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## Shared Knowledge\*

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#### Abstract

Agents use their knowledge on the history of the economy in order to choose what is the optimal action to take at any given moment of time, but each individual observes history with some noise. This paper shows that the amount of information available on the past evolution of the economy is an endogenous variable, and that this leads to overconcentration of the investment, which can be interpreted as underinvestment in research.

It presents a model in which agents have to invest at each period in one of K sectors, each of them paying an exogenous return that follows a well defined stochastic path. At any moment of time each agent receives an unbiased noisy signal on the payoff of each sector. The signals differ across agents, but all of them have the same variance, which depends on the aggregate investment in that particular sector (so that if almost everybody invests in it the perceptions of everybody will be very accurate, but if almost nobody does the perceptions of everybody will be very noisy). The degree of hetereogeneity across agents is then an endogenous variable, evolving across time determining, and being determined by, the amount of information disclosed.

As long as both the level of social interaction and the underlying precision of the observations are relatively large agents behave in a very precise way. This behavior is unmodified for a huge range of informational parameters, and it is characterized by an excessive concentration of the investment in a few sectors. Additionally the model shows that generalized improvements in the quality of the information that each agent gets may lead to a worse outcome for all the agents due to the overconcentration of the investment that this produces.

### 1 Introduction

Economic agents have to make predictions. They have to allocate scarce resources among different possible uses, so they would like to know the payoffs of these uses.

In making predictions they will use two different sets of information:

- 1. Their knowledge of the structure of the economy. In particular of its links across time, the way in which what happened in the past determines what will happen in the future.
- 2. Their knowledge of what actually happened in the past. As long as temporal links do exist, the knowledge of the past evolution of the economy is a crucial piece of information.

In this paper I am going to argue that the amount of available information on the past evolution of the economy is *not* an exogenous variable, but an endogenous one. We will see that the information disclosed to the agents at any moment of time is determined by the heterogeneity of their beliefs, a variable that will be modeled as endogenous to the economy. This will allow us to make strong predictions on the collective behavior of the agents. The following example intends to clarify what this paper is about.

# 1.1 The endogenous accuracy of the perceptions of the past.

In 1972 few, if any, companies were making research in energy-efficient bulbs. Everybody had a prior on what return such an investment would pay, and the commonly shared view considered it quite a foolish investment.

Nowadays all the major firms in the sector are producing new and every time more efficient lamps. This is so even if the price of energy is not that much different, in real terms, today than what it was before the oil crisis. This is an indication that 25 years ago investment in energy efficient products would have been as wise as it is today. Nevertheless, the recession of the seventies was necessary in order for the companies to learn it.

The reason is that the priors that everybody had were not being tested. This kept the priors unchanged, which prevented them to be tested, et cetera...

The investment in research on low consumption bulbs had a very good rate of return, but this particular piece of information lied outside the information set of the agents. The rate of return that such an investment would have had during 1971 was, in 1972, part of the past evolution of the economy, but the information that the agents had on it was very inaccurate.

The interesting point is that this information was very inaccurate due to endogenous reasons. We can speculate, for a moment, that if a group of crazy outliers made the investment in 1971, assuming this investment was successful, and this success was publicized, then everybody would have tested their priors. In those circumstances most agents would have placed investments in such technologies before the '73 shock hit; but, you need the outliers, their success and its publicity.

At this stage it is probably clear that we are going to talk a lot about informational externalities. The hypothetical outliers, by engaging in an apparently foolish investment, would have produced a big positive externality on the whole of the economy. Had they invested, and had their actions been observed, they would have greatly enriched the information available to everybody else.

So in this context, agents produce informational externalities, but the structure of these externalities differs from most papers related to the topic. Note that I am assuming that agents not only observe the actions that other people took, but also the payoffs that these actions generated. In this respect the paper departs from the body of literature on the topic.

# 1.2 Externalities on the second moments of informational variables

We are going to assume that at any given moment of time, everybody gets an unbiased signal on the value of each of the relevant variables. The actions that agents take affect the variance of these signals, their accuracy, but not their expected value. The summation of all the signals will always reveal the truth.

What I am saying is that in 1972 every bulb manufacturer was receiving a signal on the value of investment in energy efficient technology, and that the expected value of each of these signals said the truth. Nevertheless the accuracy of these signals was very low, and so they scarcely amounted for information.

Imagine that there actually was a lonely outlier and that the perceived rates of return on an investment in efficient bulbs is known for everybody to be very volatile. This is because there is a lot of idiosyncratic noise, or simply because nobody can ever be 100% certain that what he perceives that happens to others is what really happened to them. Well, our outlier made the investment and it was a success. Even in these circumstances the bulb manufacturers would not have tested their priors substantially. They would have attributed the outlier's perceived success to just a lot of luck, or simply to inaccurate information. The high volatility of the perceived return with respect to the well established prior would have made the value of the first practically nil.

On the other hand, the priors would have been tested if instead of a single outlier there would have been a group of them. Lots of successful outliers cannot be lucky, they must be right.

The presence of externalities that affect the second moments of the informational variables is sufficient to generate herds of people not investing in energy-efficient technology. It is not necessary to assume that agents do not observe the outcomes of the actions taken by others: a noisy perception of the outcomes is enough to generate a very interesting collective behavior.

Typically informational externalities are modeled as affecting directly the first moment of the random variables that define an agent's view of the world. That is, when an agent takes an action, he changes the beliefs that other agents have by affecting the expected value that they assign to the different random variables which they face. The most clear example of this is the literature on herding (Banerjee[2] and Bikchandani et al.[3] for instance).

There agents observe the actions taken by other agents, but they do not observe their outcome. Some agents have private information (a noisy signal) on which is the best action to take, otherwise they start with flat priors on them. If you observe that most agents are taking a given action, not knowing how happy they are with their decisions, you would think that it is very likely that they are right even if you have a signal that says otherwise. You would then 'herd', do as they did, and by doing so you would be inducing more and more people to join the herd.

The key of this 'they might know better' type of herding is that agents do receive biased information from other agents, because they are unable to observe the outcome of other agents' actions. The summation of the signals that they receive from other people would not reveal the truth. If they were capable of knowing the satisfaction that other agents get from their actions, the whole construction would fall and herding would disappear. There would be fast, optimal, learning.

The key assumption is the unobservability of other people's outcomes, because the biased information that this induces. Not being an innocuous assumption, we should examine whether it is far away from reality, a task which I will not attempt to do. Nevertheless, it seems to me that, at least in what respects to macroeconomics, the value of such assumption is dubious. After all, firms make their books public, and any interested investor knows which firms are having abnormally high benefits and which ones are incurring in shameful losses.

In this paper I present a model in which agents do observe the outcomes, but with some noise. The information that they interchange will be unbiased. In spite of it, their collective behavior could rightly be categorized as 'herding'.

### 1.3 Links across time

A few pages ago I made reference to two sets of information that the agents use: information on the structure of the economy and information on its past behavior. So far I have talked only about the second, as it is what the model intends to explain: but the first is not less important and some of the assumptions that I make on it are not completely innocuous.

We will assume that the return that different actions pay is independent of the activity of the agents, completely exogenous. This intends to bring both simplicity and clarity: simplicity because otherwise the model would be orders of magnitude more complex, and clarity because we are interested in the study informational effects, not physical ones. In our context the actions that agents take affect the payoffs that they will get in the future because they affect their view of the world, not the world in itself.

Additionally we will build on the hypothesis that agents know the structure of the economy. They will not need to expend time and resources in learning how what actually happened in the past affects the events that will happen in the future. All their effort will be concentrated in learning what happened in the past. I may agree that this is quite a courageous assumption, but it greatly simplifies the model, allowing me to write the paper, and helps to focus it in its intended object of study: the endogeneity of perceptions.

There is a last assumption that is worth mention here which refers to the particular form of the links across time that agents will face. I will assume that the rates of return in the different sectors are random variables, the realizations of well defined stationary stochastic processes with a positive autocorrelation. Additionally, and this is not so important, this processes will be assumed to be independent among them.

The past rate of return in any sector will have predictive power on what will happen with this rate of return in the future, because the autocorrelation is different from zero. Additionally, the fact that the processes are supposed to be stationary, mean reverting, implies that agents will always expect the rates of return to move towards their long run average. After a period in which the disclosure of information on this variable was minimal the priors will converge to the long run mean of the variable. This turns out to be crucial for the conclusions that I will present. Even if the model itself allows for the presence of non stationary processes, its qualitative results would differ. To understand its solution in such a case would require further work.

### 1.4 Heterogeneity across agents

The amount of information disclosed on the return of a sector depends on the number of agents that are investing in it. If all the agents of the economy share the same opinions, they will all invest in the same sector because they will all think that is the one promising the highest payoff. The direct con-

sequence of it is that no new information gets disclosed on any of the other sectors. In our bulb manufacturing example we started by saying that all the agents had, in 1972, a *commonly* shared prior.

There is an intimate relationship between heterogeneity and information disclosure. If everybody invests in the same sector, the information disclosed on that sector will be very accurate. In other words, everybody will observe approximately the true value of the variable, and so all the agents will share very similar views of the world. In the same manner, if no information is disclosed, the priors of all the agents will converge to the long run mean of the variable, producing very homogeneous beliefs.

If beliefs across agents are heterogeneous, the aggregate investment will be diversified across the different sectors: consequently, there will be information disclosure in all the sectors. If they are very homogeneous, the amount of information disclosed will be high in one sector and minimal in the rest.

This dynamic relationship between information disclosure and homogeneity of beliefs is in the core of the explanation of the economy's behavior.

In the next section I present the model that we will use as the thinking tool in the rest of the paper. After that, the following three sections will pave the road towards the solution of the model. Section 3 studies how the perceptions differ across agents, the evolution of the heterogeneity of the perceptions; section 4 focuses on how the distribution of investment will be determined, and section 5 studies the dynamics of the model.

Section 6 will finally solve the model using numerical simulations. It may seem surprising now, but in section 7 we will use the model to help us understand things as diverse as why Japan's MITI is so successful, why the mileage of the cars was so low in 1972, and why it took so long for the Soviet Union to break down.

We will conclude by summarizing the findings and establishing the conclusions that they imply.

# 2 Set up of the Model

There is a continuum of risk neutral agents in the interval (0,1). Each agent has an unit of investment good available at each period. They have to invest it in one of K investment opportunities. At time t sector k ( $k = \{1, 2, ..., K\}$ ) pays an exogenous amount  $R_t^k$  per unit of investment. We do not need to deal with intertemporal decision problems, because there is no accumulation (the good is perishable) and the informational structure rules out experimentation (see bellow).

The rates of return of each sector follow independent AR(1) processes, all of them with the same autocorrelation and variance of the perturbations:

$$R_{t}^{k} = \rho R_{t-1}^{k} + e_{t}^{k} \quad where \quad e_{t}^{k} \quad N(0, \sigma)$$
 (1)

We will call  $x_t^k$  to the amount of aggregate investment in sector k at time t.

# 2.1 Structure of the information; Externalities.

Before playing at time t, the information set of an individual consists of the perception that he has on the performance of each sector at all previous times. He has been collecting information since the beginning of time. This information consists in what he believes that each sector paid at each previous time, his perceptions coming in the form of signals. With all this information they establish priors on what is going to be the return on each sector at t.

The information that an agent receives (the signals) is independent of what he chooses to do. This rules out experimentation. Agents have no incentive to sacrifice some income (not maximizing expected income) in order to get information. By assuming this we will substantially simplify the model and make it tractable.

### 2.1.1 Perceptions of the past.

The perception that an individual has of the past does not depend exclusively on his own experience. Individuals observe how happy other individuals are with the actions they took, and communicate with each other. In all economies information on past events flows from one individual to another. The perceptions that any agent has on the state of the economy depends not only on what he observes, but also in what others observe and how this information is transmitted to him. In other words the new information that an agent receives in a given period of time depends on:

- 1. The information that he receives because he invests in some place and gets a return from it.
- 2. The information that other agents receive because they too are investing in some sector.
- 3. The way that they interchange this information.

To model realistically all the micro characteristics of a process of information diffusion is a task well beyond the reach of this paper. Nevertheless, I will try to capture what I believe are the relevant characteristics of such a process by using a, hopefully reasonable, reduced form. First I will list the characteristics of the information diffusion process that I would like to capture. Later I will propose a reduced form that includes them.

1. An agent's perception must have idiosyncratic characteristics. That is, it must be different for different agents. This is so because the rate

of return that each agent observes will have idiosyncratic components (people are subject to idiosyncratic shocks in the rates of return). Additionally, the information that any individual gets from others will also depend on who he is because, for instance, who are the agents that exchange information with a given agent depends on his location.

2. I will assume that the perception that agents have is always unbiased, so that if they were able to pool their information they will get the truth out of it. This is not necessarily the case in the real world, but by doing this I am making things harder to the model. I am taking away the most obvious effect of informational externalities. We will see that there is herding even in this case.

Additionally, as long as we believe that agents interchange information on the *outcomes* of their actions this seems to be the natural assumption; we are ruling out the 'they-might-know-better' effect, and doing so we do not have 'informational cascades'.

- 3. One would think that the precision in the perception that individuals have on what happened during the previous period depends on the number of individuals investing in any given sector. If nobody is investing in a sector, agents are going to receive no new information on the rate of return in that sector during that period. On the contrary, if all the agents are investing in a sector we should assume that the level of precision in the agents' perceptions on what happens in that sector is relatively high, because there is a big pool of information before they start interchanging it.
- 4. It is difficult to know how much the aggregate investment level in a sector affects the precision of the observations.

If an agent gets his information by sampling from the pool of agents, we would have a linear relationship, so that an increase of a one percent in the number of agents investing in k increases the precision of the perceptions of the sector k's return by a one percent.

But that is not the only possible world. In a 'yellow journalism' society, the bigger the aggregate investment in a given sector, the bigger the attention that this sector receives from the news media, and so the more information on this sector that is reported, increasing exponentially the precision of the information that everybody gets (there is a bigger pool of information, and it is reported much more extensively). An increase of a one per cent in the aggregate investment produces an increase in the precision of more than a one per cent.

Alternatively, the media could be making 'structural' research, focusing on the processes themselves, and not relaying that much in the experiences of the agents. In such a case, an increase of a one percent in the number of agents that engage in a certain activity would increase the level of precision in less than a one percent.

The moral is that on this respect we should be quite open. Not knowing how the real world is, we should not burn our bridges by doing a too restrictive modelization.

In order to capture all the points expressed above I will assume that after playing at any time t each agent receives K signals, one for each sector. Each signal is the summation of the true rate of return in the corresponding sector plus some noise that is independent across sectors, individuals and time.

Call this signal  $S_i^k(i)$ , then:

$$S_t^k(i) = R_t^k(i) + \frac{1}{\sqrt{P_t^k}} \epsilon_t^k(i)$$
 (2)

Where  $\epsilon_t^k(i)$  comes from a standard normal independent across sectors, individuals and time.  $\frac{1}{P_t^k}$  is the variance of the noise:  $P_t^k$  the precision of the signals.

The elasticity of the precision of the signals with respect to the aggregate investment in a sector is an exogenous constant, that depends on the information diffusion process of the society; in the 'sampling' case would be one, in a 'yellow journalism' society it would be bigger than one, and in the 'structural information' case smaller than one:

$$P_t^k = A \left(x_t^k\right)^p \tag{3}$$

A high p represents a high level of social interaction, a situation where there are big informational externalities. In such a society an agent's decision to invest in k and not in j, has important consequences on the accuracy of the information that all the agents get on both sectors.

If the elasticity of the precision with respect to the aggregate investment were zero (p=0) there would be no informational externality. The information would be independent of the evolution of the economy, and we would have an economy with no social interaction.

All values of p between 0 and infinity are admissible, the higher the value the bigger the level of social interaction in the economy.

The parameter A represents the level of 'underlying precision'. The maximum possible level of precision, attainable when all the agents are investing in the same sector.

Given that the amount of investment in any sector is not bigger than one, the precision is always decreasing in p. This implies that higher levels of social interaction have a direct effect in lowering the precision. In order to

compensate for this effect, when we make comparative statics we will have to move not only p, but also the level of 'underlying precision'.

### 2.2 Generating a prior about the future.

Each agent builds his belief about the future based on the information available to him: his perception of the history of the economy. This is, the collection of all the signals that he has received.

The signals are normally distributed, as a consequence his priors will also be normally distributed. In order to see this let's go back to the first time he plays.

At the origin of time he had no information other than the knowledge of the stochastic process driving the returns (equation 1), so his prior on the return in any sector would be normally distributed with mean zero and variance  $\frac{\sigma}{1-\rho^2}$ . After receiving a signal on what happened at t=1, he would update his prior on the realization of the returns at t=1, and after that, and based in his knowledge of equation 1, he will establish a prior on the value of the returns at t=2.

We will now see that if the prior on the realization of the returns at any time t that an agent has before playing at t is normally distributed, then the prior on the returns at t+1 after playing at t will also be normally distributed. This implies that all the priors are going to be normally distributed at all times, because at t=1 they already were so.

Assume that at t, before investing, individual i believes that  $R_t^k$  is distributed from a normal with a mean  $\mu_t^k(i)$  and a variance  $V_t^k$ :

$$R_t^k(i) \sim N\left(\mu_t^k(i), V_t^k\right)$$
 (4)

Independently of what he chooses to do, after playing he gets an unbiased signal on the rate of return in each sector with a (known) precision  $P_t^k$ .

When the prior is updated (using Bayes law) the posterior on  $R_t^k$  is also a normal distribution

$$R_t^{k+}(i) \sim N\left(\theta_t^k \mu_t^k(i) + \left(1 - \theta_t^k\right) S_t^k(i) , \theta_t^k V_t^k\right) \tag{5}$$

where:

$$\theta_t^k = \frac{1}{1 + P_t^k V_t^k} \tag{6}$$

The effect of a change in the level of social interaction  $\left(\frac{\partial P_t^k}{\partial p}\right)$  is an exercise of comparative statics, *not* an externality.

<sup>&</sup>lt;sup>1</sup>The effect of the informational externality is on the change in the precission derived by a change in the number of agents investing:  $\frac{\partial P_t^k}{\partial x_i^k}$ .

is the weight that agents assign to the prior when updating their belief. If  $\theta_t^k$  is close to one (because the signal is very noisy or the prior very accurate) the agents put all the weight on the priors. If it is close to zero, they put all the weight on the signal.

Now let's go back to equation 1: from the point of view of i before playing at t+1  $R_t^k$  is normally distributed, and he knows that  $\epsilon_t^k$  is so too. Thus given his information he will perceive  $R_{t+1}^k$  as a normally distributed random variable.

$$R_{t+1}^k(i) \sim N\left(\mu_{t+1}^k(i), V_{t+1}^k\right) \tag{7}$$

with mean:

$$\mu_{t+1}^{k}(i) = \rho \left( \theta_t^k \, \mu_t^k(i) + \left( 1 - \theta_t^k \right) \, S_t^k(i) \right) \tag{8}$$

and variance:

$$V_{t+1}^{k} = \rho^{2} \frac{V_{t}^{k}}{1 + P_{t}^{k} V_{t}^{k}} + \sigma = \rho^{2} \theta_{t}^{k} V_{t}^{k} + \sigma$$
(9)

The priors change across agents because they depend on the whole history of signals that the agents receive, and these signals are different for different agents. They have different perceptions of the past, and this induces different beliefs about the future.

On the other hand the variance of the beliefs depends only in the past variance of the signals, something that the agents know and that is common for all of them.  $P_t^k$  and  $V_t^k$  are common to all the agents. The priors differ only in their mean. Note that the variance of the priors is bounded by above by  $\frac{\sigma}{1-\rho^2}$  (unconditional variance) and by bellow by  $\sigma$  (because always there is uncertainty on future events, even if you know the past).

# 3 Distribution of beliefs across agents

We have seen how the beliefs are generated, and that they may differ across agents; now we will see how much do they differ. We will see that the expected return is always distributed as a normal, and we will determine how this distribution does evolve. The strategy is equal that in the previous section, first we show that at the origin of time the expected return is normally distributed across agents; then that if at any moment of time the distribution is a normal, it will always be so afterwards. Doing this we will also identify the stochastic differential equations that drive the beliefs of the agents.

Let's go back again to the first time that they played. They had common flat priors, and then they updated them using their signals, as a consequence

the prior that an individual i has on the value of the return in sector k at t=2 is a normal with mean:

$$\mu_2^k(i) = \rho \ (1 - \theta_1^k) \ S_1^k(i) \tag{10}$$

Across agents  $S_1^k(i)$  is a normal with mean  $R_1^k$  (what actually happened) and variance  $\frac{1}{P_1^k}$ . So, across agents  $\mu_2^k(i)$  will also be a normal, with mean

$$\rho \left(1 - \theta_1^k\right) R_1^k \tag{11}$$

and variance:

$$\rho^2 \left(1 - \theta_1^k\right)^2 \frac{1}{P_1^k} \tag{12}$$

Now let's assume then that at some time t (that is, immediately before playing at t) the expected return was normally distributed across agents:

$$\mu_t^k(i) \sim N\left(\bar{\mu}_t^k, M_t^k\right)$$
 (13)

Where  $\bar{\mu}_t^k$  is the average expectation on  $R_t^k$ , the 'consensus' expected return, and  $M_t^k$  measures the degree of heterogeneity in the beliefs of  $R_t^k$ , the dispersion in these beliefs.

The expected return at t+1 for agent i is

$$\mu_{t+1}^{k}(i) = \rho \left( \theta_t^k \, \mu_t^k(i) + \left( 1 - \theta_t^k \right) \, S_t^k(i) \right) \tag{14}$$

Given that  $\mu_t^k(i)$  and  $S_t^k(i)$  are independently distributed normals (across agents), then  $\mu_{t+1}^k(i)$  will also be a normal,

$$\mu_{t+1}^k(i) \sim N\left(\bar{\mu}_{t+1}^k, M_{t+1}^k\right)$$

where its mean is

$$\bar{\mu}_{t+1}^{k} = \rho \ \left(\theta_{t}^{k} \,\bar{\mu}_{t}^{k} + \left(1 - \theta_{t}^{k}\right) \,R_{t}^{k}\right) \tag{15}$$

and its variance

$$M_{t+1}^{k} = \rho^{2} \theta_{t}^{k} (\theta_{t}^{k} M_{t}^{k} + (1 - \theta_{t}^{k}) V_{t}^{k})$$
(16)

Thus across agents the expected return is always distributed as a normal, because we know that this was already the case at time t = 2.

Equations 15 and 16 (together with 9) are probably the most important for understanding the solution to the model, so later we will return to then for a more careful study, but so far let's remark some of their implications.

When the signals are much more accurate than the priors so that the agents weigh the signal much more than the prior ( $\theta_t^k$  is close to zero), the

'consensus' belief tracks the truth very well.  $(\bar{\mu}_{t+1}^k \simeq \rho R_t^k = E(R_{t+1}^k \mid R_t^k))$  and the dispersion of the beliefs is minimal, because everybody trusts what it 'sees', and everybody 'sees' more or less the same signal.

If the opposite happens (the priors are much more accurate than the signals) everybody moves its expected value towards zero (the long run mean), and so the range of beliefs decreases. Consequently both the average belief and the dispersion move towards zero independently of the realizations of  $R_t^k$ .

# 4 Aggregate Investment

Each individual has zero weight, so they are unable to change the variance of the signals by investing in one sector or another. This makes the individual decision problem quite trivial. They are risk neutral and they have no incentive to experiment, so they will invest everything in the sector from where they expect to get the highest return.

So the aggregate investment in sector k ( $x_t^k$ ) will be the number of agents for whom:

$$\mu_t^k(i) > \mu_t^j(i) \quad ; \ \forall j \neq k \tag{17}$$

Given the distribution of the beliefs across agents and the fact that there is a continuum of agents,  $x_t^k$  will be exactly the probability that the previous condition holds.

 $\mu_t^k(i)$  depends on the history of signals that i received and the true rate of return in sector k at each different moment, but all this is independent across sectors, and the signals are also independent across agents. Thus  $\mu_t^k(i)$  and  $\mu_t^j(i)$  are independent random variables, both of them normally distributed.

The probability that 17 holds ( $x_t^k$ ) is then the cumulative distribution function of a Standard multivariate normal with K-1 variables<sup>2</sup>. Calling the variables  $j = \{1, 2, ..., k-1, k+1, ...K\}$ , the integration limit of variable j is

$$\frac{\bar{\mu}_t^k - \bar{\mu}_t^j}{\sqrt{M_t^k + M_t^j}} \tag{18}$$

and the correlation between variables j and h:

$$\frac{M_t^k}{\sqrt{M_t^k + M_t^j}} \sqrt{M_t^k + M_t^h} \tag{19}$$

<sup>&</sup>lt;sup>2</sup>See appendix A

If all the agents share very similar beliefs with respect to all the sectors (so that  $M_t^k$  is close to zero for all k), then everybody does the same thing; everybody invests in the sector that offers the highest expected payoff<sup>3</sup>. This is so unless their beliefs are identical across sectors ( $\bar{\mu}_t^k = \bar{\mu}_t \quad \forall k$ ), in which case the aggregate investment will also be identical across sectors.

On the other hand, if the beliefs are not homogeneous across agents, the distribution of investment will follow an extremely non linear function of their averages and dispersions, allowing for diversification.

## 5 Dynamics

The state of nature is defined by the real aggregate shocks on the return of the sectors. These rates of return are the only exogenous variable. The noise in the signals is whipped out by the law of the large numbers, its only purpose being to generate heterogeneity in the beliefs of the agents and, by doing so, the possibility of diversification of the investment. It has an effect through its variance, because it affects the weight that agents put on their signals, and in doing so, the dispersion of the beliefs.

The dynamic structure of the model is the following:

Given the averages and dispersion of the beliefs referring to the returns at t, the investment in each sector is determined:

$$x_t^k = \Phi(\{\bar{\mu}_t^j, M_t^j\} \ \forall j) \tag{20}$$

This level of investment induces the precision with which each agent will observe the returns at t:

$$P_t^k = A \left(x_t^k\right)^p \tag{21}$$

The precision of the signals and the variance of the priors at t determines the share of the prior in the update for each sector, how much do they trust their priors:

$$\theta_t^k = \frac{1}{1 + P_t^k V_t^k} \tag{22}$$

With it we can calculate the variance of the priors the following period:

<sup>&</sup>lt;sup>3</sup>In such a case the denominator of 18 is always zero, and given that it is sufficient to have a single integration limit close to minus infinity to have a probability close to zero; then the probability will be almost zero in all the sectors where the numerator is not always positive, and in the sector where it is always positive, it will be one.

$$V_{t+1}^k = \rho^2 \,\theta_t^k V_t^k + \dot{\sigma} \tag{23}$$

The only exogenous variable is the rate of return at t, and this enters exclusively through the average belief at t+1

$$\bar{\mu}_{t+1}^{k} = \rho \ \left(\theta_{t}^{k} \, \bar{\mu}_{t}^{k} + \left(1 - \theta_{t}^{k}\right) \, R_{t}^{k}\right) \tag{24}$$

Finally, the variance of the priors and the past value of the dispersion will generate the dispersion of beliefs at t+1:

$$M_{t+1}^{k} = \rho^{2} \theta_{t}^{k} \left( \theta_{t}^{k} M_{t}^{k} + (1 - \theta_{t}^{k}) V_{t}^{k} \right)$$
 (25)

The function  $\Phi$  has no closed form (there is no closed form for the CDF of a multivariate normal); consequently, it is not possible to find an analytical solution to the model. So we have to run simulations and see how the results change when the level of 'social interaction' p and the 'underlying precision' A change. Before doing so it is convenient to have a careful look at the previous equations and make an exercise that will help us understand the results.

Let's take equations 22, 23 and 25; additionally let's assume that the precision of the observations is fixed. All this accounts for a very non linear system of differential equations. The solutions of M as a function of the level of precision (assuming  $\rho = 0.75$  and  $\sigma = 1$ ) appear in figure 1.

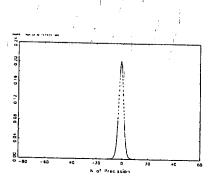


Figure 1: Solution for M of equations 22, 23 and 25; assuming that the precision of the observations is fixed.

There it is clear that when the precision of the observations is either very small or very large, the dispersion of the beliefs becomes zero rather fast. We have already seen the intuition for this.

- If the precision of the observations is quite good, all the weight of the posteriors falls on the signals (the minimum variance of the priors is σ, while the signals might have a variance as low as ½). The very fact that the precision is high implies that all the signals will be quite similar, their value very close to what actually happened. Thus agents will have very similar posteriors.
- If the precision is very low, they will put all the weight on the priors (the maximum variance of the priors is  $\frac{\sigma}{1-\rho^2}$  while the variance of the signals is not bounded by above), but by doing so the range of beliefs gets smaller: the heterogeneity decreases, eventually being zero.

So, the agents have very homogeneous beliefs if in each sector the precision is either very high or very low. As we saw, this implies that the investment will be very concentrated in one sector.

The degree of precision in the signals of any sector k depends on the underlying precision A, the level of social interaction p, and the aggregate investment in the sector. Even if the investment is very low (but positive) the precision level can be quite high, provided that either A is very big and/or p very small. Conversely, even if almost everybody is investing in a sector the precision might be very small.

Now let's imagine 3 polar cases:

1. A is quite big and p quite small, so that even if the number of agents investing in a sector is very small, the precision is quite big.

In this case the dispersion of the beliefs will always be almost zero in all the sectors. This implies that almost everybody will invest in the sector that in the immediate past produced the highest payoff.

There will always be a few outliers that make 'mistakes'. They are very useful for the society because they generate signals with quite a high level of precision indicating the evolution of the non-optimal sectors. Thus almost all the members of the society are continuously keeping track of the evolution of the payoffs in all the sectors, and the aggregate investment is always close to one in the sector with the highest expected return.

2. A is quite small and p quite big, so that even if almost everybody were investing in a sector the precision in the signals of that sector would be very small.

This means that they cannot see anything, they will never update their signals significatively. They start with a common prior (zero, the long run distribution of the returns), and they stay with it forever. The dispersion of the beliefs is almost zero, but the investment is distributed evenly across sectors because the expected return is zero in all of them.

3. A and p are such that if the number of agents investing is very small, the precision is quite small (at the left of the peak in figure 1): and if the investment is close to one, quite big (at the right of the peak in figure 1).

Imagine that almost everybody is investing in the right place, the dispersion of beliefs will be nil in all the sectors, so they will keep concentrating the investment in one sector. They are able of keeping a good track of what happens in that sector, but they are unable to get information from the other sectors. The beliefs in all the other sectors will converge to zero (the long run mean) rather fast.

Almost everybody will invest in the chosen sector as long as it is paying above the long run average, even if other sectors are paying more. There is an obvious overconcentration of the investment in one of the sectors, excluding the possibility of learning. Only when the rate of return of the 'herd' is below the long run average will they diversify their investment and begin learning what is the best thing that they can do.

Assume that when they diversify they invest  $\frac{1}{K}$  in each sector. They will learn fast if doing so the precision of the signals is quite big. and again overconcentrate their investment in one of the sectors. On the other hand if when they diversify the precision in each sector is not so high, say that is in the 'peak' of figure 1, the learning process will be much slower, the recession longer.

### 6 Solution

As I said before, the solution of the model comes from the hand of simulations. All the simulations presented are done with parameters  $\rho=.75$  and  $\sigma=1$ , but changing these parameters does not change the qualitative results at all. Due to computational reasons all the simulations are done with K=4, but as it will be clear this will provide us with enough information to adequately discuss how the solutions would be if there were more sectors. The simulations are the product of averaging the results for 25 different histories of 1000 observations each.

Each figure shows the result for an endogenous variable as a function of the logs of A and p. This allows for comparisons when there are wide changes in the informational parameters. In figure 2 the graphs in the top are the 3-dimensional representation, while the figures in the bottom are the contour map of the respective surfaces. The column representation of beliefs production, the average Herfindal Index and the average dispersion of beliefs respectively.

To say that the model is non-linear is an understatement, a glimpse to the simulations is proof enough of it. Things change suddenly for small changes

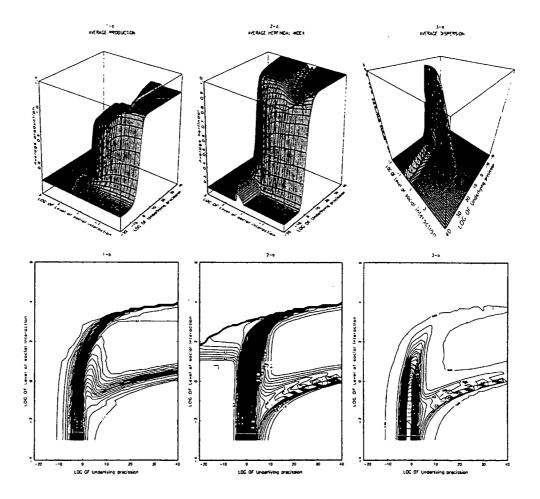


Figure 2: Averages from 25 simulations of 1000 observations each.  $\sigma = 1; \quad \rho = 0.75$ 

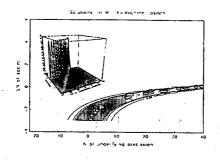
in the parameters and remain steady for very large ranges of them. In order to understand the model we have to take a careful look at the figures and think about them having in mind the arguments given in the previous section.

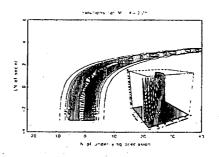
To a first approximation we can divide the space defined by A and p in three regions that correspond with the 3 polar cases that we already saw. Inside these regions changes in the informational parameters do not induce significant changes in the expected return.

Now is when it comes in handy to make some exercises of comparative statics.

# 6.1 Increasing the level of social interaction

The first polar case exposed in the previous section corresponds to the region where the average payoff is maximum (see the first colum of figure 2). In this area the precision of the signals is relatively high even if almost nobody is investing in a sector. This induces very homogeneous beliefs (see the second colum), and thus big concentration of the investment (see the third).





(a)  $x_t^k$  is a very small positive number (machine epsilon)

(b)  $x_t^k = 0.25$ , (diversifivation)

Figure 3: Solution for M of equations 22, 23 and 25; assuming differnt fixed values of  $x_t^k$ 

Figure 3(a) clarifies why is this so. There we represent the solution for M in the system of differential equations that we saw in the previous section, but instead of being a function of the precision (as in figure 1), here is a function of its components (A and p) when the investment is very close to zero<sup>4</sup>. It is clear that this first region corresponds with the area of the space  $A \times p$  where the beliefs are very homogeneous even if a minimal number of

<sup>&</sup>lt;sup>4</sup>Actually 'very close to zero' in this context means the machine epsilon, the smallest

agents is investing in the sector. The variance of the priors is always relatively high, because of the stochastic structure of the returns (see equation 23, the variance of the priors is always bigger than  $\sigma$ ), its precision always much smaller than the precision of the signals (even if an extremely small number of individuals is investing), so in doing the update almost all the weight is given to the signals, to the new information. In this area the agents keep very good track of the ranking of the sectors from best to worse. There is always a very small number of agents making 'mistakes', because the returns are continuously changing and to learn takes time. If the rates of return were constant, eventually everybody would be investing in the 'right' stuff. The noisy structure of the model induces some people to take the wrong decisions, but doing so they generate a big amount of information that the society ,as a whole, uses.

If the level of social interaction were higher, the precision would decrease, both when almost everybody is investing and when almost nobody is doing it: but the decrease will be bigger when x is close to zero:

$$\frac{\partial \frac{A x^p}{A y^p}}{\partial p} = \left(\frac{x}{y}\right)^p \log\left(\frac{x}{y}\right) > 0 \quad \iff \quad x > y$$

So, going back to figure 1, before increasing p the precision is always far away and to the right of the peak. As we go increasing p and the precision decreases, we move towards the peak faster for small values of x. There are no substantial changes as long as the precision is at the right of the peak for small values of x, because the signals are always much more accurate than the priors. The investment keeps concentrated in the right stuff.

Eventually the precision when the number of investors is very small arrives to the peak of figure 1 (or figure 3(a)). Here, if the investment were very concentrated in one sector the heterogeneity of the beliefs in the other sectors would be quite high. However, this implies that the investment is not going to be very concentrated in any sector to start with (the denominators of the integration limits in equation 20 will not be close to zero). So in this range of parameters we should expect smaller concentration levels, more heterogeneity in the beliefs and a decrease in average output, and that is exactly what happens.

The 'peak' in figure 3(a) corresponds with the increase in average heterogeneity (see the second colum of figure 2) and the 'valley' in the average concentration index (see third colum). It also corresponds with the big fall in average output from the 'high' to the 'middle' plateaus. This can be seen more clearly in the contour 'maps', we are talking about the clearly defined lower 'arm' in all of them.

number bigger than zero that the computer recognizes as different from zero. This is a really small number

In this small region most of the agents 'do the right thing', but there is a substantial number of agents investing in sectors that did not do well in the immediate past. Again, if there was no change in the returns, they would eventually learn and concentrate their investment, but the optimal action changes continuously, and this allows for heterogeneity in the beliefs.

If p increases the average output falls into the 'middle' plateau. In this region when almost nobody invests the precision of the observations is at the left of the peak in figure 1 (above it in figure 3(a)), while it is still at the right when almost everybody does it. If the investments were very diversified among the sectors the precision of the signals would be at the right of the peak. This can be seen in figure 3(b), which represents the solution of the already familiar system of differential equations that defines M, this time for an investment level of  $\frac{1}{4}$ .

Imagine that the informational parameters fall inside this region and people have flat priors on what is the best thing to do. They will diversify their investment. Doing so they will get very accurate signals, and in the next period they would concentrate their investment in the more promising sector. From then on, and until things change, they will get very accurate information about what is happening with that sector, but they will be getting no practical information on the evolution of the other sectors. The beliefs get very homogeneous, because all the weight is given to the signals in the 'chosen' sector (and the signals have low variance) and to the prior in the other sectors (and so the priors converge to zero because the payoffs are mean reverting). Homogeneity reinforces the situation, because the investment gets concentrated, and this produces homogeneous beliefs, et cetera.

The individuals will diversify their investment only when the return of the sector that they observe falls bellow the long run average. Then they will learn fast, and again concentrate their signals, so that the situation remains the same, perhaps with the investment concentrated in a different sector, but extremely concentrated nevertheless. So the average Herfindal Index (second colum of figure 2) is very high, while the average dispersion of beliefs is very low (third colum).

The average output (first colum of figure 2) in this region is substantially lower than in the first 'plateau' because the society only keeps track of what happens in one sector. Any of the other sectors could get a series of positive shocks and start paying above the one that carries almost all the investment. Actually this will happen most of the time, but the number of people that observes it is too small for the society as a whole to notice.

If the world were not changing continuously, the information that these outliers generate would be used sooner or later. Little by little more people would move to the sector with the highest payoff. A stationary stochastic world prevents this from happening because:

1. Mean reversion implies that the priors are continuously moved towards the long run mean, and the priors have a huge weight if the precision of

the observations is low. The two effects work in the same direction, inducing the agents to believe that the sector is close to its unconditional expected value and minimizing heterogeneity.

2. The amount of information that the outliers generate is extremely low, so the learning process would be very slow in any case. The probability of the event 'the sector where everybody invest goes back to its mean before agents learn that another sector is paying more is close to one.

It is in this region where we can talk properly of herding. Agents 'follow the herd' because it is paying above the long run average, so they are not doing too badly, and they are unable to see what happens in the rest of the world. This is a conformist society where investment in research is nil. Experimentation could produce higher returns for the economy as a hole, but nobody has the incentives to sacrifice expected income in order for everybody to learn.

Imagine that the level of social interaction increases even further, so that when agents diversify their investment the precision of the observations is small, generating substantially heterogeneous beliefs (i.e., when there is diversification the precision of the observations is in the peak of figure 1). We are now in the area that produces the peak in figure 3(b), and that corresponds with the 'upper arm' in all the contour maps.

Here when the agents concentrate their investment the heterogeneity would be quite small, and so far they would be behaving as in the previous region. But sooner or later the sector will start paying below the long run average, inducing diversification of the investment. Now the learning process will be much slower than it was in the previous region, and the beliefs will be heterogeneous for a longer period.

This accounts for the sharp falls in average production and concentration, and the increase in heterogeneity observable in this area.

The smaller the precision at the diversification stage, the slower the learning process. If we keep increasing p the learning process becomes all too slow.

If at the diversification stage the precision of the signals is very low, the weight in the update falls overwhelmingly on the priors. The agents do not receive new information, and very soon all of them will use as a prior the long run distribution of the rates of return. In this area, beyond the 'upper arm', agents never learn. The beliefs are homogeneous, but the investment is equally distributed among all the sectors because they expect all of them to produce the same return (the numerator of the integration limits is zero).

# 6.2 Increasing the underlying precision of the observations.

One can think of two effects due to an increase in A.

- 1. The standard: people have better information, so they will do better.
- 2. The perverse: everybody has better information, so more people will do the right thing; but doing so the society may be overconcentrating their investment and not learning about the evolution of most sectors.

The most graphic way of seeing the second effect is in figure 2.

There, when A is very small the amount of information that agents receive is nil; the beliefs are very homogeneous because all the weight is put on the priors. If we increase A so that the precision level falls inside the 'peak' of figure 1, people begin to learn. To learn means to make use of the signals, and given that they differ across individuals, there is an increase in the degree of heterogeneity.

If A increases even further, the signals get more precise, and as a consequence the view of the world that agents have gets more homogeneous. So we observe a sharp fall in M.

For relatively small values of p the level of precision is mainly determined by A:

$$\lim_{p \to 0} A x^p = A$$

In this region a decrease in heterogeneity, associated with an increase in A. does not have negative consequences (the first effect dominates). Both if almost nobody invests, or if everybody does it, the increase in precision of the observations is roughly the same. The second effect is very small because the informational externalities are almost non-existent.

On the other hand if the level of social interaction is relatively large, the negative effects of overconcentration are apparent. Say that p and A are such that we are on top of the 'arm rest' in the firstcolum of figure 2. There diversification of the investment produces a relatively high level of heterogeneity in the beliefs (see figure 3(b)). If A increases there will be an increase in the concentration of the investment. Additionally the increase in the information generated in the 'minority' sectors is not as big as the increase generated in the one where most people are investing:

$$\frac{\partial A x^p}{\partial A} = x^p$$

There is a net loss in information, because the number of agents in 'minority' sectors decreases when homogeneity increases. Herds 'begin' to be formed, eventually the economy falls in the second 'plateau' where further increases in A have an effect only if they make the precision in the minority sectors to fall in the 'peak' of figure 3(a), because there the homogeneity of the beliefs increases and 'bad herds' are broken relatively fast.

## 7 Some Examples

As in most macroeconomics papers that end up being mostly theoretical, one of the most difficult moments appears when the model has to be taken from the theoretical Valhalla and brought down to the crude world of reality.

Evidence of the kind of effects on which I have commented is, at most, scant. Nevertheless this should not be too discouraging. On one hand such a simple model does not intend to be a precise description of a very complex reality. On the other there is enough evidence how to allow me to make some points, both normative and positive.

The model gives support to dirigiste industrial policies, such as the ones profusely employed by Japan's MITI. These policies, as much praised by their supporters as criticized by their detractors, are based in the belief that firms, if left to themselves, will not generate an optimal allocation of resources among the different possible investments. The government gives incentives to the existence of firms in a very wide range of sectors, even if most companies would opt otherwise for not being in them. The resulting economy is quite flexible to get in different fields as they open; it discovers new fields of action with more flexibility than it would if it were directed exclusively by market forces. From a strictly orthodox point of view these policies should be rapidly discarded, but few people will dare to doubt the success story that is post war Japan.

Another, more direct, piece of evidence is provided by the oil crisis of the seventies, and the posterior surge in energy-efficient products. Previously to the crisis it would have seemed bizarre to place big investments in order to decrease the appetite for energy of most products. During the crisis it became imperative. In the car industry, for instance, most firms found themselves with an obsolete product portfolio<sup>5</sup>. A new range of energy efficient products was developed. New fields of research were born, ranging from recycling to mere efficiency improvement. After the crisis the price of energy returned to pre-1973 levels, but the new industry did not disappear. If any, it expanded. Nowadays in the Ruhr region there are more workers in 'green' industries than in coal and steel activities. There was probably a market for all of these new activities previously to the recession, but few people noticed it, and the ones who did were not perceived by the rest of the world; a big shock was necessary for most of us to see it.

In a somewhat more esoteric way the model also explains why it took so long for the Soviet Union to disappear.

Even if any moderately neutral observer could have perceived in the mid sixties that the Soviet way of doing things was vastly more inefficient than the Western one, a surprisingly large number of people did not. Actually, the people that really mattered did not. Surprisingly as it may seem the

<sup>&</sup>lt;sup>5</sup>Some remarkable exceptions come to mind, in particular the *japanese* car industry.

Soviet intelligentsia believed in what it preached (see Hosking[5]), even a good number of outside observers did not see the system as inherently inferior<sup>6</sup>. People tend to disregard an opponent's achievements, that is in the human character: so one should not be surprised that the Soviet intelligentsia did not compare their system with the Western one. More surprising is the fact that they did not pay due attention to the Hungarian experience of economic liberalization.

The explanation that the model gives is that, compared with before the revolution, the Soviet Union was not faring so badly. The standard of living was improving substantially, and that was so in most countries that adopted their system. They had no incentives to experiment with new policies. Only when the system fell by its own weight -be the shock called Gorvachov, or Afghanistan, or whatever- did the Hungarian experience take a new light. An experiment that had been successful for quite a long time was rediscovered as the 'new' promising path to be taken.

The system fell from within, all the new establishment (the people that took power after the revolution of 1990) was part of the former Soviet intelligentsia. This are people that even well into the eighties still professed an almost blind trust in the old orthodoxy. It was necessary a big crash for them to see that the system was plainly inefficient, something that would have been obvious if the Hungarian experience could have been perceived before.

Even after these events, the range of disagreement on what is the best path to take is quite surprising, both among different countries of the former Soviet Block and within most of then. One is tempted to interpret this as the heterogeneity in beliefs, and consequent diversification, that the model predicts after a sharp fall.

### 8 Conclusions

The main moral of this paper is that the presence of informational externalities may lead to overconcentration of the investment activity in a few sectors, even if the externalities affect only the second moment of the information that agents receive.

We have seen that for a wide range of informational parameters the behavior of the agents, and so their payoff, does not change with changes in these parameters, and that can be rightly categorized as 'herding'. In this environment herding is not a consequence of people thinking that other agents 'might-know-better'. Everybody does the same thing because different courses of action are not being explored, nobody has incentives to explore

<sup>&</sup>lt;sup>6</sup>See Salisbury ed. [7] for an amusing contemporaneous description of Soviet life in the mid sixties by a bunch of New York Times reporters.

the unbeaten path. In our context herding takes the form of overconcentration of the investment, something that can be interpreted as underinvestment in research.

This overconcentration of the investment appears when the level of underlying precision of the information that the individuals receive (A) is relatively high and at the same time there exists a substantial amount of social interaction (high p). The high level of A allows the signals to be quite accurate when most people are investing in one sector, thus inducing a high degree of homogeneity in the perception that the members of the society have of that sector. Additionally, the high p implies that, in order to be heard, the number of agents investing in a sector has to be big. If only a few outliers invest in some sector, the beliefs of everybody on the evolution of that sector become more homogeneous because all the weight of the updating is given to the priors, and the processes that drive the rates of return are mean reversing. Thus one effect feeds the other and we end up with a very homogeneous society: everybody sees what happens in one sector and assumes that all the others are near their long run average. Not until the sector in which they are investing starts paying below the long run average do they diversify their investment, and really learn what happens in the rest of the world.

That the results are not linear comes as no surprise, that they are so much non linear it is probably more so. This is due to the big nonlinearity in the heterogeneity of the priors. When the perceptions are either bad or good, relatively to the priors, the agents will tend to share very similar views on the evolution of the return. When they are bad because agents expect the sector to go back to its long run average. When they are good because all the agents observe the same thing, and so they all share basically the same information.

It is only for a relatively small range of values in the precision of the signals that heterogeneity is generated; only when the precision of the signals is roughly of the same magnitude as the precision of the priors. In any case the evolution of both variables is endogenous to the economy; what happens in one sector affects the view that individuals have of other sectors, at least by default, by quitting experimentation.

We have also seen that across-the-board improvements in the accuracy of the information that agents receive may have quite perverse consequences. More information for everybody is not always good if we share that information.

In a world of short-sighted people, individuals are going to be constantly insecure of what is the best thing to do, and their views on it are necessarily going to differ; as a consequence, new paths are continuously going to be explored, and new roads found. In a world where agents posses almost  $20 \times 20$  vision, they are going to be overconfident on what is the best course to take, nobody is going to explore off the beaten paths, and the society will be defenseless when exogenous shocks hit the chosen road.

There are only a few remarks left to make. I cannot deny the criticism that I have been using the mechanism of information diffusion as a black box. Undoubtedly the mechanism should be endogenous to the economy, and infinitely more complex than I have assumed. However reduced forms should be acceptable if they accurately represent a complex reality in a simplified way. By allowing for a very flexible functional form, and by not assuming any range of parameters. I have insured that whatever reality is, it is captured by some point in the space defined by A and p.

If the reader is very worried about the reasons why the agents of the model have limited information on the past events, he might try to interpret this as a result of incomplete markets (of information) or (even better) to bounded rationality.

Most of the paper is a long exercise of comparative statics. It determines the effects of changes in the informational parameters. Nevertheless, whatever the values of the parameters, in the real world they are most likely to be constant.

As long as we believe that both A and p are relatively large, we should expect agents to behave in a very precise way. This behavior is unmodified for a huge range of parameters and it is characterized by excessive concentration of the investment in a few sectors. Thus, it results in underinvestment in experimentation for the society as a whole.

# A Aggregate Investment

Agents will invest in the sector that has the highest expected return. Aggregate investment in sector k is the number of agents for whom:

$$\mu_t^k(i) - \mu_t^j(i) > 0 \qquad \forall j \neq k \tag{26}$$

It is clear that

$$x_{t}^{k} = \Pr\left\{\mu_{t}^{k}(i) - \mu_{t}^{j}(i) > 0 \quad \forall j \neq k\right\} =$$

$$\Pr\left\{\frac{\left(\mu_{t}^{k}(i) - \mu_{t}^{j}(i)\right) - \left(\bar{\mu}_{t}^{k} - \bar{\mu}_{t}^{j}\right)}{\sqrt{M_{t}^{k} + M_{t}^{j}}} > -\frac{\left(\bar{\mu}_{t}^{k} - \bar{\mu}_{t}^{j}\right)}{\sqrt{M_{t}^{k} + M_{t}^{j}}}\right\} =$$

$$\Pr\left\{\frac{\left(\mu_{t}^{j}(i) - \bar{\mu}_{t}^{j}\right) - \left(\mu_{t}^{k}(i) - \bar{\mu}_{t}^{k}\right)}{\sqrt{M_{t}^{k} + M_{t}^{j}}} < \frac{\left(\bar{\mu}_{t}^{k} - \bar{\mu}_{t}^{j}\right)}{\sqrt{M_{t}^{k} + M_{t}^{j}}}\right\}$$
(27)

Both,  $\mu_t^k(i)$  and  $\mu_t^j(i)$  are independent random variables, both of them normally distributed:

$$\mu_t^k(i) \sim N\left(\bar{\mu}_t^k, M_t^k\right)$$

$$\mu_t^j(i) \sim N\left(\bar{\mu}_t^j, M_t^j\right)$$

$$covariance\left(\mu_t^k(i), \mu_t^j(i), \right) = 0$$
(28)

so:

$$\mu_t^k(i) - \mu_t^j(i) \sim N\left(\bar{\mu}_t^k - \bar{\mu}_t^j : M_t^k + M_t^j\right)$$
 (29)

Define  $\nu^j$  as:

$$\nu^{j} = \frac{\left(\mu_{t}^{j}(i) - \bar{\mu}_{t}^{j}\right) - \left(\mu_{t}^{k}(i) - \bar{\mu}_{t}^{k}\right)}{\sqrt{M_{t}^{k} + M_{t}^{j}}} = \frac{\sqrt{M_{t}^{j}}}{\sqrt{M_{t}^{k} + M_{t}^{j}}} \frac{\left(\mu_{t}^{j}(i) - \bar{\mu}_{t}^{k}\right)}{\sqrt{M_{t}^{k}} - \sqrt{M_{t}^{k}}} \frac{\sqrt{M_{t}^{k}}}{\sqrt{M_{t}^{k}}} \frac{\left(\mu_{t}^{k}(i) - \bar{\mu}_{t}^{k}\right)}{\sqrt{M_{t}^{k}}}$$

Then:

$$\nu^j \sim N(0,1) \tag{30}$$

and,  $\forall j, h \neq k$ :

$$E\left(\nu^{j} \cdot \nu^{h}\right) = \frac{M_{t}^{k}}{\sqrt{M_{t}^{k} + M_{t}^{j}} \sqrt{M_{t}^{k} + M_{t}^{h}}}$$
(31)

Define now the vector v:

$$v = \left\{ \nu^{1}, \nu^{2}, ..., \nu^{k-1}, \nu^{k+1}, ..., \nu^{K} \right\}$$
 (32)

and the vetor l:

$$l = \left\{ \frac{(\bar{\mu}_t^k - \bar{\mu}_t^j)}{\sqrt{M_t^k + M_t^j}} \right\} \qquad j = \{1, 2, ..., k - 1, k + 1, ...K\}$$
 (33)

Then:

$$x_t^k = \Pr\left\{v < l\right\} \tag{34}$$

The joint CDF of v is a normal a standard multivariate normal with a matrix of variances defined by the correlations in equation 31.

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