest rates at zero for longer than what the Taylor rule would imply in the baseline, then the same fiscal expansion does not produce an upward shift in the nominal interest rate path relative to the no-expansion reference scenario. As a result, forward guidance strengthens the effects of the fiscal expansion, i.e. a positive synergy arises. By contrast, a supply-side measure such as a structural reform has the opposite effect on monetary policy. As shown by Figure

\[ \text{See Erceg and Lindé (2014) for an in-depth analysis of the effects of government spending shocks at the ZLB when the lift-off date is endogenous to the size of such shocks.} \]
Figure 9: GDP synergies between country-specific policies and forward guidance
Figure 10: Effects of country-specific policies on nominal interest rates and inflation
10, the product market reform (i.e. the 1% reduction in desired price markups) is not deflationary enough to delay the lift-off date, but it does moderate the magnitude of the nominal interest rate increase once the latter exits the ZLB, which strengthens the expansionary effect of the reform. Following the same logic as before, forward guidance partially undoes the positive effect from the reform. We may refer to this channel as the *lift-off effect*.\(^{30}\)

In light of these two channels, we can better understand the difference in the sign and size of synergies between forward guidance and different country-specific policies. In the case of the fiscal expansion in the Core, the lift-off date effect is particularly important in generating positive synergies with forward guidance; whereas the discounting effect is relatively less important as the expansionary effects are rather short-lived. The product market reform features both the (positive) discounting effect and a negative lift-off effect. Quantitatively, the second effect dominates, giving rise to the negative synergies with forward guidance. Finally, the labor market reform features a similar discounting effect but essentially no lift-off effect, because it is much less deflationary than the comparable product market reform.\(^{31}\) As a result, it generates positive synergies.

## 6 Concluding Remarks

We have provided a general equilibrium framework for analyzing the effects of supply and demand side policies, the associated cross-country spillovers, and the potential synergies between such policies, in an asymmetric monetary union that faces a liquidity trap and a slow deleveraging process in its ‘periphery’. The set of policies that we consider is inspired by the current situation in the EMU. On the demand side, we

\(^{30}\)Strictly speaking, neither the product nor the labor market reform delay the lift-off *date* relative to the baseline. Therefore, the lift-off effect in this case refers to the intensity of the nominal rate increase once outside of the ZLB. For larger price markup reductions than the one assumed here (1%), the product market reform does delay the lift-off date. Results are available upon request.

\(^{31}\)As emphasized by Andrés, Arce and Thomas (2015), reductions in desired wage markups must overcome a *double* layer of nominal rigidities (first wages, then prices) before affecting actual production prices.
analyze (i) the effects of forward guidance about the future path of nominal policy interest rates, as a means of alleviating the constraints imposed by a binding ZLB on short-term rates; and (ii) those of a fiscal expansion in the ‘core’, i.e. in those countries in the union with sufficient fiscal capacity to implement such an expansion. On the supply side, we study the role of pro-competition structural reforms in the periphery.

In terms of spillovers, we find that the effects of national policies on other countries depend crucially on whether monetary policy is constrained by the ZLB. Thus, deflationary structural reforms in the periphery tend to create (small) contractionary effects in the ‘core’ when the monetary authority cannot accommodate such a deflationary pressure. On the contrary, a fiscal expansion in the core may benefit the periphery provided the monetary authority is stuck at the ZLB and hence does not react to the resulting inflationary pressure.

As regards the synergies across these policies, we find potentially sizable short-run economic gains from their joint implementation. Thus, forward-guidance reinforces the expansionary effects of country-specific policies, and the latter in turn improve the effectiveness of forward guidance. Two prominent channels through which these synergies take places are the following. First, forward guidance lowers long-run real interest rates and hence increases the present-discounted value of the future output and consumption gains produced by national stimulus policies, thus fostering investment and consumption already in the short-run. Second, under our implementation of forward guidance, the latter reinforces the expansionary effects of demand-side policy stimuli, such as a fiscal expansion, by avoiding the upward shift in the nominal interest rate path that such stimuli would otherwise produce.

It should be stressed that our results are conditional on our assumed form of forward guidance. Exploring the synergies between country-specific policies and forward guidance for alternative formulations of the latter is an important avenue for further research.
References


Appendix

A. Equilibrium conditions

In order to express all equations in terms of stationary variables, we define $p_{H,t} \equiv P_{H,t}/P_t$, $\pi_{H,t} \equiv P_{H,t}/P_{H,t-1}$ (PPI inflation), $\tilde{p}_t \equiv P_{H,t}/P_{F,t}$ (terms of trade), $\tilde{\pi}_t \equiv P_{H,t}/P_{H,t}$; $w_t \equiv W_t/P_t$, $\tilde{w}_t \equiv \tilde{W}_t/W_t$, $\pi_{wt} \equiv W_t/W_{t-1}$; analogously for the Foreign economy: $p_{F,t}^* \equiv P_{F,t}^*/P^*_t$, $\pi_{F,t} \equiv P_{F,t}^*/P_{F,t-1}^*$, $\tilde{p}_t^* \equiv P_{F,t}/P_{H,t}$, etc.

Home country

- Unconstrained household budget constraint and first-order conditions ($c^u_t$, $d_t$, $h^u_t$),

$$\lambda^u_t = \frac{\zeta_t}{c^u_t},$$

$$c_t^u + d_t + p^h_t [h_t^u - (1 - \delta_h) h_{t-1}^u] = \frac{R_{t-1}}{\pi_t} d_{t-1} + (1 - \tau_w) w_t n_t^u - T_t,$$  \hspace{1cm} (17)

$$\lambda^u_t = \beta^u E_t \frac{R_t}{\pi_{t+1}} \lambda^u_{t+1},$$

$$\lambda^u_t p^h_t = \frac{\zeta_t}{h_t^u} + \beta^u E_t \lambda^u_{t+1} (1 - \delta_h) p^h_{t+1}.$$  \hspace{1cm} (18)

- Constrained household budget constraint, debt constraints, and first-order conditions ($c^c_t$, $b_t$, $h_t$),

$$\lambda^c_t = \frac{\zeta_t}{c^c_t},$$

$$c_t^c + \frac{R_{t-1}}{\pi_t} b_{t-1} + p^h_t [h_t - (1 - \delta_h) h_{t-1}] = b_t + (1 - \tau_w) w_t n_t^c - T_t,$$  \hspace{1cm} (19)

$$b_t \leq \begin{cases} R_t^{-1} m_t E_t \pi_{t+1} p^h_{t+1} h_t, & \text{if } m_t R_t^{-1} E_t \pi_{t+1} p^h_{t+1} h_t \geq \gamma b_{t-1}/\pi_t, \\ \gamma b_{t-1}/\pi_t, & \text{if } m_t R_t^{-1} E_t \pi_{t+1} p^h_{t+1} h_t < \gamma b_{t-1}/\pi_t, \end{cases}$$  \hspace{1cm} (20)

$$\lambda^c_t = \beta E_t \frac{R_t}{\pi_{t+1}} \lambda^c_{t+1} + \xi_t 1 (\chi_t \geq 0) + \mu_t 1 (\chi_t < 0) - \beta \gamma E_t \frac{\mu_{t+1}}{\pi_{t+1}} 1 (\partial_{t+1} < 0),$$  \hspace{1cm} (21)
Entrepreneur budget constraint, debt constraints, and first-order conditions

\[
\lambda_t^e p_t^h = \frac{\zeta_t \partial}{h_t} + \beta E_t \lambda_{t+1}^e (1 - \delta_h) p_{t+1}^h + \xi_t 1 (\pi_t \geq 0) \frac{m_t}{E_t} E_t \pi_{t+1} p_{t+1}^h,
\]  

(25)

where \( \mu_t \) is the Lagrange multiplier on constraint (7) in the text, \( 1(\cdot) \) is the indicator function and \( \pi_t \equiv R_t^{-1} m_t E_t \pi_{t+1} p_{t+1}^h h_t - \gamma b_{t-1}/\pi_t \).

- Entrepreneur budget constraint, debt constraints, and first-order conditions

\[
\lambda_t^e = \frac{\zeta_t}{c_t^e},
\]  

(26)

\[
c_t^e = m c_t k_{t-1}^{\alpha_h} (h_{t-1}^e)^{\alpha_h} (n_t^e)^{1-\alpha_h-\alpha_k} - w_t n_t^e - \rho_t \left[ h_t^e - (1 - \delta_h) h_{t-1}^e \right] + b_t^e - \frac{R_{t-1} b_{t-1}}{\pi_t} - q_t \left[ k_t - (1 - \delta_k) k_{t-1} \right] + \Pi_t^e + \Pi_t^h + \Pi_t^k,
\]  

(27)

\[
b_t^e \leq \left\{ \begin{array}{ll}
R_{t-1}^{-1} m_t^e E_t \pi_{t+1} p_{t+1}^h h_t^e & \text{if } m_t^e R_{t-1}^{-1} E_t \pi_{t+1} p_{t+1}^h h_t^e \geq \gamma e b_{t-1}^e/\pi_t, \\
\gamma e b_{t-1}^e/\pi_t, & \text{if } m_t^e R_{t-1}^{-1} E_t \pi_{t+1} p_{t+1}^h h_t^e < \gamma e b_{t-1}^e/\pi_t,
\end{array} \right.
\]  

(28)

\[
\lambda_{t+1}^e = \beta E_t \frac{R_t}{\pi_{t+1}} \lambda_{t+1}^e + \xi_t^e 1 (\pi_t \geq 0) + \mu_t^e 1 (\pi_t < 0) - \beta \gamma e E_t \frac{H_{t+1}}{\pi_{t+1}} 1 (\pi_{t+1} < 0),
\]  

(29)

\[
\lambda_{t+1}^e p_{t+1}^h = \beta E_t \lambda_{t+1}^e \left[ m c_{t+1} \alpha_h k_{t-1}^{\alpha_h} (h_{t-1}^e)^{\alpha_h} (n_{t+1}^e)^{1-\alpha_h-\alpha_k} + (1 - \delta_h) p_{t+1}^h \right] + \xi_t^e E_t \pi_{t+1} p_{t+1}^h 1 (\pi_{t+1} \geq 0),
\]  

(30)

\[
w_t = m c_t (1 - \alpha_h - \alpha_k) k_{t-1}^{\alpha_h} (h_{t-1}^e)^{\alpha_h} (n_{t+1}^e)^{1-\alpha_h-\alpha_k},
\]  

(31)

\[
\lambda_t^e q_t = \beta E_t \lambda_{t+1}^e \left[ m c_{t+1} \alpha_h k_{t-1}^{\alpha_h-1} (h_t^e)^{\alpha_h} (n_{t+1}^e)^{1-\alpha_h-\alpha_k} + (1 - \delta_k) q_{t+1} \right],
\]  

(32)

where \( \mu_t^e \) is the Lagrange multiplier on constraint (7) in the text, and \( \pi_t^e \equiv R_t^{-1} m_t^e E_t \pi_{t+1} p_{t+1}^h h_t^e - \gamma e b_{t-1}^e/\pi_t \).

- Retailers’ optimal price decision, and aggregate profits,

\[
E_t \sum_{s=0}^{\infty} \left( \beta \tau_p \right)^s \frac{\lambda_{t+s}^e}{\lambda_t^e} \left[ \frac{(1 - \tau_p) \tilde{p}_t}{\Pi_{t+j}^s} \pi_{H,t+s} - \frac{\varepsilon p}{\varepsilon p - 1} m c_{t+s} \right] \left( \frac{\Pi_{j=1}^s \pi_{H,t+j}}{\tilde{p}_t} \right)^{\varepsilon p} y_{t+s} = 0,
\]  

(33)

\[\Pi_t^e = y_t \left( (1 - \tau_p) p_{H,t} - mc_t \Delta t \right),\]  

(34)
• Dynamics of PPI inflation and price dispersion,
\[
1 = (1 - \theta_p) \tilde{p}_t^{1-\varepsilon_p} + \theta_p \pi_{H,t}^{\varepsilon_p-1}, \tag{35}
\]
\[
\Delta_t \equiv (1 - \theta_p) \tilde{p}_t^{-\varepsilon_p} + \theta_p \pi_{H,t}^{\varepsilon_p} \Delta_{t-1}. \tag{36}
\]

• Construction firm output, first order conditions \((n_t^h, i_t^h)\), and profits,
\[
I_t^h = (n_t^h)^{\omega} \left\{ \frac{i_t^h}{i_{t-1}^h} \left[ 1 - \frac{\Phi_h}{2} \left( \frac{i_t^h}{i_{t-1}^h} - 1 \right)^2 \right] \right\}^{1-\omega}, \tag{37}
\]
\[
w_t = p_t^h (n_t^h)^{\omega-1} \left\{ \frac{i_t^h}{i_{t-1}^h} \left[ 1 - \frac{\Phi_h}{2} \left( \frac{i_t^h}{i_{t-1}^h} - 1 \right)^2 \right] \right\}^{1-\omega}. \tag{38}
\]
\[
1 = p_t^h (n_t^h)^{\omega} (1 - \omega) \left\{ \frac{i_t^h}{i_{t-1}^h} \left[ 1 - \frac{\Phi_h}{2} (d_{it}^h)^2 \right] \right\}^{-\omega} \left[ 1 - \frac{\Phi_h}{2} (d_{it}^h)^2 - \Phi_h (d_{it}^h) \frac{i_t^h}{i_{t-1}^h} \right] \\
+ \beta \frac{\lambda_{t+1}^e}{\lambda_t^e} \tilde{p}_{t+1}^h (n_{t+1}^h)^{\omega} (1-\omega) \left\{ \frac{i_{t+1}^h}{i_{t-1}^h} \left[ 1 - \frac{\Phi_h}{2} (d_{i_{t+1}^h})^2 \right] \right\}^{-\omega} \Phi_h d_{i_{t+1}^h} \left( \frac{i_{t+1}^h}{i_{t-1}^h} \right)^2, \tag{39}
\]
with \(d_{i_t}^h \equiv i_t^h / i_{t-1}^h - 1\).

• Equipment capital producers output, first order condition \((i_t)\), and profits,
\[
I_t = i_t \left[ 1 - \frac{\Phi_k}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \right], \tag{41}
\]
\[
1 = q_t \left[ 1 - \frac{\Phi_k}{2} (d_{it})^2 - \Phi_k (d_{it}) \frac{i_t}{i_{t-1}} \right] + E_t \frac{\lambda_{t+1}^e}{\lambda_t^e} q_{t+1} \Phi_k d_{i_{t+1}} \frac{i_{t+1}^2}{i_t^2}, \tag{42}
\]
\[
\Pi_t^k = q_t I_t - i_t, \tag{43}
\]
with \(d_{i_t} \equiv i_t / i_{t-1} - 1\).
• Optimal wage decision,

$$\sum_{x=c,u} E_t \sum_{s=0}^{\infty} (\beta^x \theta_w)^s \left[ \frac{1-\tau_w}{\prod_{j=1}^{s} \pi_{t+j}^w} \frac{1}{\xi_{t+s}} - \frac{\zeta_{t+s} \chi (n_{t+s}^c)}{\xi_{t+s} / \xi_w} \left( \frac{\bar{w}_t}{\prod_{j=1}^{s} \pi_{t+j}^w} \right)^{-\varepsilon_w \varphi} \right] \left( \frac{\sum_{j=1}^{s} \pi_{t+j}^w}{\bar{w}_t} \right)^{\varepsilon_w} n_{t+s}^x = 0,$$

(44)

with $\beta^c = \beta$.

• Dynamics of wage inflation and wage dispersion,

$$1 = (1 - \theta_w) \bar{w}_t^{1-\varepsilon_w} + \theta_w \pi_{wt}^{\varepsilon_w - 1},$$

(45)

$$\Delta_{t}^{w,n} = (1 - \theta_w) \bar{w}_t^{\varepsilon_w} + \theta_w \pi_{wt}^{\varepsilon_w} \Delta_{t-1}^{w,n}.$$  

(46)

• Fiscal authority’s budget constraint,

$$\tau_w w_t (n_t^c + n_t^u) + \tau_p p_{H,t} y_t + 2T_t = p_{H,t} g_t.$$  

(47)

• Aggregate employment,

$$N_t^c = n_t^c \Delta_t^{w,n},$$  

(48)

$$N_t^u = n_t^u \Delta_t^{w,n},$$  

(49)

$$N_t = N_t^c + N_t^u.$$  

(50)

• Export demand,

$$x_t = \frac{1 - \omega^u}{s} (1 - \omega^u) (\bar{p}_t p_{F,t}^*)^{\varepsilon_F} \left( c_t^{e*} + c_t^{i*} + i_t^* + h_t^* \right).$$  

(51)

• Intermediate good market clearing,

$$y_t \Delta_t = h_{t-1}^{e} \left( h_{t-1}^{e} \right)^{\alpha_h} (n_t^e)^{1-\alpha_h-\alpha_k},$$  

(52)
• Labor market clearing,
\[ n^e_t + n^u_t = n^e_t + n^h_t. \] (53)

• Consumption goods basket market clearing,
\[ y_t = c^H_H + c^u_H + c^e_H + i^H + i^h_H + g_t + x_t. \] (54)

• Real estate market clearing,
\[ h_t + h^u_t + h^e_t = I^h_t + (1 - \delta_h) (h_{t-1} + h^u_{t-1} + h^e_{t-1}). \] (55)

• Equipment capital market clearing,
\[ k_t = (1 - \delta_k) k_{t-1} + I_t. \] (56)

• Real wages,
\[ w_t = w_{t-1} \frac{\pi_{wt}}{\pi_t}, \] (57)

• Terms of trade,
\[ \hat{p}_t = \hat{p}_{t-1} \frac{\pi_{H,t}}{\pi_{F,t}}. \] (58)

• Relative demand for domestic goods,
\[ z_{H,t} = \omega_H p^{-e_H}_{H,t} z_t, \quad z = c^e, c^u, c^e, i, i^h \] (59)

• Relative demand for constrained/unconstrained household labor,
\[ (1 - \mu) n^e_t = \mu n^u_t, \] (60)

where \( \mu \equiv \mu_e = \mu_h. \)
• Relative domestic producer prices,

\[ p_{H,t}^{\epsilon H-1} = \omega_H + (1 - \omega_H) \hat{p}^{\epsilon H-1}_t. \]  

(61)

• CPI inflation,

\[ \pi_t^{1-\epsilon H} = \frac{\omega_H \left(p_t^s\right)^{1-\epsilon H}}{\omega_H \left(p_t^s\right)^{1-\epsilon H} + 1 - \omega_H} + \frac{1 - \omega_H}{\omega_H \left(p_t^s\right)^{1-\epsilon H} + 1 - \omega_H} \pi_{F,t}^{1-\epsilon H}, \]  

(62)

• Real (PPI-deflated) GDP,

\[ gdp_t = y_t + \frac{1}{p_{H,t}} \left( q_t I_t - i_t \right) + \frac{1}{p_{H,t}} \left( p_t^b I_t^h - i_t^h \right), \]  

(63)

• Gross nominal interest rate,

\[ R_t = R_t^{MU} \exp \left( -\psi \frac{d_t - b_t - b_t^s}{p_{H,t} gdp_t} \right). \]  

(64)

Foreign country

• Zero net supply of international bonds,

\[ s (d_t - b_t^s - b_t^* t) + (1 - s) \frac{p_{H,t}}{p_{F,t}} \left( d_t^s - b_t^* - b_t^{s*} \right) = 0, \]  

• Unconstrained household first-order conditions \((c_t^{u*}, q_t^s, h_t^{u*})\),

\[ \lambda_t^{u*} = \frac{\zeta_t}{c_t^{u*}}, \]  

(65)

\[ \lambda_t^{u*} = \beta_t^{u*} E_t \frac{R_t^*}{\pi_{t+1}^{u*}} \lambda_{t+1}, \]  

(66)

\[ \lambda_t^{u*} p_t^{h*} = \frac{\zeta_t}{h_t^{u*}} + \beta_t^{u*} E_t \lambda_{t+1}^{u*} (1 - \delta_t^h) p_{t+1}^{h*}, \]  

(67)

54
• Constrained household budget constraint, debt constraints, and first-order conditions \((c_t^*, b_t^*, h_t^*)\),

\[ c_t^* = \frac{\zeta_t}{c_t^*}, \]  

\[ c_t^* + \frac{R_t}{\pi_t} b_{t-1}^* + p_{t-1} h_t^* \left[ h_t^* - (1 - \delta_h^*) h_{t-1}^* \right] = b_t^* + (1 - \tau_w^*) w_t^* - T_t^*, \]  

(68)

\[ b_t^* \leq \begin{cases} 
R_t^{t-1} m_t^* E_t \pi_{t+1}^* p_{t-1}^{h_t^*} h_t^*, & \text{if } m_t^* R_t^{t-1} E_t \pi_{t+1}^* p_{t+1}^{h_t^*} h_t^* \geq \gamma^* b_{t-1}^*/\pi_t^*, \\
\gamma^* b_{t-1}^*/\pi_t^*, & \text{if } m_t^* R_t^{t-1} E_t \pi_{t+1}^* p_{t+1}^{h_t^*} h_t^* < \gamma^* b_{t-1}^*/\pi_t^*, 
\end{cases} \]  

(69)

\[ \lambda_t^* = \beta^* E_t \frac{R_t}{\pi_t} \lambda_{t+1}^* + \xi_t^* \mathbf{1}(\xi_t^* \geq 0) + \mu_t^* \mathbf{1}(\xi_t^* < 0) - \beta^* \gamma^* E_t \frac{\mu_{t+1}^*}{\pi_t^*} \mathbf{1}(\xi_{t+1}^* < 0), \]  

(70)

\[ \lambda_t^* p_t^{h_t^*} = \frac{\zeta_t}{h_t^*} + \beta^* E_t \lambda_{t+1}^* \left( 1 - \delta_h^* \right) p_{t+1}^{h_t^*} + \xi_t^* \mathbf{1}(\xi_t^* \geq 0) \left( m_t^* E_t \pi_{t+1}^* p_{t+1}^{h_t^*} h_t^* - \gamma^* b_{t-1}^*/\pi_t^* \right) \]  

(71)

where \(\mu_t\) is the Lagrange multiplier on constraint (7) in the text, \(\mathbf{1}(\cdot)\) is the indicator function and \(R_t^* \equiv R_t^{t-1} m_t^* E_t \pi_{t+1}^* p_{t+1}^{h_t^*} h_t^* - \gamma^* b_{t-1}^*/\pi_t^*\).

• Entrepreneur budget constraint, debt constraints, and first-order conditions \((c_t^*, b_t^*, h_t^*, n_t^*, k_t^*)\),

\[ c_t^* = \frac{\zeta_t}{c_t^*}, \]  

(72)

\[ c_t^* = m c_t^* \left( k_{t-1}^* \right)^{\alpha_h^*} \left( h_{t-1}^* \right)^{\alpha_h^*} \left( n_{t-1}^* \right)^{1-\alpha_h^*} \left( k_{t-1}^* \right)^{1-\alpha_h^*} - w_t^* n_t^* - p_t^{h_t^*} \left[ h_t^{h_t^*} - (1 - \delta_h^*) h_{t-1}^{h_{t-1}^*} \right] + b_t^{h_t^*} - \frac{R_t^{t-1} b_t^{h_t^*} q_t^{\left[ k_{t-1}^* - (1 - \delta_h^*) k_{t-1}^* \right]} + \Pi_t^* + \Pi_t^{k_t^*} + \Pi_t^{h_t^*}, \]  

(73)

\[ b_t^{h_t^*} \leq \begin{cases} 
R_t^{t-1} m_t^* E_t \pi_{t+1}^* p_{t+1}^{h_t^*} h_t^{h_t^*}, & \text{if } m_t^* R_t^{t-1} E_t \pi_{t+1}^* p_{t+1}^{h_t^*} h_t^{h_t^*} \geq \gamma^* b_{t-1}^*/\pi_t^*, \\
\gamma^* b_{t-1}^*/\pi_t^*, & \text{if } m_t^* R_t^{t-1} E_t \pi_{t+1}^* p_{t+1}^{h_t^*} h_t^{h_t^*} < \gamma^* b_{t-1}^*/\pi_t^*, 
\end{cases} \]  

(74)

\[ \lambda_t^{h_t^*} = \beta^* E_t \frac{R_t}{\pi_t} \lambda_{t+1}^{h_t^*} + \xi_t^{h_t^*} \mathbf{1}(\xi_t^* \geq 0) + \mu_t^{h_t^*} \mathbf{1}(\xi_t^* < 0) - \beta^* \gamma^* E_t \frac{\mu_{t+1}^*}{\pi_t^*} \mathbf{1}(\xi_{t+1}^* < 0), \]  

(75)
\[ \lambda'_t p^*_t = \beta^* E_t \lambda^*_t \left[ mc^*_t \alpha_k (k^*_t)^\alpha_k \left( h^*_t \right)^{\alpha_k-1} \left( n^*_t \right)^{1-\alpha_k} + (1 - \delta^*_k) p^*_t \right] + \xi^*_t \left( m^*_t R^*_t \right) E_t \pi^*_t p^*_{t+1} 1 (\pi^*_t \geq 0) , \]

(77)

\[ w^*_t = mc^*_t (1 - \alpha^*_k) (k^*_t)^\alpha_k \left( h^*_t \right)^{\alpha_k} \left( n^*_t \right)^{-\alpha_k} , \]

(78)

\[ \lambda^*_t q^*_t = \beta^* E_t \lambda^*_t \left[ mc^*_t \alpha_k (k^*_t)^\alpha_k - (h^*_t)^{\alpha_k} \left( n^*_t \right)^{1-\alpha_k} + (1 - \delta^*_k) q^*_t \right] , \]

(79)

where \( \mu^*_t \) is the Lagrange multiplier on constraint (7) in the text, and \( \pi^*_t \equiv R_t^{-1} m^*_t E_t \pi^*_{t+1} h^*_t - \gamma^* u^*_{t-1}/\pi_t^* \).

- Retailers’ optimal price decision, and aggregate profits,

\[ E_t \sum_{s=0}^{\infty} \left( \beta^* \theta_p^* \right)^s \frac{\lambda^*_t}{\lambda^*_t} \left[ \frac{1-\tau^*_p}{\pi_{F,t}^*} p^*_{F,t+s} - \frac{\varepsilon^*_p}{\varepsilon^*_p - 1} mc^*_t \right] \left( \frac{\Pi^*_{j=1} \pi_{F,t+j}^* \varepsilon^*_p}{\pi^*_t} \right) y^*_{t+s} = 0 , \]

(80)

\[ \Pi^*_t = y^*_{t} \left[ 1 - \tau^*_p (p^*_{F,t} - mc^*_t) \right] , \]

(81)

- Dynamics of PPI inflation and price dispersion,

\[ 1 = (1 - \theta^*_p) \left( \tilde{p}^*_t \right)^{1-\varepsilon^*_p} + \theta^*_p \left( \pi_{F,t}^* \right)^{\varepsilon^*_p-1} , \]

(82)

\[ \Delta^*_t \equiv (1 - \theta^*_p) \left( \tilde{p}^*_t \right)^{-\varepsilon^*_p} + \theta^*_p \left( \pi_{F,t}^* \right)^{\varepsilon^*_p} \Delta^*_{t-1} . \]

(83)

- Construction firm output, first order conditions \( (n^*_t, i^*_t) \), and profits,

\[ P^*_t = (n^*_t)^{\omega^*} \left\{ i^*_t \left[ 1 - \Phi^*_h \left( \frac{i^*_t}{i^*_t} \right)^2 \right] \right\}^{1-\omega^*} , \]

(84)

\[ w^*_t = P^*_t \omega^* (n^*_t)^{\omega-1} \left\{ i^*_t \left[ 1 - \Phi^*_h \left( \frac{i^*_t}{i^*_t} \right)^2 \right] \right\}^{1-\omega^*} , \]

(85)
\[ 1 = p_t^{h*} (n_t^{h*})^{\omega^*} (1 - \omega^*) \left\{ i_t^{h*} \left[ 1 - \frac{\Phi_h^*}{2} (d_t^{h*})^2 \right] \right\}^{-\omega^*} \left[ 1 - \frac{\Phi_h^*}{2} (d_t^{h*})^2 - \Phi_h^* \left( \frac{d_t^{h*}}{i_t^{h*}} \right) \right], \]

for \( d_t^{h*} = i_t^{h*}/i_{t-1}^{h*} - 1 \).

- Equipment capital producers output, first order condition (\( i_t^* \)), and profits,

\[ I_t^* = i_t^* \left[ 1 - \frac{\Phi_k^*}{2} \left( \frac{i_t^*}{i_{t-1}^*} - 1 \right)^2 \right], \]

\[ 1 = q_t^* \left[ 1 - \frac{\Phi_k^*}{2} (d_t^*)^2 - \Phi_k^* \left( \frac{i_t^*}{i_{t-1}^*} \right) \right] + E_t^{\lambda^*_t} \frac{q_t^*}{\lambda^*_t} \left( \frac{i_{t+1}^*}{i_t^*} \right)^2, \]

\[ \Pi_t^{k*} = q_t^* I_t^* - i_t^*, \]

for \( d_t^* = i_t^*/i_{t-1}^* - 1 \).

- Optimal wage decision,

\[ \sum_{x=c,u} E_t \sum_{s=0}^{\infty} (\beta^w \theta_w^*)^s \left[ \frac{(1 - \tau^*_w) \bar{w}_t^* w_{t+s}^*}{\prod_{j=1}^{s} \tau_{t+j}^*} \frac{1}{\lambda_t^{x+s}} \right] \left( \frac{w_t^*}{\prod_{j=1}^{s} \tau_{t+j}^*} \right) \left[ \frac{\varepsilon_w^*}{\varepsilon_w^* - 1} \right] n_{t+s}^{x*} = 0, \]

with \( \beta^{c*} = \beta^* \).

- Dynamics of wage inflation and wage dispersion,

\[ 1 = (1 - \theta_w^* \bar{w}_t^* \varepsilon_w^* + \theta_w^* \bar{\pi}_{w*} \varepsilon_w^* - 1), \]

\[ \Delta_t^{w*} = (1 - \theta_w^* \bar{w}_t^* \varepsilon_w^* + \theta_w^* \bar{\pi}_{w*} \varepsilon_w^* \Delta_{t-1}^{w*}). \]
• Fiscal authority’s budget constraint,

\[ \tau_w w_t (n_t c^* + n_t^u*) + \tau p_F t y_t^* + 2T_t^* = p_F t g_t^*. \] (94)

• Aggregate employment,

\[ N_t^c* = n_t^c* \Delta_t^u*; \] (95)
\[ N_t^u* = n_t^u* \Delta_t^w*; \] (96)
\[ N_t^* = N_t^c* + N_t^u*. \] (97)

• Export demand,

\[ x_t^* = \frac{s}{1 - s} (1 - \omega_H) (\hat{H}_t^* P_{H_t})^{-\varepsilon_F} (c_t^c + c_t^u + c_t^c + i_t + i_t^h). \] (98)

• Intermediate good market clearing,

\[ y_t^* \Delta_t^* = (k_{t-1}^*)^{\alpha_h} (h_{t-1}^c)^{\alpha_h} (n_t^c^*)^{1 - \alpha_h^* - \alpha_h^*}, \] (99)

• Labor market clearing,

\[ n_t^c* + n_t^u* = n_t^c* + n_t^h*. \] (100)

• Consumption goods basket market clearing,

\[ y_t^* = c_{F,t}^c + c_{F,t}^u + c_{F,t}^c + i_{F,t}^c + i_{F,t}^h + g_t^* + x_t^*. \] (101)

• Real estate market clearing,

\[ h_t^* + h_t^u* + h_t^c* = I_t^h* + (1 - \delta_h^*) (h_{t-1}^* + h_{t-1}^u* + h_{t-1}^c*). \] (102)

• Equipment capital market clearing,

\[ k_t^* = (1 - \delta_k^*) k_{t-1}^* + I_t^*. \] (103)
• Real wages,
\[ w_t^* = w_{t-1}^* \frac{\pi_{mt}}{\pi_t^*}, \] (104)

• Terms of trade,
\[ \hat{p}_t^* = 1/\hat{p}_t. \] (105)

• Relative demand for domestic goods,
\[ z_{F,t}^* = \omega_F^* \left( p_{F,t}^* \right)^{1-\varepsilon_F} z_t^*, \quad z = c^e, c^u, c^e, i, i^h \] (106)

• Relative demand for constrained/unconstrained household labor,
\[ (1 - \mu^*) n_t^c = \mu^* n_t^u, \] (107)
where \( \mu = \mu_c = \mu_h. \)

• Relative domestic producer prices,
\[ \left( p_{F,t}^* \right)^{1-\varepsilon_F} = \omega_F^* + \left( 1 - \omega_F^* \right) \left( \hat{p}_t^* \right)^{1-\varepsilon_F}. \] (108)

• CPI inflation,
\[ (\pi_t^*)^{1-\varepsilon_F} = \omega_F^* \left( \hat{p}_{t-1}^* \right)^{1-\varepsilon_F} + \frac{1 - \omega_F^*}{\omega_F^* \left( \hat{p}_{t-1}^* \right)^{1-\varepsilon_F} + 1 - \omega_F^*} \pi_{F,t}^{1-\varepsilon_F} + \frac{1 - \omega_F^*}{\omega_F^* \left( \hat{p}_{t-1}^* \right)^{1-\varepsilon_F} + 1 - \omega_F^*} \pi_{H,t}^{1-\varepsilon_F}, \] (109)

• Real (PPI-deflated) GDP,
\[ gdp_t^* = y_t^* + \frac{1}{p_{H,t}^*} \left( q_t^* I_t^* - i_t^* \right) + \frac{1}{p_{H,t}^*} \left( p_{h,t}^* i_t^h - i_t^h \right), \] (110)
• Gross nominal interest rate,

\[
R^*_t = R^{MU}_t \exp \left( -\psi^* \frac{d^*_t - b^*_t - \psi^*_t}{p^{H,t}_{t} g d^*_t} + \psi^*_0 \right).
\] (111)

Union-wide variables

• Nominal policy interest rate,

\[
R^{MU}_t = (\bar{R}^{MU})^{1-\rho_R} (\bar{R}^{MU}_{t-1})^{\rho_R} (\bar{\pi}^{MU}_t)^{\rho_s (1-\rho_R)}.
\]

• Union-wide CPI inflation rate,

\[
\bar{\pi}^{MU}_t = s \pi_t + (1-s) \pi^*_t.
\]

B. Equilibrium unemployment

Following Galí (2011), we assume that each representative household consists of a unit squared of individuals indexed by \((i,j) \in [0,1] \times [0,1]\), where \(i\) represents the variety of labor service provided by the individual and \(j\) indexes her disutility from working, given by \(\chi^j \varphi^j\). Let \(n^x_t (i)\) denote the number of variety-\(i\) workers in household \(x = c, u\) employed at time \(t\). Total household disutility from working is given by

\[
\chi \int_0^1 \int_0^{n^x_t (i)} \varphi^j dj di = \chi \int_0^1 \frac{n^x_t (i)^{1+\varphi}}{1+\varphi} di,
\]

for \(x = c, u\). Given the type-specific wage \(W_t (i)\), the number of type-\(i\) workers that each household would like to send to work is given by

\[
\arg \max_{n^x_t (i)} \left\{ \frac{\lambda^x_t W_t (i) - n^x_t (i)}{P_t} - \zeta^x_t \frac{n^x_t (i)^{1+\varphi}}{1+\varphi} \right\} = \left( \frac{\lambda^x_t W_t (i)}{\zeta^x_t P_t} \right)^{1/\varphi} \equiv l^x_t (i),
\]

for \(x = c, u\), where \(\lambda^x_t \equiv 1/c^x_t\). Unemployment in the market for type-\(i\) labor is just the number of workers willing to work at the going wage minus effective labor
demand: \( u_t(i) \equiv \sum_{x=c,u} l^x_t(i) - \sum_{x=c,u} n^x_t(i) \). Let

\[
l^x_t \equiv \int_0^1 l^x_t(i) \, di = \left( \frac{\lambda^x_t \, W_t}{\zeta_t \, P_t} \right)^{1/\phi} \int_0^1 \left( \frac{W_t(i)}{W_t} \right)^{1/\phi} \, di = \left( \frac{\lambda^x_t \, W_t}{\zeta_t \, P_t} \right)^{1/\phi} \Delta^w_{t}^x;
\]

\[
N^x_t \equiv \int_0^1 n^x_t(i) \, di = n^x_t \int_0^1 \left( \frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} \, di = n^x_t \Delta^w_{t}^n_x,
\]

denote total household-specific labor supply and labor demand, respectively, for \( x = c, u \), where \( \Delta^w_{t}^x \equiv \int_0^1 (W_t(i)/W_t)^{1/\phi} \, di \) and \( \Delta^w_{t}^n \equiv \int_0^1 (W_t(i)/W_t)^{-\varepsilon_w} \, di \) are indexes of wage dispersion. Then aggregate unemployment is

\[
u_t \equiv \int_0^1 u_t(i) \, di = l_t - N_t,
\]

where \( l_t \equiv \sum_{x=c,u} l^x_t \) and \( N_t \equiv \sum_{x=c,u} N^x_t \) are aggregate labor supply and labor demand, respectively. Finally, the unemployment rate is \( \overline{u}_t \equiv u_t/l_t \).