Betting on Exports: Trade and Endogenous Heterogeneity*

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Abstract

We study the equilibrium determinants of firm-level heterogeneity in a model in which firms can choose between different probability distributions when drawing productivity at the entry stage and explore the implications in closed and open economy. One novel result is that export opportunities, by increasing payoffs in the tail, induce firms to draw technology from riskier distributions. When more productive firms also pay higher wages, trade amplifies wage dispersion by inducing firms to take more risk \textit{ex-ante} and hence making them more unequal \textit{ex-post}. Our model is consistent with new evidence on how firm-level heterogeneity varies across U.S. industries.

\textbf{JEL Classification:} F12, F16, E24.

\textbf{Keywords:} Firm Heterogeneity, Productivity Dispersion, Wage Inequality, International Trade.

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1 Introduction

Current research in international trade puts firm-level heterogeneity at a center stage. As documented by a growing empirical literature, firms differ in size and productivity even within narrowly defined industries and these differences vary systematically with trade participation. In particular, exporters are bigger and more productive than nonexporters, and they pay higher wages. Firm heterogeneity also has crucial implications for other macroeconomic outcomes, such as aggregate efficiency. Yet, despite the growing attention that firm-level productivity differences are attracting, we still have a limited understanding of the theoretical and empirical underpinnings of this heterogeneity.

Although the distribution of the entire population of existing firms has some common characteristics that have been documented extensively, these aggregate statistics mask significant heterogeneity across sectors and even between countries. For example, Helpman, Melitz and Yeaple (2004) shows that cross-sector variation in measures of firm heterogeneity has important effects on firm strategies. Poschke (2014) and Bartelsman, Haltiwanger and Scarpetta (2009) document instead differences in the firm-size distribution across countries. For instance, firm size is typically found to be larger but also more dispersed in the United States than in Spain or Italy. Moreover, given that more productive firms pay higher wages, firm heterogeneity is likely to map into wage dispersion, and wage inequality varies significantly across countries.\footnote{Dunne et al. (2004) and Faggio, Salvanes and Van Reenen (2010) show that a large fraction of the observed wage dispersion is between firms.}

Besides these scant observations, systematic evidence and theoretical explanations for differences in firm heterogeneity are still missing. The primary goal of this paper is to take a first step towards filling this gap.

We start our analysis by documenting some overlooked facts regarding how a synthetic measure of firm heterogeneity, the standard deviation of the log of sales, varies across sectors and time in the U.S. economy. We find that this measure of dispersion can differ by a factor of two between 4-digit NAICS industries, that it has increased on average by 25.9 per cent between 1997 and 2007, and that it correlates with industry characteristics such as average sales and export intensity. Motivated by this evidence, we propose one possible explanation based on the idea that the observed heterogeneity stems
from the choice of risk in the innovation strategies of entering firms. More precisely, we propose a model where firms can choose between different probability distributions when drawing productivity at the entry stage and we explore the implications for the equilibrium distribution of firms and wages in closed and open economies. Leading models of heterogeneous firms take the probability distribution from which firms draw productivity as given and characterize the resulting distribution of firm-level characteristics through the dynamics of entry and exit, and sometimes innovation by incumbents. Prominent example are Melitz (2003), Luttmer (2010) and more recently Jones and Kim (2014). The aim of this project is to take a complementary and perhaps more fundamental approach, namely, to recognize that firms can affect directly the expected dispersion of productivity by choosing the riskiness of their initial entry investment.

Although the success in starting a new enterprise is inherently uncertain, firms can deliberately choose between small projects with relatively safe returns and large projects with risky payoffs. Such a trade off is very familiar to anyone pursuing academic research, but is also common in the world of business. For instance, designing and assembling a new variety of laptop PCs, which mostly requires the use of established technologies, is safer and less costly than developing an entirely new product, such as tablet computers. In fact, the first tablet-like products date back to the 1980s, but did not reach success until the release of the iPad in 2010. After decades of research, Apple’s investment was rewarded with the sale of more than 200 million units over a period of four years only.

We formalize these ideas in a multi-industry model à la Melitz (2003) in which firms can draw a random productivity level upon paying a fixed entry cost and there is a fixed export cost. We modify this setup by allowing firms to choose the variance of the probability distribution from which to draw their productivity. Since expected profits are increasing in the dispersion of productivity, assuming that draws from riskier distributions are more costly delivers a well-defined trade-off. A first result of the paper is to show how the optimal point in this trade-off depends on industry-level characteristics, such as fixed costs of production and the elasticity of demand, in a way consistent with the patterns found in the data. A second key result is that export opportunities induce firms to draw technology from a riskier distribution. The reason is that trade reallocates profits in favor of the most productive firms, thereby increasing the payoffs in the right tail. Thus, export opportunities increase the returns to risky investment.
Finally, we extend the model to show how firm heterogeneity can map into income and wage inequality (for instance, as in Helpman, Itskhoki and Redding, 2010). When more productive firms pay higher wages, we obtain a third novel result: trade amplifies wage dispersion by inducing firms to take on more risk \textit{ex-ante} and hence making them more unequal \textit{ex-post}.

To the best of our knowledge, the choice of the riskiness of innovation, and its aggregate implications, has received little attention. Papers on technological change sometimes consider the distinction between radical and incremental innovation (e.g., Acemoglu and Cao, 2011). But these types of innovations differ more in the degree to which they replace or complement existing technologies, rather than in the variance of the potential outcomes. In any case, studying how different types of innovations affect the distribution of firms and income is an underexplored and promising area of research.

The large literature on trade with heterogeneous firms started by Melitz (2003) does study the implications of export opportunities for the distribution of existing firms.\footnote{See Melitz and Redding (2014) for an excellent survey.} As it is well-known, trade can make firms more unequal by reallocating profits and workers from the least to the most productive firms. This effect is however very different from the one we emphasize, in that it abstracts from the possibility that trade changes the fundamental reason why firms are different, i.e., the unconditional productivity distribution. Moreover, the focus of our paper is on measures of dispersion of firms’ attributes, such as the log of sales, that are scale invariant rather than other characteristics, such as average size or the productivity cutoff for exit, that have been studied more extensively. Similarly, several papers have shown, both theoretically and empirically, that trade impacts wage inequality because exporters pay higher wages.\footnote{See, for example, Helpman, Itskhoki and Redding (2010), Helpman et al. (2014) and other papers surveyed in section ten of Melitz and Redding (2014).} In our model, however, the effect of trade works not only through the exporters’ wage premium, but also by making the entire wage schedule steeper, with different implications. Finally, some recent papers endogenize productivity via \textit{ex-post} decisions on product scope, innovation and changes in organization.\footnote{See papers surveyed in section nine of Melitz and Redding (2014).} These models show that trade liberalization can raise firm-level productivity, but do not focus on its dispersion. Yet, combining our \textit{ex-ante}
choice of innovation risk with *ex-post* decisions that can affect an initial realization of productivity seems a natural step forward to develop a comprehensive theory of how productivity differences emerge and evolve.

The remainder of the paper is organized as follows. In Section 2, we document some novel stylized facts regarding how the dispersion of log-sales varies across sectors and time in the U.S. economy. Motivated by these empirical observations, in Section 3 we propose a closed-economy model where differences in the variance of firm-level outcomes stem from the possibility of choosing the probability distribution from which to draw productivity at the entry stage. Section 4 adds costly trade and shows that more export opportunities induce firms to draw their productivity from riskier distributions, thereby generating more heterogeneity in equilibrium. In Section 5 we consider the implications of the model for income and wage inequality. Section 6 concludes.

## 2 Motivating Evidence

In this section, we document how the dispersion of sales of U.S. firms varies across sectors and time, and how it correlates with a number of sector characteristics. First, we show that the dispersion of sales differs significantly across sectors and it has increased over time. Second, we report panel regressions suggesting that higher dispersion at the industry level is systematically associated with larger scale in terms of sales, higher demand elasticity, export intensity and capital intensity.

Our measures of dispersion are the variance and standard deviation of the logarithm of sales at firm level. We focus on sales because they are an easy-to-observe, synthetic measure of overall size, and we take the log to make the variance scale invariant. We compute these variables using data from the U.S. Census of Manufacturing for years 1997 and 2007. Data on (receipts of) sales and number of firms and establishments are available at 6-digit NAICS industry level for the universe of U.S. firms, aggregated into sales-size categories. Since unfortunately we do not have access to firm-level data, we follow Helpman, Melitz and Yeaple (2004) in assuming that all firms falling within the same sales-bin have the same value as the group mean, and using the number of firms in

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5 The lower bin contains firms making less than 50 thousand US$ in sales and the highest bin contains firms making more than 100 millions.
<table>
<thead>
<tr>
<th>NAICS code</th>
<th>Industry description</th>
<th>SD mean</th>
<th>SD min</th>
<th>SD max</th>
<th>SD %Δ</th>
<th>#firms mean</th>
</tr>
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<tbody>
<tr>
<td>315</td>
<td>Apparel and Manufacturing</td>
<td>2.291</td>
<td>2.158</td>
<td>2.453</td>
<td></td>
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<tr>
<td>332</td>
<td>Fabricated Metal Product Manufacturing</td>
<td>2.258</td>
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<td>2.750</td>
<td>14.5</td>
<td>9868</td>
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<tr>
<td>334</td>
<td>Computer and Electronic Product Manufacturing</td>
<td>2.397</td>
<td>1.952</td>
<td>3.285</td>
<td>-0.6</td>
<td>3311</td>
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<td>314</td>
<td>Textile Product Mills</td>
<td>2.404</td>
<td>2.115</td>
<td>2.558</td>
<td>23.4</td>
<td>2993</td>
</tr>
<tr>
<td>323</td>
<td>Printing and Related Support Activities</td>
<td>2.452</td>
<td>2.452</td>
<td>2.452</td>
<td>24.1</td>
<td>31655</td>
</tr>
<tr>
<td>339</td>
<td>Miscellaneous Manufacturing</td>
<td>2.454</td>
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<td>2.694</td>
<td>51.0</td>
<td>14705</td>
</tr>
<tr>
<td>335</td>
<td>Electrical Equipment, Appliance and Component Manufacturing</td>
<td>2.477</td>
<td>1.883</td>
<td>2.861</td>
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<tr>
<td>313</td>
<td>Textile Mills</td>
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<td>2.668</td>
<td>-0.3</td>
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<tr>
<td>337</td>
<td>Furniture and Related Product Manufacturing</td>
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<td>2.700</td>
<td>37.5</td>
<td>10510</td>
</tr>
<tr>
<td>333</td>
<td>Machinery Manufacturing</td>
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<td>2.865</td>
<td>26.4</td>
<td>3541</td>
</tr>
<tr>
<td>326</td>
<td>Plastics and Rubber Products Manufacturing</td>
<td>2.564</td>
<td>2.477</td>
<td>2.940</td>
<td>22.2</td>
<td>8080</td>
</tr>
<tr>
<td>327</td>
<td>Nonmetallic Mineral Product Manufacturing</td>
<td>2.626</td>
<td>2.027</td>
<td>3.553</td>
<td>40.2</td>
<td>3604</td>
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<tr>
<td>321</td>
<td>Wood Product Manufacturing</td>
<td>2.694</td>
<td>2.232</td>
<td>3.033</td>
<td>42.7</td>
<td>6190</td>
</tr>
<tr>
<td>316</td>
<td>Leather and Allied Product Manufacturing</td>
<td>2.737</td>
<td>2.534</td>
<td>2.992</td>
<td>27.7</td>
<td>502</td>
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<tr>
<td>331</td>
<td>Primary Metal Manufacturing</td>
<td>2.848</td>
<td>2.699</td>
<td>3.012</td>
<td>34.8</td>
<td>843</td>
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<tr>
<td>322</td>
<td>Paper Manufacturing</td>
<td>2.875</td>
<td>2.519</td>
<td>3.302</td>
<td>29.4</td>
<td>1790</td>
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<tr>
<td>325</td>
<td>Chemical Manufacturing</td>
<td>3.015</td>
<td>2.618</td>
<td>3.252</td>
<td>20.8</td>
<td>1459</td>
</tr>
<tr>
<td>336</td>
<td>Transportation Equipment Manufacturing</td>
<td>3.095</td>
<td>2.178</td>
<td>3.357</td>
<td>30.9</td>
<td>1837</td>
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<tr>
<td>312</td>
<td>Beverage and Tobacco Product Manufacturing</td>
<td>3.101</td>
<td>2.885</td>
<td>3.572</td>
<td>29.2</td>
<td>2349</td>
</tr>
<tr>
<td>311</td>
<td>Food Manufacturing</td>
<td>3.135</td>
<td>2.486</td>
<td>3.553</td>
<td>36.8</td>
<td>2660</td>
</tr>
<tr>
<td>324</td>
<td>Petroleum and Coal Products Manufacturing</td>
<td>3.302</td>
<td>3.302</td>
<td>3.302</td>
<td>28.0</td>
<td>1189</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>2.844</td>
<td>1.667</td>
<td>3.572</td>
<td>25.9</td>
<td>4227</td>
</tr>
</tbody>
</table>

Note: SD is the standard deviation of the log of sales. Δ denotes percentage changes between 1997 and 2007. Statistics are computed on data at 4-digit industry level. Source: US Census.
each category as weights. In particular, we consider each bin in a 6-digit NAICS industry as a single observation, and compute the variance of log-sales aggregating appropriately the observations at 4-digit industry level. Helpman, Melitz and Yeaple (2004) show that this methodology to compute dispersions yields results that are highly correlated with direct measures based on the entire population. Table 1 reports some descriptive statistics. For 3-digit manufacturing sectors, it shows the average standard deviation of log-sales in 2007, its minimum and maximum in each 4-digit subindustry, and the average percentage change over the previous ten years.\footnote{All averages are weighted by sales.} For convenience, industries are ordered by increasing dispersion. The first three columns show that dispersion varies significantly across industries, ranging from a minimum of 1.667 (in Furniture Related Products) to a maximum of 3.572 (in Tobacco Products). The last column also reports the average number of firms in the 4-digit subindustries. Comparing the first and the last columns reassures that dispersion in an industry is not mechanically driven by sample size. Finally, the second-last column points out that the standard deviation of log-sales increased remarkably in nearly all sectors, on average by 25.9 per cent, between 1997 and 2007.

To further describe the data, we now exploit the variation across 4-digit industries and over time to study how dispersion in revenues correlates with a number of sector characteristics. Table 2 reports panel regressions where the dependent variable is the variance of the log of sales. All specifications are estimated both with random and industry-specific fixed effects, and standard errors are clustered by industry. Columns 1 and 2 show that our measure of dispersion is positively and significantly correlated with sales per firm, while it correlates negatively with employment per firm. There is no correlation, instead, with the number of firms in the sector. This suggests that dispersion is higher in sectors where firms are larger in terms of revenue. In columns 3 and 4, we add export intensity, measured as the ratio of exports to sales, and the demand elasticity estimated by Broda and Weinstein (2006).\footnote{The demand elasticity is estimated over the period 1990-2001 and has no time variation. Hence, it is dropped in fixed-effects specifications.} The coefficients for both variables are positive and significant, meaning that sales are more dispersed in more export-oriented sectors producing less differentiated goods. Finally, in columns 5 and
Table 2. Dispersion of Sales - Panel Regressions

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sales per firm</td>
<td>0.014***</td>
<td>0.017***</td>
<td>0.013***</td>
<td>0.017***</td>
<td>0.006***</td>
<td>0.011**</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.002)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>employment p.f.</td>
<td>-5.710***</td>
<td>-7.575***</td>
<td>-5.329***</td>
<td>-7.390***</td>
<td>-0.450</td>
<td>-2.394</td>
</tr>
<tr>
<td></td>
<td>(0.944)</td>
<td>(1.544)</td>
<td>(0.986)</td>
<td>(1.514)</td>
<td>(1.180)</td>
<td>(1.857)</td>
</tr>
<tr>
<td>export intensity</td>
<td>2.348**</td>
<td>6.487*</td>
<td>3.263***</td>
<td>6.737**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.063)</td>
<td>(3.704)</td>
<td>(0.923)</td>
<td>(3.868)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>demand elasticity</td>
<td>0.131**</td>
<td>-</td>
<td>0.193***</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td></td>
<td>(0.043)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>capital intensity</td>
<td></td>
<td></td>
<td>10.161***</td>
<td>19.078***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1.567)</td>
<td>(5.967)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>skill intensity</td>
<td></td>
<td></td>
<td>-11.341***</td>
<td>-10.489</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(4.380)</td>
<td>(6.839)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.240</td>
<td>0.230</td>
<td>0.264</td>
<td>0.220</td>
<td>0.386</td>
<td>0.343</td>
</tr>
<tr>
<td>Industry-FE</td>
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<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
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<tr>
<td>Observations</td>
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<tr>
<td>Industries</td>
<td>84</td>
<td>84</td>
<td>84</td>
<td>84</td>
<td>84</td>
<td>84</td>
</tr>
</tbody>
</table>

Note: the dependent variable is the variance of the log of sales. The number of firms is expressed in millions, sales per firm are in US$ millions, employment per firm is in thousand workers. Export intensity is computed as exports over sales, capital intensity is computed as the non-labor share in value added, skill intensity as non-production workers per firm. The sample period includes years 1997 and 2007. Robust standard errors, clustered by industry, are reported in parenthesis. Significance at 1, 5 and 10 per cent is denoted by ***, **, and *, respectively.

6, we also include capital intensity, computed as the non-labor share in value added, and skill intensity, proxied by the number of non-production workers per firm.\(^8\) The coefficients suggest that log-sales are more dispersed in capital-intensive sectors, while the correlation with skill intensity is not always significant. The coefficients for sales per firm, export intensity and demand elasticity remain positive and significant across all specifications.\(^9\) Although these correlations do not establish any causal relationship between the covariates, they uncover some new systematic patterns in the data.\(^10\) In the rest of the paper, we propose one possible theory that can help explaining these

\(^8\)Data on sales, value added, employment and non-production workers are obtained from the NBER Manufacturing Industry Database. The dependent variable and the number of firms are computed with data from the U.S. Census. Export data, from the Center for International Data and NBER, refer to years 1997 and 2006. Since we control for employment, non-production workers measures skill intensity in the workforce.

\(^9\)In the worst case, export intensity is significant at the 8% level.

\(^10\)Exploring more in detail these interesting patterns goes beyond the scope of this paper and is left for future research.
observations by endogenizing the dispersion in productivity across firms at the sector level.

3 Closed-Economy Model

We now build a multi-sector, one factor, model of monopolistic competition between heterogeneous firms along the lines of Melitz and Redding (2014). After paying a fixed entry cost, firms draw their productivity from a distribution and exit if they cannot profitably cover a fixed cost of production. Differently from Melitz (2003), we allow firms to affect the “riskiness” of their entry investment by choosing the distribution from which to draw their productivity. In this section we study the determinants of entry risk and how it affects the equilibrium distributions in a closed economy. We defer to the next section the case in which firms can engage in costly trade. For simplicity, we consider a static model in which entry and production decisions are all simultaneous.

3.1 Preferences

Consider an economy populated by a unit measure of identical households of size $L$ with quasi-linear preferences over consumption of a homogenous good $q_0$ and differentiated goods produced in $I$ industries:

$$U = q_0 + \sum_{i=1}^{I} \alpha_i \frac{X_i^{\zeta_i}}{\zeta_i}, \quad \zeta_i \in (0, 1), \quad \alpha_i > 0$$

Each industry $i \in \{1, ..., I\}$ produces differentiated varieties and preferences over these varieties take the constant elasticity of substitution form:

$$X_i = \left[ \int_{\omega \in \Omega_i} x_i(\omega)^{\frac{\sigma_i-1}{\sigma_i}} \, d\omega \right]^{\frac{\sigma_i}{\sigma_i-1}}, \quad \sigma_i > 1$$

where $x_i(\omega)$ is consumption of variety $\omega$, $\Omega_i$ denotes the set of varieties produced in sector $i$ and $\sigma_i$ is the elasticity of substitution between varieties within an industry. We denote by $p_i(\omega)$ the price of variety $\omega$ in industry $i$ and by $P_i$ the ideal price of the
consumption basket $X_i$:

$$P_i = \left[ \int_{\omega \in \Omega_i} p_i(\omega)^{1-\sigma} \, d\omega \right]^{1/(1-\sigma_i)}.$$

The demand for the differentiated basket $X_i$ is $X_i = (\alpha_i/P_i)^{1/(1-\zeta_i)}$ and the demand for each individual variety is

$$x_i(\omega) = X_i \left( \frac{P_i}{p_i(\omega)} \right)^{\sigma_i}.$$  \hspace{1cm} (1)

The demand for the homogenous good $q_0$ is residual. We assume that income of each household is sufficiently high to always guarantee a positive consumption of the homogenous goods, which is chosen as the numeraire. In the remainder of the paper, we focus on a single sector and derive results that do not depend on general equilibrium effects. For this reason, and to save notation, from now on we remove the index $i$ with the understanding that all parameters can potentially vary across sectors.

3.2 Problem of the Firm

Recall that we now focus on the industry equilibrium of a single sector $i \in \{1, \ldots, I\}$. Within each sector, every variety $\omega$ is produced by monoplisitically competitive firms that are heterogeneous in their labor productivity, $\varphi$. Since all firms with the same productivity behave symmetrically, we index firms by $\varphi$. There are fixed costs of production, $f$, and of entry, $F$, in units of the numeraire good. Upon entry, a firm can choose to randomly draw its productivity from a menu of distributions differing in the riskiness of their realizations and pays the corresponding entry cost. Next, the firm faces standard production and pricing decisions. We solve the problem backwards: first, we describe the strategy of a firm with a given productivity and then solve the problem of choosing the productivity distribution given rational expectation on the industry equilibrium.

Given a productivity $\varphi$ and a marginal cost of $w/\varphi$, where $w$ is the wage, the firm will choose its price and whether to exit so as maximize profit, $\pi(\varphi)$, subject to a downward-sloping demand curve with elasticity $\sigma$. The first-order conditions for this problem imply
that firms set prices equal to a constant markup over the marginal cost,

\[ p(\varphi) = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi}, \tag{2} \]

and exit if \( \pi(\varphi) < 0 \). Using (1) and (2), we can express profit as a function of productivity:

\[ \pi(\varphi) = A \varphi^{\sigma - 1} - f \tag{3} \]

where \( A = \left( \frac{\sigma w}{\sigma - 1} \right)^{1-\sigma} \frac{X \rho}{\sigma} \). Since profits are increasing in \( \varphi \), the firm will exit whenever its productivity is below the cutoff \( \varphi^* = (f/A)^{1/(\sigma - 1)} \).

Combining the pricing and exit decision, we can write \textit{ex-ante} expected profit of a firm drawing its productivity from a distribution with cumulative distribution \( G(\varphi) \) as:

\[ \mathbb{E}[\pi] = \int_0^\infty \pi(\varphi) \, dG(\varphi) = \int_{\varphi^*}^{\infty} \left( A \varphi^{\sigma - 1} - f \right) \, dG(\varphi). \tag{4} \]

To study the incentives for firms to choose the “riskiness” of their initial entry investment, we assume that \( G(\varphi) \) belongs to a family of Pareto distributions with different shape parameters:

\[ G(\varphi) = 1 - (\varphi_{\text{min}}/\varphi)^{1/v}, \quad v \in [\underline{v}, \bar{v}] \]

where \( \varphi_{\text{min}} > 0 \) is the lower bound of the support and the shape parameter is \( 1/v \). Written in this way, \( v \) can be interpreted as an index of the variance of the distribution. More precisely, \( v \) is equal to the standard deviation of the log of \( \varphi \) and will be one of the key determinants of the equilibrium distributions of the log of firm characteristics, such as sales. The bounds \( \underline{v} > 0 \) and \( \bar{v} < 1 \) rule out the possibility of a degenerate distribution and ensure a finite mean for \( v \). Note also that the mean of \( \varphi \) is \( \varphi_{\text{min}} (1 - v)^{-1} \) which increases with \( v \). Thus, our assumption that firms can choose between distributions with different \( v \) embeds the notion that high expected payoffs are associated to more risk. As we will show shortly, this property is not strictly needed for many of the results of the paper. Yet, it seems a very natural property and we conform to it.

There are several reasons for focusing on Pareto distributions. Besides being tractable and widely used, it has been shown that Pareto distributions approximate well some observed firm-level characteristics (especially in the right tail). Thus, it is an empirically
reasonable assumption. Moreover, the Pareto distribution has the useful property that power functions of $\varphi$ are also Pareto distributed, although with a different shape parameter. This helps to map the model to the data because it will allow us to obtain closed-form solutions for the measure of dispersion computed in Section 2.

Substituting $A(\varphi^*)^{\sigma-1} = f$ into (4), assuming $\varphi^* > \varphi_{\min}$ (so that there is selection) and using $G(\varphi)$, we can solve for expected profits:

$$
\mathbb{E}[\pi] = f \int_{\varphi^*}^{\infty} \left[ \left( \frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] dG(\varphi) = \frac{f\varsigma}{1/\varsigma - \varsigma} \left( \frac{\varphi_{\min}}{\varphi^*} \right)^{1/\varsigma}.
$$

(5)

where it proves convenient to define $\varsigma \equiv \sigma - 1$ and we assume $\nu < 1/\varsigma$ for $\mathbb{E}[\pi]$ to be finite. It is easy to see that expected *ex-ante* profits are increasing in $\nu$:

$$
\frac{\partial \ln \mathbb{E}[\pi]}{\partial \ln \nu} = \frac{1}{1 - \nu\varsigma} + \ln \left( \frac{\varphi^*}{\varphi_{\min}} \right)^{1/\nu} > 0.
$$

(6)

There are three reasons why a higher $\nu$, and hence more dispersion in the distribution of productivity draws, implies higher expected profits. First, as already seen, a higher $\nu$ raises average productivity directly. Second, given the shape of the Pareto distribution, it increases the probability of drawing a productivity above the exit cutoff $\varphi^*$.\(^{11}\) Third, even in the absence of the previous effects, more dispersion increases expected profits whenever the profit function is convex in prices and hence in $\varphi$. As equation (3) shows, this is the case when $\sigma > 2$ (i.e., for $\varsigma > 1$). To see this, suppose now that $\varphi_{\min} = \varphi^* (1 - \nu)$ so that the mean of the distribution is constant at $\bar{\varphi}$ and an increase in $\nu$ corresponds to a mean-preserving spread. Then:

$$
\frac{\partial \ln \mathbb{E}[\pi]}{\partial \ln \nu} = \frac{1}{1 - \nu\varsigma} - \frac{1}{1 - \nu} + \ln \left( \frac{\varphi^*}{\varphi_{\min}} \right)^{1/\nu}
$$

which is necessarily positive when $\varsigma > 1$ ($\sigma > 2$), even in the absence of selection effects (i.e., when $\varphi^* \rightarrow \varphi_{\min}$). The intuition is that firms can expand to take advantage of good realizations of productivity and contract to insure against bad realizations, making them potentially risk loving. This is a well-known result, sometimes referred to as the

\(^{11}\)An increase in $\nu$ raises the density at any $\varphi > \varphi^*$. As shown below, this is true even holding constant the mean of the Pareto distribution.
Hartman (1972) and Abel (1983) effect.

Having characterized the value of drawing productivity from riskier distributions, we need to specify its cost. In order to have a well defined trade-off, we assume that the entry cost $F$ is an increasing and convex function of $v$: $F'(v) > 0$ and $F''(v) > 0$. The fact that this entry cost is increasing in $v$ (at a sufficient rate) is not just needed to prevent firms from choosing trivially the distribution with the highest dispersion; it also captures the sensible notion that risky projects with high expected payoffs require bigger investments. All in all, although our description of the entry risk faced by firms is admittedly stylized, it accords well with common sense.\footnote{Equivalently, we could have assumed that firms can choose some other variable $s$ which in turn affects positively $v$. If $v(s)$ is a sufficiently convex function, we would obtain a non-increasing marginal benefit of raising $s$.}

We are now in the position to solve the entry stage. The problem is greatly simplified by the fact that all firms in a given sector are \textit{ex-ante} identical and therefore face the same problem of choosing $v$ so as to maximize expected profits minus the entry cost:

$$\max_{v \in [\bar{v}, \bar{v}]} \{ \mathbb{E}[\pi] - F(v) \}.$$ 

To ensure that the maximand is concave, we impose $\partial^2 \mathbb{E}[\pi] / \partial v^2 < F''(v)$. Then, the first-order condition for $v$ is

$$\frac{\mathbb{E}[\pi]}{v} \left[ \frac{1}{1 - v} + \ln \left( \frac{\varphi^*}{\varphi_{\min}} \right)^{1/v} \right] \geq F'(v).$$

(7)

Concavity and implicit differentiation allow us to sign the comparative statics for $v$. If interior, the unique equilibrium choice of $v$ is increasing in the elasticity of substitution, $\varsigma$, average profit, $\mathbb{E}[\pi]$, and the exit cutoff, $\varphi^*/\varphi_{\min}$. However, both $\mathbb{E}[\pi]$ and $\varphi^*/\varphi_{\min}$ are endogenous and to solve for them we now turn to the industry equilibrium.
Free entry implies that \textit{ex-ante} expected profits must be equal to the entry cost: \( E[\pi] = F(v) \). Substituting (5) into this condition, we can solve for the exit cutoff:

\[
\left( \frac{\varphi^*}{\varphi_{\min}} \right)^{1/v} = \frac{f}{F(v)} \frac{s}{1/v - s}.
\]  

(8)

To make sure that \( \varphi^*/\varphi_{\min} > 1 \), we impose \( f > \max_{v \in [v_1, v]} \{ F(v) (1/s v - 1) \} \). Next, using \( E[\pi] = F(v) \) and (8), we can rewrite the first-order condition for \( v \) in the case of an interior solution (7) as:

\[
\ln \left( \frac{f}{F(v)} \frac{s}{1/v - s} \right) + \frac{1}{1 - v s} = \frac{v F'(v)}{F(v)}
\]  

(9)

Under regularity conditions that we take for granted, equation (9) has an interior solution over the relevant range \( v \in [v_1, \bar{v}] \).\(^\text{13}\) Moreover, provided that \( F(v) \) is sufficiently convex, the solution will be unique. Although the possibility of multiple equilibria is interesting, exploring it goes outside the scope of this paper and we therefore disregard this possibility.\(^\text{14}\)

We can now study the equilibrium determinants of \( v \). A higher fixed cost of production, \( f \), increases the exit cutoff and hence raises the benefit of choosing a more dispersed distribution. A higher elasticity of substitution raises the value of \( v \) by making profits more convex in productivity and by increasing the exit cutoff. Moreover, since \( E[\pi] = F(v) \) and \( F'(v) > 0 \), the model predicts a positive association between average profits (or sales) and the variance of productivity.

The choice of \( v \) affects the equilibrium distribution of firm characteristics. Consider the distribution of revenues, which matches closely the variable documented in Section 2. It is easy to show that revenues are a power function of productivity:

\[ r(\varphi) = r(\varphi^*) (\varphi/\varphi^*)^{\zeta}. \]

Then, from the properties of the Pareto distribution, \( r(\varphi) \) is

\(^\text{13}\)Sufficient conditions are \( F'(v) = 0 \) with \( F(v) > 0 \) and \( F'(\bar{v}) \to \infty \) with \( \bar{v} < 1/s \).

\(^\text{14}\)One can imagine a situation in which the expectation of high profits in equilibrium induces firms to choose a high initial investment, which in turn confirms the initial expectation that firms be large. Similarly to Bonfiglioli and Gancia (2014), this multiplicity may help explain cross-country differences in the prevalence of small and large firms and other outcomes.
also Pareto distributed with c.d.f. \( G_r(r) = 1 - (r_{\text{min}}/r)^{1/\nu} \), for \( r > r_{\text{min}} = \sigma f \).\(^{15}\) Hence, the log of revenue is exponential with a standard deviation equal to \( \nu \). This immediately implies that differences in the choice of entry risk across sectors will translate into differences in the equilibrium distributions of firm characteristics as summarized in the following Proposition.

**Proposition 1** Assume that the solution to (9) is unique and interior. Then, the equilibrium dispersion of firm productivity and revenue, as measured by the variance of the log of \( \varphi \) and \( r(\varphi) \), is larger in sectors with a higher fixed cost and higher elasticity of substitution between varieties.

These results are consistent with the empirical correlations documented in Section 2.\(^{16}\)

### 4 Trade and Equilibrium Firm Heterogeneity

We now extend the model by adding the possibility for firms to export their varieties subject to fixed and variable costs. This will lead to the familiar results that only the most productive firms export and that trade forces the least productive firms out. This *ex-post* reallocation of revenues will have new implications for the *ex-ante* entry stage: by increasing the payoffs in the tail, trade will induce firms to take on more risk and draw their productivity from more dispersed distributions.

Consider a world economy composed, for simplicity, of two symmetric countries. To serve the foreign market, firms must incur a fixed cost \( f_x \) in units of the numeraire and an iceberg variable cost such that \( \tau > 1 \) units must be shipped for one unit to arrive at destination. The presence of a fixed trade cost implies that only the most productive firms choose to serve the foreign market. Formally, notice that, in analogy to (3), profits from exporting are \( \pi_x(\varphi) = A(\varphi/\tau)^{\sigma-1} - f_x \). These profits would be negative for firms

\(^{15}\)If \( \varphi \) follows a Pareto(\( \varphi^*, z \)), then \( x = \ln(\varphi/\varphi^*) \) is distributed as an exponential with parameter \( z \). Then, any power function of \( \varphi \) of the type \( A\varphi^B \), with \( A \) and \( B \) constant, is distributed as a Pareto(\( A(\varphi^*)^B, z/B \)), since \( A\varphi^B = A(\varphi^*)^B e^{Bz} \) with \( Bx \sim \text{Exp}(z/B) \), by the properties of the exponential distribution.

\(^{16}\)Although there is no capital in the model, one could interpret the fixed cost of production, which is in units of the numeraire, as a proxy for capital expenditures. Notice also that a higher elasticity of substitution, \( \zeta \), increases the variance of log revenues both directly and through its impact on \( \nu \).
with productivity $\varphi < \varphi^*_x = \tau (f_x / A)^{1/\kappa}$. As usual, we restrict attention to the space of parameters such that $\varphi^*_x / \varphi^* = \tau (f_x / f)^{1/\kappa} > 1$, so that there is a range of firms with $\varphi \in [\varphi^*, \varphi^*_x]$ operating in the domestic market only, while the most productive firms also export.

Under these assumptions, \textit{ex-ante} expected profits are:

$$
\mathbb{E} [\pi] = \int_{\varphi^*}^{\infty} \left[ \left( \frac{\varphi}{\varphi^*} \right)^{\varsigma} - 1 \right] \mathrm{d}G(\varphi) + f_x \int_{\varphi^*_x}^{\infty} \left[ \left( \frac{\varphi}{\varphi^*_x} \right)^{\varsigma} - 1 \right] \mathrm{d}G(\varphi),
$$

(10)

where the two terms represent expected profits from the domestic and the foreign market. Solving the integrals yields:

$$
\mathbb{E} [\pi] = \frac{\varsigma}{1/v - \varsigma} \left[ f \left( \frac{\varphi^*_x}{\varphi^*} \right)^{1/v} + f_x \left( \frac{\varphi^*_x}{\varphi^*} \right)^{1/v} \right].
$$

To study how export opportunities affect the value of drawing productivity from a riskier distribution, we compute again the elasticity of expected profits to $\upsilon$:

$$
\frac{\partial \ln \mathbb{E} [\pi]}{\partial \ln \upsilon} = \frac{1}{1 - \upsilon \varsigma} + \ln \left( \frac{\varphi^*_x}{\varphi^*} \right)^{1/v} + \frac{\ln \left( \varphi^*_x / \varphi^* \right)^{1/v}}{(\varphi^*_x / \varphi^*)^{1/v} f_x / f + 1},
$$

(11)

Comparing this derivative to (6), we see that choosing a riskier distribution yields now a new advantage: conditional on surviving, it increases the probability of reaching the export cutoff, $\varphi^*_x$. Moreover, as it is well known and we show next, $\varphi^*_x / \varphi^*_{\min}$ is higher with trade.

As in autarky, we solve for the equilibrium $\upsilon$ by imposing the free-entry condition, $\mathbb{E} [\pi] = F(\upsilon)$. This condition allows us to find the exit cutoff:

$$
\left( \frac{\varphi^*_x}{\varphi^*_{\min}} \right)^{1/v} = \frac{\varsigma}{1/v - \varsigma} f + f_x (\varphi^*_x / \varphi^*)^{-1/v} F(\upsilon).
$$

(12)

As expected, the exit cutoff is higher than in autarky and is increasing in the barriers to export. For convenience, we now define $\rho \equiv \varphi^*_x / \varphi^*_{\min} = (f / f_x)^{1/\kappa} / \tau$ and use it as a synthetic measure of trade openness. This index, which varies between zero and one, only depends on exogenous parameters and determines the fraction of exporting firms, which is equal to $\rho^{1/v}$. Using this notation and (12) into (11), we can show how trade affects
the elasticity of expected profits to $v$, and hence the incentive to draw productivity from riskier distribution:

$$\frac{\partial^2 \ln \mathbb{E} [\pi]}{\partial \ln v \partial \rho} = \frac{f/f_x}{\rho^{1+1/v} v (\rho^{-1/v} f/f_x + 1)^2} \ln \rho^{-1/v} > 0.$$ (13)

In words, more openness raises unambiguously the return from riskier productivity draws. This result is intuitive: trade offers new profitable opportunities, but only to the most productive firms and hence reallocates profits to the right tail of the distribution. In turn, a higher $v$ (a lower shape parameter of the distribution of productivity) increases the probability mass in that tail. This is one of the main results of the paper: the chance of winning the extra prize of exporting induces firms to take a riskier bet at the entry stage. Notice also that even in the extreme case in which all firms export ($\varphi^*/\varphi_x^* \to 1$), $v$ will be higher in the trading equilibrium than in autarky because of the higher fixed cost faced by each firm ($f + f_x$ instead of $f$).\footnote{This result is true as long as trade is costly. Of course, the case of costless trade (e.g., if $f$ is not desinantion specific, $f_x = 0$ and $\tau = 1$) would be equal to autarky.}

Following the same steps as in autarky, the equilibrium $v$ is implicitly determined by:

$$\frac{1}{1 - v\varsigma} + \ln \left( \frac{\varsigma}{1/v - \varsigma} \frac{f + f_x \rho^{1/v}}{F(v)} \right) + \frac{\ln \rho^{-1/v}}{\rho^{-1/v} f/f_x + 1} = \frac{vF'(v)}{F(v)}.$$ (14)

Since the left-hand side is increasing in openness (this follows from 13), and assuming again the solution to be unique and interior, more openness leads to a higher equilibrium $v$ and hence more productivity dispersion.

These results are summarized in the following Proposition.

**Proposition 2** An increase in openness, as measured by the fraction of exporters, induces firms to choose riskier productivity draws (higher $v$) and raises the equilibrium dispersion of firm productivity, as measured by the variance of the log of $\varphi$.

An additional interesting implication of this model is that trade has a new effect of productivity. Since a higher $v$ also raises the mean of $\varphi$, export opportunities induce firms to choose riskier technologies with higher expected returns. As a result, in an equilibrium with trade firms will be more productive, not just because of the usual
selection effect, but also because firms choose more costly, but on average more efficient, technologies. This prediction is also of intuitive appeal: the higher premium for success in the global economy makes firms more “ambitious” by choosing a bigger (and riskier) investment in the entry stage.

Of course, the analytical results derived in this section partly hinge on functional form assumptions and on the convenient properties of Pareto distributions. Yet, we expect the main mechanism to hold more in general. In particular, as long as trade reallocates profits in favor of exporters and exporting firms are a minority, trade will increase the payoff in the tails and hence raise the return from taking a riskier bet on productivity.

5 From Firm Heterogeneity to Income Inequality

We now explore the implications of our theory for income and wage inequality. This as a natural step: the distribution of productivity is likely to be a major determinant of the distribution of wages because in the data more productive firms pay higher wages. We therefore extend the model to allow for differences in wages across firms. This will yield two main results: first, it will highlight a new channel through which trade can increase wage inequality and, second, it will identify some additional variables affecting the choice of risk at the entry stage.

In principle, our theory can be used to study top-income inequality. An immediate way of doing this is to draw a link between profits and entrepreneurial income. For example, one could assume that there is a class of agents, entrepreneurs, who are the only ones who can enter and start new firms. These agents may be able to finance part of the entry cost externally and will be the residual claimants on a share of profits. Recent models along these lines include Jones and Kim (2014) or Grossman and Helpman (2014). Since trade increases the dispersion of profits, it will also make entrepreneurial income more unequal. Several contributions in corporate finance, such as Gabaix and Landier (2008), have indeed shown that CEO compensations are proportional to firm size and that this can explain why they have increased so much in recent decades. Our theory can then help rationalize some of the changes in the firm size distribution behind this phenomenon.
Another possibility, that we consider more in detail, is to extend the model to study implications for wage dispersion. In the literature, there are several ways of linking firm productivity to wages. With competitive labor markets, wages can vary because of differences in workforce composition across firms (e.g., Sampson, 2014, Monte, 2011, Yeaple, 2005). Alternatively, workers could be paid different wages due to labor market frictions (e.g., Helpman et al. 2010, Amiti and Davis, 2012, Egger and Kreickemeier, 2009, Felbermayr, Impullitti and Prat, 2014). For example, in Helpman, Itskhoki, and Redding (2010, HIR henceforth) workers matched randomly with heterogeneous firms draw a match-specific ability which is not observed and firms can invest in costly screening. In equilibrium, more productive firms screen workers more intensively to exclude those with lower ability. As a result, they have workforces of higher average ability and pay higher wages. These models yield an exporter wage premium and have been found to have considerable empirical support (e.g., Helpman et al. 2014). We therefore now borrow the framework of HIR to study the implications of our theory for wage dispersion. One key advantage of HIR is that it preserves the main equations of the basic Melitz model, thereby allowing us to apply our previous results in a relatively straightforward manner.

We briefly derive the equations of HIR that are relevant for our purpose and refer the reader to the original article for more details. For ease of comparison, we try to follow the original notation whenever possible. Production depends on the productivity of the firm, \( \varphi \), the measure of hired workers, \( h \), and the average ability of these workers, \( \bar{a} \):

\[
y = \varphi h^{\gamma} \bar{a},
\]

where \( \gamma \in (0, 1) \) implies diminishing returns to hired workers. Two important properties of this production function are the complementarity between firm productivity and average worker ability and a trade-off between the quantity and quality of hired workers. Workers’ ability is assumed to be independently distributed and drawn form a Pareto distribution with shape parameter \( k > 1 \) and c.d.f. \( G_a(a) = 1 - (a_{\min}/a)^{-k} \). Search frictions in the labor market imply that a firm has to pay \( bn \) units of the numeraire to be matched randomly with a measure \( n \) of workers. Ability is unknown. However, once the match is formed, the firm can use a screening technology to identify workers with
ability below $a_c$ at the cost of $ca_c^\delta/\delta$ units of the numeraire, with $c > 0, \delta > k$. Given the
distribution of ability, a firm matched with $n$ workers and screening at the cutoff $a_c$ will
hire a measure $h = n (a_{min}/a_c)^k$ of workers with an average ability of $\bar{a} = a_c k / (k - 1)$.
Following the notation in HIR, we define $\beta \equiv 1 - 1/\sigma$. Then, total revenue of a firm
with productivity $\varphi$ can be written as

$$r(\varphi) = (1 + \Pi r^{1-\sigma})^{1-\beta} PX^{1-\beta}(\varphi \bar{a})^{\beta} h^{\beta \gamma}$$

where $\Pi$ is an indicator function taking value 1 if the firm decides to export and zero
otherwise.

Wages are determined through strategic bargaining between the firm and workers,
after the firm has paid all the costs. HIR show that the outcome is that the firm retains
a fraction of revenues equal to the Shapley value, $1/(1+\beta \gamma)$, and pays the rest to the
workers. Thus, the profit maximization problem of the firm is:

$$\pi(\varphi) = \max_{n,a_c^*,1} \left\{ \frac{r(\varphi)}{1+\beta \gamma} - bn - \frac{ca_c^\delta}{\delta} - f - \Pi f_x \right\},$$

and the first-order conditions for $n$ and $a_c$ are

$$\frac{\beta \gamma}{1+\beta \gamma} r(\varphi) = bn(\varphi)$$
$$\frac{\beta(1-\gamma k)}{1+\beta \gamma} r(\varphi) = ba_c(\varphi)^\delta.$$

Inspection reveals immediately that firms with higher revenue sample more workers
(higher $n$) and screen more intensively (higher $a_c$). Assuming $\delta > k$ also ensures that
firms with higher revenue hire more workers.

Substituting the first-order conditions for $n$ and $a_c$ into the profit function yields
$\pi(\varphi) = \Gamma r(\varphi) f - \Pi f_x$, with $\Gamma \equiv 1 - \beta \gamma - (1 - \gamma k) \beta / \delta$. Since revenues are increasing in
productivity, the fixed costs implies that firms with $\varphi < \varphi^*$ exit (where $\pi_{1=0}(\varphi^*) = f$)
and firms with $\varphi > \varphi_x^*$ export (where $\pi_{1=0}(\varphi_x^*) = \pi_{1=1}(\varphi_x^*)$). Moreover, the relative
revenue of any two firms only depends on their relative productivity and export status:

$$r(\varphi) / r(\varphi^*) = (1 + \Pi r^{1-\sigma})^{(1-\beta)/\Gamma} (\varphi/\varphi^*)^{\beta/\Gamma}.$$ 

Combining these results, we find an expression for ex-ante expected profits, $E[\pi]$, which turns out to be identical to the one in the
previous section (equation 10) after the redefinition of the parameter $\zeta = \beta/\Gamma$ (instead of $\sigma - 1$). The ratio of the cutoffs is now:

$$\rho = \frac{\varphi^*}{\varphi_x^*} = \left(\frac{f}{f_x}\right)^{1/\zeta} \left[(1 + \tau^{1-\sigma})^{(1-\beta)/\beta} - 1\right]^{1/\zeta}.$$  

which is still increasing in $\zeta$.

The equilibrium $v$ depends on $\zeta$, $f$ and $\rho$ as implied by equation (14) and, in particular, it is increasing in $\zeta$. The difference, however, is that $\zeta$ corresponds now to a combination of more parameters, $\zeta = \left[\beta^{-1} - \gamma - (1 - \gamma k)/\delta\right]^{-1}$, so that in this extended version of the model there are more determinants of $v$. In particular, through their impact on $\zeta$, an increase in $\gamma$ or a fall in $k$ and $\delta$ leads firms to draw from more dispersed distributions. These results are intuitive. As already discussed, more risk taking is optimal for the firm when profits are more convex in productivity. In the simpler version of the model, convexity only depends on $\sigma$. Now, instead, the profit function is more convex also when there are weaker diminishing returns (high $\gamma$) and when screening - which is disproportionately beneficial to more productive firms - is more effective, i.e., when worker ability is more dispersed and the screening cost not too elastic.

**Proposition 3** The dispersion of firm productivity, as measured by the variance of the log of $\varphi$, is larger in sectors with more ability dispersion and weaker decreasing returns to scale.

What are the implications for wages? Using the definition of wages as a share of revenue per hired worker yields:

$$w(\varphi) \equiv \frac{\beta \gamma}{1 + \beta \gamma} r(\varphi) h(\varphi) = b \left[\frac{a_c(\varphi)}{a_{\min}}\right]^k.$$  

Since $a_c(\varphi)$ is increasing in productivity, more productive firms pay higher wages. Due to the complementarity in production between average worker ability and productivity, more productive firms have a stronger incentive to be more selective, hire workers with higher ability and pay them higher wages. Moreover, since wages are proportional to revenue, which jumps at the export cutoff $\varphi = \varphi_x^*$, the model implies an exporter wage
premium. More precisely, the wage paid by firms with productivity \( \varphi \) can be written as

\[
w(\varphi) = \left(1 + \Pi r^{1-\sigma} \right)^{\frac{k(1-\beta)}{\sigma r}} \varphi^{\frac{\beta k}{\sigma r}} w(\varphi^r).
\]

Finally, since employment, \( h(\varphi) \), is also a power function of productivity, the wages of workers employed by domestic firms and exporters are Pareto distributed with shape parameter:

\[
1 + \delta[(v\varsigma)^{-1} - 1]/k,
\]

which is decreasing in \( v \). Thus, heterogeneity in productivity maps into wage dispersion. This allows us to state the following proposition on the impact of trade on wage inequality.

**Proposition 4** More openness raises unambiguously sectoral wage dispersion among workers employed by domestic firms and among workers employed by exporters. Conditional on not changing export status, more openness increases wage inequality between workers employed by any pair of firms with different productivity.

Before concluding, it is important to highlight the qualitative and quantitative differences between our result and HIR. In HIR and some other existing models, trade affects wage dispersion through the exporter wage premium. The sign of the effect then depends on the fraction of exporters. As long as exporters are a minority, trade increases wage dispersion by raising the share of firms paying high wages. Once exporters are a majority, instead, trade decreases wage dispersion by pushing low-wage domestic firms to exit and making the surviving firms more equal. Thus, the overall effect of trade on inequality is inverted-U shaped. This effect is present also in our model. But there is now another, potentially more powerful, force: by making all firms more unequal, trade is changing the slope of the entire wage schedule. This second effect, which is absent in HIR, implies that trade now increases wage inequality within exporters, within nonexporters, and also between the two groups of firms. It follows that, as stated in Proposition 4, openness raises unambiguously some measures of wage inequality. Other measures of inequality, such as the Gini coefficient, will instead depend on the combination of the exporter wage premium, as in HIR, and the steeper wage schedule. For those measures,
depending on which effect dominates, the effect of trade on inequality may or may not be ambiguous.

6 Conclusions

In this paper, we made several contributions to the literature. First, we have taken a first step at uncovering some overlooked facts regarding how the distribution of firms varies across sectors and over time. We have found that the extent of heterogeneity, measured by the standard deviation of log sales, changes systematically with industry characteristics and has increased significantly over time. Second, we have proposed one possible explanation, based on the idea that firms can choose the risk of their random productivity draw at the entry stage. The model formalizes the hypothesis that firms can choose between larger and riskier projects with high expected payoffs, and smaller but safer projects with lower expected returns. Third, we have found that export opportunities, by reallocating profits to the most productive firms, increase the return to risk. Finally, we have explored the implication for wage inequality and found a new channel through which trade liberalization can affect the entire wage distribution and increase its dispersion: export opportunities induce more risk taking and this translates into a higher equilibrium heterogeneity both in productivity and wages. As we discussed, this mechanism differs in important respects from those already emphasized in existing models.

In many ways, however, this paper raises more questions than answers. Our theory explores only one out the many forces shaping the equilibrium distribution of firms. For example, to focus on one mechanism and preserve tractability, we left firm dynamics and innovation by incumbent firms out of the analysis. Within our theory, we also restricted the attention to positive implications. Yet, the model suggests interesting normative questions: do firms take too much or too little risk, especially if workers are risk averse and insurance markets are imperfect? Does international trade introduce new externalities in the technology choice at the entry stage? Finally, our first look at the data is just a scratch on the surface. Much remains to be done to document extensively how firm heterogeneity varies across sectors and time, for instance using alternative measures including estimates of firm-level productivity, and how it affects
wage inequality. Although we found that more dispersion is associated to a higher export share, causality remains to be established. In our theory, export opportunity leads to firm heterogeneity, but it is also true that more dispersion in productivity increases the fraction of exporters. Hence, by exploiting exogenous sources of variation, it would be interesting to test the relative strength of both arrows of causality. In conclusion, we hope that the last contribution of this paper will be to stimulate more research on these questions.

References


