DISCUSSION PAPER SERIES

No. 10140

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Atsushi Inoue, Chun-Hung Kuo
and Barbara Rossi

INTERNATIONAL MACROECONOMICS

ISSN 0265-8003

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Atsushi Inoue, Vanderbilt University and Tohoku University
Chun-Hung Kuo, International University of Japan
Barbara Rossi, ICREA-Universitat Pompeu Fabra, Barcelona GSE and CREI

Discussion Paper No. 10140
September 2014

Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 10140

September 2014

ABSTRACT

Identifying the Sources of Model Misspecification*

In this paper we propose empirical methods for detecting and identifying misspecifications in DSGE models. We introduce wedges in a DSGE model and identify potential misspecification via forecast error variance decomposition (FEVD) and marginal likelihood analyses. Our simulation results based on a small-scale DSGE model demonstrate that our method can correctly identify the source of misspecification. Our empirical results show that the medium-scale New Keynesian DSGE model that incorporates features in the recent empirical macro literature is still very much misspecified; our analysis highlights that the asset and labor markets may be the source of the misspecification.

JEL Classification: C32 and E32
Keywords: DSGE models, empirical macroeconomics and model misspecification

Atsushi Inoue
Department of Economics
Vanderbilt University
Nashville, TN 37235
USA

Email: atsushi.inoue@vanderbilt.edu

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Chun-Hung Kuo
Department of Economics
International University of Japan
777 Kokusai-cho
Minami Uonuma-shi
Niigata 949-7277
JAPAN

Email: ckuo2@ncsu.edu

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Barbara Rossi
Department of Economics and
Business
Universitat Pompeu Fabra
Ramon Trias Fargas 25-27
08005 Barcelona
SPAIN

Email: barbara.rossi@upf.edu

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*The first and third authors acknowledge financial support from the National Science Foundation through grants 102159 and 1022125, respectively. The authors thank the conference and seminar participants at the Applied Time Series Econometrics Workshop at the Federal Reserve Bank of St. Louis, Chung-Hua Institution for Economic Research, International University of Japan, National Taipei University and the Triangle Econometrics Conference for helpful comments on earlier versions of the paper.

Submitted 03 September 2014

1 Introduction

In this paper, we examine the empirical importance of model misspecification in dynamic stochastic general equilibrium (DSGE) models. DSGE models are commonly used in academia and central banks as standard tools for analyzing macroeconomic policies – see Del Negro and Schorfheide (2012)) among others. However, DSGE models are often highly parameterized to better fit the data, which raises the concern that they may be misspecified. In a recent paper, Del Negro, Schorfheide, Smets, and Wouters (2007) raise this very issue and show that model misspecification cannot be ignored in policy analyses. While model misspecification may be widespread, macroeconomists are often left wondering whether their models are misspecified and, if they are, what parts of the models are misspecified. In this paper, we propose an empirical framework for addressing exactly these issues.

To tackle the issue of model misspecification, it is beneficial to reexamine economists' view about the sources of business cycles. When studying business cycles using DSGE models, economists usually incorporate into models several exogenous stochastic processes, which are called structural shocks. These shocks are treated as structural because they assume that economic agents understand the exact nature of these shocks. Hence, when some shocks happen, economic agents rationally adjust their behavior. Under this viewpoint, the business cycle phenomena are fully driven by fluctuations of structural shocks included. That is, the underlying uncertainty of the real economy are well captured by the structural shocks included. The above view is commonly held by New Keynesian theorists. See Justiniano, Primiceri, and Tambalotti (2010) and Smets and Wouters (2007), for examples.

There appears to be no consensus among macroeconomists how to model and interpret structural shocks, however. For example, real-business-cycl theorists tend to model Hicks-neutral and investment-specific technology shocks in the production function and capital accumulation equations, respectively, while new Keynesian macroeconomists tend to include so-called price and wage markup shocks in their imperfect competition models. Chari, Kehoe, and McGrattan (2009) argue that the elasticity of substitution among differentiated labors fluctuates with a huge volatility in the model of Smets and Wouters (2007) and it may be due to the wage markup shock.

To examine model misspecification issues in DSGE models, we propose to introduce two types of exogenous processes into a model. The first type of exogeneous shocks are structural and are interpreted in the conventional way; the second type of exogenous shocks are not structural and are labeled as *wedges*. We incorporate these time-varying wedges into the agents' optimization problem (such as the households' budget constraints) to allow deviations in the relative prices of relevant goods because the model misspecification will eventually lead to distorted relative prices. These wedges can be used to measure both the nature and importance of misspecification. In fact, by estimating the wedges, we are able to assess: (i) where misspecification might be located (that is, which parts of the model are more affected by the misspecification); and (ii) how qualitatively important it is. While wedges could be interepreted as another structural shock, we include wedges only after reasonably many structural shocks are modeled.

To provide more intuition and better illustrate our framework, we consider a medium-scaled

DSGE model, embedded with most New-Keynesian features. If we treat the structural shocks as the integral part of the model, our model is mildly misspecified in the sense that (i) the cross equation and equilibrium restrictions imposed by the model do not exactly hold in every time period; and (ii) these deviations from equilibrium are zero on average. We assume that the economic agents in the model (both firms and households) take into account the exogenous stochastic processes of the deviations from equilibrium when solving their optimization problems. We interpret the variances of the deviations as a measure of the degree of misspecification of the New Keynesian DSGE model. To examine where and how large the misspecification is, we conduct impulse response analyses, forecast error variance decompositions (FEVDs) and marginal likelihood comparison.

Our empirical application to a state-of-the-art DSGE model *á la* Smets and Wouters (2003, 2007) and Christiano et al. (2005) highlights two interesting findings. First, our technique points to severe misspecification in the labor demand component of the model. Second, the bond market also shows evidence of misspecification, which is persistent in its nature. Our findings suggest that further work in these areas would be beneficial for the models.

Our method is related to several recent contributions in the literature. Our paper is closely related to Chari, Kehoe, and McGrattan (2007) who introduce time-varying "wedges" into a macroeconomic model to capture deviations from equilibrium conditions. There are two main differences between our work and theirs. One difference is that they consider a neoclassical stochastic growth model with wedges (what they call the benchmark prototype economy) while we consider a New Keynesian DSGE model with wedges. Our New Keynesian DSGE model is based on Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007) and incorporates several frictions. Thus, our distortions reflect model misspecification that cannot be accounted for by frictions that are built in the model. The other difference is that their analysis is based on calibrated parameter values, while ours is based on a fully estimated model, where the estimation takes into account possible misspecification. Another closely related work is Del Negro and Schorfheide (2009). Using the method developed in their earlier work (Del Negro and Schorfheide (2004)), they develop a framework for Bayesian estimation of possibly misspecified DSGE models. Specifically, they use DSGE-implied parameters as a prior for vector autoregressive (VAR) models. Their framework allows for model misspecification and produces the posterior distribution of structural parameters as well as the posterior structural impulse responses implied by DSGE-implied priors.[1] Our framework complements theirs in that we can identify which parts of the model are misspecified.

The rest of the paper is organized as follows: Section 2 reports results based on simulation exercises to demonstrate that the proposed method is capable to identify the sources of model misspecification in a small New Keynesian model. In Section 3 we describe our empirical model, inspired by state-of-the-art New Keynesian models. Section 4 discusses the estimation results on the sources of model misspecification. Section 5 concludes.

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[1] It should be noted that they assume that DSGE models have a VAR representation in their implementation. Many DSGE models have VARMA representations and we do not need to assume a VAR representation although the error due to VAR approximations may be small if the lag is sufficiently large.

2 Simulation Analysis

In this section, we provide some simulation evidence to demonstrate how to use our proposed method to figure out whether or not a model is misspecified and, in case it is misspecified, how the source of model misspecification can be detected. To do so, we first lay out a simple New Keynesian model and use it as the data generating process for hours worked, real output, nominal interest rate, and inflation rate. We then use these observables to estimate three misspecified models and one correctly specified model. For each of these models, we report forecast error variance decomposition (FEVD) analyses and calculate the marginal likelihood of the estimated models. The two measures are useful for detecting the sources of model misspecification.

2.1 The Data-Generating Process (DGP)

We consider a simple New Keynesian model without the capital accumulation decision. It consists of a representative final good firm, a continuum of intermediate good firms, a representative household and a monetary and fiscal authority. Because similar models have been considered in the literature (e.g., An and Schorfheide, 2007), we describe the model only briefly.

A representative final good firm transforms a continuum of intermediate goods $Y_{j,t}$ into a homogeneous final good Y_t by the following technology:

$$Y_t = \left[\int_0^1 \left(Y_{j,t} \right)^{\frac{1}{\lambda_f}} \, \mathrm{d}j \right]^{\lambda_f} . \tag{1}$$

Taking the prices of intermediate goods $(P_{j,t})$ and that of the final good (P_t) as given, the final good firm solves the profit maximization problem:

$$\max_{\left\{Y_{j,t}\right\}_{j\in[0,1]}} \quad P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} \mathrm{d}j,$$

subject to equation (1).

The intermediate goods firms are monopolistic providers of differentiated goods. They are, hence, able to set their own product prices. Following Calvo (1983), we assume that in each period of time only a fraction $1 - \xi$ of intermediate goods firms are able to set their desired optimal price. As a consequence, when an intermediate good firm is able to re-optimize its price in period t, its pricing problem can be expressed as follows:[2]

$$\begin{aligned} \max_{\tilde{P}_t} \quad & \mathrm{E}_t \sum_{l=0}^{\infty} (\beta \xi)^l \, v_{t+l} \left\{ \tilde{P}_t - P_{t+l} s_{t+l} \right\} \tilde{Y}_{t+l} \\ \text{s.t.} \quad & \tilde{Y}_{t+l} = \left(\frac{\tilde{P}_t}{P_t} \right)^{-\frac{\lambda_f}{\lambda_f - 1}} Y_{t+l} \end{aligned} \tag{2}$$

The constraint is the intermediate good demand function in period $t + l$ with the price set in period

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[2] Note that there is no need to specify the firm subscripts in this pricing problem since firms, which are able to re-optimize their problem, face the same problem and make the same decision.

t. s_{t+l} is the real marginal cost in period $t+l$. v_{t+l} is the marginal value of per unit of money for households at time $t+l$, because households hold equities of intermediate good firms. Lastly, the presence of ξ^l in the objective function indicates the probability of not being able to re-optimize the price after l periods.

Intermediate goods firms only use labor to produce their products, and their production functions are assumed linear: $Y_{j,t} = z_t L_{j,t}$. In terms of the deviation from the steady state, the stochastic technology shock z_t follows an AR(1) process

$$\hat{z}_{t+1} = \rho_z \hat{z}_t + \sigma_z \varepsilon_{z,t+1}, \quad \varepsilon_{z,t+1} \overset{\text{i.i.d.}}{\sim} N(0,1).$$

Taking the nominal interest R_t and nominal wage rate W_t as given, each intermediate good firm $j \in [0,1]$ determines its labor input by solving the following cost minimization problem:

$$\min_{L_{jt}} W_t L_{jt}$$

subject to its production function.

A representative household receives utility from consuming final goods and disutility from providing its labor services to intermediate goods firms. The representative household maximizes the expected discounted lifetime utility function:

$$\max_{\{C_t, L_t\}_{t=0}^{\infty}} \mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{\chi_t L_t^{1+\varphi}}{1+\varphi} \right\},$$

subject to its budget constraint:

$$P_t C_t + B_{t+1} = R_{t-1} B_t + W_t L_t + \mathsf{D}_t + \mathsf{T}_t.$$

The household has several sources of income in each time period. First, the household collects return from holding government bonds with gross nominal interest rate R_t. Second, by providing labor services, the household obtains labor income at the overall nominal wage rate W_t. Third, the household receives dividends D_t from holding equities of intermediate goods firms. Finally, there exists a lump-sum transfer (or taxation) T_t from the government sector. The household uses its income to buy C_t units of the final good under the competitive price P_t and to hold government bond B_{t+1}, which facilitates consumption smoothing across time. There exists a labor supply shock χ_t. In terms of log deviation from its steady state value, the labor supply shock (χ_t) follows:

$$\hat{\chi}_{t+1} = \rho_\chi \hat{\chi}_t + \sigma_\chi \varepsilon_{\chi,t+1}, \quad \varepsilon_{\chi,t+1} \overset{\text{i.i.d.}}{\sim} N(0,1)$$

The government is the sole provider of bond assets, and it consumes final goods for non-productive purposes. We assume that the government spending G_t is stochastic, and governed by a stochastic process ζ_t such that $G_t = \zeta_t Y_t$. To simplify the equilibrium conditions of the model, we reparametrize ζ_t as $g_t \equiv \frac{1}{1-\zeta_t}$, and assume that in terms of log deviation, g_t follows an AR(1) process:

$$\hat{g}_{t+1} = \rho_g \hat{g}_t + \sigma_g \varepsilon_{g,t+1}, \quad \varepsilon_{g,t+1} \overset{\text{i.i.d.}}{\sim} N(0,1).$$

In terms of log deviation from steady state values, the Central Bank determines the nominal interest rate by the Taylor rule:

$$\hat{R}_t = \rho_1 \hat{R}_{t-1} + \rho_2 \hat{R}_{t-2} + (1 - \rho_1 - \rho_2) \left\{ \gamma_\pi \hat{\pi}_t + \gamma_Y \hat{Y}_t \right\} + \hat{\nu}_t.$$

The two lag terms of the nominal interest rate capture the persistence in the short-term nominal rate. The random variable $\hat{\nu}_t$ captures the non-systematic part of the nominal rate decision and follows:

$$\hat{\nu}_{t+1} = \rho_\nu \hat{\nu}_t + \sigma_\nu \varepsilon_{\nu,t+1}, \quad \varepsilon_{\nu,t+1} \overset{\text{i.i.d.}}{\sim} N(0,1).$$

The linearized equilibrium conditions of the above model are list in Appendix A.[3]

2.2 Fitted Models

In order to demonstrate the usage of the proposed method, we estimate four different models to reflect some possible model misspecifications. Let us call them $\mathcal{M}_0$, $\mathcal{M}_l$, $\mathcal{M}_m$, and $\mathcal{M}_g$, respectively:

- $\mathcal{M}_0$: The model is correctly specified as the data-generating process.
- $\mathcal{M}_l$: The model is misspecified in the sense that the labor supply shock $\hat{\chi}_t$ is excluded from the set of structural shocks. In other words, we assume that there are only three structural shocks in this model.
- $\mathcal{M}_m$: The model is misspecified in sense that the non-systematic part of the the nominal rate decisions, $\hat{\nu}_t$, is assumed as an *iid* random variable rather than following the AR(1) process as it should be.
- $\mathcal{M}_g$: The model is misspecified in that the government spending shock $\hat{g}_t$ is excluded from the set of the structural shocks. Thus, there are only three structural shocks left in the model.

It is worth to note that, in the above models, we exactly know whether or not they are misspecified and the sources of model misspecification. In contrast to actual empirical studies, such valuable information is too luxury for researchers to have. In order to mimic such circumstance, we assume for a moment that we do not know which models are misspecified and introduce some wedges into the above four models. Specifically, we introduce a time-varying labor wedge $\tau_{l,t}$ into the cost minimization problem of each intermediate good firm such the problem can be modified as:

$$\min_{L_{j,t}} \tau_{l,t} W_t L_{j,t},$$

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[3] For the derivation of these conditions, we refer to the technical appendix of the paper, which is available form the authors upon request.

Table 1: The DGP Parameter Values and the Prior Setting of Simulation Exerecises

			Prior		
Parameter	Description	DGP	Type	Mean	STD
β	Subjective discount factor	0.99	-	-	-
γ	Risk aversion coefficient	1.50	-	-	-
φ	inverse Frisch elasticity	1.00	-	-	-
ξ	Calvo parameter	0.67	B	0.70	0.05
γ_π	Interest rate policy rule: inflation	1.80	G	2.00	0.05
γ_y	Interest rate policy rule: output	0.50	G	0.40	0.05
ρ_r	Persistence coef. of interest rate	0.70	B	0.75	0.10
ρ_z	Persistence coef. of technology shocks	0.70	B	0.75	0.10
ρ_ν	Persistence coef. of monetary shocks	0.70	B	0.75	0.10
ρ_g	Persistence coef. of spending shocks	0.70	B	0.75	0.10
ρ_χ	Persistence coef. of labor supply shocks	0.70	B	0.75	0.10
σ_z	Std. Of innovation of technology shocks	0.10	I	0.30	2.00
σ_ν	Std. Of innovation of monetary shocks	0.10	I	0.30	2.00
σ_g	Std. Of innovation of spending shocks	0.10	I	0.30	2.00
σ_χ	Std. Of innovation of labor supply shocks	0.10	I	0.30	2.00
ρ_l	Persistence coef. of labor wedges ($\tau_{l,t}$)	-	B	0.50	0.20
ρ_c	Persistence coef. of consumption wedges ($\tau_{c,t}$)	-	B	0.50	0.20
ρ_b	Persistence coef. of bond wedges ($\tau_{b,t}$)	-	B	0.50	0.20
σ_l	Std. Of innovation of labor wedges	-	I	0.10	2.00
σ_c	Std of innovation of consumption wedges	-	I	0.10	2.00
σ_b	Std of innovation of bond wedges	-	I	0.10	2.00

subject to its production function. Moreover, we introduce a final good wedge $\tau_{c,t}$ and a bond demand wedge $\tau_{b,t}$ into the household budget constraint, and thus, the constraint is modified to:

$$\tau_{c,t}P_tC_t + B_{t+1} = \tau_{b,t}R_{t-1}B_t + W_tL_t + \mathsf{D}_t + \mathsf{T}_t.$$

We assume that these wedges all follow independent AR(1) processes. That is, for $x \in \{l, c, b\}$

$$\hat{\tau}_{x,t+1} = \rho_x\hat{\tau}_{x,t} + \sigma_x\varepsilon_{x,t+1}, \quad \varepsilon_{x,t+1} \overset{\text{i.i.d.}}{\sim} N(0,1).$$

{I feel we need to explain the philosophy of our method such as why we introduce the above 3 wedges in the particular place.}

2.3 Simulation Results

Using the data simulated from the DGP in subsection 2.1, we estimate the four model mentioned above by the typical Bayesian estimation method. The simulated observables consist of hours worked $\hat{L}_t$, real output $\hat{Y}_t$, inflation rate $\hat{\pi}_t$, and the nominal interest rate $\hat{R}_t$.[4] The sample size of simulated data is 100, which is in the neighborhood of usual sample sizes adopted in business cycle studies based on quarterly data. The parameter values used to simulate the data are summarized in Table 1, and basically, these values are set within the widely accepted range.

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[4] As one can see from the notations of the observables, these pseudo-true data are expressed in terms of log deviation from the corresponding steady state values.

To conduct the Bayesian estimation method, we adopt the random walk Metropolis Hasting algorithm (An and Schorfheide (2007)) to characterize the posterior distribution. The prior settings for these four models are basically the same and listed in Table 1. However, it is worth noting that depending on a particular model at hand, some parameters are not necessary to be estimated. Specifically, in the model $\mathcal{M}_l$, where the labor supply shocks are excluded, ρ_χ and σ_χ are hence set to 0; in the model $\mathcal{M}_g$, where government spending shocks are absent, ρ_g and σ_g are set to 0; in the model $\mathcal{M}_m$, where the non-systematic part of the Taylor rule is misspecified, ρ_ν is set to 0. Moreover, to highlight how this wedges-based method can help us identify the sources of model misspecification and to reduce some potential noises due to estimation, there parameters are not estimated across the 4 considered models. They are the subjective discount factor β, the risk aversion coefficient γ, and the inverse elasticity of labor supply φ. These parameters are fixed in the true parameter values of the DGP. In our estimation exercises, the number of MCMC draws are 120,000, and the first 60,000 draws are discarded.

The first tool of investigating the sources of model misspecification is the forecast error variance decomposition (FEVD). Traditionally, econometricians use FEVD to evalulate the contribution of different structural shocks on the variation of interested variable. See, for example, Ireland (2001) and Justiniano, Primiceri, and Tambalotti (2010). Usually, researchers assume the model is correctly specified such that all of variations are fully explained by the included structural shocks. In contrast to this traditional assumption, we take a rather conservative stand. That is, we think all the models at hand are potentially misspecified. Thus, it is worth including various wedges into the model and to examine the contribution of these wedges in terms of FEVD.

The FEVDs of the considered models are summarized in Table 2.[5] The FEVD analysis of model $\mathcal{M}_0$ in the panel (a) shows that the structural shocks, such as the labor supply shock, explain the most of FEVD in this economy as expected. Since the model $\mathcal{M}_0$ is correctly specified, the three wedges are essentially redundant. The FEDV contributions of these wedges are indeed negligible for any of the four variables, which indicate that the model is correctly specified.

In model $\mathcal{M}_l$, the FEVD results in panel (b) show that the contribution of the labor wedge (τ_l) is substantial for all the four observables. Comparing these results with those in panel (a), we can easily observe that the role of the labor supply shock in FEVD is almost replaced by the labor wedge. This is because the omission of the labor supply shock in the utility function distorted the wage in the labor market of model $\mathcal{M}_l$.

Examining panel (c), where the FEVD of model $\mathcal{M}_m$ is summarized, one can see that the bond wedge explains the majority of the forecast error decompositions of all variables. The labor supply and final good wedges explain almost nothing.

Compared with panel (a), it is clear that the final good wedge somehow replaces the role of government spending shock g in panel (d), even though the absoulte magnitude of the contribution is not very large compared to the other two cases ($\mathcal{M}_l$ and $\mathcal{M}_m$). The latter may be due to our

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[5] We only report the variance decomposition of 12 periods ahead forecast errors. The results for longer and shorter forecast periods are more or less the same. These unreported results are available upon request from the authors.

Table 2: Forecast Error Variance Decomposition of Models

Panel (a): Model $\mathcal{M}_0$

Variable	z	ν	g	χ	τ_l	τ_c	τ_b
Hours	9.15	57.78	10.06	21.41	0.44	0.89	0.26
Output	9.64	57.47	10.01	21.29	0.44	0.89	0.26
Inflation Rate	3.56	87.81	0.10	8.17	0.29	0.00	0.07
Interest Rate	21.21	27.99	1.20	47.53	1.16	0.21	0.70

Panel (b): Model $\mathcal{M}_l$

Variable	z	ν	g	χ	τ_l	τ_c	τ_b
Hours	9.40	60.67	8.93	–	19.73	1.13	0.15
Output	9.51	60.59	8.92	–	19.70	1.13	0.15
Inflation Rate	3.39	88.29	0.06	–	8.23	0.01	0.03
Interest Rate	21.25	30.04	1.51	–	46.62	0.27	0.31

Panel (c): Model $\mathcal{M}_m$

Variable	z	ν	g	χ	τ_l	τ_c	τ_b
Hours	11.96	14.87	8.55	22.18	0.50	0.95	40.99
Output	9.66	15.26	8.77	22.76	0.51	0.97	42.06
Inflation Rate	6.54	22.84	0.19	15.10	0.62	0.02	54.69
Interest Rate	2.94	3.81	0.06	6.97	0.14	0.02	86.06

Panel (d): Model $\mathcal{M}_g$

Variable	z	ν	g	χ	τ_l	τ_c	τ_b
Hours	9.46	59.89	–	22.49	0.51	6.59	1.05
Output	9.04	60.17	–	22.60	0.51	6.63	1.06
Inflation Rate	3.37	87.25	–	8.66	0.33	0.04	0.36
Interest Rate	18.64	27.56	–	47.04	1.23	1.19	4.33

[*] Values in the table are all in terms of percentage. The results are based on 12 periods ahead forecast error.

Table 3: Marginal Likelihood Values of Models

	$\mathcal{M}_0$	$\mathcal{M}_l$	$\mathcal{M}_m$	$\mathcal{M}_g$
full wedges	-1412.3	-1415.4	-1497.1	-1412.2
remove τ_l	-1413.0	-1429.1	-1497.6	-1412.6
remove τ_c	-1411.8	-1414.4	-1496.4	-1413.3
remove τ_b	-1411.7	-1414.6	-1506.7	-1412.6

setting of DGP where the role of government spending is relatively small. The lack of the government spending in model $\mathcal{M}_g$ distorts the resource constraint which in turn causes distortion in the final good market.

Table 3 reports the marginal likelihood of the models with the three wedges as well as the likelihood of the models with one wedge removed at-a-time. The first column shows that when the correctly specified model is fitted ($\mathcal{M}_0$), the marginal likelihood does not change much regardless of which wedge is dropped. This shows that the marginal likelihood provides another way to see the wedges are negligible in the correctly specified model. When the labor supply shock is neglected ($\mathcal{M}_l$), however, the model without the labor wedge has the lowest marginal likelihood. Similarly, when the process for the monetary policy shock (model $\mathcal{M}_m$) is misspecified, the marginal likelihood is lowest when the bond demand shock is removed. When government spending is omitted from the model ($\mathcal{M}_g$), removing the final good wedge has the largest effect although it is not very large in absolute value.

To provide some intuition about why our methodology works, let's examine the linearized equilibrium conditions in Appendix A. In model $\mathcal{M}_l$, the only equation affected by the misspecification is the New Keynesian Phillips curve, where the labor supply shock is missing, and where two of the wedges ($\tau_{c,t}, \tau_{l,t}$) show up. Note that the IS curve is not affected by the mis-specification, and that two wedges ($\tau_{c,t}, \tau_{b,t}$) show up in the IS curve. Thus, the misspecification in the Phillips curve must be captured by the $\tau_{l,t}$ wedge, which is the only wedge that appears in the NKPC but not in the IS curve. This is exactly what we find.

Conversely, in model $\mathcal{M}_g$, the mis-specification affects both the NKPC and the IS equations; the only wedge that enters in both is ($\tau_{c,t}$), which will thus capture the misspecification. In model $\mathcal{M}_m$, the only equation affected by the misspecification is the Taylor rule, hence the nominal interest rate. Since the nominal interest rate appears in the IS curve and not in the NKPC, the mis-specification will be captured by the wedge that appears in the IS but not in the NKPC, that is $\tau_{b,t}$.

The lessons we learn from this simulation exercises is that by introducing some wedges into the model, it has potential to capture some possible missing channel. For example, if there are some important structural shocks are missing, it will be revealed in the wedge process.

To summarize, the FEVD and marginal likelihood analyses provide a useful tool for detecting and identifying model misspecification. When a model is misspecified, the contribution of a wedge to the FEVD is substantial at least in one variable and the type of the wedge tends to be related to that of the misspecification, thus signaling the possible cause of the misspecification. Also, removing the wedge that is most related to the type of misspecification has the largest impact on the marginal likelihood.

2.4 Suggestions for Practitioners

In practice, one does not know what parts of a model are misspecified and may wonder in which parts of the model he/she should introduce wedges. In theory, one can introduce a wedge in every market in the model but that might yield an overparameterized model even for Bayesian estimation methods. We suggest two approaches. One is to introduce wedges in markets which the macroeconomist suspects are misspecified. Another is to introduce wedges everywhere, but impose tight priors on the wedge AR(1) parameters and innovation variances if misspecification is unlikely. The first approach corresponds to the case in which the researcher has a strong view on which parts of the model are correctly specified and which parts are potentially misspecified. The second approach is more agnostic about misspecification and can incorporate prior views on misspecification.

3 The Medium-Scale New Keynesian Model

In this section, we lay out a medium-scale New Keynesian DSGE model for identifying the sources of model misspecification. Essentially, the model is a stochastic neoclassical growth model with various real and nominal frictions. The frictions that we incorporate comprise imperfect competition in the intermediate goods and labor markets, sticky price and wage, habit formation in consumption, investment adjustment cost and variable capital utilization. The model is broadly inspired by the state-of-the-art New Keynesian models, such as Smets and Wouters (2003), Smets and Wouters (2007) and Christiano, Eichenbaum, and Evans (2005). Nonetheless, these models may be still misspecified, and in our model we incorporate various time-varying wedges to reflect the potential model misspecification. Although this type of models has been widely investigated, we cannot simply list the equilibrium conditions of the model. The reason is that we treat the wedges as integral parts of agents' (including households and firms) decision problems. Thus, we have to specify the environment explicitly to show how these wedges affect agent's decisions. The implied linearized equilibrium conditions are listed in Appendix B.

3.1 Final Good Firms

Perfectly competitive final good firms produce homogeneous goods Y_t from a continuum of intermediate goods $Y_{j,t}$'s by the technology:

$$Y_t = \left[\int_0^1 (Y_{j,t})^{\frac{\eta_{p,t}-1}{\eta_{p,t}}} \mathrm{d}j\right]^{\frac{\eta_{p,t}}{\eta_{p,t}-1}}, \tag{3}$$

where $\eta_{p,t}$ governs the elasticity of substitution among different intermediate goods in period t and follows

$$\log \eta_{p,t} = (1-\rho_p)\log \eta_p + \rho_p \log \eta_{p,t-1} + \sigma_p \varepsilon_{p,t}, \quad \varepsilon_{p,t} \overset{\text{i.i.d.}}{\sim} N(0,1). \tag{4}$$

Since $\eta_{p,t}$ affects the desired markup of intermediate good firms, it is the price markup shock.[6]

Taking the final good price P_t and intermediate good prices $P_{j,t}$'s as given, final good firms solve the profit maximization problem:

$$\max_{\{Y_{j,t}\}_{j\in[0,1]}} \quad P_t Y_t - \int_0^1 \tau_{y,t} P_{j,t} Y_{j,t} \mathrm{d}j$$

subject to the final good production function (3). Notice that we introduce a time-varying intermediate good wedge $\tau_{y,t}$ into this problem, and it follows

$$\log \tau_{y,t} = (1-\rho_y)\log \tau_y + \log \tau_{y,t} + \sigma_y \varepsilon_{y,t}, \quad \varepsilon_{y,t} \overset{\text{i.i.d.}}{\sim} N(0,1), \tag{5}$$

with $\tau_y = 1$ in the steady state. The value of $\tau_{y,t}$ is assumed to be known for final good firms when they make their decisions. The reason of introducing $\tau_{y,t}$ is to capture the model misspecification in demanding intermediate goods. To see this, consider the demand function of intermediate good:

$$Y_{j,t} = \left(\frac{\tau_{y,t} P_{j,t}}{P_t}\right)^{-\eta_{p,t}} Y_t,$$

which can be easily derived from the first order condition of the above maximization problem. The intermediate good demand $Y_{j,t}$ is not only affected by the relative price $\frac{P_{j,t}}{P_t}$ but also by the intermediate good wedge $\tau_{y,t}$. This wedge is designed to capture the contribution of factors other than $\frac{P_{j,t}}{P_t}$ that affect final firms decisions. Thus, it can be interpreted as a proxy to reflect the ignorance of model builder since in reality firms consider more things than merely the prices. Moreover, $\tau_{y,t}$ helps to relax the tight restriction between intermediate good prices and the final good price. To see this, consider the following equilibrium condition:

$$P_t = \tau_{y,t} \left[\int_0^1 (P_{jt})^{1-\eta_{p,t}} \mathrm{d}j\right]^{\frac{1}{1-\eta_{p,t}}}, \tag{6}$$

which can be obtained by substituting the intermediate good demand function into (3). (6) implies that if the intermediate good prices $P_{j,t}$ cannot characterize well the the dynamics of overall price level $P_{j,t}$, the deviation between them are captured by $\tau_{y,t}$. Besides, our assumption of $\tau_y = 1$ in the steady state indicates that we take the stand that relationship between intermediate goods and final good prices is, on average, correctly specified.

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[6] Add explanation somewhere about the dubious of price markup shocks. See Chari et al (2009).

3.2 Intermediate Goods Firms

Following the tradition of New Keynesian models, we assume that there exist a continuum of intermediate goods firms, which are operating in imperfectly competitive markets. Thus, they are able to set their desired prices. A two-stage procedure involving cost minimization and profit maximization can be used to describe the intermediate good firms' optimal behavior.

Cost Minimization In period t, the j^{th} intermediate goods firm rents capital services $K_{j,t}$, and *homogeneous* labor services, $L_{j,t}$ from perfectly competitive factor markets. Taking the nominal capital service rental rate $R_{k,t}$, and the nominal wage rate W_t as well as two time-varying wedges ($\tau_{k,t}$ and $\tau_{l,t}$) as given, the firm solves the following problem:[7]

$$\begin{aligned} \min_{\{K_{j,t},L_{j,t}\}} \quad & \tau_{k,t} R_{k,t} K_{j,t} + \tau_{l,t} W_t L_{j,t} \\ \text{s.t.} \quad & Y_{j,t} = \max\left\{ z_t \left(K_{j,t}\right)^{\alpha} \left(L_{j,t}\right)^{1-\alpha} - \Phi, 0 \right\}. \end{aligned}$$

Here, α is the capital share, Φ is the fixed production cost, $R_{k,t}$ is the nominal rental rate of capital, and W_t is the nominal wage rate. In each period, all intermediate goods firms face a common neutral technology shock z_t, and it follows:

$$\log z_{t+1} = (1-\rho_z)\log z + \rho_z \log z_t + \sigma_z \varepsilon_{z,t+1}, \quad \varepsilon_{z,t+1} \overset{\text{i.i.d.}}{\sim} N(0,1). \tag{7}$$

Two time-varying wedges ($\tau_{k,t}$ and $\tau_{l,t}$) are introduced to handle the model misspecification in the cost minimization problem. The first one is the capital market wedge $\tau_{k,t}$, and the second one is the homogeneous labor market wedge $\tau_{l,t}$. They are both assumed to be known for the intermediate good firms at time t. To be parsimonious, we assume that they both follow AR(1) processes:

$$\log \tau_{k,t+1} = (1-\rho_k)\log \tau_k + \rho_k \log \tau_{k,t} + \sigma_k \varepsilon_{k,t+1}, \quad \varepsilon_{k,t+1} \overset{\text{i.i.d.}}{\sim} N(0,1), \tag{8}$$

$$\log \tau_{l,t+1} = (1-\rho_l)\log \tau_l + \rho_l \log \tau_{l,t} + \sigma_l \varepsilon_{l,t+1}, \quad \varepsilon_{l,t+1} \overset{\text{i.i.d.}}{\sim} N(0,1). \tag{9}$$

The interpretation of these two wedges is similar to that of the intermediate good wedge $\tau_{y,t}$. That is, they represent some missing factors affecting capital and labor demand. As in the case of the intermediate good wedge $\tau_{y,t}$, the steady state values of these two wedges are assumed to be one.

Profit Maximization Since the intermediate good firms have monopoly power on their own products, they seek to maximize the present value of expected future profit flow. Following Calvo (1983), for all intermediate good firms, the probability of being able to re-optimize their prices is $1-\xi_p$, which is independent across firms and common across time. Thus, we can view this uncertainty as idiosyncratic shocks facing intermediate good firms. For the j^{th} firm which is not able to re-optimize

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[7] Following Christiano, Eichenbaum, and Evans (2005), we assume that there is no exit nor entry decisions for intermediate good firms.

its desired price in period t, it resets its product price by a partial indexation scheme:

$$P_{j,t} = \pi_{t-1}^{\iota_p} \pi^{1-\iota_p} P_{j,t-1}.$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes the period t gross inflation rate, and π is the steady-state inflation rate. This partial indexation scheme has been widely adopted in New Keynesian models, and is believed as a convenient mean for capturing the persistence property of inflation rate observed the data. See Smets and Wouters (2007) and Justiniano and Primiceri (2008) for instance.

For any intermediate good firms who are able to set their optimal price in period t, their optimal pricing decision can be expressed by the following optimization problem:[8]

$$\begin{aligned} \max_{\tilde{P}_t} \quad & \mathrm{E}_t \left\{ \sum_{l=0}^{\infty} (\beta \xi_p)^l \lambda_{t+l} \left[\tilde{P}_t F_{t,l}^p - P_{t+l} s_{t+l} \right] \tilde{Y}_{t+l} \right\} \\ \text{s.t.} \quad & \tilde{Y}_{t+l} = \left(\frac{\tau_{y,t+l} \tilde{P}_t F_{t,l}^p}{P_{t+l}} \right)^{-\eta_{p,t+l}} Y_{t+l}. \end{aligned}$$

Here, s_{t+l} is the real marginal cost of the firm.[9] λ_t is the household's marginal value of nominal income in period t, since we assume that households hold equities of intermediate goods firms. The constraint is intermediate good demand period $t+l$, when the optimal price of the intermediate good is set in period t. $F_{t,l}^p$ is defined as

$$F_{t,l}^p = \begin{cases} \prod_{k=1}^{l} \pi_t^{\iota_p} \pi^{1-\iota_p} & \text{if} \quad l \geq 1; \\ 1 & \text{if} \quad l = 0, \end{cases}$$

which captures the compound effect of partial indexation between period t to period $t+l$. Note as well that the intermediate good wedge $\tau_{y,t}$ facing final good firms affects the pricing decision of intermediate goods firms.

3.3 Households

There exists a continuum of infinitely living households denoted by $i \in [0,1]$. Following Erceg, Henderson, and Levin (2000) and Christiano, Eichenbaum, and Evans (2005), we assume that these households are monopolistic differentiated labor suppliers, and hence, are able to set their own wage rates $W_{j,t}$'s. The i^{th} household's budget constraint is

$$B_{t+1} + \tau_{c,t} P_t C_t + P_t I_t + P_t a(u_t) \bar{K}_t = \tau_{b,t} R_{t-1} B_t + R_{k,t} u_t \bar{K}_t + W_{i,t} h_{i,t} + \mathsf{D}_{i,t} + \mathsf{F}_t + \mathsf{A}_{i,t}. \quad (10)$$

The right hand side of (10) reflects various sources of the household's income in period t. $W_{i,t} h_{i,t}$ is the household's wage income, which is determined by the wage setting decision to be discussed later. The household receives the dividend $\mathsf{D}_{i,t}$ from intermediate good firms and the lump-sum

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[8] Since all firms which are able to set the desired optimal prices face the same problem, we do not distinguish them and drop the firm index accordingly.

[9] The real marginal cost can be obtained from the cost minimization problem of the intermediate good firm.

transfer $\mathsf{F}_{i,t}$ from the government. Following Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), we assume that there exists a complete set of arrow securities, and $\mathsf{A}_{i,t}$ is the return on holding such securities. This complete market assumption implies the all households make identical decisions on consumption C_t, investment I_t, capital $\bar{K}_t$, and bonding holding B_t, while their labor income are heterogeneous.[10]

We assume that the household can determine the utilization rate of capital, u_t. Hence, the effective capital used in the production of intermediate goods is $K_t = u_t \bar{K}_t$, and the nominal return of providing capital to intermediate goods firms is $R_{k,t} u_t \bar{K}_t$. By buying government bond B_t in period $t-1$, the household receives (gross) interest income $\tau_{b,t} R_{t-1} B_t$ in period t, where R_{t-1} is the nominal interest rate in period $t-1$. Note that we introduce a time-varying bond market wedge $\tau_{b,t}$, which is used to capture any potentially missing factors that might affect the demand of the government bond. Similar to other wedges in our model, it is assumed to follow an AR(1) process in terms of the logarithm:

$$\log \tau_{b,t+1} = (1-\rho_b) \log \tau_b + \rho_b \log \tau_{b,t} + \sigma_b \varepsilon_{b,t+1}, \quad \varepsilon_{b,t+1} \overset{\text{i.i.d.}}{\sim} N(0,1). \tag{11}$$

It is worth to note that depending on the view of the model builder, $\tau_{b,t}$ can also be treated as a structural shock. For instance, Smets and Wouters (2007) call $\tau_{b,t}$ the risk premium shock.

The left hand side of (10) reflects the allocation of the household income for various purposes. First of all, B_{t+1} is the bonding holding to be carried to period $t+1$. Second, the household spends $P_t I_t$ to accumulate physical capital. The evolution of physical capital can be expressed as

$$\bar{K}_{t+1} = (1-\delta) \bar{K}_t + \mu_t \left[1 - \mathcal{S}\left(\frac{I_t}{I_{t-1}} \right) \right] I_t,$$

where $S(\cdot)$ reflects the investment adjustment cost and is restricted to be $\mathcal{S}(1) = \mathcal{S}'(1) = 0$ in the steady state.[11] Following Justiniano and Primiceri (2008), we introduce a investment shock μ_t, which follows:

$$\log \mu_{t+1} = (1-\rho_\mu) \log \mu + \rho_\mu \log \mu_t + \sigma_\mu \varepsilon_{\mu,t+1}, \quad \varepsilon_{\mu,t+1} \overset{\text{i.i.d.}}{\sim} N(0,1) \tag{12}$$

Moreover, the household faces capital utilization cost $P_t a(u_t) \bar{K}_t$, which varies with different level of capital utilization rate u_t. Finally, the household uses its income to consume the final good C_t with price P_t. When the household makes its consumption decision, it also considers the effects of a time-varying consumption wedge $\tau_{c,t}$, which is designed to capture model misspecification in the consumption decision. While we treat $\tau_{c,t}$ as a wedge, it can also be viewed as a structural shock. For example, Leeper et al. (2010) interpret $\tau_{c,t}$ as the taxation shock on consumption. The consumption wedge $\tau_{c,t}$ follows

$$\log \tau_{c,t+1} = (1-\rho_c) \log \tau_c + \rho_c \log \tau_{c,t} + \sigma_c \varepsilon_{c,t+1}, \quad \varepsilon_{c,t} \overset{\text{i.i.d.}}{\sim} N(0,1), \tag{13}$$

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[10] Thus, there is no need to add the firm index subscript i with respect to consumption, investment, capital, and bond holding in the budget constraint

[11] See SU (200x) for the rationale of such setting.

and as before we assume $\tau_c = 1$.

The i^{th} household receives utility from consuming the final good C_t and disutility from providing differentiated labor $h_{i,t}$. Thus, its behavior can be characterized by maximizing the following expected lifetime utility function:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t d_t \left\{ \frac{(C_t - bC_{t-1})^{1-\gamma}}{1-\gamma} - \psi_h \frac{(h_{i,t})^{1+\varphi}}{1+\varphi} \right\}.$$

subject to its budget constraint (10). Essentially, the household makes a sequence of decisions on consumption, capital accumulation, capital utilization, bond holding and idiosyncratic desired wage. In this utility maximization problem, β is the subjective discount factor, γ the risk aversion coefficient, φ is the inverse of Frisch elasticity of labor supply, and ψ_h controls the weight on labor disutility. The parameter b governs the extent of consumption habit formation.[12] We adopt a CRRA consumption utility function, rather than log utility, because our empirical study focuses on the business cycle, and we do not need a balanced growth path. Following Justiniano et al. (2010), we assume that there exists a intertemporal preference shock d_t such that:

$$\log d_{t+1} = (1-\rho_d)\log d + \rho_d \log d_t + \sigma_d \varepsilon_{d,t+1}, \quad \varepsilon_{d,t} \overset{\text{i.i.d.}}{\sim} N(0,1). \tag{14}$$

Labor-Packer Firms A notable feature of New Keynesian models is that households provide differentiated labor labor service $h_{i,t}$, but intermediate good firms are using homogenous labor services $L_{j,t}$. Following CEE(2005) and SW(2007), we introduce perfectly competitive labor-packer firms which are convenient to reconcile the above distinction. The representative labor-packer firm hires household labor services $h_{i,t}$'s and transforms them to the homogeneous labor service L_t by the following technology:

$$L_t = \left[\int_0^1 (h_{i,t})^{\frac{\eta_{w,t}}{\eta_{w,t}-1}} \mathrm{d}i \right]^{\frac{\eta_{w,t}-1}{\eta_{w,t}}}. \tag{15}$$

Similar to the final good firms problem, we assume $\eta_{w,t}$ to be time-varying and it follows

$$\log \eta_{w,t+1} = (1-\rho_w)\log \eta_w + \rho_w \log \eta_{w,t} + \sigma_w \varepsilon_{w,t+1}, \quad \varepsilon_{w,t} \overset{\text{i.i.d.}}{\sim} N(0,1). \tag{16}$$

Since $\eta_{w,t}$ governs household's monopoly power in determining the desired wage rate, it can be interpreted as the wage markup shock.

Taking the market wage rate W_t and idiosyncratic wage rates $W_{i,t}$ of households as given, the labor packer firm solves the following problem:

$$\max_{\{h_{i,t}\}_{i\in[0,1]}} W_t L_t - \int_0^1 \tau_{h,t} W_{i,t} h_{i,t} \mathrm{d}i$$

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[12] Christiano et al. (2005) show that the introducing habit formation helps to explain the hump-shaped impulse response function of consumption with respect to monetary policy shocks.

subject to the transform technology (15). Here, we introduce a time-varying wedge $\tau_{h,t}$ as follows

$$\log \tau_{h,t+1} = (1-\rho_h)\log \tau_h + \rho_l \log \tau_{h,t} + \sigma_l \varepsilon_{h,t+1}, \quad \varepsilon_{h,t+1} \overset{\text{i.i.d.}}{\sim} N(0,1). \tag{17}$$

The first order condition of the above problem implies that the demand for the i^{th} household's labor service is:

$$h_{i,t} = \left(\frac{\tau_{h,t} W_{i,t}}{W_t}\right)^{-\eta_{w,t}} L_t.$$

Wage Setting of Households Following Erceg, Henderson, and Levin (2000), we assume that not all households are able to set the desired optimal wage rates in period of time. Specifically, the probability of being able to set the optimal wage rate in each period is $1-\xi_w$, which is independent across households and constant over time. If the i^{th} household is not able to set the optimal wage rate, it resets its wage rate by the following partial indexation scheme:

$$W_{i,t} = \pi_{t-1}^{\iota_w} \pi^{1-\iota_w} W_{i,t-1}.$$

Thus, the households' wage setting decision can be represented by

$$\begin{aligned} \max_{\tilde{W}_t} \quad & E_t \left\{ \sum_{s=0}^{\infty} (\beta\xi_w)^s \left[-d_{t+s}\psi_h \frac{\left(\tilde{h}_{t+s}\right)^{1+\varphi}}{1+\varphi} + \lambda_{t+s}\left(\tilde{W}_t F_{t,s}^w\right) \tilde{h}_{t+s} \right] \right\} \\ \text{s.t.} \quad & \tilde{h}_{t+s} = \left(\frac{\tau_{h,t+s} \tilde{W}_t F_{t,s}^w}{W_{t+s}} \right)^{-\eta_{w,t+s}} L_{t+s}. \end{aligned}$$

Here, $\tilde{h}_{t+s}$ is the period $t+s$ household labor demand if the optimal wage is determined in period. $\tilde{W}_t F_{t,s}^w$ is the effective wage rate in period $t+s$, and

$$F_{t,s}^w = \begin{cases} \prod_{k=1}^{s} \pi_{t+k-1}^{\iota_w} \pi^{1-\iota_w} & \text{if} \quad s \geq 1, \\ 1 & \text{if} \quad s = 0, \end{cases}$$

captures the compound effect of partial indexation from period t to period $t+s$. Note as well how the $\tau_{h,t}$ enters the problem. Moreover, λ_{t+s} is current value of the marginal utility in period $t+s$, which is exactly the Lagrange multiplier of the household budget constraint.

3.4 The Government Sector

In the model, the government consumes a proportion of final output for non-productive purposes, $G_t = g_t Y_t$. The proportion g_t of government consumption is stochastic and follows

$$\log g_{t+1} = (1-\rho_g)\log g + \rho_g \log g_t + \sigma_g \varepsilon_{g,t+1}, \quad \varepsilon_{g,t+1} \overset{\text{i.i.d.}}{\sim} N(0,1). \tag{18}$$

Besides, the government finance its spending by lump-sum taxes and issuing government bonds. Since the government's behavior is Ricardian, there is no need to explicitly specify its fiscal policy rule.

We assume that the government controls the nominal interest rate R_{t+1} by the following Taylor's rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{\gamma_y}\right]^{1-\rho_r} \nu_t.$$

where π is the target (gross) inflation rate set by the central bank, and R and Y are steady-state values of nominal interest rate and aggregate real output. ν_t is the monetary policy shock, which follows an AR(1) process:

$$\log \nu_{t+1} = \rho_\nu \log \nu + (1-\rho_\nu) \log \nu_t + \sigma_\nu \varepsilon_{v,t+1}, \quad \varepsilon_{\nu,t+1} \overset{\text{i.i.d.}}{\sim} N(0,1), \tag{19}$$

and ρ_r is a smoothing parameter introduced to capture the persistence of the nominal interest rate. While there exist various specifications of the Taylor rule in the existing literature, our specification is in line with Del Negro et al. (2007) and Justiniano and Primiceri (2008), for instance.

3.5 Aggregation

Because of the complete markets assumption, the aggregation of the model economy is straightforward, even though the households and intermediate goods firms face idiosyncratic shocks due to the Calvo pricing. By directly integrating the production function of intermediate goods firms, we have the following relation between the aggregate output and two aggregate factor inputs:

$$Y_t = z_t K_t^\alpha L_t^{1-\alpha} - \Phi.$$

Since our model is a closed economy, the total output of the economy can only be distributed to four outlets - the household and government consumptions, investment, and the capital utilization cost:

$$C_t + I_t + G_t + a(u_t)\bar{K}_t = Y_t.$$

4 Estimation

We estimate the structural parameters of the model as well as the time-varying wedges processes by Bayesian methods. Following most of the DSGE literature, we first linearize the equilibrium conditions of the model around the steady state. After linearization, the model can be represented by a system of 26 equations, which consist of 13 equilibrium conditions, 7 exogenous structural shocks processes, and 6 exogenous wedges processes. Correspondingly, there are 26 variables in the system. The linearized equations are listed in Appendix B. We use the method proposed by Klein (2000) to solve for the linearized policy functions of the model and then express the model in the canonical state space from such that we can use the Kalman filter to evaluate the likelihood function of the observed data. By imposing proper prior distributions to be discussed later, we are able to evaluate the values of the posterior function in the parameters domain. Then, we adopt the random-walk

Metropolis (RWM) algorithm to obtain the MCMC draws with respect to the posterior function.[13] Our estimation are based on 120,000 MCMC draws, and the first 60,000 draws are discarded.

Following Smets and Wouters (2007), we utilize seven quarterly US aggregate time series to estimate the model. Specifically, we use per captia real output, real consumption, real investment, hours worked, inflation rate, and the Federal fund rate as our observables. The construction of these series are discussed as follows: Real output and consumption per capita result from nominal GDP and nominal aggregate consumption, both divided by the population and the price indices. Nominal aggregate consumption are defined as the sum of personal expenditure on non-durable goods and services. Nominal investment is the sum of the private domestic investment and personal expenditure on durable goods. Dividing nominal investment by population and the price index, we have the real investment per capita. The total hours worked in non-farm business sector, divided by the population, gives us the hours worked per capita. Real wage is obtained through dividing the non-farm business sector hourly compensation by the price index. When constructing above variables, we use the civilian non-institutional population as our population measure. The price index of this study is the GDP deflator, and the quarterly inflation rate are constructed accordingly. Besides, the Federal fund rate are transformed to the quarterly based. Since we focus on only the business cycle dynamics rather than the long run growth, we remove the trend components of above variables by the two-sided Hodrick-Prescott filter with smooth parameter $\lambda = 1600$.

Our benchmark dataset cover 1984:I to 2006:IV, which is viewed as the post-great moderation period. The choice of the starting period is based on Smets and Wouters (2007)'s finding that the behavior of aggregate series before and after 1984 are quite different. The choice of ending point is motivated by 2007 financial crisis. By focusing on this relatively stable periods of time, we are able to exam the performance of our model but prevent the disturbance due to structural breaks. As a comparison, we also estimate our model based on an extended dataset which covers 1984:I to 2011:IV.

4.1 Priors

It is well known that some of structural parameters are not well-identified in this type of DSGE models unless more observed variables are included. Thus, in this study we calibrate these parameters by the widely accepted values. Specifically, we set the subjective discount factor $\beta = 0.995$, which implied that the annual real interest rate is approximately equal to 2 percent. Following Christiano et al. (2010), the quarterly depreciation rate of installed capital is calibrated to $\delta = 0.025$, and the capital share is set to $\alpha = 0.36$. By imposing the zero profit condition, the fixed cost of production, Φ, is endogenously determined. We normalize the steady-state value of hours worked to 1, and consequently the disutility parameter ψ_h is endogenously determined as well. Finally, as explained by Smets and Wouters (2007), it is difficult to identified the steady-state government spending ratio g unless the government spending series is included as one of the observables, and hence we set

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[13] An and Schorfheide (2007) provide detail discussion about the usage of the RWM algorithm.

$g = 0.22$.

The prior distributions of the estimated parameter are summarized in Table 4. In principle, we impose the loose priors, and these priors are broadly line with the typical choices of the literatures. See, for examples, Christiano et al. (2010) and Levin et al. (2006). Following most existing literatures and for the sake of simplicity, we set the prior distributions mutually independent from each others.[14]

4.2 Empirical Results

The estimation results of structural parameters as well as the parameters for the wedge processes are summarized in the right hand side of Table 4, which reports the posterior median of the parameters. In what follows, we discuss how our estimates differ from those in the existing literature.

Table 4 shows that our estimate are broadly similar to the ones reported in the existing literature, with a few exceptions. The estimate of habit formation parameter is $b = 0.56$, which is slightly smaller than the one reported in Christiano et al. (2005) and Smets and Wouters (2007). The point estimate of labor supply disutility parameter is $\varphi = 0.76$, which implies a Frisch elasticity of labor supply around 1.32, which is slightly bigger than that reported in the micro literature. However, it is close to the usual estimates reported in the macro literature.[15]

Our point estimate of the Calvo parameter is $\xi_p = 0.85$, implying that intermediate good firms, on average, reset their optimal prices every 6.6 quarters. Comparing to the empirical findings of BK(2004), the ξ_p estimate is a bit higher than what expect, but it is not absurd. For example, LOWW(2005)'s estimate with respect to the same parameter is 0.83.[16] As for the Calvo parameter of sticky wage, our estimate is $\xi_w = 0.50$, which implies that average duration of a wage contract is around 2 quarters. In line with LOWW(2005) and Christiano et al. (2005), but in contrast to Smets and Wouters (2007), we find that the sticky price feature plays a more important role than sticky wages. Moreover, our results reveal that the extent of partial indexation on previous inflation and the wage rate is moderate: the point estimates are $\iota_p = 0.50$ and $\iota_w = 0.43$, respectively. This results are broadly in line with Smets and Wouters (2007).

It is well known that technology shocks play a crucial role in explaining fluctuations in economic variables, and several researchers have devoted efforts in studying its magnitude and consequences. **[See for example, Ireland (2004, RES), technology shocks in NK model].** Most of empirical studies show that the technology shock is quite persistent. For instance, the persistency parameter estimate in Smets and Wouters (2007) and LOWW(2005) is 0.95 and 0.96, respectively. In contrast to these studies, our point estimate of the persistence of the technology shock parameter, ρ_z, is around 0.60. Moreover, the estimated standard deviation of technology innovation is 0.45 percent, which is more or less similar to the existing findings. Our results imply that the unconditional variance of the technology shock is much smaller than traditional estimates. [model misspecification][Solow][Del

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[14] For imposition of dependent prior distributions, please refer to DS(2008, JME, Forming priors for DSGE models....).

[15] See, for example, Guerron-Quintana (2011). This estimate is substantially different from the one in Galí, Smets, and Wouters (2012), who explicitly takes into account the friction in the labor market.

[16] Altig, Christiano, Eichenbaum, and Linde (2011) explain that once the feature of firm-specific capital is included, the implied duration of price stickiness will reduce.

Table 4: Prior and Posterior Distribution

		Prior			Posterior		
Parameter Description		Type	Mean	Std. Dev.	Mode	Pct(5)	Pct(95)
γ	Risk aversion coef.	Gamma	2.00	0.50	1.99	1.35	2.79
φ	Inverse Frisch elasticity	Gamma	1.20	0.50	0.76	0.41	1.26
η_p	Elasticity parameter: goods	Gamma	6.00	0.50	5.27	4.51	6.09
η_w	Elasticity parameter: labors	Gamma	21.00	1.00	21.19	19.59	22.87
b	Habit formation	Beta	0.66	0.10	0.56	0.42	0.69
ξ_p	Calvo parameter: goods	Beta	0.38	0.10	0.85	0.78	0.90
ξ_w	Calvo parameter: labors	Beta	0.38	0.10	0.57	0.44	0.69
ι_p	Partial indexation: goods	Beta	0.50	0.10	0.50	0.35	0.66
ι_w	Partial indexation: labors	Beta	0.50	0.10	0.43	0.28	0.60
$\varkappa_a$	Utilization adjustment para.	Gamma	1.00	0.75	2.57	1.45	4.30
$\varkappa_s$	Investment adjustment para.	Gamma	5.00	5.00	0.33	0.17	0.71
γ_π	Monetary rule: inflation	Gamma	1.50	0.10	1.44	1.29	1.58
γ_y	Monetary rule: output	Gamma	0.50	0.10	0.60	0.46	0.76
ρ_r	Interest rate smooth para.	Beta	0.80	0.10	0.77	0.72	0.82
Structural Shock Processes: Persistence Coefficients							
ρ_z	Technology shocks	Beta	0.75	0.10	0.60	0.49	0.70
ρ_ν	Monetary shocks	Beta	0.50	0.10	0.44	0.32	0.58
ρ_g	Spending shocks	Beta	0.75	0.10	0.92	0.88	0.95
ρ_d	Preference shocks	Beta	0.75	0.10	0.75	0.57	0.88
ρ_μ	Investment shocks	Beta	0.50	0.10	0.50	0.34	0.67
ρ_p	Price markup shocks	Beta	0.75	0.10	0.77	0.58	0.90
ρ_w	Wage markup shock	Beta	0.75	0.10	0.76	0.56	0.90
Structural Shock Processes: Standard Deviations of Innovations							
σ_z	Technology shocks	Inverse Gamma	0.10	2.00	0.34	0.30	0.39
σ_ν	Monetary shocks	Inverse Gamma	0.10	2.00	0.11	0.09	0.12
σ_g	Spending shocks	Inverse Gamma	0.10	2.00	1.50	1.33	1.70
σ_d	Preference shocks	Inverse Gamma	0.10	2.00	0.07	0.02	0.19
σ_μ	Investment shocks	Inverse Gamma	0.10	2.00	0.05	0.02	0.15
σ_p	Price markup shocks	Inverse Gamma	0.10	2.00	0.05	0.02	0.19
σ_w	Wage markup shocks	Inverse Gamma	0.10	2.00	0.11	0.02	0.47
Wedge Processes: Persistence Coefficients							
ρ_l	Type 1 labor wedges	Beta	0.50	0.10	0.50	0.34	0.68
ρ_k	Capital wedges	Beta	0.50	0.10	0.50	0.33	0.66
ρ_c	Consumption wedges	Beta	0.50	0.10	0.35	0.23	0.48
ρ_b	Bond wedges	Beta	0.50	0.10	0.65	0.53	0.75
ρ_h	Type 2 labor wedges	Beta	0.50	0.10	0.65	0.57	0.73
ρ_y	Intermediate good wedges	Beta	0.50	0.10	0.75	0.62	0.83
Wedge Processes: Standard Deviations of Innovations							
σ_l	Type 1 labor wedges	Inverse Gamma	0.10	2.00	0.07	0.02	0.26
σ_k	Capital wedges	Inverse Gamma	0.10	2.00	0.05	0.02	0.13
σ_c	Consumption wedges	Inverse Gamma	0.10	2.00	2.67	1.48	6.35
σ_b	Bond wedges	Inverse Gamma	0.10	2.00	0.65	0.37	1.35
σ_h	Type 2 labor wedges	Inverse Gamma	0.10	2.00	0.62	0.55	0.71
σ_y	Intermediate good wedges	Inverse Gamma	0.10	2.00	0.16	0.14	0.19

Negro and Schorfheide (2009).

The AR coefficient estimates for technology shocks ($\rho_z = 0.754, 0.757$) are much smaller than the typical estimate. Del Negro and Schorfheide (2009) also found that the persistence and standard deviation of the technology shock process become smaller once misspecification is taken into account.

Forecast Error Variance Decomposition (FEVD) As shown in the simulation results, FEVDs can help identify the sources of model misspecification. We report the FEVD of our empirical model in Table 5. The Table reports, for every horizon $H = 4, 12, 100$ quarters, the contribution of each of the shocks (listed in the columns) to the overall variance of the observable variables (Output, Consumption, Investment, etc.). For convenience, Table XX reports a summary of the notation corresponding to all the wedges and shocks in the model.

Table 5 shows that the capital and the homogenous labor market wedges are very small for all the variables of interest. In contrast, the bond market, consumption, household labor market and wage wedges contribute to explain the variability of several observables, sometimes substantially. For example, the bond market wedge is crucial in explaining the variability of the interest rate, and it is also important to explain variability in output, investment and hours at most horizons. The consumption wedge is very important to explain the variability of consumption. The household labor wedge is crucial to explain the variability of the real wage and the wedge in the intermediate good production function is crucial to explain the variability of inflation (in addition to marginally explaining output and the real wage). Among the shocks, the ones that explain the most variability in the data are the technology and monetary policy shocks, which are important to explain hours, the monetary policy shock, which is important to explain the interest rate and output, and the government spending shock, which is important to explain investment at all horizons and consumption at very long horizons.

Comparing our results to the Smets and Wouters (2007), they found a larger contribution of monetary policy shocks to the interest rate at short horizons than we do, although the long-run contribution is similar. Regarding inflation, they find a large role played by the price markup shock, which we do not find: instead, inflation is driven mainly by the wedge in the intermediate goods production in our estimates. Finally, they found a large contribution of the government spending and risk premia shocks to the variability of output, whereas we find a large contribution of the bond risk premia and monetary policy shocks.

Table 5: FEDV of the Medium-Scale New Keynesian Model

Horizon	z	ν	g	d	μ	η_p	η_w	τ_l	τ_k	τ_c	τ_b	τ_h	τ_y
Output													
H = 4	0.56	24.94	3.97	0.00	0.12	0.00	0.00	0.01	0.00	1.56	62.88	0.72	5.24
H = 12	0.79	24.77	4.02	0.01	0.12	0.00	0.00	0.01	0.00	1.59	62.14	1.04	5.52
H = 100	0.79	24.72	4.20	0.01	0.12	0.00	0.00	0.01	0.00	1.59	62.01	1.04	5.52
Consumption													
H = 4	0.05	0.98	10.30	0.40	0.02	0.00	0.00	0.00	0.00	85.57	2.51	0.06	0.10
H = 12	0.08	1.31	26.94	0.41	0.02	0.00	0.00	0.00	0.00	68.52	2.53	0.09	0.10
H = 100	0.10	1.66	47.83	0.29	0.02	0.00	0.00	0.00	0.00	47.36	2.53	0.12	0.08
Investment													
H = 4	0.51	16.95	31.47	0.03	0.13	0.00	0.00	0.01	0.00	4.70	42.62	0.73	2.85
H = 12	0.55	12.81	47.37	0.04	0.09	0.00	0.00	0.00	0.00	3.90	32.06	0.79	2.38
H = 100	0.55	12.81	47.37	0.04	0.09	0.00	0.00	0.00	0.00	3.90	32.06	0.79	2.38
Hours													
H = 4	23.17	19.11	3.28	0.00	0.10	0.00	0.00	0.01	0.00	1.27	49.07	0.82	3.17
H = 12	22.03	18.53	5.00	0.00	0.10	0.00	0.00	0.01	0.00	1.27	48.45	1.07	3.54
H = 100	19.90	17.01	13.47	0.01	0.09	0.00	0.00	0.01	0.00	1.28	44.02	1.01	3.21
Real Wage													
H = 4	0.21	0.22	0.96	0.01	0.00	0.00	0.00	0.01	0.00	0.03	0.30	89.00	9.26
H = 12	0.47	0.62	6.54	0.01	0.00	0.00	0.00	0.01	0.00	0.15	0.63	82.31	9.24
H = 100	0.44	1.09	24.09	0.01	0.01	0.00	0.00	0.01	0.00	0.39	1.11	65.50	7.35
Inflation Rate													
H = 4	6.34	0.85	1.66	0.03	0.00	0.01	0.00	0.04	0.00	0.01	0.38	8.03	82.65
H = 12	6.17	0.83	3.66	0.05	0.01	0.01	0.00	0.04	0.00	0.03	0.68	7.74	80.78
H = 100	6.13	0.84	4.14	0.05	0.01	0.01	0.00	0.04	0.00	0.04	0.70	7.72	80.30
Interest Rate													
H = 4	1.75	9.77	1.53	0.02	0.12	0.00	0.00	0.01	0.00	1.30	80.52	2.14	2.85
H = 12	1.56	8.29	1.71	0.03	0.11	0.00	0.00	0.01	0.00	1.12	81.48	1.91	3.79
H = 100	1.55	8.29	2.17	0.03	0.11	0.00	0.00	0.01	0.00	1.13	81.03	1.90	3.77

Table XX. Wedges and Shocks: Summary Table

Panel A. Wedges

$\tau_{y,t}$	Wedge in the intermediate good production function	eq.(5)
$\tau_{k,t}$	Capital market wedge	eq.(8)
$\tau_{l,t}$	Homogeneous labor market wedge	eq.(9)
$\tau_{b,t}$	Bond market wedge	eq.(11)
$\tau_{c,t}$	Consumption wedge	eq.(13)
$\tau_{h,t}$	Household labor wedge	eq.(17)

Panel B. Shocks

z_t	Technology shock	eq. (7)
ν_t	Monetary policy shock	eq. (19)
g_t	Government spending shock	eq. (18)
d_t	Preference shock	eq. (14)
μ_t	Investment shock	eq. (12)
$\eta_{p,t}$	Price markup shock	eq. (4)
$\eta_{w,t}$	Wage markup shock	eq. (16)

The importance of investment shock Tables 3 and 4 report the posterior medians and 95% Bayesian credible intervals of the parameters for the sub-sample periods 1984:I-2006:IV and 2007:I-2011:IV, respectively. While many of our estimates are similar to those in the existing literature, others are substantially different. For example, our estimates of the inverse of Frisch elasticity of labor supply are 0.424 and 0.508 and are substantially different from the one in Gali et al. (2012). Our estimates of $\varkappa_s$ (investment adjustment cost) are 0.518 and 0.575. These values are much smaller than those in the existing literature (e.g., Justiniano et al. (2010)). The AR coefficient estimates for technology shocks ($\rho_z = 0.754, 0.757$) are much smaller than the typical estimate. Del Negro and Schorfheide (2009) also found that the persistence and standard deviation of the technology shock process are smaller once misspecification is taken into account. Comparing the two sets of estimates, the effect of the variable utilization rate on the rate of return on physical capital ($\varkappa_a$) decreased, the intemporal preference shock (d_t) became more persistent and volatile, and the government spending shock became more volatile after the crisis. Figures 5 and 6 reports the median impulse responses as well as the 95% credible intervals.

While the labor wedge was more volatile before the crisis, the captal market wedge became more volatile after the crisis.

Tables 5 and 6 report the FEVD for the two sub-samples. The bond wedge explains about 60% of the output variation before the crisis and almost a half after the crisis. The monetary policy shock is less important and the contribution of the technology shock is trivial. This finding is sharply different from most of the existing literature. This result is even more extreme than that of Galí and

Table 6: Prior and Posterior for Parameters of the Medium-Scale New Keynesian Model without any wedges

		Prior			Posterior		
Parameter Description		Type	Mean	Std. Dev.	Mode	Pct(5)	Pct(95)
γ	Risk aversion coef.	Gamma	2.00	0.50	1.71	1.18	2.41
φ	Inverse Frisch elasticity	Gamma	1.20	0.50	0.64	0.32	1.18
η_p	Elasticity parameter: goods	Gamma	6.00	0.50	5.23	4.50	6.08
η_w	Elasticity parameter: labors	Gamma	21.00	1.00	20.91	19.40	22.53
b	Habit formation	Beta	0.66	0.10	0.60	0.48	0.71
ξ_p	Calvo parameter: goods	Beta	0.38	0.10	0.84	0.80	0.87
ξ_w	Calvo parameter: labors	Beta	0.38	0.10	0.32	0.23	0.43
ι_p	Partial indexation: goods	Beta	0.50	0.10	0.27	0.17	0.41
ι_w	Partial indexation: labors	Beta	0.50	0.10	0.47	0.31	0.64
$\varkappa_a$	Utilization adjustment para.	Gamma	1.00	0.75	2.82	1.62	4.70
$\varkappa_s$	Investment adjustment para.	Gamma	5.00	5.00	0.89	0.45	1.77
γ_π	Monetary rule: inflation	Gamma	1.50	0.10	1.46	1.32	1.63
γ_y	Monetary rule: output	Gamma	0.50	0.10	0.52	0.39	0.68
ρ_r	Interest rate smooth para.	Beta	0.80	0.10	0.78	0.73	0.83
Structural Shock Processes: Persistence Coefficients							
ρ_z	Technology shocks	Beta	0.75	0.10	0.64	0.53	0.74
ρ_ν	Monetary shocks	Beta	0.50	0.10	0.43	0.31	0.56
ρ_g	Spending shocks	Beta	0.75	0.10	0.95	0.91	0.96
ρ_d	Preference shocks	Beta	0.75	0.10	0.41	0.29	0.54
ρ_μ	Investment shocks	Beta	0.50	0.10	0.45	0.34	0.56
ρ_p	Price markup shocks	Beta	0.75	0.10	0.47	0.33	0.62
ρ_w	Wage markup shock	Beta	0.75	0.10	0.39	0.27	0.51
Structural Shock Processes: Persistence Coefficients							
σ_z	Technology shocks	Inverse Gamma	0.10	2.00	0.35	0.31	0.40
σ_ν	Monetary shocks	Inverse Gamma	0.10	2.00	0.10	0.09	0.12
σ_g	Spending shocks	Inverse Gamma	0.10	2.00	1.48	1.31	1.69
σ_d	Preference shocks	Inverse Gamma	0.10	2.00	1.63	1.17	2.27
σ_μ	Investment shocks	Inverse Gamma	0.10	2.00	1.85	1.18	3.09
σ_p	Price markup shocks	Inverse Gamma	0.10	2.00	14.96	8.87	26.97
σ_w	Wage markup shocks	Inverse Gamma	0.10	2.00	119.83	69.68	221.33

Rabanal (2005), where the contribution of technology shock is also small, but not as small as ours. Consumption is mainly explained by two "demand" shocks, g_t and d_t. The variation of investment is mainly explained by the spending shock (g_t), the bond wedge ($\tau_{B,t}$), and the monetary policy shock (ν_t). Unlike output, consumption and investment, the technology shock plays an important role in explaining the variation of hours worked, about 20%-30%. The bond wedge still provides the largest contribution, explaining more than 40% of variation in hours worked.

In the short run, more than 80% of the variation in real wages is explained by the labor wedge, τ_l. Except for the government spending shock, the other structural shocks cannot explain the variation of the real wage rate. This result suggests that the channel that was proposed for explaining real wage dynamics in New Keyensian models, (**??**), may be problematic. The technology shock can explain 16% of the variation of inflation. However, the most important factor affecting inflation variation is τ_y. Recalling 6, this result suggests that more than 35% of the variation of inflation is explained by something that was not included in the model. The second important contribution to inflation variation is τ_L, which indicates that the real marginal cost of the model cannot fully explain the actual cost facing intermediate good firms, especially in hiring labor. In explaining the variation of inflation, the four structural shock can explain less than 20%. In the long-run about 75% of nominal interest rate variation is explained by bond wedges.

Overall, our results show that the bond wedge is the most important source of misspecification, especially for the quantity variables (Y, C, I, L) and the fed funds rate. We also find that the variability in the real wage (w) and inflation (π) rates are explained by τ_l and τ_y. The technology shock mainly affects hours worked, but not other variables. Our results suggest that the bond market and labor market may be severely misspecified, at least in the medium-scale New Keyensian model we considered.

The Model Without Wedges {DELETE: How would the estimates in a model without wedges be different from those in the model with wedges? In order to understand why the technology shock is not that persistent, we estimate a model without containing any wedges.}

In order to understand whether including wedges has substantial consequences on the parameter estimates, we report parameter estimates for the model without wedges. The parameter estimates in the models estimated with and without wedges are very similar, with the exception of the investment adjustment parameter, whose value in the model with no wedges is about a third of that in the model with wedges. We also note that the persistence of some of the shocks in the model estimated without wedges is smaller and their variance is larger than that of a model with wedges. **{THE VARIANCE OF SOME OF THE SHOCKS IN THE MODEL WITHOUT WEDGES ARE QUITE LARGE...}**

		Prior			Posterior		
Parameter Description		Type	Mean	Std. Dev.	Mode	Pct(5)	Pct(95)
γ	Risk aversion coef.	Gamma	2.00	0.50	1.71	1.18	2.41
φ	Inverse Frisch elasticity	Gamma	1.20	0.50	0.64	0.32	1.18
η_p	Elasticity parameter: goods	Gamma	6.00	0.50	5.23	4.50	6.08
η_w	Elasticity parameter: labors	Gamma	21.00	1.00	20.91	19.40	22.53
b	Habit formation	Beta	0.66	0.10	0.60	0.48	0.71
ξ_p	Calvo parameter: goods	Beta	0.38	0.10	0.84	0.80	0.87
ξ_w	Calvo parameter: labors	Beta	0.38	0.10	0.32	0.23	0.43
ι_p	Partial indexation: goods	Beta	0.50	0.10	0.27	0.17	0.41
ι_w	Partial indexation: labors	Beta	0.50	0.10	0.47	0.31	0.64
$\varkappa_a$	Utilization adjustment para.	Gamma	1.00	0.75	2.82	1.62	4.70
$\varkappa_s$	Investment adjustment para.	Gamma	5.00	5.00	0.89	0.45	1.77
γ_π	Monetary rule: inflation	Gamma	1.50	0.10	1.46	1.32	1.63
γ_y	Monetary rule: output	Gamma	0.50	0.10	0.52	0.39	0.68
ρ_r	Interest rate smooth para.	Beta	0.80	0.10	0.78	0.73	0.83
Structural Shock Processes: Persistence Coefficients							
ρ_z	Technology shocks	Beta	0.75	0.10	0.64	0.53	0.74
ρ_ν	Monetary shocks	Beta	0.50	0.10	0.43	0.31	0.56
ρ_g	Spending shocks	Beta	0.75	0.10	0.95	0.91	0.96
ρ_d	Preference shocks	Beta	0.75	0.10	0.41	0.29	0.54
ρ_μ	Investment shocks	Beta	0.50	0.10	0.45	0.34	0.56
ρ_p	Price markup shocks	Beta	0.75	0.10	0.47	0.33	0.62
ρ_w	Wage markup shock	Beta	0.75	0.10	0.39	0.27	0.51
Structural Shock Processes: Persistence Coefficients							
σ_z	Technology shocks	Inverse Gamma	0.10	2.00	0.35	0.31	0.40
σ_ν	Monetary shocks	Inverse Gamma	0.10	2.00	0.10	0.09	0.12
σ_g	Spending shocks	Inverse Gamma	0.10	2.00	1.48	1.31	1.69
σ_d	Preference shocks	Inverse Gamma	0.10	2.00	1.63	1.17	2.27
σ_μ	Investment shocks	Inverse Gamma	0.10	2.00	1.85	1.18	3.09
σ_p	Price markup shocks	Inverse Gamma	0.10	2.00	14.96	8.87	26.97
σ_w	Wage markup shocks	Inverse Gamma	0.10	2.00	119.83	69.68	221.33

By comparing the forecast error variance decompositions of the models estimated with and without wedges, it is clear that, in several cases, by including wedges in the model, the latter tend to explain a large component of the variance of the observables that, in the model without wedges, was

Table 7: FEVD of the Medium-Scale New Keynesian Model with No Wedge

Horizon	z	ν	g	d	μ	η_p	η_w
Output							
$H = 4$	0.53	18.76	6.52	4.42	61.61	7.14	1.02
$H = 12$	0.95	18.56	7.18	4.83	57.08	8.29	3.13
$H = 100$	0.93	18.25	8.70	4.75	56.14	8.15	3.08
Consumption							
$H = 4$	0.12	1.14	11.32	78.79	7.95	0.24	0.43
$H = 12$	0.17	1.09	27.41	62.21	8.10	0.29	0.72
$H = 100$	0.17	1.05	48.07	39.67	9.97	0.35	0.72
Investment							
$H = 4$	0.51	9.31	20.21	6.13	59.36	3.04	1.43
$H = 12$	0.69	7.76	33.20	6.74	45.97	3.03	2.60
$H = 100$	0.65	7.26	37.44	6.31	43.08	2.84	2.43
Hours							
$H = 4$	26.42	13.19	5.84	2.78	45.37	4.16	2.23
$H = 12$	23.37	12.37	9.33	2.83	43.44	4.54	4.10
$H = 100$	19.35	10.43	21.84	2.63	38.38	3.86	3.51
Real Wage							
$H = 4$	0.03	1.61	2.89	5.30	0.40	7.04	82.73
$H = 12$	0.03	1.91	9.99	4.62	0.76	7.33	75.35
$H = 100$	0.06	1.74	23.85	4.01	3.02	6.05	61.26
Inflation Rate							
$H = 4$	4.89	1.29	9.40	1.56	1.79	61.55	19.51
$H = 12$	4.46	1.18	17.79	1.51	3.06	53.30	18.70
$H = 100$	4.26	1.13	21.42	1.46	3.06	50.81	17.85
Interest Rate							
$H = 4$	3.22	17.63	1.10	7.50	43.66	15.09	11.81
$H = 12$	3.03	14.48	9.03	6.90	41.43	12.62	12.50
$H = 100$	2.92	13.96	11.74	6.70	40.50	12.14	12.04

explained by structural shocks. For example, while in the model with no wedges, the investment shock explains about 60% of the variability of output, and about 50% of the variability of investment, hours and the interest rate, in the model with wedges the investment shock explains almost none of the variability in these observables. Similarly, the model without wedges attributes most of the variability of the real wage to the wage markup shock, whereas the model with wedges attributes that variability mainly to the household labor wedge. **{I FIND IT HARD TO INTERPRET THESE RESULTS}**

Marginal Likelihood Values Please see the following table for the marginal likelihood values.

5 Conclusion

In this paper we propose empirical methods for detecting and identifying misspecification in DSGE models. Our approach is based on analyzing FEVD and marginal likelihoods of DSGE models aug-

Table 8: Marginal Likelihood Values

Model	logML	Rank
Inculde All Wedges	-365.425	3
Remove τ_l wedge	-363.129	1
Remove τ_k wedge	-363.572	2
Remove τ_c wedge	-365.520	4
Remove τ_b wedge	-373.434	5
Remove τ_h wedge	-376.056	7
Remove τ_y wedge	-372.875	6
Remove all Wedges	-407.599	8

mented with wedges, where the wedges are introduced to capture possible misspecification. Our simulation results demonstrate that our method can correctly identify the source of the misspecification. Our empirical results show that the medium-scale New Keynesian DSGE model that incorporates features in the recent empirical macro literature is still severely misspecified, and that the asset market and labor market may be the source of misspecification.

We should note that there are two potential issues with implementing our method: exogeneity of the wedges and over-parameterization. First, because the wedge processes are assumed to be exogenous, our method might not correctly identify the location of misspecification if the misspecification were endogenous. One way to get around this is to let the wedges depend on state variables. Second, when many markets are included in the model, there may be too many locations for introducing wedges and for forming priors for these processes. We suggest to either use fewer wedges when the prior on the misspecification location is strong (e.g. there is no misspecification in some parts of the model, but misspecification in others, and the researcher has strong opinions about where the misspecification is potentially located), or introduce many wedges and impose prior information when the misspecification location is more uncertain (i.e. every part of the model can potentially be misspecified). When neither is possible, another approach is to use a Bayesian model averaging approach to take into account many wedges, and let the Bayesian model average procedure provide information on the location of the wedges. We leave these extensions to future research.

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A Supplementary Material for Simulation Exercises

In this section, we provide supplementary materials of our simulation exercises.

A.1 The Data-Generating Process

The DGP of our simulation exercises can be summarized by the following linearized equilibrium condition

$$\begin{aligned}
\hat{Y}_t &= \mathrm{E}_t\left\{\hat{Y}_{t+1}\right\} - \frac{1}{\gamma}\left(\hat{R}_t - \mathrm{E}_t\hat{\pi}_{t+1}\right) + \left(\hat{g}_t - \mathrm{E}_t\hat{g}_{t+1}\right), && (20)\\
\hat{\pi}_t &= \kappa\left\{(\gamma+\varphi)\,\hat{Y}_t - (\varphi+1)\,\hat{z}_t - \gamma\hat{g}_t + \hat{\chi}_t\right\} + \beta\mathrm{E}_t\left\{\hat{\pi}_{t+1}\right\}, && (21)\\
\hat{R}_t &= \rho_r\hat{R}_{t-1} + (1-\rho_1-\rho_2)\left\{\gamma_\pi\hat{\pi}_t + \gamma_y\hat{Y}_t\right\} + \hat{\nu}_t, && (22)\\
\hat{Y}_t &= \hat{z}_t + \hat{L}_t, && (23)
\end{aligned}$$

where (20) is the dynamics IS curve, (21) is the New Keysian Phillips curve, (22) is the government monetary policy rule, and (23) is the linearized aggregate production function. Note that in the New Keynesian Phillips curve, $\kappa = \frac{(1-\beta\xi)(1-\xi)}{\xi}$.

For the sake of convenience, we list the structural shocks processes below:

$$\begin{aligned}
\hat{z}_{t+1} &= \rho_z\hat{z}_t + \sigma_z\varepsilon_{z,t+1}, \quad \varepsilon_{z,t+1} \overset{i.i.d.}{\sim} N(0,\sigma_z^2), && (24)\\
\hat{\nu}_{t+1} &= \rho_\nu\hat{\nu}_t + \sigma_\nu\varepsilon_{\nu,t+1}, \quad \varepsilon_{\nu,t+1} \overset{i.i.d.}{\sim} N(0,\sigma_\nu^2), && (25)\\
\hat{g}_{t+1} &= \rho_g\hat{g}_t + \sigma_g\varepsilon_{g,t+1}, \quad \varepsilon_{g,t+1} \overset{i.i.d.}{\sim} N(0,\sigma_g^2), && (26)\\
\hat{\chi}_{t+1} &= \rho_\chi\hat{\chi}_t + \sigma_\chi\varepsilon_{\chi,t+1}, \quad \varepsilon_{\chi,t+1} \overset{i.i.d.}{\sim} N(0,\sigma_\chi^2). && (27)
\end{aligned}$$

A.2 Models Containing Wedges

The wedges processes of the four estimated models are

$$\begin{aligned}
\hat{\tau}_{l,t+1} &= \rho_l\hat{\tau}_{l,t} + \sigma_l\varepsilon_{l,t+1}, \quad \varepsilon_{l,t+1} \overset{i.i.d.}{\sim} N(0,\sigma_l^2). && (28)\\
\hat{\tau}_{c,t+1} &= \rho_c\hat{\tau}_{c,t} + \sigma_c\varepsilon_{c,t+1}, \quad \varepsilon_{c,t+1} \overset{i.i.d.}{\sim} N(0,\sigma_c^2). && (29)\\
\hat{\tau}_{b,t+1} &= \rho_b\hat{\tau}_{b,t} + \sigma_b\varepsilon_{b,t+1}, \quad \varepsilon_{b,t+1} \overset{i.i.d.}{\sim} N(0,\sigma_b^2). && (30)
\end{aligned}$$

Equilibrium Conditions of the $\mathcal{M}_0$ model Since the $\mathcal{M}_0$ model is basically correctly specified, the equilibrium conditions of the model $\mathcal{M}_0$ is highly similar to those of the DGP. The only difference is that the dynamics IS curve and the New Keynesian Phillips curve contain three time-varying wedges. The corresponding equilibrium conditions are listed below:

$$\begin{aligned}
\hat{Y}_t &= \mathrm{E}_t\left\{\hat{Y}_{t+1}\right\} - \frac{1}{\gamma}\left(\hat{R}_t - \mathrm{E}_t\hat{\pi}_{t+1}\right) + \left(\hat{g}_t - \mathrm{E}_t\hat{g}_{t+1}\right) - \frac{1}{\gamma}\left(\hat{\tau}_{c,t} - \mathrm{E}_t\hat{\tau}_{c,t+1}\right) - \frac{1}{\gamma}\mathrm{E}_t\left\{\tau_{b,t+1}\right\}, && (31)\\
\hat{\pi}_t &= \kappa\left\{(\gamma+\varphi)\hat{Y}_t - (\varphi+1)\hat{z}_t - \gamma\hat{g}_t + \hat{\chi}_t\right\} + \beta\mathrm{E}_t\left\{\hat{\pi}_{t+1}\right\} + \kappa\left(\hat{\tau}_{c,t} + \hat{\tau}_{l,t}\right), && (32)\\
\hat{R}_t &= \rho_r\hat{R}_{t-1}\left(1-\rho_r\right)\left\{\gamma_\pi\hat{\pi}_t + \gamma_y\hat{Y}_t\right\} + \hat{\nu}_t, && (33)\\
\hat{Y}_t &= \hat{z}_t + \hat{L}_t, && (34)
\end{aligned}$$

In sum the $\mathcal{M}_0$ model is charactriz by

- four equilibrium conditions: (31), (32), (33), and (34);
- three exogeneous structural shocks process: (24), (25), (26), and (27)
- three exogeneous wedge process process: (28), (29), and (30).

Equilibrium Conditions of the $\mathcal{M}_l$ model Comparing to the model $\mathcal{M}_0$, model $\mathcal{M}_l$ does not contain labor supply shocks. Thus, the New Keynesian Phillips curve has to be modified as

$$\hat{\pi}_t = \kappa\left\{(\gamma+\varphi)\hat{Y}_t - (\varphi+1)\hat{z}_t - \gamma\hat{g}_t\right\} + \beta\mathrm{E}_t\left\{\hat{\pi}_{t+1}\right\} + \kappa\left(\hat{\tau}_{c,t} + \hat{\tau}_{l,t}\right). \tag{35}$$

In sum, the M_l are characterized by

- four equilibrium conditions: (31), (35), (33), and (34);
- three exogeneous structural shocks process: (24), (25), and (26);
- three exogeneous wedge process process: (28), (29), and (30).

Equilibrium Conditions of the $\mathcal{M}_m$ Model In this model, the only source of model misspecification is due the specification of monetary shock. That is, the monetary shocks are treated as identical and indepent rather than serial correalted:

$$\hat{\nu}_{t+1} = \sigma_\nu\varepsilon_{\nu,t+1}, \quad \varepsilon_{\nu,t+1} \overset{i.i.d.}{\sim} N(0,\sigma_\nu^2). \tag{36}$$

In sum, the $\mathcal{M}_m$ are characterized by

- four equilibrium conditions: (31), (32), (33), and (34);
- three exogeneous structural shocks process: (24), (36), (26), and (27);
- three exogeneous wedge process process: (28), (29), and (30).

Equilibrium Conditions of the $\mathcal{M}_g$ Model In this model, since the government spending shocks are excluded, this exclusion affects both the dynamics IS curve and the New Keynesian Phillips curve. They are replaced by

$$\begin{aligned}
\hat{Y}_t &= \mathrm{E}_t\left\{\hat{Y}_{t+1}\right\} - \frac{1}{\gamma}\left(\hat{R}_t - \mathrm{E}_t\hat{\pi}_{t+1}\right) - \frac{1}{\gamma}\left(\hat{\tau}_{c,t} - \mathrm{E}_t\hat{\tau}_{c,t+1}\right) - \frac{1}{\gamma}\mathrm{E}_t\left\{\tau_{b,t+1}\right\}, && (37)\\
\hat{\pi}_t &= \kappa\left\{(\gamma+\varphi)\hat{Y}_t - (\varphi+1)\hat{z}_t + \hat{\chi}_t\right\} + \beta\mathrm{E}_t\left\{\hat{\pi}_{t+1}\right\} + \kappa\left(\hat{\tau}_{c,t} + \hat{\tau}_{l,t}\right). && (38)
\end{aligned}$$

In sum, the M_g are characterized by

- four equilibrium conditions: (37), (38), (33), and (34);
- three exogeneous structural shocks process: (24), (25), and (27);
- three exogeneous wedge process process: (28), (29), and (30).

B Equilibrium Conditions for the Medium-Scale New Keyensian Model

The Endogenous Part In this appendix, all variables are expressed in log-deviation from their steady-state values, and thus we add a hat (ˆ) on the top of variables. There are 13 endogenous variables in the model, containing

$$\hat{Y}_t,\ \hat{C}_t,\ \hat{I}_t,\ \hat{L}_t,\ \hat{w}_t,\ \hat{\pi}_t,\ \hat{R}_t,\ \hat{s}_t,\ \hat{u}_t,\ \hat{r}_k,\ \hat{\bar{K}}_{t+1},\ \hat{q}_t,\ \hat{\psi}_t.$$

Accordingly, there are 13 equilibriums conditions as follows:

- The 1^{st} equation is

$$\hat{s}_t + \hat{z}_t = (1-\alpha)\left(\hat{w}_t + \hat{\tau}_{l,t}\right) + \alpha\left(\hat{\tau}_{k,t} + \hat{r}_{k,t}\right).$$

- The 2^{nd} equation is

$$\left(\hat{u}_t + \hat{\bar{K}}_t\right) - \hat{L}_t = \left(\hat{\tau}_{l,t} + \hat{w}_t\right) - \left(\hat{\tau}_{k,t} + \hat{r}_{k,t}\right).$$

- The 3^{rd} equation is

$$\begin{aligned}\hat{\pi}_t \quad = \quad & \frac{\iota_p}{(1+\beta\iota_p)}\hat{\pi}_{t-1} + \frac{\beta}{(1+\beta\iota_p)}E_t\hat{\pi}_{t+1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\iota_p)\,\xi_p}\left(\hat{s}_t + \frac{1}{1-\eta_p}\hat{\eta}_{p,t}\right) \\ & -\frac{1}{(1+\beta\iota_p)}\hat{\tau}_{y,t-1} + \frac{1+\beta\xi_p^2}{\xi_p\,(1+\beta\iota_p)}\hat{\tau}_{y,t} - \frac{\beta}{1+\beta\iota_p}E_t\hat{\tau}_{y,t+1},\end{aligned}$$

- The 4^{th} equation is

$$\hat{\bar{K}}_{t+1} = (1-\delta)\,\hat{\bar{K}}_t + \delta\left(\hat{I}_t + \hat{\mu}_t\right).$$

- The 5^{th} equation is

$$\begin{aligned}\hat{C}_t = & \frac{b}{1+\beta b^2}\hat{C}_{t-1} + \frac{\beta b}{1+\beta b^2}\mathrm{E}_t\left\{\hat{C}_{t+1}\right\} - \frac{(1-b)(1-\beta b)}{\gamma\,(1+\beta b^2)}\left(\hat{\tau}_{c,t} + \hat{\psi}_t\right) \\ & + \frac{1-b}{\gamma\,(1+\beta b^2)}\hat{d}_t - \frac{(1-b)\beta b}{\gamma\,(1+\beta b^2)}\mathrm{E}_t\left\{\hat{d}_{t+1}\right\}.\end{aligned}$$

- The 6^{th} equation is

$$\hat{\psi}_t = \mathrm{E}_t\left\{\hat{\tau}_{b,t+1} + \hat{\psi}_{t+1} + \hat{R}_t - \hat{\pi}_{t+1}\right\}.$$

- The 7^{th} equation is

$$\hat{r}_{k,t} = \varkappa_a \hat{u}_t.$$

Note that $\varkappa_a \equiv {}^{a''}/_{a'}$.

- The 8th equation is

$$\hat{q}_t + \hat{\mu}_t = \varkappa_s E_t \left\{ \left(\hat{I}_t - \hat{I}_{t-1} \right) - \beta \left(\hat{I}_{t+1} - \hat{I}_t \right) \right\}.$$

- The 9th equation is

$$\hat{q}_t + \hat{\psi}_t = E_t \left\{ \beta(1-\delta)\hat{q}_{t+1} + \hat{\psi}_{t+1} + \beta r_k \hat{r}_{k,t+1} \right\}.$$

- The 10th equation is

$$\begin{aligned} \hat{w}_t \quad = \quad & \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\left(\hat{w}_{t+1}\right) + \frac{\iota_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\iota_w}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}E\left(\hat{\pi}_{t+1}\right) \\ & + \frac{\left(1-\beta\xi_w\right)\left(1-\xi_w\right)}{\phi_w \xi_w \left(1-\beta\right)} \left(\hat{d}_t + \frac{1}{1-\eta_w}\hat{\eta}_{w,t} - \hat{\psi}_t + \varphi \hat{L}_t + \tau_{h,t} - \hat{w}_t \right) \\ & - \frac{1}{1+\beta}\hat{\tau}_{h,t-1} + \hat{\tau}_{h,t} - \frac{\beta}{1+\beta}E\left(\hat{\tau}_{h,t+1}\right) \end{aligned}$$

where $\phi_w = (1 + \varphi\eta_w)$

- The 11th equation is

$$\hat{Y}_t = \phi \left[z_t + \alpha \left(\hat{u}_t + \bar{K}_t \right) + (1-\alpha)\hat{L}_t \right]$$

- The 12th equation is

$$\left(1 - \frac{G}{Y}\right)\hat{Y}_t = \left(\frac{C}{Y}\right)\hat{C}_t + \left(\frac{I}{Y}\right)\hat{I}_t + \left(\frac{G}{Y}\right)\hat{g}_t + \left(\frac{r_k \bar{K}}{Y}\right)\hat{u}_t$$

- The 13th equation is

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1-\rho_r)\left[\gamma_\pi \hat{\pi}_t + \gamma_y \hat{Y}_t\right] + \hat{\nu}_t$$

Structural Shocks In terms of log deviations from the steady state values, there are 7 structural shocks, containing (i) the technology shock $\hat{z}_t$, (ii) the monetary shock $\hat{\nu}_t$, (iii) the government spending shock $\hat{g}_t$, (iv) the preference shock $\hat{d}_t$, (v) the investment shock $\hat{\mu}_t$, (vi) the price markup shock $\hat{\eta}_{p,t}$, and (vii) the wage markup shock: $\hat{\eta}_{w,t}$.

Wedges Processes In terms of log deviations, there are 6 wedges in our model, containing (i) the labor demand wedge $\hat{\tau}_{l,t}$, (ii) the capital demand wedge $\hat{\tau}_{k,t}$, (iii) the consumption wedge $\hat{\tau}_{c,t}$, (iv) the bond market wedge: $\hat{\tau}_{b,t}$, (v) the labor market wedge $\hat{\tau}_{h,t}$, and (vi) the intermediate good wedge $\hat{\tau}_{y,t}$.