HB
34
U474
1993

ECONOMICS WORKING PAPERS SERIES

Economics Working Paper 33

The Shakeout

Hugo A. Hopenhayn*
Universitat Pompeu Fabra

April 1993

UNIVERSITAT POMPEU FABRA

Balmes, 132
Telephone (343) 484 97 00
Fax (343) 484 97 02
08008 Barcelona
e-mail econwp@upf.es

Abstract

Prior to maturity, industries exhibit a period in wich the number of firms is significantly reduced: the shakeout. This is not a period of decay of the industry, but rather a period where output continues to expand at a considerably high rate. This regularity has been also evidenced in the evolution of organizational populations and is central to the emerging field of organizational ecologies in sociological studies. This paper develops an economic model of industry evolution which provides some explanations for this observed shakeout.

1 Introduction.

Prior to maturity, industries exhibit a period in which the number of firms is significantly reduced: the shakeout. This is not a period of decay of the industry, but rather a period where output continues to expand at a considerably high rate. This regularity has been also evidenced in the evolution of organizational populations and is central to the emerging field of organizational ecologies in sociological studies. This paper develops an economic model of industry evolution which provides some explanations for this observed shakeout.

In a study of the evolution of 46 new products, Gort and Klepper (1982) find an average rate of shakeout of firms - measured by the number of firms after the decrease relative to the peak- of roughly 40%. This shakeout occurs after a short period of relative stability which in turn follows a period of sharp increase in the size of the industry. In turn, sociological studies of several industries and populations of organizations, such as trade unions, argue that as these populations mature, competitive forces dominate and the survival of members of these populations decreases.[1]

In this paper we develop a dynamic model of the evolution of a competitive industry. We consider two alternative forces of industry expansion. The first one is given by demand expansion and the second one by cost reducing technological change. As we find, both of these provide a rationale for the shakeout, but for quite different reasons. Consider first demand expansion. The typical evolution of an industry suggests that demand growth is not constant and that after a period of take-off the rate of growth slows down. The period of high demand growth is a period of high entry. Limited by scale, incumbents cannot satisfy all the demand increase. But if the average size of incumbents tends to increase through time, as demand growth slows down this source of supply expansion may exceed the demand increase. As a consequence, the less efficient firms are squeezed out of the market.

We consider two sources of growth in the average size of firms. The first is stochastic evolution, a variant of stochastic learning by doing. The second one is selection: as good firms are sorted from bad ones, the average quality (or productivity) of firms in the industry increases. As demand slows down

[1] As indicated by Hannan and Carroll, "Organizational populations initially grow slowly from zero, increase very rapidly over a brief period, reach a peak, and the often decline moderately before stabilizing for some, usually extended, period."

and entry decreases, the age distribution of firms starts shifting away from the younger firms. These are in turn the smaller firms, as is now widely documented in the empirical studies of establishment growth (see for example Dunne, Roberts and Samuelson (1989) and Davis and Haltiwanger (1991)). This change in the age distribution implies an increase in the average size of firms.

Theories of firm and industry evolution based on the idea of stochastic growth and selection have been recently developed, following the original work of Jovanovic.[2] The model used in this paper is based on Hopenhayn (1992b). The implications of stochastic evolution are developed in section 3.1. Selection is studied in section 3.2. We establish that (1) industries with faster diffusion processes will tend to exhibit a larger shakeout; (2) industries where selection is more important, will also exhibit a larger shakeout, and (3) the larger is the gap between the average size of firms in the industry and the average size of entrants, the larger the shakeout will be. To test the quantitative significance of this theory, section 3.4 discusses a method to assign values to the parameters in the model and provides some numerical results. These results suggest that the theory can explain a sizable degree of shakeout.

The theories of industry expansion based on cost reduction are analyzed in section 4. Cost reduction increases the optimal scale of firms and thus the supply of incumbent firms. In a competitive market with free entry, these lower costs are translated into lower prices. As a consequence demand expands. How much of this expansion can be met by the larger output of incumbent firms, will determine whether the total number of firms will increase or decrease. This depends on the elasticities of demand and supply. The evidence from Gort and Klepper suggest that demand elasticity decreases as the industry expands. The period of shakeout would then correspond to a stage with lower elasticity, where demand growth falls short of the supply increase, so some incumbents are crowded out. This model is developed in section 4.1 and the corresponding numerical calculations developed in section 4.2.

While the technological change considered in the previous paragraph is exogenous to the firms, section 4.3 considers a situation in which firms can, in addition, make investments in improvements. These improvements are specific to a technology. As the pace of exogenous technological change decreases, the incentives for firms to invest in these specific improvements increases.

[2]See for example Pakes and Ericson(1990) and Hopenhayn (1992a,1992b).

Those firms that are more successful with these projects obtain a competitive advantage, while the less successful ones exit the industry.

While the importance of costs and demand considerations have been quite studied and are well understood as determinants of market structure, very little theoretical work has been done in providing a link to industry evolution.[3] As important exceptions, two recent contributions to this field deserve particular consideration. Jovanovic and MacDonald (1992) develop a theoretical model to study the evolution of the tire industry, which exhibited a path for the number of firms quite characteristic of the Gort and Klepper type. In their model the shakeout is produced as some firms develop cost advantages and consequently a much larger efficient scale, crowding out the firms with higher costs. Petrakis, Rasmusen and Roy (1993) model more explicitly the cost reducing investments through learning by doing. They show that if demand and economies of scale are such that the market will not support all firms in the long run, some firms will invest more in learning by doing than others and consequently survive longer. The model we develop in section 4.3 shares some of the features of these two contributions.

The paper is organized as follows. Section 2 discusses in more detail the evidence from Gort and Klepper. Section 3.1 develops the demand based theories. Section 4 develops the theories based on cost reduction. Finally, section 5 concludes.

2 The Evidence.

In a study of the history of 46 new products, Gort and Klepper (1982) identify 5 stages of evolution. Figure 1 reproduces a plot included in their paper which gives the evolution of the number of firms for a 'representative' industry.[4] The 5 stages I-V are defined in terms of net entry. Gort and Klepper (GK) characterize these stages as follows:

> Stage I encompasses the interval in which the number of producers in the market remains relatively small (usually between one and three). Stage II is the interval from the "take-off" point of net

[3]The selection models developed following Jovanovic (1982) have focused mostly on firm dynamics, without much consideration for the implications on industry evolution.

[4]It should be noted that the variance of the 46 industry experiences is quite high.

entry to the time that net entry decelerates drastically. Stage III is the ensuing period of low or zero net entry, and Stage IV is the subsequent period of negative net entry. Stage V represents the new equilibrium in the number of producers that coincides with the maturity of the product market and continues until some new fundamental disturbance generates a change in market structure.

In this paper we focus mostly on stages II-IV, where most of the interesting dynamics takes place. More specifically, we will concentrate on stage IV, the period of shakeout. In any case, some of the characteristics of the remaining stages will be important in evaluating the theories developed. Table 1 summarizes the information for stages II-V which are mostly relevant to our analysis.

Table 1: Entry, Growth and Price Evolution.

Stage	Mean Duration	Mean annual entry rate [a]	mean annual growth rate	mean annual price decrease
II	9.7 years	24.8%	35%	-13%
III	7.5 years	0.2%	12%	-7%
IV	5.4 years	-9%	8%	-9%
V	-	-0.5%	1%	-5.2%

[a]Computed from the data presented in Gort and Klepper

Stage II, the period of take-off, is characterized by high rates of entry, output growth and price decrease. In Stage III output continues to grow and prices decrease, but at more moderate rates. Most importantly, the number of firms remains constant. Stage IV, the shakeout, is characterized by a reduction in the number of firms. Note that this period does not correspond to the decay of the industry, since output continues to grow. It seems to be more a stage of consolidation of the industry towards its maturity. Stage V exhibits almost no change in the number of firms and very slow output growth with a moderate rate of price decrease.

3 Demand Growth: Stochastic Evolution and Selection.

In this section we consider demand growth as the source of expansion of the industry. Throughout the section we take the process of demand growth for the product as exogenous.

Studies of product development in the marketing literature seem to agree that the growth of a market is well approximated by a logistic diffusion curve. After a takeoff, there is a period of sustained growth. Yet market growth tends to slow down thereafter, reaching a peak at some point in the history of the product which varies across experiences. In this section we show that a demand process with these characteristics combined with very plausible types of firm heterogeneity can explain a period of shakeout following a decrease in the rate of demand growth.

Section 3.1 emphasizes the role of stochastic growth of firms as a source of heterogeneity. Section 3.2 emphasizes the role of selection - the sorting of good and bad firms by entry and exit - that results from heterogeneity. Finally, Section 3.4 considers some numerical computations, where parameters are assigned values to match observations corresponding to manufacturing industries. The model described in these sections is based on Hopenhayn (1992a,1992b).

3.1 Stochastic Evolution: An example.

Consider the following simple model of a competitive industry. Firms in the industry produce a homogeneous product according to a variable cost function $c(s,x)$, where x denotes the output of the firm and s a productivity shock which takes values $\{s_1, s_2, s_3\}$, where $s_1 > s_2 > s_3$. Assume that $\partial c/\partial s < 0$ and $\partial c/\partial s\partial x < 0$, so firms with higher s will be more profitable and choose higher output. The productivity shock is entirely firm specific and for each firm follows a markov process with transition function

$$P = \begin{pmatrix} \lambda & 1-\lambda-\theta & \theta \\ 1-\lambda-\theta & \lambda & \theta \\ \theta & 1-\lambda-\theta & \lambda \end{pmatrix}$$

and initial distribution $\nu = (\nu_1, \nu_2, \nu_3)$. A firm that enters the industry can be thought of as making a draw for its initial parameters $s(0)$ from the distribution ν. If it decides to stay in the industry its productivity state for the next period will evolve according to the transition matrix P. The parameter λ measures the degree of persistence of a productivity shock and, as established below, θ corresponds to the death rate of firms.

All entrants bear a cost of entry c_e which is made prior to observing $s(0)$ and a fixed or opportunity cost c_f per period while the firm remains in the industry. We will assume that the optimal scale of firms is small relative to aggregate industry demand so at any point in time there will be a large number (continuum) of firms in the industry. Aggregate industry demand is given by a strictly decreasing inverse demand function $p_t(Q_t)$ where Q_t is the aggregate output of the industry in period t.

The timing of decisions is as follows. At the beginning of a period, incumbent firms observe their shocks and first decide whether to stay in the industry or leave. Likewise, those potential entrants who decide to enter the industry pay their entry costs and draw their initial productivity shocks. All the firms that decide to stay in the industry will be producers during that period. Prices are determined competitively. Firms collect their profits and the period is over.

It is convenient to consider the case where the equilibrium price remains constant through time at a level p^e. This characterizes a stationary equilibrium. Suppose that for any price $p > 0$ there exist well defined profit functions $\pi_i(p)$ and supply functions $q_i(p)$ for $i = 1, 2, 3$. Note that profits are net of the fixed cost, so for some i can (and will) be negative. The value of a firm of type s_i is given by

$$V_i(p) = \max\left(0, \pi_i(p) + \delta \sum_j p_{ij} V_j(p)\right) \tag{1}$$

where the value of exiting the industry is normalized to zero. Assume that the parameter values are such that $V_1 > V_2 > V_3 = 0$, so only firms with productivity shocks s_3 exit (later we provide conditions under which this holds true). The values of the other two types of firms will be

$$V_1(p) = \pi_1(p) + \delta\left(\lambda V_1(p) + \eta V_2(p)\right) \tag{2}$$

and

$$V_2(p) = \pi_2(p) + \delta\left(\lambda V_2(p) + \eta V_1(p)\right), \tag{3}$$

where $\eta = 1-\lambda-\theta$. It immediately follows that $V_1(p) > V_2(p) > V_3(p)$. Finally, $V_3(p)$ will be equal to zero provided that

$$\pi_3(p) + \delta\eta V_2(p) \leq 0. \tag{4}$$

The value of an entrant will be given by

$$V^e(p) = \nu_1 V_1(p) + \nu_2 V_2(p). \tag{5}$$

In a free entry equilibrium, $V^e(p) - c_e \leq 0$ and if the inequality is strict, there is no entry. It can be easily established that there is a unique value p^e such that $V^e(p^e) = c_e$. (See Hopenhayn, 1992b).

Consider the evolution of a cohort of entrants with total mass one. In the first period, after exit takes place, only a fraction $\nu_1 + \nu_2$ of the initial entrants will stay, with probability weights given by $\mu_0 = \left(\frac{\nu_1}{\nu_1+\nu_2}, \frac{\nu_2}{\nu_1+\nu_2}, 0\right)$. At the beginning of the following period, a fraction θ exit and the probability distribution of those firms of age 1 that stay is given by

$$\mu_1 = \left(\frac{\lambda\nu_1 + \eta\nu_2}{(1-\theta)(\nu_1+\nu_2)}, \frac{\eta\nu_1 + \lambda\nu_2}{(1-\theta)(\nu_1+\nu_2)}, 0\right). \tag{6}$$

So letting $\mu_t(i)$ denote the fraction of firms of age t with shock s_i and ρ_t the proportion of firms in the cohort that are still in the industry after t periods,

$$\rho_t = (1-\theta)^t \rho_0, \text{ where } \rho_0 = \nu_1 + \nu_2 \tag{7}$$

and

$$\mu_{t+1}(2) - \mu_{t+1}(1) = \frac{\lambda - \eta}{\lambda + \eta}\left(\mu_t(2) - \mu_t(1)\right) \tag{8}$$

These distributions converge to the limiting distribution $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$. It follows from (8) that if $\nu_1 < \nu_2$, $\mu_t(1)$ increases while $\mu_t(2)$ decreases. This we shall assume.

Consider now the special case in which $c(s,q) = \left(\frac{q^2}{2s}\right)$. Given a price p for the good in the industry, the corresponding supply and profit functions are

$q(s,p) = sp$ and $\pi(s,p) = \frac{p^2 s}{2}$. The value functions

$$V_1(p) = \frac{p^2 s_1}{2} - c_f + \delta\left(\lambda V_1(2) + \eta V_2(p)\right) \tag{9}$$

$$V_2(p) = \frac{p^2 s_2}{2} - c_f + \delta\left(\lambda V_2(p) + \eta V_1(p)\right) \tag{10}$$

and

$$V_1(p) - V_2(p) = \frac{\frac{p^2}{2}(s_1 - s_2)}{1 - \delta(\lambda - \eta)}. \tag{11}$$

Since $s_1 > s_2$, it follows that $V_1(p) > V_2(p)$.

We will describe an equilibrium where there is positive entry (and exit) every period.[5] The expected value of an entrant, $V^e(p) = \nu_1 V_1(p) + \nu_2 V_2(p)$ must then equal the cost of entry. For convenience, we set the cost of entry so that $V(1) = c_e$ and c_f, s_1, s_2 and s_3 so that $V_2(1) > V_3(1) = 0$.

In any period, total industry output will be the sum of the output of all firms in the industry. Let m_t denote the number (mass) of new firms that enter in period t. After k periods, $\rho_k m_t$ firms in this cohort will still be in the industry with an *average* output $q_k = \mu_k(1)s_1 + \mu_k(2)s_2$, where we have used the fact that $p = 1$. Total output in the industry in period t must satisfy

$$\sum_{i=1}^{t} m_{t-i}\rho_i q_i = \rho_0 \sum_{i=1}^{t} m_{t-i}(1-\theta)^i q_i = Q_t, \tag{12}$$

where Q_t satisfies $p_t(Q_t) = 1$.

Consider an industry where demand grows each period at a rate $\gamma - 1 \geq 0$, that is $p_t(Q) = p(\frac{Q}{\gamma^t})$ for all t. The long run equilibrium for this economy can be constructed as follows. Conjecture that at this long run equilibrium the number of entrants will grow at the rate γ, so $m_t = \gamma^t m_0$ for some fixed $m_0 > 0$ and $Q_t = \gamma^t Q_0$ where $p(Q_0) = 1$ and thus $p_t(Q_t) = 1$ for all t. Then as $t \to \infty$

$$Q_0 = \frac{Q_t}{\gamma^t} = \rho_0 m_0 \sum_{j=0}^{\infty}\left(\frac{1-\theta}{\gamma}\right)^j q_j. \tag{13}$$

This is well defined even in the case $\gamma = 1$ since q_j is bounded above by its limiting value $(s_1 + s_2)/2$ and $0 < \theta < 1$. Letting M_t denote the total number

[5] This is characteristic of most US industries. For manufacturing, the average rates of entry and exit exceed 40Most of this entry and exit is not accounted for by intersectoral movements but occurs within fairly disaggregated product classes (4 digit SIC codes).

of firms in the industry in period t, as $t \to \infty$

$$\frac{M_t}{\gamma^t} = \rho_0 m_0 \sum_{j=0}^{\infty} \left(\frac{1-\theta}{\gamma}\right)^j. \tag{14}$$

For a fixed market size, the number of firms is inversely related to the average size of firms. Changes in the average size of firms will play a crucial role in producing the shakeout. We now analyze some key determinants of average firm size. As $t \to \infty$, average output per firm converges to

$$\frac{Q_t}{M_t} = \frac{\sum_{j=0}^{\infty} \left(\frac{1-\theta}{\gamma}\right)^j q_j}{\sum_{j=0}^{\infty} \left(\frac{1-\theta}{\gamma}\right)^j}. \tag{15}$$

As shown in the appendix there is a simple solution to the above, given by

$$\frac{Q_t}{M_t} = q_0 + \frac{[(1-\theta)-\lambda]\,(s_1 - s_2)\,(\mu_0(2) - \mu_0(1))}{\gamma - (2\lambda - (1-\theta)} \tag{16}$$

where q_0 is the average size of entering firms.

From equation (16) it follows that the average size of firms decreases with θ and γ: higher death rates or higher demand growth increases the weight of younger firms, which are smaller. It also follows that for fixed θ, average size decreases with λ. Thus average firm size will increase with the degree of mobility η.

The empirical evidence on diffusion curves indicates that the rate of expansion of markets is decreasing. With demand based expansion, this means that demand grows at a decreasing rate. Consider as an extreme the situation where demand grows at a constant rate γ and ceases to expand after some period T. If T is large, average firm size at T will be approximately equal to the value indicated by equation (15). But since growth ceases afterwards, as $t \to \infty$ average size converges to

$$\frac{Q_\infty}{M_\infty} = \frac{\sum_{j=0}^{\infty}(1-\theta)^j q_j}{\sum_{j=0}^{\infty}(1-\theta)^j} = q_0 + \frac{[(1-\theta)-\lambda]\,(s_1 - s_2)\,(\mu_0(2) - \mu_0(1))}{1 - (2\lambda - (1-\theta)}, \tag{17}$$

which is larger than the average size at T. Since total output remains constant after period T, the number of firms decreases to converge to its asymptotic value. This is the shakeout.

Dividing equation (16) by equation (17) we obtain an expression for M_∞/M_T. The magnitude of the shakeout depends on characteristics of the demand process and the stochastic process for firm shocks. The following proposition gives some comparative statics results.

Proposition 1 *Let* $sk = 1 - \frac{M_\infty}{M_T}$. *Then*

1. $\partial sk/\partial\gamma > 0$,
2. $\partial sk/\partial\theta < 0$
3. *For* q_0 *fixed,* $\partial sk/\partial(s_1 - s_2) > 0$ *and* $\partial sk/\partial(\mu_0(2) - \mu_0(1)) > 0$.

Industries with a faster diffusion process will tend to exhibit a larger shakeout. Industries with larger attrition rate will exhibit a lower shakeout. The larger is $(s_1 - s_2)$ or $\mu_0(2) - \mu_0(1)$, the larger the gap between the average size of firms and the average size of entrants, and also the larger the shakeout will be. Consider now the effect of persistence. If $\lambda = (1-\theta)$, there is no mobility, the average size of firms is constant and thus there is no shakeout. For $\lambda < 1$, there is a positive shakeout. This indicates that, at least for values of λ close to 1, there is a negative relationship between persistence and the degree of shakeout.

3.2 The Selection Effect.

The example considered above emphasized the role of mobility. The example presented here emphasizes the role of *selection,* the process by which firms are sorted in the market. Consider a transition matrix of the following type:

$$P = \begin{pmatrix} \lambda_1 & 0 & 1-\lambda_1 \\ 0 & \lambda_2 & 1-\lambda_2 \\ 0 & 0 & 1 \end{pmatrix}$$

The state s_3 is absorbing and there is no mobility between the other two states. The parameter λ_i, $i = 1, 2$ measures the rate of survival for firms of type i. We will assume that $\lambda_1 > \lambda_2$, so that the rate of survival is higher for larger firms, which is also a well established empirical fact. This implies that $V_1 > V_2 > V_3$. As in the previous section we will assume that $V_3 = 0$.

The difference in survival rates implies that $\mu_t(1)$, the fraction of firms of type 1 increases over time. Consequently the average size of firms in a given age cohort j

$$q_j = \frac{\lambda_1^j \mu_0(1)s_1 + \lambda_2^j \mu_0(2)s_2}{\lambda_1^j \mu_0(1) + \lambda_2^j \mu_0(2)} \tag{18}$$

also increases with the age of a cohort.

Consider a market in which demand grows at a constant rate $\gamma - 1 \geq 0$ and conjecture that along the equilibrium path entry grows at the same rate while price remains constant at one. It is quite simple to establish that as $t \to \infty$, the average size of firms in the industry

$$\frac{Q_t}{M_t} \to \frac{\mu_0(1)(\gamma - \lambda_2)s_1 + \mu_0(2)(\gamma - \lambda_1)s_2}{\mu_0(1)(\gamma - \lambda_2) + \mu_0(2)(\gamma - \lambda_1)}. \tag{19}$$

It follows easily that average size is decreasing in γ, increasing in λ_1 and decreasing in λ_2.

As in the previous section, we can derive the ratio between the limiting number of firms and the number at the peak, for an industry that exhibits constant demand growth up to period T and stable demand thereafter. This is given by

$$\frac{M_\infty}{M_T} = \left(\frac{\mu_0(1)(\gamma - \lambda_2)s_1 + \mu_0(2)(\gamma - \lambda_1)s_2}{\mu_0(1)(\gamma - \lambda_2) + \mu_0(2)(\gamma - \lambda_1)}\right) / \left(\frac{\mu_0(1)(1 - \lambda_2)s_1 + \mu_0(2)(1 - \lambda_1)s_2}{\mu_0(1)(1 - \lambda_2) + \mu_0(2)(1 - \lambda_1)}\right). \tag{20}$$

The following Proposition provides comparative static results similar to those given in the previous section.

Proposition 2 *Let* $sk = 1 - \frac{M_\infty}{M_T}$. *Then*

1. $\partial sk/\partial\gamma > 0$,
2. $\partial sk/\partial\lambda_1 > 0$ *and* $\partial sk/\partial\lambda_2 < 0$,
3. $\partial sk/\partial(s_1 - s_2) > 0$ *and* $\partial sk/\partial(\mu_0(2) - \mu_0(1)) > 0$.

These results are consistent with those obtained before. Part (1) implies that industries with faster diffusion processes will tend to exhibit a larger shakeout. Part (2) implies that industries where selection is more important, will also exhibit a larger shakeout. Finally, part (3) implies that the larger is the gap between the average size of firms in the industry and the average size of entrants, the larger the shakeout will be.

3.3 Discussion.

The analysis carried out in the last two sections generalizes to a much broader setup. Two ingredients were key to generating the shakeout: (a) the increase in average firm size as a function of age; (b) the decrease in the rate of growth of aggregate demand. Both of these seem well supported by the data. The rise in average firm size is a well established empirical fact (references). The empirical evidence on the diffusion of new products suggests that diffusion curves are typically log concave, thus exhibiting decreasing expansion rates(references).

The results provided above correspond to the comparative static analysis of long run equilibria. We now show that under a mild regularity condition, we can in fact *construct* the entire equilibrium path. This method is the one used for the computations discussed in the following section.

Assumption 1 *The sequence $\rho_j q_j$ is decreasing in the age j of a firm cohort.*

If m_t firms enter in period t, the total output of that cohort j periods after entry will be $m_t \rho_j q_j$. So this assumption implies that the total output contributed by a given cohort of firms decreases over time. According to the data provided by Dunne, Roberts and Samuelson this seems to hold for manufacturing, taking periods of 5 years as the unit time interval. The role of this assumption is to guarantee that the equilibrium is interior, *i.e.* that $m_t > 0$ for all t, so the equilibrium price remains constant.

For a fairly general class of industry equilibrium models (see Hopenhayn 1992a) there is a unique price p^e such that $V^e(p^e) = c_e$. Assuming that the inverse demand function is strictly decreasing in total output, define the output sequence Q_t by setting $p^e = p_t(Q_t)$. Suppose that Q_t is a non decreasing sequence. This is obviously the case of interest here. Let $\tilde{q}_j = \rho_j q_j$. Choose m_0 so that $m_0 \tilde{q}_0 = Q_0$. Given a sequence of entries $m_0, m_1, ..., m_{t-1}$ choose m_t so that

$$m_t \tilde{q}_0 + \sum_{j=0}^{t-1} m_j \tilde{q}_{t-j} = Q_t. \tag{21}$$

By Assumption 1 $\tilde{q}_j$ is a decreasing sequence so

$$\sum_{j=0}^{t-1} m_j \tilde{q}_{t-j} < \sum_{j=0}^{t-1} m_j \tilde{q}_{t-j-1} = Q_{t-1}. \tag{22}$$

where a' is a constant depending on a, p and α and ϵ'_{it} is also normally distributed with zero mean and variance equal to $\left(\frac{1-\alpha}{\alpha}\right)^2 \sigma_\epsilon^2$. Given estimates of equation (25) we can derive the persistence and the variance of innovations for the process for $\ln(s_{it})$.

Estimates for ρ and σ_ϵ^2 were obtained for a panel consisting of all establishments recorded in both the 1972 and 1977 Census of Manufactures.[6] These values are .93 and 0.53, respectively. The time period of 5 years is too long for the purpose of our analysis. The numerical results presented below are done considering a time period of 1 year, instead. We annualize the above process in the natural way, making the assumption that the yearly process is also an AR1 process with normal innovations. The level of persistence we use is $(.93)^{1/5}$ and the variance for the yearly innovations obtained in the obvious way. Unfortunately, this procedure is subject to a selection bias that results from the exit option: survivors are more likely to have received good news in the past. Consequently, we may interpret this persistence parameter as an upper bound.

The values for the rest of the parameters were chosen as described in Hopenhayn (1992), by matching the model's predicted values to the data on entry/exit rates, average size of firms and size distribution for entrants.

Demand Process. To model demand expansion we consider the following diffusion process which is widely used in the literature:

$$Q(t) = e^{\frac{r_0}{c}(1-e^{-ct})} \tag{27}$$

with parameters r_0 and c. The asymptotic level $\ln(Q(\infty)) = \frac{r_0}{c}$, the growth rate is

$$\frac{d\ln(Q_t)}{dt} = r_0 e^{-ct}, \tag{28}$$

with value r_0 at $t = 0$ and decay given by c. Keeping r_0/c constant, the speed of diffusion depends on this single parameter c. For simplicity, we fix $r_0/c = 1$. To choose values for c, note that

$$\ln\left(\frac{Q(t)}{Q(\infty)}\right) = -\frac{r_0}{c}e^{-ct} = -e^{-ct}. \tag{29}$$

Different values for c where chosen so that demand reaches maturity (defined

[6] I am grateful to John Haltiwanger for providing these estimates.

This guarantees that $m_t > 0$ and thus the equilibrium is interior. Furthermore, using Theorem 2 in Hopenhayn (1990) it can be established that this is the unique equilibrium.

3.4 Numerical Results.

In the analysis of the previous sections, the precise characteristics of the process of demand growth and stochastic evolution of firms play a critical role. We have established that a model based on demand growth and firm heterogeneity can generate a shakeout. But how much of a shakeout can it *realistically* explain? This section attempts to provide a preliminary answer to this question. We discuss a method to assign parameter values and provide numerical computations of the shakeout.

The Technology of Firms. The cost function considered above can be derived from a homogeneous production function

$$q = f(s, n) = \frac{s^\alpha n^\alpha}{\alpha} \tag{23}$$

for $\alpha = \frac{1}{2}$, taking n to be the amount of a single homogeneous input (labor) with a market price which is given to the industry and normalized to one. For a constant price, this function gives a labor demand function of the form:

$$\ln\ n(p, s) = \frac{1}{1-\alpha}\ln(p) + \frac{\alpha}{1-\alpha}\ln(s). \tag{24}$$

This formulation has the convenient feature that, assuming prices are constant (which is true in the model), one can calibrate the process for $\ln(s_t)$ from evidence on employment growth of firms.

As described in detail in Hopenhayn and Rogerson (1991) and Hopenhayn (1992b), assuming the ln(employment) of firms follows an AR1 process with normally distributed innovations of the form

$$\ln(n_{it}) = a + \rho \ln(n_{it-1}) + \epsilon_{it}, \tag{25}$$

where ρ is the persistence parameter and ϵ_{it} is normally distributed with zero mean and variance σ_ϵ^2, then the corresponding process for $\ln(s_{it})$ is given by

$$\ln(s_{it}) = a' + \rho \ln(s_{it-1}) + \epsilon'_{it}, \tag{26}$$

Table 2: Numerical Results.

t(90%)	Period at peak	Shakeout
5	5	66.3%
10	9	59.1%
15	13	52.8%
20	17	46.8%
30	23	36.0%
40	30	26.2%

here as 90% of its asymptotic value) in periods $\{5, 10, 15, 20, 30, 40\}$.

Before discussing our results it is important to point out that our intention here is not to provide an exhaustive empirical test of the model. We are combining very different data sources and considering values which are averages of the histories of many diverse industries. Our purpose is rather to get some orders of magnitude to see whether the selection hypothesis may be an alternative worth considering when studying industry evolution, growth and consolidation.

Results. The results are presented in Table 2 and Figure 2. Table 2 gives the year at which the number of firms is at its maximum and the amount of shakeout for the demand diffusion processes considered above. The shakeout is considerable in all cases and is larger the faster the diffusion process is. This is consistent with the results obtained in the previous sections. Note also that the peak in the number of firms occurs in all cases before demand reaches 90% of its limiting value, but more so when diffusion is slower. Figure 2 plots the diffusion curve and the evolution of the number of firms for the case where diffusion is the slowest (40 years). It is worth emphasizing that it not only matches the shakeout, but also the other characteristics of the evolution of the number of firms discussed in section 2.

4 Market Expansion by Cost Reduction.

In this section we explore the implications of cost reduction as a determinant of market expansion. Aggregate demand for the industry is given by the time invariant inverse demand function $p(Q_t)$. As before, the cost of production of an individual firm may depend on its productivity shock s and a market-wide technology level γ_t as given by the cost function $c(s, x/\gamma_t)$, where γ_t is a nondecreasing sequence. In this section all technological progress is exogenous to the firm and accessible by both, incumbents and potential entrants. Section 4.3 considers firm specific cost reduction.

4.1 Demand Elasticity and Crowding out.

Though in this section demand does not play an *active* role in the expansion of market size, it has important implications on the evolution of the number of firms. As production costs are lowered, the market equilibrium price decreases and total demand in the industry expands. There are two sources for the supply expansion, the increase in the output of incumbent firms and the increase in the number of firms. The elasticity of demand will determine what the total expansion of demand is, and whether the first source of supply expansion suffices or not.

To illustrate the mechanics of market expansion, consider a special case where there is no idiosyncratic productivity shock, so all firms are identical. Let $\pi(\gamma, p)$ and $q(\gamma, p)$ denote the individual profit and supply functions of firms. To begin, assume that entry is free ($c_e = 0$), so the unique equilibrium price in each period satisfies $\pi(\gamma_t, p_t^*) = 0$. Assume that the inverse demand function is strictly decreasing and define the total output sequence Q_t to be the unique solution to $p_t^* = p(Q_t)$. The number of firms producing in the industry is given by $m_t^* = Q_t/q(\gamma_t, p_t^*)$.

We now derive more explicitly these equilibrium sequences using the specification of cost function and technological progress given above. The output function is obtained by equating marginal cost to price, i.e.

$$p_t^* = c_2(s, x_t/\gamma_t)\frac{1}{\gamma_t}, \tag{30}$$

where s is common to all firms and constant throughout. The zero profit condition for each firm requires that

$$p_t^* x_t - c(s, x_t/\gamma_t) - c_f = 0. \tag{31}$$

Normalizing $\gamma_0 = 1$, let x_0 be the unique solution to (30) and p_0^* the unique price that makes net profits zero in the initial period. Let $p_t^* = p_0^*/\gamma_t$ and $x_t = x_0\gamma_t$. It is easy to check that these are the unique values which satisfy the above two equations. It thus follows that

$$\frac{p_t^*}{p_{t+1}^*} = \frac{\gamma_{t+1}}{\gamma_t} = \frac{q(\gamma_{t+1}, p_{t+1}^*)}{q(\gamma_t, p_t^*)}. \tag{32}$$

Equation (32) summarizes all the relevant information about the equilibrium path: prices must drop at the same rate at which the output of incumbent firms expands. If demand elasticity exceeds one, total demand will increase at a faster rate and positive net entry will occur. Alternatively, if demand elasticity is less than one, net entry will be negative and the number of firms will decrease. Letting e_t denote the absolute value of demand elasticity in period t, then

$$\frac{m_{t+1} - m_t}{m_t} \cong (e_t - 1)\frac{\gamma_{t+1} - \gamma_t}{\gamma_t} \cong \left(1 - \frac{1}{e_t}\right)\frac{Q_{t+1} - Q_t}{Q_t}. \tag{33}$$

If elasticity decreases as price goes down, there can be a period of *net entry* to the industry followed by a period of *net exit*. This is indeed the case with a linear demand curve: as the market reaches half of its maximum potential size, demand growth fails to be enough to absorb the increasing supply of incumbent firms and crowding out occurs.

From the information contained in Table 1 and assuming a time invariant demand function, an estimate of demand elasticity in each period can be obtained. Denoting by e_{II}, e_{III} and e_{IV} the elasticities for periods II, III and IV, respectively, the values thus obtained are: $e_{II} = 2.7$, $e_{III} = 1.7$ and $e_{IV} = 0.9$. So the elasticities are aligned correctly around 1 to match the qualitative implications of the cost reduction explanation. To obtain a quantitative estimate we can replace output growth and the elasticity figure in (33) to obtain predicted values for the rate of entry. The values obtained this way are 18%, 5% and −1% for periods II, III and IV, respectively. The true values are given in Table 1. The shakeout turns out to be largely underestimated.

Consider now the more general case where firms face idiosyncratic uncertainty and there is a positive entry cost. The sequence of entries $\{m_t\}$ can be obtained by modifying slightly equation (21) as follows. As before, let q_j denote the average output of the cohort of firms of age j for $\gamma = \gamma_0 = 1$ and let ρ_j be the proportion of firms in that cohort that have survived j periods.

Then

$$m_t \gamma_t \tilde{q}_0 + \gamma_t \sum_{j=0}^{t-1} m_j \tilde{q}_{t-j} = Q_t, \tag{34}$$

where Q_t is defined by $p(Q_t) = p_0/\gamma_t$ and p_0 is the stationary equilibrium price for an industry with no technological change and $\gamma = \gamma_0$. Provided that

$$\gamma_t \sum_{j=0}^{t-1} m_j \tilde{q}_{t-j} < Q_t, \tag{35}$$

the nonnegativity constraint on entry is satisfied and the allocations thus constructed give the unique equilibrium for the industry.

4.2 Some more numerical computations.

Using the calibrated model described in Section 3.4 and the procedure for constructing the equilibrium entry sequence described above, we can obtain a predicted series for the number of firms. To derive the sequence γ_t, we use the data on evolution of average real prices given in Table 7 of Gort and Klepper (GK) . We also use the numbers they provide on average output growth (Table 5 of GK) to construct the sequence Q_t.

Figure 3 plots the results obtained. The simulations where made starting at the beginning of stage II (year 15). Since GK only provide average values for output growth and price decrease for each of the stages, two alternative approaches were followed. As a first alternative, we constructed the series Q_t and γ_t keeping growth and price decay constant and equal to the mean rate for each stage. The broken line gives the evolution of the number of firms for this case. Alternatively, we used a smooth approximation to interpolate the series Q_t and γ_t, which resulted in a smooth evolution of the number of firms as given by the solid line.

The predicted average growth rate of the number of firms in stage II is approximately 18% for both cases, somewhat below the average rate for the GK data (24.8%). For stage III the model predicts a very small expansion in the number of firms, averaging roughly 1% per year. The corresponding rate in the GK data is 0.2%. In stage IV (the shakeout) we obtain a rate of decrease in the number of firms of approximately 9%, which is exactly the value in the GK data. After that period, though the rate of growth of output is very small (1% per year), prices decrease at a fairly fast rate (5%). This implies a very low elasticity of demand (0.2) and consequently a sharp decrease in the number of firms (leaving aside heterogeneity , an annual decrease of 4%). As a consequence, the model predicts that the number of firms converges to zero, which is clearly not supported by the GK data.

4.3 Technological Change and Cost Reducing Investments

In the model discussed above, all cost reduction is exogenous to the firm. This may reflect an important component of technological change, particularly in the first stages of development of an industry, but leaves aside firm specific investments. In this section we consider the situation in which firms may, at any point in time, invest in *improvements* to the existing technology. Those firms that are successful in making an improvement obtain a cost advantage.

In order to explain the patterns of industry evolution obtained from the case studies, Gort and Klepper suggest the following hypothesis: For the first part of an industry's evolution, most innovations are accessible by all firms and entrants, while at a later stage the innovations tend to be more proprietary to the firms. The initial stage would then induce an expansion in the number of firms. In contrast, in the later stage those firms that are successful in their investments obtain a cost advantage, crowding out the unsuccessful ones as the number of firms decreases. While there is truly something compelling about this story and it fits stages II and IV, it leaves unexplained the causes of the shift in the innovation process.

The model discussed here provides a slightly different story. At all times, firms have available the possibility of engaging in cost reducing investments. However, these cost reducing investments are specific to the technology process at use. If new and superior technologies are expected to arrive soon, it may not be worth for a firm to incur these costs as the improvements are likely to become obsolete. In consequence, investment behavior is affected by the expectations of future technical advance. Technological change is high in the initial years but becomes less likely thereafter. As firms' assessed probabilities of major future changes decrease, investment in proprietary cost reduction will increase. At this point, Gort and Klepper's story continues in place.[7]

[7] The idea of random sucess in developing a cost advantage was used by Jovanovic and MacDonald (1992) as a mechanism to generate the shakeout. Our model has basically the

We now turn to the description of the model. For convenience, we consider here a continuous time model. The cost of producing output flow x is given by a cost function $c(\frac{x}{s\gamma})$, where $s \in \{\theta, 1\}$ is a firm specific cost component and $\theta > 1$; $\gamma \in \{\gamma_1, \gamma_2, ...\}$ is a technology parameter common to all firms. The function c is strictly increasing and strictly convex. Firms pay a fixed cost per period c_f. For simplicity we assume that there are no costs of entry and all potential entrants are identical.

When a new technology γ_j arrives, all firms -and potential entrants- have free access to this technology with no proprietary enhancements. Consequently, if a firm decides to use this new technology, its cost of production will be given by $c(\frac{x}{\gamma_j})$. Alternatively, if such firm had developed an improvement to the previous technology and decides *not* to use the new technology, its cost of production will be given by $c\left(\frac{x}{\theta\gamma_{j-1}}\right)$. For simplicity we assume that $\theta\gamma_{j-1} < \gamma_j$, i.e. technological change is sufficiently large so that even a firm that has developed an enhancement to the previous technology will decide to switch.

In any period, firms can invest in enhancements specific to the current technology. The probability that an enhancement is obtained depends on the level of investment. More precisely, enhancements have Poisson arrivals, with arrival rate $\alpha(z)$, where z is the firm's investment flow. We assume α is strictly increasing and strictly concave. Note that since all cost reducing investments are specific to a technology, firms will be reluctant to invest unless the probability of a change in technology is perceived to be low.

The technology level γ evolves according to a Poisson process with arrival rates $\lambda > 0$.[8] However, there is also an independent event, which causes technological change to cease forever after. The arrival time for this event is exponentially distributed with constant hazard rate ρ. We will say that this event leads to the *no change* regime. Firms only observe whether a new technology has arrived or not and use bayes rule to update their prior probability of being at the no change regime.

This distinction between *change* and *no change* regimes, albeit arbitrary,

same mechanism, with two differences: i) firms can choose the level of investment in cost reduction, while in Jovanovic and MacDonald it is fixed. Higher investment results in a higher likelihood of obtaining the cost advantage; ii) The level of investment -and thus the rate of cost reduction- is affected by expectations of technological change (and therefore potential obsolescence of investments), which is absent in their model.

[8]Consequently, times of arrival have an exponential distribution with parameter λ.

tries to capture a common characteristic observed in the life cycle of new products or processes. In the early stages, industries go through rapid changes both in production technologies and in product standards. But over time, the industry settles down while products and processes tend to become more standardized. In our model the product is homogeneous, so the industry evolution is governed by the process of cost reduction.

The demand for the industry's output is given by the inverse demand function $p(Q_t)$. To isolate the features that are specific to this story from those considered in the previous section, we assume that the demand function has constant elasticity, so $p(Q_t) = Q_t^{-\eta}$ where $0 < \eta$.

The equilibrium implies a stochastic process for prices, output and the number of firms in the industry. As in the previous section, every time a new technology arrives price will drop so that $p_j \gamma_j = p_{j-1} \gamma_{j-1}$, where p_j denotes the price *in the period* where technology γ_j appears. Because entry cost is zero, once a new technology arrives incumbents lose any advantage they may have previously acquired . When this occurs, the value of a firm will be zero.[9] This simplifies the analysis considerably, by breaking the link between periods with different technology levels. When a technological change arrives it is as if a new industry supersedes the existing one. To derive the equilibrium allocations we only need to consider as representative stage the case $\gamma = 1$. Because of the homogeneity built into the model, we may derive the equilibrium allocations for other values of γ as multiples of these.

We will now derive the equilibrium conditions. Let t denote the time elapsed since the last technological change. Let $\pi_\theta(t)$ denote the profits of upgraded firms and $\pi(t)$ the profits of those firms with no upgrade. The value of an upgraded firm satisfies the following differential equation

$$rV(t) = \pi_\theta(t) + V'(t) + \lambda(t)\,(0 - V(t))\,. \tag{36}$$

That is, the flow equivalent of the value of a firm consists of: i) the profit flow $\pi_\theta(t)$, ii) the change in the value of the firm, and iii) the capital loss that results when a new technology arrives and the firm loses its acquired cost advantage, where $\lambda(t)$ is the hazard rate associated to this event and is given

[9]With a positive entry cost, the value of the firm would then be equal to this cost.

by

$$\lambda(t) = \frac{(\rho + \lambda)\,\lambda\, e^{-(\rho+\lambda)t}}{\rho + \lambda\, e^{-(\rho+\lambda)t}}. \tag{37}$$

This hazard rate is decreasing in t, reflecting the increasing likelihood of the no change regime. Equation (36) can be more conveniently rewritten as

$$(r + \lambda(t))\, V(t) = \pi_\theta(t) + V'(t). \tag{38}$$

Consider now the situation of a firm with no improvements. Because the cost of entry is zero, if entry is positive at t the value of this firm will satisfy:

$$rW(t) = \pi(t) + \max_x \{\alpha(x)V(t) - x\} = 0. \tag{39}$$

In particular, if the optimal choice is not to invest at all, then $\pi(t) = 0$.

Given $V(t)$, equation (39) determines uniquely $\pi(t)$ and thus $p(t)$ and $\pi_\theta(t)$. Thus, equations (38) and (39) define an ordinary differential system for $V(t)$. Provided that the number of firms with no upgrades is positive throughout the equilibrium path, $p(t) \to p(\infty)$ and $V(t) \to V(\infty)$, where these values can be solved uniquely from these two equations. With this boundary condition, using the above differential system a unique solution for $V(t)$ is obtained. In the appendix we establish the following proposition.

Proposition 3 *There exists a unique interior solution $m(t), V(t)$ corresponding to the system of differential equations (38) and (39). In this solution $V(t)$ is strictly increasing and $p(t)$ decreasing. There is a time $T \geq 0$ such that $x(t) = 0$ and $m'(t) = 0$ for all $t < T$ and $x'(t) > 0$ afterwards. If $T < \infty$, then as $t \to \infty$, $m(t) \to 0$.*

Note that the number of firms is constant in periods $[0, T]$. This corresponds to the situation described by Stage III in Gort and Klepper. In our model, this is a *waiting* stage. Higher values of $\lambda(t)$ and r increase the duration of this waiting stage. Note, however, that both the initial and final number of firms (and thus the shakeout) are independent of ρ and λ. The final number of firms is affected by the investment technology. The higher the chances of success are, the lower price will be in the limit and thus the larger the number of firms will be. Conversely, lower chances of success will imply a larger shakeout.

We now provide some numerical computations of the model. Assuming that $\gamma_j/\gamma_{j-1} = \gamma$ is constant, the expected rate of technological change is $\lambda(\gamma - 1)$. This corresponds to the rate of price decrease in stage II, which is 13%. Without further information on the process for price change, we cannot identify λ and γ. For illustrative purposes, we choose $\lambda \in \{0.5, 1\}$ with an average duration of a technology of 2 and 1 years, respectively. The value for γ is set accordingly. Demand elasticity for the region of shakeout was set to 0.9, the value corresponding to stage IV. θ was set equal to γ, the highest value that satisfies the assumptions. This is the value that maximizes the shakeout.

Stage II ends when the no change regime starts. The average duration of stage II is approximately 10 years, so we set $\rho = 0.1$. For the cost function we use $c(q) = \frac{c}{2}q^2$, setting $c = 1$ and choosing c_f so that the initial equilibrium price is equal to one. The probability function used is $\alpha(x) = \ln(1 + x)$. This implies that $x(t) = V(t) - 1$ if $V(t) \geq 1$ and zero otherwise, and that $\alpha(x(t)) = \ln(V(t))$ when $x(t) > 0$.

Figures 4 and 5 give the results obtained. The solid line corresponds to $\lambda = 0.5$ ($\gamma = 1.26$) and the dashed line to $\lambda = 1$ ($\gamma = 1.13$). The initial number of firms is normalized to 100. Investment starts after 1.7 periods in the case where $\lambda = 0.5$ and after 2.6 periods in the case where $\lambda = 1$. After that point, the number of firms decreases monotonically when $\lambda = 0.5$, while in the other case it decreases monotonically but after an initial period of increase. The limiting number of firms is approximately 88% of the number at the peak. Figure 5 exhibits the fraction of upgraded firms, which follows a logistic diffusion curve.

The annual rates of output growth implied by the model in the phase of shakeout (stage IV) are just 1.7% and 0.7% for cases $\lambda = 0.5$ and $\lambda = 1$, respectively. The corresponding rates of price decrease are only 1.9% and 0.8%, respectively. These low figures and the relatively small shakeout result from the fact that average size of firms increases over time, but much less than what is exhibited in the data. This suggests that a larger firm specific cost reduction (higher θ) would be needed to account for this data.[10]

[10]Recall that we have limited the value of θ so that $\theta < \gamma$ in order to simplify the equilibrium analysis.

5 Conclusions.

This paper considered economic models of industry evolution which can explain the empirical observations on firm shakeout. The theories developed provide other testable implications and point to further evidence which may be relevant to the study of industry evolution. In particular, the theories based on demand growth and firm heterogeneity predict a larger shakeout for industries where (1) the demand diffusion is faster, (2) selection is a more important force and (3) the difference between the average size of incumbent firms and entrants is larger.

The theories of market expansion based on cost reduction have focused on the effect of technological change on the supply behavior of incumbent firms and on demand elasticity. An important ingredient in producing the shakeout here is the decrease in demand elasticity as the industry expands. Given that the final size of a market is limited, this decrease in demand elasticity is very plausible. As markets approach a point of saturation and demand becomes very inelastic, cost reducing technological change results in the crowding out of firms.

The evidence from Gort and Klepper suggests that cost reduction must have been an important force in the development of these industries and that the crowding out effect discussed above has probably played a crucial role. Yet our analysis also suggests that there may be an important residual which is unexplained by cost reduction alone, and that selection is a good candidate hypothesis to consider.

Firm heterogeneity underlies the theory of selection. It thus seems important for the study of industry evolution to better understand the characteristics of industries that lead to more or less heterogeneity and the process by which this differentiation takes place. The model developed in section 4.3 emphasizes the role of randomness in the outcomes of private investments in cost reduction and it jointly determines industry evolution and the investment behavior of firms. There are obviously other important sources of firm differentiation, such as the outcome of R&D or market positioning leading to product differentiation. Our analysis suggests that these factors can be important not only to understand the long run performance of an industry but also for explaining its evolution.

Appendix

1 Derivation of Formula for Average Size.

Since $q_t = \mu_t(1)s_1 + \mu_t(2)s_2$ it follows that

$$q_{t+1} - q_t = (s_1 - s_2)\left(\mu_{t+1}(1) - \mu_t(1)\right) = (s_1 - s_2)\frac{\eta}{\lambda + \eta}\left(\mu_t(2) - \mu_t(1)\right), \quad (40)$$

where the last equality follows by using $\mu_{t+1}(1) = \frac{\lambda\mu_t(1)+\eta\mu_t(2)}{\lambda+\eta}$. From equation (8) it follows that

$$\mu_t(2) - \mu_t(1) = \phi^t\left(\mu_0(2) - \mu_0(1)\right) \quad (41)$$

where $\phi = \frac{\lambda-\eta}{\lambda+\eta}$. Letting $\tau = \frac{\eta}{\lambda+\eta}(s_1 - s_2)$ it follows that

$$q_{t+1}(1 - L) = \tau\phi^t\left(\mu_0(2) - \mu_0(1)\right), \quad (42)$$

where L is the lag operator. Let $z = (1 - \theta)/\gamma$. Then

$$\frac{\sum_{t=0}^{\infty} z^t q_t}{\sum_{t=0}^{\infty} z^t} = (1 - z)\, L(1 - L)^{-1}\tau\frac{(\mu_0(2) - \mu_0(1))}{1 - z\phi}. \quad (43)$$

From here it follows that

$$\frac{\sum_{t=0}^{\infty} z^t q_{t+1} - \sum_{t=0}^{\infty} z^t q_t}{\sum_{t=0}^{\infty} z^t} = (1 - z)\,\tau\frac{(\mu_0(2) - \mu_0(1))}{1 - z\phi}. \quad (44)$$

But since

$$\sum_{t=0}^{\infty} z^t q_{t+1} = z^{-1}\left(\sum_{t=0}^{\infty} z^t q_t - q_0\right) \quad (45)$$

it follows that

$$\frac{\sum_{t=0}^{\infty} z^t q_t}{\sum_{t=0}^{\infty} z^t} = q_0 + z\tau\frac{(\mu_0(2) - \mu_0(1))}{1 - z\phi}. \quad (46)$$

Replacing for z and τ and using $\lambda + \eta = 1 - \theta$ equation (16) is obtained.

2 Proof of Proposition 3.

We first establish that $V'(t) > 0$. Suppose towards a contradiction that $V'(t) \leq 0$ for some t. From equation (38) it follows that

$$V''(t) = (r + \lambda)V'(t) - \pi'_\theta(t) + \lambda'(t)V(t).$$

By hypothesis the first term in the right hand side is negative and since $\lambda'(t) < 0$ the last term is also negative (unless $V(t) = 0$). As for the second term, if $m_t = 0$ then $\pi'(t) = 0$ which in turn implies $p'(t) = 0$ and $\pi'_\theta(t) = 0$. If $m_t > 0$ and $x_t = 0$ then $\pi(t) = 0$ and thus $\pi'(t) \geq 0$ so $\pi'_\theta(t) \geq 0$. Finally, if $x(t) > 0$ then by the envelope theorem $\pi'(t) = -\alpha(x(t))V'(t) > 0$, so $\pi'_\theta(t) > 0$.This proves that $V''(t) < 0$, but this implies that $V(t) \to -\infty$. Hence $V'(t) > 0$.

We will now establish that $m(t) > 0$ for all t. Suppose there exists some T such that $m(t_1) > 0$ for some $t_1 < T$ and $m(t_2) = 0$ for some $t_2 > T$. Without loss of generality let $m(t) > 0$ for all $t \in (t_1, T)$ and $m(t) = 0$ for all $t \in [T, t_2]$. It follows that $p(T) = p(t_2)$ and thus $\pi(T) = \pi(t_2)$. Furthermore, since $V'(t) > 0$ it follows that $V(t_2) > V(T)$. By continuity $W(T) = 0$ and thus it follows that $x(t) = 0$ for all $t < T$, since otherwise $W(t_2) > 0$. But then $z(t) = 0$ and thus $\pi(T) > \pi(t_1) = 0$, which cannot occur by free entry. This establishes the contradiction.

Since $m(t) > 0$ and $V'(t) > 0$ it follows that $\pi'(t) \leq 0$ and consequently $p'(t) \leq 0$, with strict inequality when $x(t) > 0$. Since V is increasing an α strictly concave, it follows that $x'(t) \geq 0$. Furthermore, if $x(t) > 0$ then $x'(t) > 0$. This completes the proof of the Proposition.

References

DAVIS, S. AND J. HALTIWANGER (1991) "Gross Job Creation, Gross Job Destruction and Employment Reallocation," Working Paper Series, Research Department, Federal Reserve Bank of Chicago.

DUNNE, T., M. ROBERTS AND L. SAMUELSON (1989) "The Growth and Failure of US Manufacturing Plants," *Quarterly Journal of Economics,* 104, 671-698.

GORT, M., AND S. KLEPPER (1982) "Time Paths in the Diffusion of Product Innovations," *Economic Journal,* 92, 630-53.

HANNAN, M.T. AND G. R. CARROLL (1992) *Dynamics of Organizational Populations.* Oxford University Press, Oxford 1992.

HOPENHAYN, H. (1990) "Industry Equilibrium Dynamics: A General Competitive Theory," Unpublished manuscript, Stanford.

HOPENHAYN, H. (1992a) "Entry, Exit and Firm Dynamics in Long Run Equilibrium," *Econometrica,* vol. 60, No. 5, 1127-1150.

HOPENHAYN, H. (1992b) "Exit, Selection and the Value of Firms," *Journal of Economic Dynamics and Control,* 16, 621-653.

HOPENHAYN, H. AND R. ROGERSON (1991) "Job Turnover and Policy Evaluation: A General Equilibrium Analysis," Unpublished manuscript, University of Minnesota *(forthcoming in the Journal of Political Economy).*

JOVANOVIC, B. (1982) "Selection and the Evolution of Industry," *Econometrica,* 50, 649-670.

JOVANOVIC, B. AND G. MACDONALD (1992) "The Life-Cycle of a Competitive Industry," Unpublished manuscript, University of Rochester.

PAKES, A. AND R. ERICSON (1990) "Empirical Implications of Alternative Models of Firm Dynamics," Working Paper, Yale University, 1990.

PETRAKIS, E., E. RASMUSEN AND S. ROY (1993) "The Learning Curve in a Competitive Industry," Unpublished manuscript, University of California at Los Angeles.

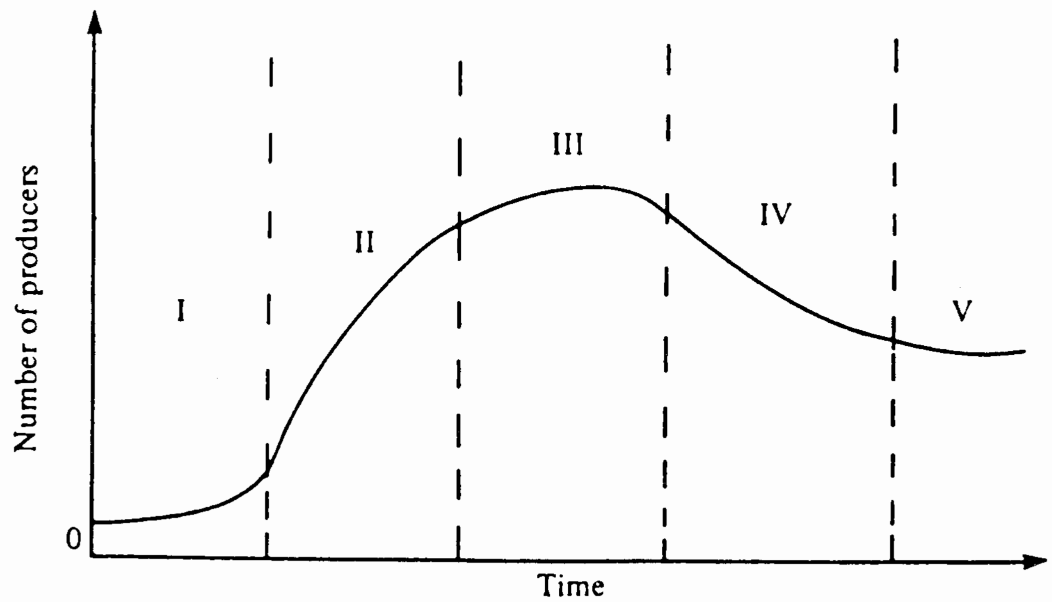


Fig. 1. The five stages of new product industries.

Figure 2. Demand Expansion and Number of Firms

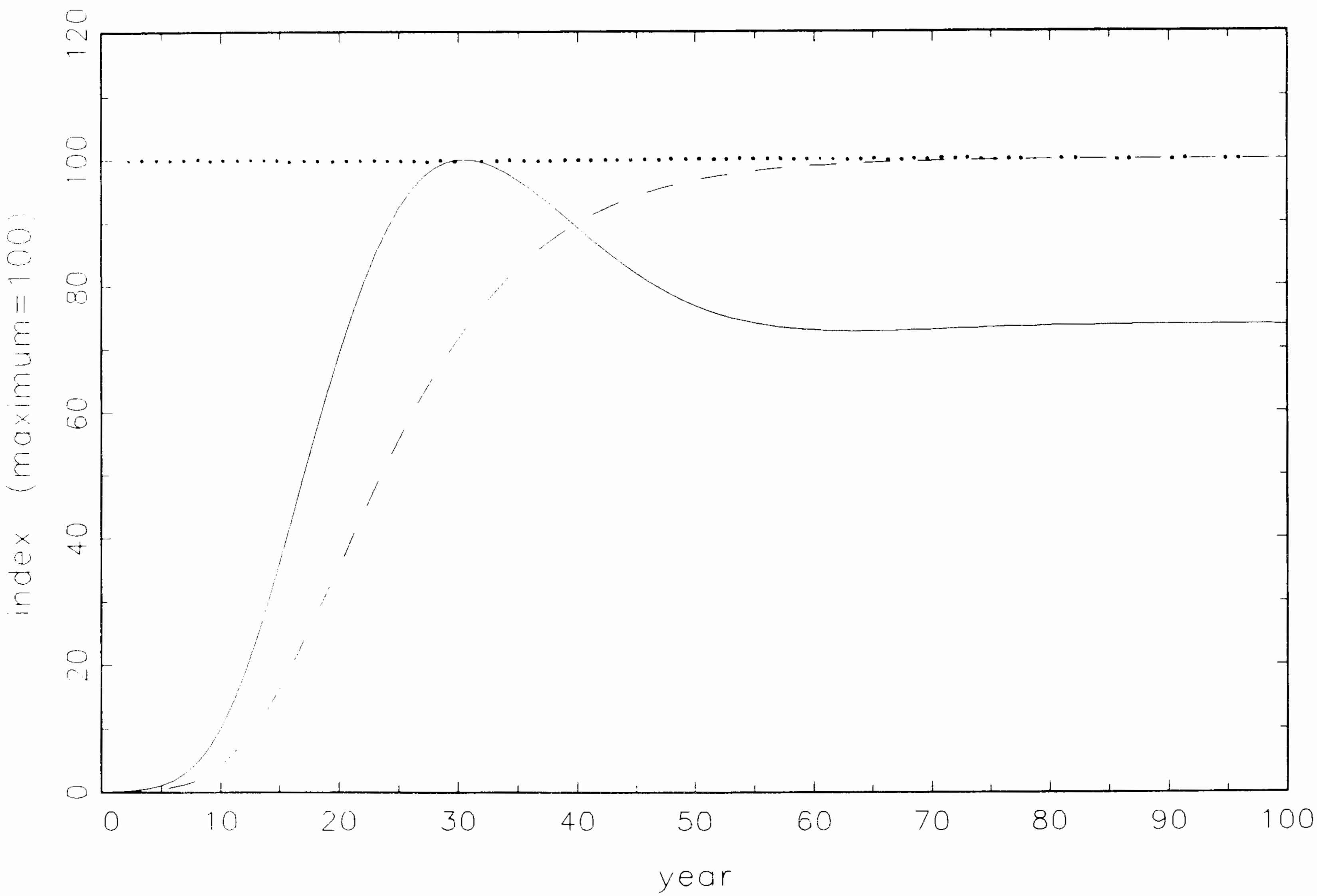


Figure 3. Cost Reduction and Number of Firms

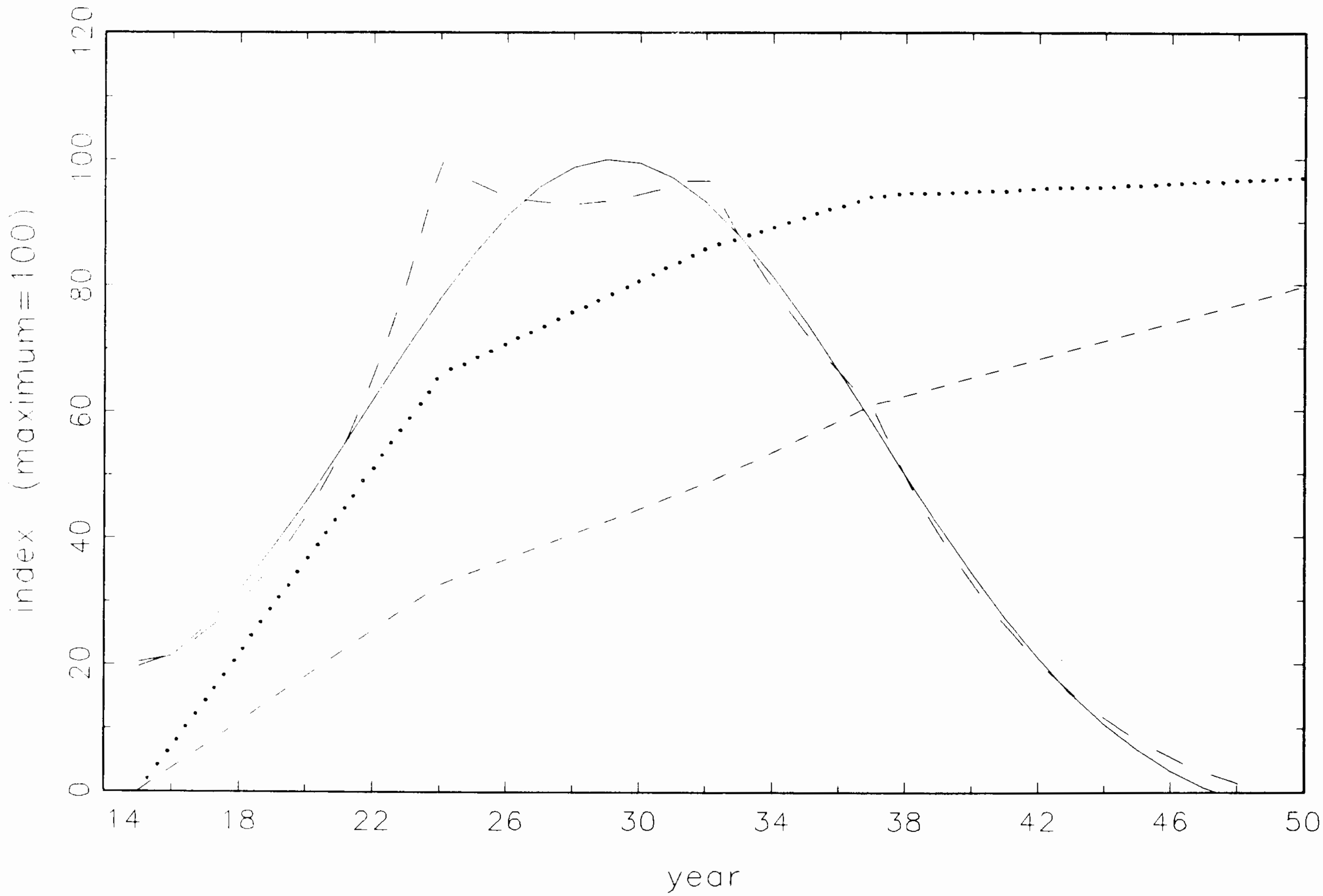


Figure 4. Number of Firms and Frequency of Change

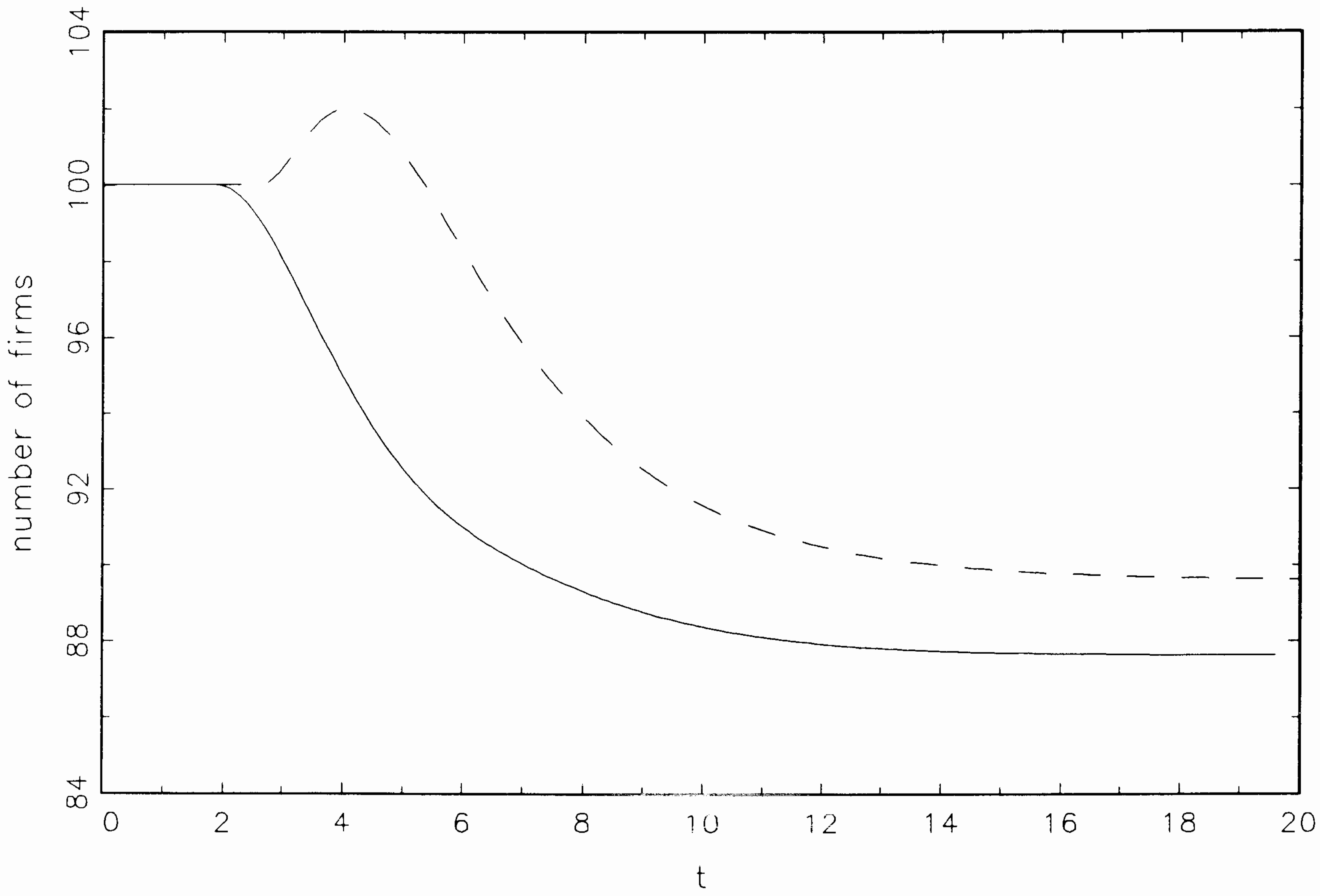
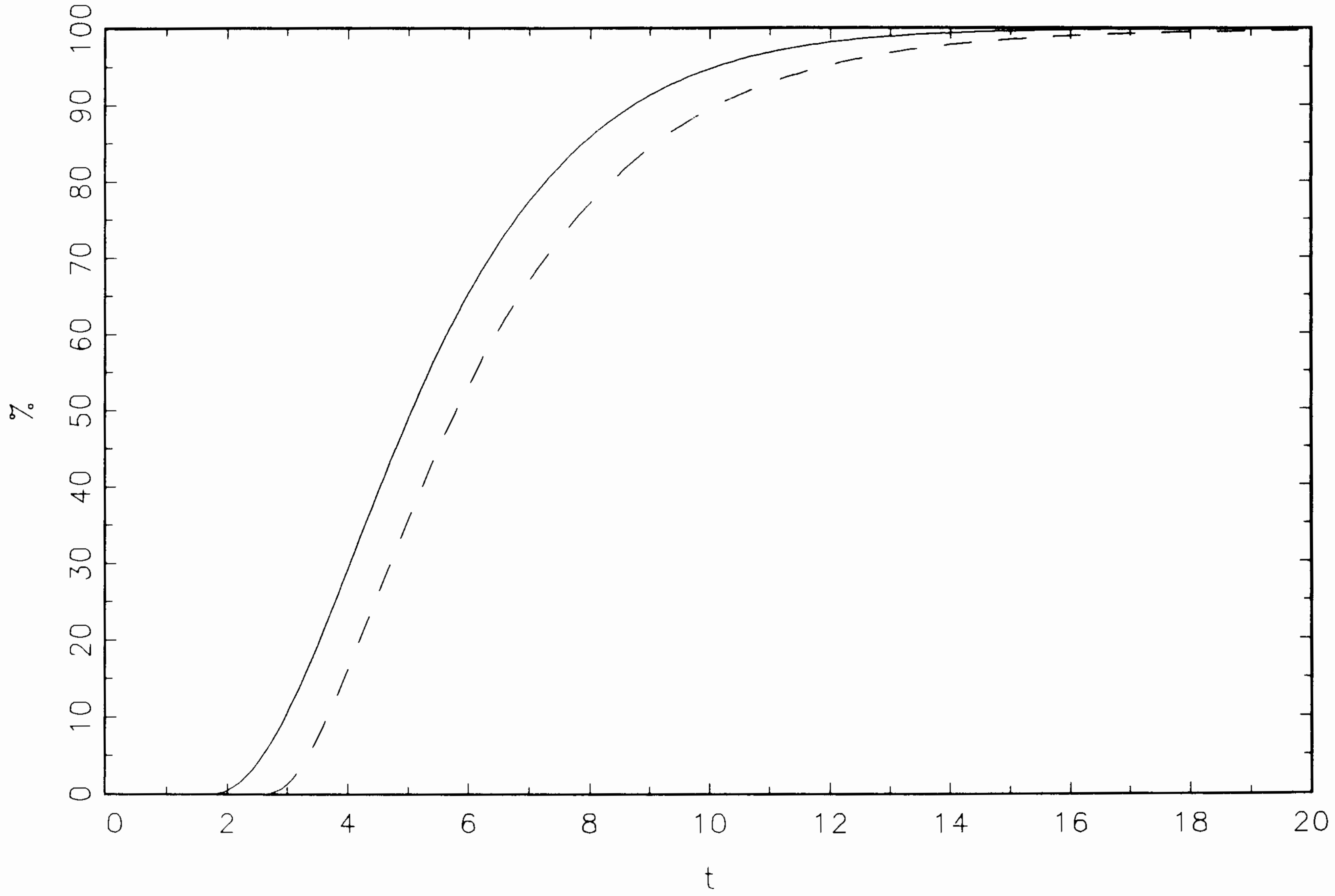


Figure 5. Evolution of Upgraded Firms (% of total)



RECENT WORKING PAPERS

1. Albert Marcet and Ramon Marimon
Communication, Commitment and Growth. (June 1991)

2. Antoni Bosch
Economies of Scale, Location, Age and Sex Discrimination in Household Demand.(June 1991)

3. Albert Satorra
Asymptotic Robust Inferences in the Analysis of Mean and Covariance Structures. (June 1991)

4. Javier Andrés and Jaume Garcia
Wage Determination in the Spanish Industry. (June 1991)

5. Albert Marcet
Solving Non-Linear Stochastic Models by Parameterizing Expectations: An Application to Asset Pricing with Production. (July 1991)

6. Albert Marcet
Simulation Analysis of Dynamic Stochastic Models: Applications to Theory and Estimation. (November 1991)

7. Xavier Calsamiglia and Alan Kirman
A Unique Informationally Efficient and Decentralized Mechanism with Fair Outcomes. (November 1991)

8. Albert Satorra
The Variance Matrix of Sample Second-order Moments in Multivariate Linear Relations. (January 1992)

9. Teresa Garcia-Milà and Therese J. McGuire
Industrial Mix as a Factor in the Growth and Variability of States'Economies. (Jannuary 1992)

10. Walter Garcia-Fontes and Hugo Hopenhayn
Entry Restrictions and the Determination of Quality. (February 1992)

11. Guillem López and Adam Robert Wagstaff
Indicadores de Eficiencia en el Sector Hopitalario. (March 1992)

12. Daniel Serra and Charles ReVelle
The PQ-Median Problem: Location and Districting of Hierarchical Facilities. Part I (April 1992)

13. Daniel Serra and Charles ReVelle
The PQ-Median Problem: Location and Districting of Hierarchical Facilities. Part II: Heuristic Solution Methods. (April 1992)

14. Juan Pablo Nicolini
Ruling out Speculative Hyperinflations: a Game Theoretic Approach. (April 1992)

15. Albert Marcet and Thomas J. Sargent
Speed of Convergence of Recursive Least Squares Learning with ARMA Perceptions. (May 1992)

16. Albert Satorra
Multi-Sample Analysis of Moment-Structures: Asymptotic Validity of Inferences Based on Second-Order Moments. (June 1992)

Special issue Vernon L. Smith
Experimental Methods in Economics. (June 1992)

17. Albert Marcet and David A. Marshall
Convergence of Approximate Model Solutions to Rational Expectation Equilibria Using the Method of Parameterized Expectations.

18. M. Antònia Monés, Rafael Salas and Eva Ventura
Consumption, Real after Tax Interest Rates and Income Innovations. A Panel Data Analysis. (December 1992)

19. Hugo A. Hopenhayn and Ingrid M. Werner
Information, Liquidity and Asset Trading in a Random Matching Game. (February 1993)

20. Daniel Serra
The Coherent Covering Location Problem. (February 1993)

21. Ramon Marimon, Stephen E. Spear and Shyam Sunder
Expectationally-driven Market Volatility: An Experimental Study. (March 1993)

22. Giorgia Giovannetti, Albert Marcet and Ramon Marimon
Growth, Capital Flows and Enforcement Constaints: The Case of Africa. (March 1993)

23. Ramon Marimon
Adaptive Learning, Evolutionary Dynamics and Equilibrium Selection in Games. (March 1993)

24. Ramon Marimon and Ellen McGrattan
On Adaptive Learning in Strategic Games. (March 1993)

25. Ramon Marimon and Shyam Sunder
Indeterminacy of Equilibria in a Hyperinflationary World: Experimental Evidence. (March 1993)

26. Jaume Garcia and José M. Labeaga
A Cross-Section Model with Zeros: an Application to the Demand for Tobacco. (March 1993)