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Communication, Commitment and Growth

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Abstract

We compute efficient accumulation mechanisms for stochastic growth economies with alternative financial structures. In particular, we compare the behavior of the economy under optimal contracts with: i)self-financing, ii) external financing with complete markets in the context of full information and full enforceability, iii) external financing with private information, and iv) external financing with limited enforcement of debt contracts. Extending the theoretical model studied in Marimon [1988] we show that when information constraints are present, there is an efficient transfer mechanism that can easily be computed using the decision rules of the full-information Pareto problem. We provide a framework for casting time inconsistent models in a recursive structure that makes their interpretation and numerical solution easier. To compute our models we apply and extend the method of "parameterizing expectations" developed by Marcet [1989]. It is shown that when there is no direct disutility of labor and risk-averse agents have access to risk-free financial markets, information constraints only affect consumption patterns and the distribution of wealth. In contrast, commitment constraints also affect investment patterns and the growth of the economy. In fact, with commitment constraints the patterns of growth can be very similar to those of an autarkic economy.

1 Introduction

Recent and past historical events show that alternative forms of social organization (mechanisms) have a crucial effect on the process of economic growth and the distribution of wealth.

The fact that social forms of organization (degree of competition, definition and enforcement of property rights, etc.) affect growth and the distribution of wealth is certainly not a new observation among economists. Adam Smith [1775], for example, takes this to be the central explanation underlying the divergent patterns of growth observed in different economies of his time, such as England and Spain. While this issue has seen a renewed interest among economic historians (e.g. North [1986], North and Weingast [1987]), it has mostly escaped the recent renewed interest on growth among economic theorists.

Endogenous growth models capture the effects of some forms of social interaction by including in the definition of the technology elements such as learning. Nevertheless, we argue that in order to model the effect on growth of some basic forms of social organization it is not enough to change the production function. One must include elements such as *the degree of communication and enforcement* in the definition of the environment.

The neoclassical growth model (e.g., Brock and Mirman [1972]), in which a single agent must solve an intertemporal optimization problem, can be seen as an extreme form of lack of commitment-communication. All investment must be self-financed and smoothing of consumption must take the form of self-insurance through the capital stock, which is the only available asset.

Alternatively, a general equilibrium growth model (e.g., Marimon [1989]) supposes a perfect commitment-communication technology. Managers can freely borrow and lend at the market interest rate and idiosyncratic risks can be smoothed out through pooling. Furthermore, agents never breach their contracts.

Most historical economies show partial forms of insurance and credit markets, and contractual opportunities might be limited by the scope of an imperfect commitment–communication technology. For example, investors might not be able to monitor all investments and agents might default on their promises. Feasible contracts (mechanisms) must take these incentive problems into account.

Recently, there have been several mechanism-anthropological studies showing that so called "primitive economies" have a fairly complex contract structure and through family and tribal ties they establish long-run relationships which resemble those predicted by the theory of dynamic mechanisms. In particular, agents can pool intravillage risks and partially smooth their consumption (intervillage pooling is usually more problematic. See, Townsend [1989], Udry [1989])

If, as exchange economies, these "primitive" economies can implement exchange allocations which are close to the Arrow-Debreu allocations, why don't we observe higher growth and/or income levels? Perhaps, implementing Arrow-Debreu investment allocations is not such an easy task in an environment with incentive constraints. But, if this is the case, which incentive constraints have a more severe impact on growth? Our work is an attempt to answer these questions.

We also expect that the theory developed here will be of some use to address the old issue *of growth and inequality.* As is known, according to standard competitive analysis the distribution of income does not affect the patterns of growth. However, the empirical evidence seems to be against this neutrality result (see for example, the recent work of Persson and Tabellini [1991]). Our work shows that, while in a perfect Arrow-Debreu world there is no relationship between wealth distribution and growth, in a limited enforcement environment this neutrality result breaks down.

Marimon [1988] studies accumulation mechanisms for stochastic growth economies with alternative forms of commitment-communication technologies. In particular, he shows that alternative mechanisms imply different wealth distributions and possibly accumulation paths. He also shows that, in an economy with limited communication and enforcement, the loss of efficiency due to incentive constraints can be made arbitrarily small if the discount factor is close enough to one. In this paper we further pursue the study of these economies, by characterizing and computing a sequentially efficient mechanism.

As in Green's [1987] exchange economy, our growth economy has a continuum of agents and a technology available to society as a whole that transforms one unit of consumption good in a given period into δ the following period, and vice versa. The social contract (mechanism) between a (risk-averse) agent and the "society" can be characterized as a contract between a risk-averse (agent) and a risk-neutral agent, the latter having possibly negative consumption (transfers). We also assume that risk-averse agents have no disutility of labor.The production technology under the control of the risk-averse agent is a fairly standard neoclassical one (i.e., a version of Brock and Mirman' technology). By simplifying our model with these assumptions, we are able to obtain a sharp picture of the effect of different constraints on growth and wealth.

In the economy *with full communication and enforcement*, the set of efficient mechanisms (contracts), can be parameterized by (λ, k, θ), where $\lambda \in R_+$ is the weight given to the risk-averse agent in the corresponding social planner's problem, k the initial capital stock, and θ the observed random shock. As is expected, risk-averse agents have a constant stream of consumption (which only depends on λ and k_0) and, if the initial capital, k_0, is low with respect to the steady-state distribution of capital stocks, then the risk-averse manager borrows heavily in the initial periods in order to finance high investment levels. In classical capital-theory terminology, we can say that "*the manager speeds up his way to the turnpike*".

We proceed with the study of an economy where investments are not observable and transfer payments can only depend on past transfers and capital stocks. We follow Abreu, Pearce and Staccetti [1987] in characterizing a contract as a prescription for each time-event of an action and a continuation payoff contingent on the observed consequences (output and capital stock) of the prescribed action. These ex-post present values can be associated with alternative λ values of the social planner's problem with full communication. In other words, the agent is rewarded/punished along each observed history by changing (the present value) of future transfers. Associated with any of these changes there is a change in the agent's marginal utility of income λ^{-1}. We call this type of mechanism a λ-transfer mechanism and show that it is a sequentially efficient mechanism.

In the full communication-commitment environment, risk-averse agents have access to risk-free financial markets. Agents can perfectly smooth their consumption and investment is independent of the weight, λ, given to the representative agent in the planner's problem. The λ-transfer mechanism for an economy with limited communication and perfect enforcement induces a less smooth pattern of consumption. Ex-ante homogeneous agents have an ex-post unequal distribution of wealth. Investment, however, is not affected by the presence of information constraints, and the process of capital accumulation is the same as in the economy with full information.

Enforcement constraints are very different from these information constraints. We study the case in which society (or the investor) has full commitment and the system of property rights establishes that, when the manager breaches the contract, he can take possession of the existing capital stock, but he will then be prevented from ever re-entering the social mechanism. That is, the present value of the autarkic solution given the current capital stock and shock is the current reservation value for the manager. With these constraints, the set of efficient mechanisms for an economy with limited enforcement and full information can be parameterized by (λ, k, M, θ), where the state variable M accounts for the past periods in which the participation constraints have been binding. The participation constraint is a non-standard constraint in dynamic programming. Nevertheless, we show how the problem can be cast in a dynamic programming framework, where the solution is given by a time-invariant function of the natural state variables and a pseudo-state variable. This approach of making recursive the characterization of the optimal contract has an independent interest since it can be applied to other non-recursive problems.

Enforcement constraints (alone) have an important effect on wealth distribution and growth. Even if agents have access to risk-free financial markets and perfect information, they cannot perfectly smooth their consumption (which now depends on (λ, k, M, θ)). Investment is also affected and, therefore, the process of capital accumulation differs from that of an economy with full enforcement. These analytical results are reinforced by our simulations showing that with limited enforcement the possibility of external financing may not be useful for growth when the initial capital is low. Then, growth will be as low as it would be under autarky and external financing is only useful for smoothing consumption against unforeseen shocks.

Phelan and Townsend [1988] have computed sequentially efficient mechanisms for stationary (non-growth) economies. They follow the approach of linearizing the sequential constraints by means of lotteries over continuation payoffs. With this approach, they can solve for the efficient mechanism by solving a large number of linear programming problems. Our approach differs from theirs in that we do not linearize the problem and, by constructing λ-transfer mechanisms, we can limit most computations to solving maximization problems without information constraints.

In order to compute the nonlinear dynamic stochastic optimization problems embedded in the computation of the efficiency frontiers, we apply the parameterized expectation approach (PEA) developed by Marcet [1989]. That is, we parameterize the conditional expectation of the optimality conditions with flexible functional forms, and we iterate on these expectations until they are the best prediction of the future in the series they generate.

Some features of the application of PEA to this problem are novel. First of all, because we are interested in the growth path of capital towards the steady state we do not use long run simulations, as in Marcet [1989], to iterate on the conditional expectation. Instead, we calculate a different policy function for the growth path by using many realizations of a few periods. Second, the participation constraints take the form of inequality constraints that involve conditional expectations and that are binding in some periods and non-binding in others. To our knowledge this is the first paper where a dynamic model with this type of constraints is solved.

The rest of the paper is organized as follows. Section 2 presents the model and the theoretical results; Section 3 describes the computational algorithm in more detail, and Section 4 presents some numerical results.

2 Alternative Growth Economies

In this section we present the four different models analyzed in this paper. What is constant in all four cases is the productive technology, the exogenous shocks, and the utility function of the agents. However, the possibilities for financing will vary form case to case. First we analyze the autarkic equilibrium with no external financing, second we study external financing under full information and commitment, third limited information but full commitment, and fourth full information but limited commitment. In all cases we analyze the optimal contracts given the constraints imposed in the type of contracts that are enforceable.

We characterize the social transfer mechanism with a representative risk averse agent (the manager), in an economy with a continuum of agents, as a transfer mechanism between a risk-averse and a risk-neutral agent (the investor). Underlying this construction is the assumption that society as a whole, but not any single agent, can use a linear technology that transforms δ units of consumption next period into one this period and vice versa.

With full information and enforcement and given an initial capital stock k_0, efficient transfer mechanisms are indexed by $\lambda \in \mathbf{R}$ and given by solutions to the following planner's problem:

$$\max\ (1-\delta)\, E_0 \left[\sum_{t=0}^{\infty} \delta^t \ [\lambda u(c_t) + (-\tau_t)] \right]$$

subject to:

$$c_t + i_t - \tau_t = f(k_t) \tag{1}$$

$$k_t = dk_{t-1} + g(i_{t-1}; \theta_t, s_t) \tag{2}$$

$$c_t \geq 0, \quad i_t \geq 0, \quad k_0 = k$$

Here, $u(\cdot)$ represents the instantaneous utility function of the risk-averse manager, $f(\cdot)$ the production function, and $g(\cdot)$ the function that converts investment goods into capital goods. The variable c_t represents consumption of the manager; τ_t transfers of the investor or, alternatively, $-\tau_t$ can be interpreted as the consumption good the risk neutral investor obtains from the contract. We assume that both agents have the same discount factor. The exogenous stochastic shocks (θ_t, s_t) affect the productivity of investment; where s_t is i.i.d., while θ_t is first order Markov. Note that the investment technology is such that at the time the investment decision is made the value of the shocks is unknown.

The following assumptions are made: *i)* the utility function $u(\cdot)$ is strictly concave, twice differentiable and satisfies the Inada conditions: $u'(c) \to +\infty$ as $c \to 0$, $u'(c) \to 0$ as $c \to \infty$; *ii)* f is concave and differentiable; *iii)* the exogenous stochastic processes (θ_t, s_t) are stationary and mutually independent; *iv)* $d \in [0,1]$; *v)* $g(\cdot;\theta,s)$ is differentiable and concave, with fixed range independent of (θ, s); if $i' > i$, then the distribution of $g(i';\cdot,\cdot)$ (second order) stochastically dominates the distribution of $g(i;\cdot,\cdot)$, and $g(\cdot;\theta,s)$ satisfies the Inada conditions described in (i), and *vi)* there exists $\beta > 0$ and $\bar{k}$ such that, for all $k \geq \bar{k}, f'(k) \leq \beta$, and, for all$\theta$ and i, if $E[g(i;\cdot,\cdot)|\theta] \geq (1-d)k$, then $\delta^{-1} \geq d + \beta E[g'(i;\cdot,\cdot)|\theta]$.

The above assumptions are relatively standard in the stochastic growth literature. The main exception is (v), which is introduced to guarantee that in a private information environment it is not possible *ex-post to* detect investment decisions with probability one from observations on the capital stock and the serially correlated shock. Assumption (vi) guarantees that present dicounted values are well defined and, as it can be seen, allows for *long-run* growth, as in the deterministic convex model of Jones and Manuelli [1990].

2.1 The environment with self-financing (AU)

The self-financing (autarkic) solution for an economy with lack of communication-commitment is obtained from the above planner's problem by having $\lambda = 1, \tau_t = 0$, for all t, and solving for an optimal investment process $\{i_t\}$. In addition to (1) and (2) the autarkic problem (AU) has the following Euler equation:

$$u'(c_t) = \delta E_t[\frac{\partial g_{t+1}}{\partial i_t} \sum_{n=0}^{\infty} (\delta d)^n u'(c_{t+n+1}) f'(k_{t+n+1})] \tag{3}$$

Throughout the paper $\frac{\partial g_{t+1}}{\partial i_t}$ will represent the derivative of the function g with respect to its first argument; notice that this derivative depends on future values of the stochastic shocks. Using standard arguments one can show the existence of a time-invariant policy function $i^a(k, \theta)$, and the corresponding consumption policy function $c^a(k, \theta)$.

2.2 The environment with full information and full enforcement (PO)

When both agents observe all shocks and contracts are perfectly enforceable, the optimal contracts are Pareto Optimal allocations.They can be found as solutions to the planner's problem described at the beginning of this section. In addition to (1) and (2), the (interior) first order conditions are:

$$1 = \delta E_t \left[\frac{\partial g_{t+1}}{\partial 1_t} \sum_{n=0}^{\infty} (\delta d)^n f'(k_{t+n+1}) \right] \tag{4}$$

$$u'(c_t) = \lambda^{-1} \tag{5}$$

It follows that the stationary policy functions $(i^*(k, \lambda, \theta), c^*(k, \lambda, \theta), \tau(k, \lambda, \theta))$ take the form $c^*(k, \lambda, \theta) = c(\lambda)$ and $i^*(k, \lambda, \theta) = i(k, \theta)$. Only transfers depend on (k, λ, θ).

The risk-averse agent is fully insured and accumulation paths $\{k_t(k_0)\}$ are independent of the relative weights in the planner's problem. Of special interest is the solution for $\lambda^*(k_0)$, where $\lambda^*(k_0)$ is the only value $\lambda \in R_+$ satisfying

$$E_0 \sum_{t=0}^{\infty} \delta^t \tau^*(\lambda^*(k_0), k_t, \theta_t)) = 0$$

That is, $\lambda^*(k_0)$ is the (inverse of) the marginal utility of expenditure in the competitive equilibrium with initial capital stock k_0. The existence and uniqueness of $\lambda^*(k_0)$ can easily be easily derived from our assumptions.

2.3 The environment with limited communication and full enforcement (PI)

We now consider the case in which investment/consumption decisions cannot be observed by outsiders. Contracts can only be contingent on observable variables such as the history of capital stocks and transfers. A few definitions are in order (We use the notation $k^t \equiv (k_0, \ldots, k_t)$)

Def: A transfer mechanism $\Gamma = \{i_t, \tau_t\}$, with observable information $z^t \equiv (k^t, \tau^t, \theta^t)$ is said to be *Sequentially Incentive Compatible* if for every time-event (t, z^t) :

$$v_1(t, z^t) = (1-\delta)\pi(i_t, \tau_t, k_t) + \delta E_{(i_t, z^t)} v_1(t+1, z^{t+1})$$

$$\geq (1-\delta)\pi(\tilde{i}, \tau_t, k_t) + \delta E_{(\tilde{i}, z^t)} v_1(t+1, z^{t+1})$$

Where $v_1(t, z^t)$ is the expected present value of utility to agent one (the risk-averse) at time-event (t, z^t) of the transfer mechanism Γ, and

$$\pi(i, \tau, k) = max_c\{u(c) : c \leq f(k) + \tau - i\}$$

Def: A *Sequentially Efficient Mechanism* is a (resource- feasible) Sequentially Incentive Compatible mechanism that is not Pareto dominated by any other (resource-feasible) Sequentially Incentive Compatible mechanism.

Now we define a transfer mechanism, called λ-transfer mechanism, and show that it is a *Sequentially Efficient Mechanism.* A dynamic mechanism or contract *ex-ante* defines contingent actions (i,τ) as functions of all past observed information. Following Abreu, Pearce and Staccetti [1987] we can simplify the characterization of the dynamic mechanism by using the Bellman decomposition. That is, in order to satisty the *Sequential Incentive Compatibility* constraints for agent one, it is enough that at every time-event (t, z^t) and value $v_1(t, z^t)$ the Γ mechanism recommends a pair of current actions (i, τ) and conditional continuation payoffs $v_1(t+1, z^t, k_{t+1})$, satisfying the corresponding incentive compatibility inequalities. For example, if the state variables are (k, θ)[1], then the above inequality simplifies to a state-inequality:

$$v_1(k, \theta) = (1-\delta)\pi(i, \tau, k) + \delta E_{(i,k,\theta)} v_1(k\prime, \theta\prime)$$

$$\geq (1-\delta)\pi(\tilde{i}, \tau, k) + \delta E_{(\tilde{i},k,\theta)} v_1(k\prime, \theta\prime)$$

The λ-transfer mechanism

The λ-transfer mechanims is a Γ mechanims that uses as state variables (k, λ, θ) where λ^{-1} is the marginal utility of expenditure at a particular time-event The λ-transfer mechanism exploits the downward sloping property of the Pareto frontier in the full information and full enforcement (PO) problem. More specifically, with a downward slopping Pareto frontier, if $v_i(k, \theta; \lambda)$ is the present value assigned to agent i in state (k, θ) when agent one has weight λ, then there is a function $\lambda(\cdot; k, \theta)$ defined by $v_1(k, \theta; \lambda(x; k, \theta) = x$. In our problem, that the risk neutral agent bears all the

[1]Transfers τ are also observable but, since in our environments transfers are fully enforced (there are no possible deviations from τ), we can simplify our presentation by not including this variable explicitely in the set of state variables.

fluctuations, $v_1(k,\theta;\lambda) = u(c(\lambda))$. We simplify notation by letting $V_1(\lambda) \equiv u(c(\lambda))$. In the PO problem, λ is constant. In contrast, in the λ-transfer mechanism λ changes in order to guarantee the incentive constraints.

The λ mechanism is recursively constructed as follows. If the current state is (k,λ,θ), then the recommended current actions are $i(k,\theta)$ and $\tau(k,\lambda,\theta)$, the efficient actions of the PO problem with λ. The particular law of motion for λ will be given by a function h(.). We will choose this function in such a way that the expected discounted utility for the manager, denoted W, will be $W(k,\lambda,\theta) \equiv V_1(\lambda)$ and the continuation payoffs will be given by $W(k',\lambda',\theta') \equiv V_1(\lambda')$, where $\lambda' = h(k,k',\lambda,\theta,\theta')$. That is, the central element of the λ-transfer mechanism is the map h(.), which defines how the present value of the contract changes with the evolution of the state. We now define h(.).

Let $\hat{v}_2(k,k',\lambda,\theta,\theta') = v_2(k',\theta';\lambda) - E_{(i(k,\theta),k,\theta)}v_2(k',\theta';\lambda)$ That is, $\hat{v}_2(\cdot)$ is agent two's deviation from the conditional expected value; conditional on the current state and the optimal investment level.

Now, $\lambda' = h(k,k',\lambda,\theta,\theta') = V_1^{-1}(V_1(\lambda) + \lambda^{-1}.\hat{v}_2(k,k',\lambda,\theta,\theta'))$.

In other words, if the current state is (k,λ,θ) - now including λ - and, after the recommendation to follow the optimal action, the observed state is (k',θ') then agent one should suffer (gain) a deviation from $V_1(\lambda)$ of $\lambda^{-1}.\hat{v}_2(k,k',\lambda,\theta,\theta')$. That is, agent one is punished/rewarded with the deviation of agent two's utility in the PO problem, properly weighted by λ.

Proposition. The λ-transfer mechanism is a Sequentially Efficient Mechanism for an economy with limited communication and full enforcement.

Proof. The mechanism is resource feasible since it defines a sequence of feasible actions (from the corresponding PO problems). We now show that it satisfies the incentive compatibility constraints. That is,

$$
\begin{aligned}
W(k,\lambda,\theta) = (1-\delta)\pi(i(k,\theta),\tau(k,\lambda,\theta),k) + \\
\delta E_{(i(k,\theta),k,\theta)}W(k',h(k,k',\lambda,\theta,\theta'),\theta') \\
\geq (1-\delta)\pi(\tilde{i},\tau(k,\lambda,\theta),k) + \delta E_{(\tilde{i},k,\theta)}W(k',h(\lambda,k,k',\theta,\theta'),\theta')
\end{aligned}
\tag{6}
$$

By construction, (6) is simply:

$$
\begin{aligned}
V_1(\lambda) \geq (1-\delta)\pi(\tilde{i},\tau(k,\lambda,\theta),k) + \\
\delta E_{(\tilde{i},k,\theta)}[V_1(\lambda) + \lambda^{-1}\hat{v}_2(k,k',\lambda,\theta,\theta')]
\end{aligned}
\tag{7}
$$

or

$$
\begin{aligned}
V_1(\lambda) \geq (1-\delta)\pi(\tilde{i},\tau(k,\lambda,\theta),k) + \delta v_1(\lambda) + \\
\lambda^{-1}\delta E_{i(\tilde{i},k,\theta)}[v_2(k',\theta';\lambda) - E_{((k,\theta),k,\theta)}v_2(k',\theta';\lambda)]
\end{aligned}
\tag{8}
$$

This can be rearranged (using the fact that $v_1(k', \theta'; \lambda) = V_1(\lambda)$) to obtain:

$$\lambda[(1-\delta)\pi(i(k,\theta), \tau(k,\lambda,\theta), k) +$$
$$\delta E_{(i(k,\theta),k,\theta)} v_1(k', \theta'; \lambda)] + [(1-\delta)(-\tau(k,\lambda,\theta)) +$$
$$\delta E_{(i(k,\theta),k,\theta)} v_2(k', \theta'; \lambda)$$

$$\geq \lambda[(1-\delta)\pi(\tilde{i}, \tau(k,\lambda,\theta), k) + \delta E_{(\tilde{i},k,\theta)} v_1(k', \theta'; \lambda)] \tag{9}$$
$$+[(1-\delta)(-\tau(k,\lambda,\theta) + \delta E_{(\tilde{i},k,\theta)} v_2(k', \theta' \lambda)$$

This last inequality is just the optimality constraint of the full information and enforcement (PO) problem with weight λ at the state (k, θ), and by optimality of $i(k, \theta)$ the inequality is satisfied.

The above argument, however, not only shows that the λ-transfer mechanism is *sequentially-incentive compatible*, but almost demonstrates its efficiency.

By Lemma II in Marimon (1988) there exists a Sequentially Efficient Mechanism. Suppose the Sequentially Incentive Compatible mechanism Γ^* Pareto dominates the λ-transfer mechanism. Let (v_1^*, v_2^*) be the present values attained through Γ^* (for a given state (k_0, θ). Let $\lambda = V_1^{-1}(v_1^*)$ and use the initial conditions $(k_0, \theta; \lambda)$ to define the λ-transfer mechanism. That is, agent one has the same present value in both mechanisms.

Therefore, Pareto dominance requires that $v_2^* > v_2(k_0, \theta; \lambda)$. However, by the above derivation of (9) we attain a contradiction with the Pareto optimality of the solution to the full information and enforcement (PO) problem with weight λ and at the state (k_0, θ).

Since in this context (with a risk-neutral agent and no disutility for labor) optimal investment plans for the full communication-commitment economy are independent of λ (i.e.,$i^*(k, \lambda, \theta) = i(k, \theta)$), and since the λ-transfer mechanism is defined in terms of alternative solutions to the full communication-commitment problem, it follows that capital accumulation paths are not affected by private information incentive constraints.

2.4 The Environment with Full Information and Limited Enforcement (PC).

Now we assume that there is a failure in the commitment technology of this economy and that at any point in time the manager can take possession of the capital stock and switch to the autarkic solution. In this environment, the contracts that can be enforced are those where the utility the manager derives from the contract is, at each point in time, at least as high as the utility from autarky, given the state variables of the period; this means that the manager will have to be compensated so as to make his utility high enough at every period.

Optimal allocations can be found by maximizing a planner's problem giving different weights to each agent, subject to participation constraints on agent 1. These constraints ensure that agent one's welfare in every period is at least as large as the welfare he would obtain from the autarkic solution. We call this the PC model. For a given $\lambda > 0$, an optimal contract in the PC economy can be found by solving

Program 1

$$max_{\{c_t,\tau_t,i_t,k_t\}_{t=0}^{\infty}}(1-\delta)E_0\sum_{t=0}^{\infty}\delta^t[\lambda u(c_t)-\tau_t]$$

$$subject \quad to: c_t - \tau_t + i_t = f(k_t) \tag{10}$$

$$k_{t+1} = dk_t + g(i_t, \theta_{t+1}, s_{t+1}) \quad and \tag{11}$$

$$(1-\delta)E_t[\sum_{i=0}^{\infty}\delta^i u(c_{t+i})] \geq V^a(k_t, \theta_t) \tag{12}$$

for all t, where V^a is the value function under autarky. Equations (10) and (11) are the technology constraints, and equation (12) is the participation constraint that makes the utility of the first agent in every period at least as large as the utility he would obtain from switching to an autarkic regime from time t onwards.

The participation constraints can be interpreted as a restriction placed on the consumption set of the agent. In the language of Arrow-Debreu modelling, this restriction can be interpreted as a survival set; clearly, it is a convex survival set, so that the usual proof of the first and second welfare theorems would hold, and we know that the planner's problem can be decentralized as a competitive equilibrium with transfers and complete markets.[2]

The dynamic programming characterization of Program 1 is not trivial, and our treatment is of independent interest to study problems with expectational constraints. To realize the special features of Program 1 let us recall that a standard dynamic program has the following form:

Program 2

$$max_{\{x_t\}_{t=0}^{\infty}}E_0\sum_{t=0}^{\infty}\delta^t r(x_t, x_{t-1}, s_t)$$

$$s.t. \quad x_t \leq \Gamma(x_{t-1}, s_t) \tag{13}$$

[2]See Kehoe and Levine [1990] for a careful treatment of these issues in a model with participation constraints but no capital growth

(see, for example, Stokey and Lucas with Prescott [1989]) where x_t is a vector of *finite* length, s_t a stochastic shock and the functions r and Γ are known and *independent* of the choice for x_t. Unfortunately, the participation constraint (12) is not

a special case of (13): even though the conditional expectation in the left side of the participation constraint is a function of past state variables, this function depends on the whole stochastic process $\{c_t\}_{t=0}^{\infty}$ [3]. In other words, the function Γ should not depend on the choice of the endogenous variables, but the conditional expectation in equation (12) does.

Now we will rewritte the model in such a way that we can still use recursive techniques by introducing constraints involving the Lagrange multipliers. This will be useful in order to characterize the form of the solution and to find the state variables of the problem. We derive some properties of the solution analytically, and we argue that the solution is time inconsistent. A very similar approach can be used to deal with many problems in which conditional expectations appear in the constraints and the solution is time inconsistent[4]

A Dynamic Program for the Problem with Participation Constraints

The solution to Program 1 is the usual saddle point of the following Lagrangean:

Program 3

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \delta^t \{\lambda u(c_t) - \tau_t +$$

$$\mu_t[E_t[\sum_{i=0}^{\infty} \delta^i u(c_{t+i})] - V^a(k_t, \theta_t)/(1-\delta)]\}$$

subject to the technology constraints (1) - (2) and

$$\mu_t \geq 0$$

where μ_t is the Lagrange multiplier of the participation constraint at time t. Using standard arguments, one can show that the solution to this problem is the solution to Program 1.

Letting

$$M_t = \sum_{j=0}^{t} \mu_{t-j}$$

[3] Note that it si not a problem if past consumptions are an argument of this function. The problem here is that the function itself is affected by the whole stochastic process for c_t

[4] See Rojas [1991] for an application to optimal taxation.

the first order conditions of the above Lagrangean are

$$\frac{\partial \mathcal{L}}{\partial c_t} = \delta^t[(\lambda + M_t)u'(c_t) - 1] = 0 \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial i_t} = \delta^t[-1 - \delta E_t[\gamma_{t+1}\frac{\partial g_{t+1}}{\partial i_t}]] = 0 \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial k_t} = \delta^t[f'(k_t) - \delta d E_t[\gamma_{t+1}] + \gamma_t - \mu_t \frac{\partial V_t^a}{\partial k_t}/(1-\delta)] = 0 \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_t} = \delta^t[E_t[\sum_{j=0}^{\infty} \delta^j u(c_{t+j})] - V^a(k_t, s_t)/(1-\delta)] \geq 0 \tag{17}$$

$$\mu_t[E_t[\sum_{j=0}^{\infty} \delta^j u(c_{t+j})] - V^a(k_t, \theta_t)/(1-\delta)] = 0 \tag{18}$$

and the technology constraints (1) and (2). In equations (14) to (18), γ_t represents the Lagrange multiplier of constraint (2).

It is convenient to write this problem in the form of a dynamic program like Program 2 (Note that Program 3 has conditional expectations in the return function so that it in not of the form of program 2.).

With this in mind, we arrive at

Program 4

$$\mathcal{L} = (1-\delta)E_0 \sum_{t=0}^{\infty} \delta^t \{(\lambda + M_{t-1})u(c_t) - \tau_t +$$
$$\mu_t[u(c_t) - V^a(k_t, \theta_t)/(1-\delta)]\}$$

subject to (1), (2), $\mu_t \geq 0$,

$$M_t = M_{t-1} + \mu_t \quad and \quad M_{-1} = 0. \tag{19}$$

Here, we have rewritten the objective function in Problem 3 by rearranging and using the law of iterated expectations to eliminate the symbols E_t . We have introduced the law of motion for M_t as contraint (19).

Notice that both the return function and the constraints here *are* of the form of standard dynamic programs like Program 2 where the feasible set at t is a known function of the past, and using standard arguments we can show that the optimal decision for the control variables at time t is a time-invariant function of the state variables (k_t, M_{t-1}, θ_t). From this characterization of the solution we see that the variables (k_t, θ_t), which would be the 'natural' state variables for the problem, are not a sufficient statistic for all past information, and that the conditional expectations in the Euler equations depend on all past Lagrange multipliers of the participation constraints. These multipliers are appropriately summarized in M_{t-1}. Program 4 has the unusual feature of having Lagrange multipliers in a constraint.[5]

Equation (19) can be viewed as a constraint that the planner imposes on himself in order to follow the optimal path. Given that only k_t and θ_t enter in the return function and in the constraints at time t of the original Program 1, it would be physically possible for the planner to re-set $M_{t-1} = 0$ at any point in time. From the point of view of the planner, it would be optimal to do that if he wants to reoptimize at time t; however, the optimal path set at the initial period calls for a scrupulous observation of (19), and resetting M_t at a later date is suboptimal. This is another version of the time-inconsistency problem of Kydland and Prescott [1977].

In general, the decision function in time inconsistent problems changes at every period, so that these problems are sometimes very difficult to solve; we have circumvented this problem by introducing a pseudo-state variable that makes the optimal allocations a time-invariant function of a few variables. The approach we use here could applied to be many other problems; the general idea is to introduce the constraints with conditional expectations in the objective function, cancel out the signs E_t, rearrange the objective function and put an initial condition on the state variables that depend on the Lagrange multipliers until the problem is recursive with lagrange multipliers as state variables. In appendix 2 we show how to apply a similar technique in models with more complicated restrictions.

The variable M_t is an accumulation of past multipliers; roughly speaking, if the participation constraint has very often been binding in the recent past, then M_t will be high. The role of M_t in Program 4 is to shift the weight λ given to agent 1 in the objective function of the planner; when the participation constraint is binding, the optimal path calls for augmenting the weight given to agent 1 in the planner's instantaneous objective function; this increase in the weight is maintained for all future periods and consumption is higher in all future periods. Therefore, whenever (12) is binding, the planner compensates agent 1 by increasing his consumption to a certain level and leaving consumption at this level until the participation constraint binds again (at which time consumption will increase again).

Kydland and Prescott [1980] also showed how some time inconsistent models could be placed in a recursive framework in a problem of optimal taxation. They also had past Lagrange multipliers determining future decisions, and they used the expression 'pseudo-state variable' to denote these Lagrange multipliers. Unlike in our case, though, their Lagrange multiplier was the one in the budget restriction of the agent ($u'(c_t)$) and their problem was only recursive after the initial period . In models with uncertainty their approach seems to lose some recursive properties and it is not clear how it would apply to a restriction on the value function as in the PC model.

Characterization of Equilibrium in the PC model

[5] See Chari and Kehoe [1990] for a similar use of these multipliers.

In this sub-section we parameterize the function that converts investment into new capital goods as:

$$g(i_t, \theta_{t+1}, s_{t+1}) = a(\theta_{t+1} + s_{t+1})i_t/(1 + i_t) + bs_{t+1}$$

.

Consumption of agent 1 satisfies

$$u'(c_t) = 1/(\lambda + M_t), \tag{20}$$

so that c_t depends only on $(\lambda + M_t)$. On the other hand, the law of motion for M_t (equation (19)) guarantees that this state variable can only grow. Finally, if we assume that the shocks have bounded support, there exists a finite constant $\bar{V}$ such that $\bar{V} \geq V^a(k_t, \theta_t)$ with probability one. Combining all these observations we see that M_t and c_t will grow until M_t reaches a level such that

$$u(c_t) \geq \bar{V}; \tag{21}$$

this inequality means that the utility of keeping consumption constant for the whole future is higher than the upper bound on autarkic utility. After M_t reaches this level, consumption will not change, since the participation constraint will never be binding again and M_t will be constant from then on.

We can now study the behavior of investment. With the above functional form for g, the first order conditions (15) and (16) reduce to

$$(1 + i_t)^2 = \delta E_t[(\theta_{t+1} + s_{t+1})a \sum_{j=0}^{\infty} (\delta d)^j \alpha k_{t+j+1}^{\alpha-1}] - \tag{22}$$

$$\delta E_t[(\theta_{t+1} + s_{t+1})a \sum_{j=0}^{\infty} (\delta d)^j \mu_{t+j+1} \frac{\partial V_{t+j+1}^a}{\partial k_{t+j+1}} /(1 - \delta)]$$

Also, we recall that the Euler equation for the case with full enforcement and full information is exactly like (22) without the second conditional expectation that depends on future $\mu's$. This tells us that investment is lower in periods when the participation constraint is likely to be binding in the near future; in this case, the second expectation in equation (22) has a high absolute value, and the left hand side must go down. Therefore, in order to know how low investment will be in this period we must determine under what conditions the participation constraint will be binding.

There are effects that determine how often the participation constraint will be binding and each works in opposite directions. First, it is clear that the lower $(\lambda + M_{t-1})$, the lower current consumption of agent 1 is, and the more likely it is that the participation constraint will be binding in the near future. On the other hand if capital is high, the value of autarky is high, so that it is more likely that the constraint will be binding.

We have seen that when M_t reaches a high enough level the participation constraint will never again be binding. Therefore, when this happens, the second expectation in the right side of (22) vanishes and investment will always be equal to the level of optimal investment with full enforcement. In the initial periods, however, when the constraints are binding and M_t is growing, we still do not know what the behavior of investment will be. To determine this we will resort to simulations of the model in section 4.

3 An Algorithm for Solving the Growth Model with Incentive Constraints

We will explain here how to obtain numerical solutions for the different models specified in the previous section. There are four models that we want to solve: autarky equilibrium (AU), Pareto Optimal with full communication and full enforcement (PO), the model with participation constraints (PC) and under private information, with incentive compatibility constraints (PI).

We use the following functional forms:

$$f(k_t) = k_t^\alpha$$

$$g(i_t, \theta_{t+1}, s_{t+1}) = a(\theta_{t+1} + s_{t+1})i_t/(1 + i_t) + bs_{t+1}$$

$$u(c_t) = c_t^{\gamma+1}/(\gamma + 1)$$

$$\log \theta_t = \rho \log \theta_{t-1} + \varepsilon_t$$

where ε_t is i.i.d. Marimon (1989) discussed how the function g makes it impossible for the investor to infer the level of investment from the capital stock series, thereby making the private information problem interesting.

3.1 Solving the autarkic equilibrium

The AU has the following first order conditions:

$$c_t^\gamma = \delta E_t[\frac{\partial g_{t+1}}{\partial i_t} \sum_{j=0}^{\infty} (\delta d)^j c_{t+j+t}^\gamma \alpha k_{t+j+1}^{\alpha-1}] \tag{23}$$

$$c_t + i_t = k_t^\alpha \tag{24}$$

$$k_{t+1} = dk_t + g(i_t, \theta_{t+1}, s_{t+1}) \tag{25}$$

To solve this model numerically we use the parameterized expectation approach (PEA). Since there is only one expectation to approximate, the model can be solved quite easily. We substitute the expectation in the right side of (23) by a parameterized function of the state variables $\psi(\beta, k_t, \theta_t)$. We choose ψ in a flexible way so as to approximate the conditional expectation arbitrarily well. In particular, we choose

$$\psi(\beta, k_t, \theta_t) = exp(P_n(\log(k_t), \log(\theta_t))$$

for a given n, here P_n is a polynomial of degree n. The parameters β are the coefficients in the polynomial. We can, in principle, increase the degree of the polynomial until we have a reasonable approximation to the conditional expectation. This functional form is convenient because it is strictly positive, as is the expression inside the conditional expectation that it intends to predict.

We want to find the parameter β_f with the following property: if agents use β_f in order to form the expectations of the Euler equation, then $\psi(\beta_f, k_t, \theta_t)$ is the best predictor among functions $\psi(., k_t, \theta_t)$.

The mechanics for finding β are the following:

-Step 1 - fix β. Substitute the conditional expectation in (23) by ψ to obtain:

$$c_t^{\gamma}(\beta) = \delta\psi(\beta, k_t(\beta), \theta_t). \tag{26}$$

-Step 2 - obtain a long series of the endogenous variables that solves (24), (25) and (26)[6] for this particular β; call this series $\{c_t(\beta), i_t(\beta), k_t(\beta)\}$

-Step 3 - for this series calculate the expressions inside the conditional expectation of (23)[7] and perform a non-linear regression of these variables on $\psi(., k_t(\beta), \theta_t)$; let $S(\beta)$ be the result of this regression.

-Step 4 - finally, use an iterative scheme to find the fixed point of S, and set $\beta_f = S(\beta_f)$ [8].

The solution for consumption, investment and capital is given by $\{c_t(\beta_f), i_t(\beta_f), k_t(\beta_f)\}$.

3.2 Solving for first periods with a low initial capital

The scheme just described can, in principle, approximate the true equilibrium arbitrarily well in the steady state distribution. However, if the economy starts at a very low capital stock k_0, the β_f from long run simulations may not be a good approximation for the first few periods, as the capital stock grows from k_0 to the steady

[6]Note that this is quite simple: $c_t(\beta)$ can be solved directly from (26), $i_t(\beta)$ from (24) and $k_{t+1}(\beta)$ from (25)

[7]Note how the sums $\sum_{j=1}^{\infty}(\delta d)^j c_{t+j}^{\gamma} \alpha k_{t+j}^{\alpha-1}$ can be calculated very efficiently using backward recursion.

[8]For more detailed description of this approach see Marcet [1989]. For details on the implementation of the algorithm in a simple growth model see den Haan and Marcet [1990].

state distribution. For example, in the first few periods marginal productivity of capital is very high and the long run simulations will not take this into account. This could be a problem for our paper because we are particularly interested in analyzing growth of the economy in the initial periods.

To avoid this problem we find a different policy function (a different β_f) for the initial periods by running many realizations of a given (short) length T, starting each realization at k_0 instead of running a long realizationb of the process. Step 2 is modified as follows:

Step 2b - Obtain a large number N of (independent) realizations of length T, that solve (24), (25) and (26); each initial capital is fixed at k_0 [9].

To obtain arbitrary accuracy in $S(\beta)$, we let $N \to \infty$. Here, T is selected to be long enough for the economy to get in the range of the steady state distribution. In our model, and for the parameters we selected, $T = 50$ was appropriate.

Then we proceed with Step 3 and 4 as before.

One final modification is needed. In the conditional expectation we find discounted sums of future variables, like

$$\sum_{i=0}^{\infty}(\delta d)^i \alpha k_{t+i+1}^{\alpha-1} c_{t+i+1}^{\gamma};$$

these are used in the non-linear regression of Step 4. Since for $t+i > T$ the model is close to the steady state distributions, we could run simulations of length $T+T'$, where T' is large so that the truncated sum is close to the infinite sum, and the k_{t+i}, for whitch $t+i > T$ are calculated with the steady state β_f . This is not a good solution, however, because it requires long simulations, as T' may have to be quite large.

Instead, we note that for $t < T$, the expectation in the right side of (23) can be rewritten as

$$E_t\Big(\frac{\partial g_{t+1}}{\partial i_t}[[\sum_{j=0}^{\infty}(\delta d)^j c_{t+j}^{\gamma} \alpha k_{t+j}^{\alpha-1}]+$$

$$(\delta d)^{T-t+1} E_{T+1}[\sum_{j=0}^{\infty}(\delta d)^j c_{T+1+j}^{\gamma} \alpha k_{T+1+j}^{\alpha-1}]]\Big)$$

The expectation conditional on information at $T+1$ involves only variables at the steady state distribution, so we can parameterize it as a polynomial function of the state variables, and find the parameters in this polynomial by running (only one) regression, with a long simulation at the steady state β_f.

So, the variable predicted in the regression of Step 3 for these periods is

[9] A similar approach was used by Marshall [1988].

$$\frac{\partial g_{t+1}}{\partial i_t}[[\sum_{j=0}^{T-t}(\delta d)^j c^{\gamma}_{t+j+1}\alpha k^{\alpha-1}_{t+j+1}] + (\delta d)^{T-t+1}\psi^{ss}(\bar{\beta}, k_{T+1}, \theta_{T+1})]$$

where ψ^{ss} is the result of the non linear regression described in the previous paragraph.

3.3 Solving the Pareto optimal equilibrium with full communication and full enforcement (PO)

The first order conditions are:

$$1 = \delta E_t[\frac{\delta g_{t+1}}{\partial i_t}\sum_{j=0}^{\infty}(\delta d)^j \alpha k^{\alpha-1}_{t+j+1}] \tag{27}$$

$$c_t + i_t - \tau_t = k_t^{\alpha} \tag{28}$$

$$k_{t+1} = dk_t + g(i_t, \theta_{t+1}, s_{t+1}) \tag{29}$$

$$\lambda c_t^{\gamma} = 1 \tag{30}$$

where λ is given.

This way of writing the first order condition is not conveniente when we try to apply PEA to this model. The reason is that in Step 2 we could not solve for all the variables in Step 2; because equation (27) does not help in solving for any variable.

This is a common situation in PEA and it can be solved by rewriting the first order conditions in a way that we can solve for the endogenous variables. In this model, noting that

$$\frac{\partial g_{t+1}}{\partial_{i_t}} = \frac{(\theta_{t+1} + s_{t+1})}{(1 + i_t)^2}a,$$

we rewrite (27) as

$$(1 + i_t)^2 = \delta E_t[a(\theta_{t+1} + s_{t+1})\sum_{j=0}^{\infty}(\delta d)^j \alpha k^{\alpha-1}_{t+1+j}] \tag{31}$$

Now we can use (31) to find investment, (30) for consumption, (28) for τ_t etc.

As in the autarkic equilibrium we have to calculate the solution in the first periods of growth, using the same approach as in section 3.2.

3.4 Solving the Problem with Full Information and Limited Enforcement.(PC)

Now we discuss how to solve the model described in section 2.4 numerically with PEA, where agent 1 (the manager) is guaranteed at least as much utility as in the autarkic equilibrium in every period, and where both agents observe all the shocks. This model is harder to solve than the previous ones because of the presence of inequality constraints that are binding in some periods and non-binding in others. Further, we now have one additional expectation to parameterize and the additional state variable M_t.

From our discussion of section 2.4 we see that the following equations have to be satisfied:

$$c_t - \tau_t + i_t = f(k_t) \tag{32}$$

$$k_{t+1} = dk_t + g(i_t, \theta_{t+1}, s_{t+1}) \tag{33}$$

$$\mu_t \left[u(c_t) + E_t[\sum_{j=1}^{\infty} \delta^j u(c_{t+j})] - V^a(k_t, \theta_t)/(1-\delta) \right] = 0, \tag{34}$$

$$u'(c_t) = 1/(\lambda + \mu_t + M_{t-1}) \tag{35}$$

$$M_t = M_{t-1} + \mu_t \tag{36}$$

$$(1+i_t)^2 = \delta E_t[(\theta_{t+1} + s_{t+1})a \sum_{j=0}^{\infty} (\delta d)^j [\alpha k_{t+j+1}^{\alpha-1} - \mu_{t+j+1} \frac{\partial V_{t+j+1}^a}{\partial k_{t+j+1}}]]; \tag{37}$$

With these equations we can solve the model following steps 1 to 4 in page ... ; only step 2 which involves solving for the endogenous series is now more cumbersome.

After parameterizing the conditional expectations in equations (34) and (35) the above system provides six equations to solve for $(c_t, \tau_t, k_t, i_t, \mu_t, M_t)$. To solve for c_t and μ_t we proceed as follows: first try the case where the participation constraint (12) is non-binding, so that $\mu_t = 0$ and c_t is given by equation (35). For this solution, we check the participation constraint is satisfied; if it is, we can go on and solve for the remaining variables; otherwise we know that $\mu_t > 0$, so that the large bracket in (34) is equal to zero, which provides an equation to solve for consumption; then we can find μ_t from (35). It can be shown that μ_t will be positive by construction.

In this model the steady state distribution for investment is the same as in the PO problem with full enforcement, so the only interesting part to solve is in the first few periods as the capital stock and M_t grow to their steady state distributions. Then the scheme we use for the initial periods described in section becomes 3.2 crucial.

Finally, we note that the expression inside the conditional expectation of (37) involves the derivative of V^a. Because the productivity of investment is not known at the time investment is realized, the usual formula for the derivative of the value function does not apply (see Lucas and Stokey [1989] for this formula). In appendix 1 we find an expression for this derivative that is easy to compute.

3.5 Solving the model with incentive compatibility constraints (PI)

In Section 2.3 we saw that in order to find the equilibrium with the incentive compatible contract at a given period, for a given value of the contract, we have to find the point in the Pareto Optimal frontier that gives the same value for the full information full enforcement model, then the manager takes the same decision as he would take at that point in the PO frontier, and the continuation payoffs are calculated using the value functions of both agents at that point of the PO frontier. Hence, we need to have the decision functions and value functions readily available at all points on the PO frontier.

We first solve the PO problem for 1000 lambdas between zero and one. At each lambda, we calculate the β_f that corresponds to the expectation involved in the Euler equation and the β_f involved in the conditional expectation of the value function to calculate

$$\hat{v}_2(k_t, k_{t+1}, \lambda, \theta_t, \theta_{t+1}) = v_2(k_{t+1}, \lambda, \theta_{t+1}) - E_t[v_2(k_{t+1}, \lambda, \theta_{t+1})]$$

This is then used to calculate the continuation payoffs with the following formula

$$W(k_{t+1}, \lambda_{t+1}, \theta_{t+1}) = W(k_t, \lambda_t, \theta_t) + (1/\lambda_t)\hat{v}_2(k_t, k_{t+1}, \lambda_t, \theta_t, \theta_{t+1}$$

After W is formed, we search again for the point in the PO frontier where $v_1(\lambda_{t+1}) = W(k_{t+1}, \lambda_{t+1}, \theta_{t+1})$, use this to find transfers and investment and we go on to the next period.

4 Characterization of Equilibria and Simulation Results

In this section we characterize de behavior of the four models. We use mainly simulations that are plotted in Figures 1 to 12 at the end of the paper; also, the main results are summarized in Table 1. Those features of the models that we could characterize analytically were described in section 2 and we will often refer to them. The series plotted in Figures 1 to 12 correspond to simulations using one particular realization of the exogenous shocks for all series.

The values of the parameters used in the simulations are the following

marginal productivity of capital $\alpha = .5$

risk aversion parameter of the manager $\gamma = -3$

discount factor $\delta = .95$

autocorrelation parameter of $\log(\theta_t) = .95$

standard deviation of innovation of $log(\theta_t) = .03$

standard deviation of $s = .03$

mean of $s = .2$

undepreciated proportion of capital $d = .9$

constant in investment function $a = .6$

Given the choice of d and δ, one period can be interpreted as one year. Most values of the parameters are within the usual range that is used in neoclassical growth models, with the exception of the standard deviations of the variances, which are higher than usual. We chose as initial capital $k_0 = 1$, in order to obtain growth rates of around 3 or 4% for the first fifteen periods, which seems reasonable for developping countries.

With our numerical results, we are interested not only in illustrating the behavior of the model, but also in detecting the magnitude of the impact on growth and utility of alternative communication and commitment environments. We also want to distinguish the impact on growth from the impact on steady state distributions. More details on how the calculations were performed and on the algorithms are given in Appendix 3.

In Figures 1 to 12, the last two letters identify the environment, so '-au' denotes autarky equilibrium, '-po' Pareto optimal allocation with full information and perfect enforcement, '-pc' participation constraints and '-pi' private information, while the first few letters identify the series that is being plotted. For example, 'kpo' denotes capital in the Pareto Optimal allocation, 'c1pc' consumption of agent 1 (the manager) in the model with participation constraints and so on. For these figures, we plot the first 50 periods as representative of the initial periods, and periods 200 to 400 as representative of the steady state distribution.

4.1 Autarky versus Full Information Full Commitment

We first compare the PO environment with an autarkic environment (AU). We already argued that consumption of the manager is constant in the PO equilibrium, while the investor absorbs all the shocks. Therefore, we do not plot consumptions for the PO equilibrium since their solution is trivial and completely determined by the weight the planner gives to the manager (λ). Also, recall that capital accumulation is unaffected by the weight λ.

When the initial capital stock is low relative to the steady state distribution, in an economy with external financing the manager can borrow heavily at the beginning to enhance his investments and attain faster growth tha he would have attained in an autarkic environment (see Figures 1 and 3). However, the mean of the steady state distribution of capital and investment (see Figures 2 and 4) is not significantly different between the two environments, although the need to use capital as the only asset for self-insurance under autarky implies that the steady state distribution has a slightly higher mean in an autarkic regime. Also, we see that investment is more volatile in the Pareto Optimal case; this then is, an example where an increase in volatility of investment is not undesirable. Figure 1 indicates that the availability of external financing has a very significant effect on growth, if there is perfect information and enforcement.

Consumption for the manager is constant in the full information - enforcement environment. In contrast, in an autarkic environment with low initial capital stock, consumption grows with the capital stock and fluctuates in response to random shocks.

4.2 Private information with full enforcement (PI) vs. autarky (AU) and vs. full information with full enforcement (PO)

We showed in Section 2 that, in our model, the λ-transfer mechanism preserves the investment decisions of the full information-enforcement environment when investment decisions are observable. Therefore, capital accumulation paths for the PI coincide with the PO-paths of Figures 1-4. Figures 5 to 8 compare the behavior of consumption and utility of the manager (respectively c1- and val1-) under autarky and under private information.

Consumption is affected by the presence of (information) incentive constraints, although the manager can smooth his consumption much more than in an autarkic environment and, therefore, attain higher payoff (Figures 5-8). Also, it is interesting to note that, even though the manager starts out with a very high utility, in the long run he can be worse off under private information than under autarky.

4.3 Limited enforcement with full information (PC) vs. autarky (AU) and vs. full enforcement with full information (PO)

In section 2 we proved that, in the *steady state distribution*, the capital and investment series under participation constraints were equal to the capital accumulation in the PO model and that consumption of agent 1 was constant; thereby, transfers absorbed all the shocks. In other words, under participation constraints it is only interesting to study the series numerically for the initial periods. We are reporting the series that correspond to a λ that makes expected discounted transfers at $t = 0$ to be equal to zero, so these series correspond to the equilibrium contract.

In the PC environment the path of capital accumulation (and investment) in the first few periods is very similar to the autarky equilibrium (see Figure 1). This is remarkable since we saw that private information did not have any effect on growth. In fact, in a given realization, the capital stock can even be lower with participation constraints for certain periods (see Figure 1) ; then, it is possible for the utility of the manager to be lower under participation constraints than in autarky in certain periods (see periods 10 to 25 in Figure 9). Notice that this does not mean that the participation constraints are violated in these periods: since capital can be smaller under participation constraints, the value for agent 1 of *moving* to autarky after a few periods is lower than if he had started out in autarky.

In the model with partial enforcement, even though borrowing from the investor does not help in growing at a faster rate, it does help the manager smooth out consumption against unforeseen shocks. Figure 10 shows how consumption of the manager grows much more smoothly under participation constraints than under autarky, even though the consumption levels are similar at any point in time. So, in the PO model, borrowing and lending was used for smoothing along the growth path and against unforeseen shocks, but in the PC model it only serves the latter purpose.

The fact that in this model external financing can be used to smooth out unforeseen shocks consumption makes it possible to have a small gain in utility with respect to autarky; the gain is equivalent to an increase in consumption of 0.1% in the first period and leaving consumption constant thereafter. Cleary, with a more risk averse utility function or increasing the randomness in the economy, it would be possible to increase the utility gain in the PC model relative to the AU model.

Figure 11 tells us that transfers are negligible (of the order of 1% the level of total consumption). Recall from section 2.4 that whenever the participation constraint is binding it causes M_t to go up, so that Figure 12 tells us that the participation constraint is binding in most periods while capital is growing.

Conclusion

In this paper we demonstrate that different communication commitment technologies have large and very different effects on growth. The results indicate that limited enforcement makes the possibility of borrowing for growth useless, although borrowing is still useful in order to smooth consumption against unforeseen shocks. On the other hand, limited information permits growth levels as high as with perfect information.

This suggests that these commitment and communication breakdowns can provide a theory for explaining different growth patterns. In this paper, we refer to growth from a low initial condition for capital to the steady state distribution.

The use of simulations is crucial in obtaining several of our results and it is one example of how simulations can be used for obtaining results that are in essence theoretical. The simulations illustrate the behavior of the economy, allow us to make quantitative statements, and enrich the analysis. For example, the fact that growth under limited enforcement is as slow as under autarky could only be discovered through simulation.

Other technical contributions of the paper are in implementing the PEA solution procedure for a model with expectations in the constraint, implementing the $\lambda - transfer$ mechanism to solve models with private information and finding a recursive framework in a time inconsistent model.

Appendix 1

Computing the Derivative of the Value Function in Autarky

In order to apply PEA to the model with participation constraints we need to calculate the values inside the conditional expectations of equation (22), so we need to calculate the derivative of V^a. It is convenient to express this derivative in terms only of conditional expectations and functions of variables of the model; we now derive such a formula based on the ideas of Benveniste & Scheinkman. In the rest of this appendix, all variables correspond to the autarky equilibrium so that the superscript 'a' on the variables is suppressed.

The Bellman equation for the autarkic problem is

$$V^a(k_t, \theta_t) = \max_{\{[c_t, i_t]\}} (1-\delta)u(c_t) + \delta E_t V^a(dk_t + g(i_t, \theta_{t+1}), \theta_{t+1})\}$$

subject to the production constraint. The first order conditions of the maximization problem in the right hand side of the Bellman equation

$$u'(c_t) = \delta E_t[V^{a'}(k_{t+1}, \theta_{t+1}) \\ [\frac{\partial g_{t+1}}{\partial i_t}], \tag{38}$$

where the primes denote derivatives with respect to the first argument of each function involved in this expression.

Letting $f(k_t, \theta_t)$ be the optimal decision function for investment under autarky, we have the following identity

$$V^a(k_t, \theta_t) = (1-\delta)u[k_t^\alpha - f(k_t, \theta_t)] + \\ \delta E_t V^a[dk_t + g(f(k_t, \theta_t)), \theta_{t+1}, s_{t+1}].$$

Differentiating both sides with respect to capital we have

$$V^{a'}(k_t, \theta_t) = (1-\delta)u'(c_t)[\alpha k_t^{\alpha-1} - f'(k_t, \theta_t)] + \delta E_t \\ [V^{a'}(k_{t+1}, \theta_{t+1})[d + g'_{t+1}(i_t)f'(k_t, \theta_t)]].$$

Using (38) this reduces to

$$V^{a'}(k_t, \theta_t) = (1-\delta)u'(c_t)\alpha k_t^{\alpha-1} + \alpha d E_t[V^a(k_{t+1}, \theta_{t+1})],$$

and, by recursive substitution we have

$$V^a(k_t, \theta_t) = (1-\delta)E_t[\sum_{j=0}^{\infty}(\delta d)^j u'(c_{t+1})\alpha k_{t+j}^{\alpha-1}],$$

which is the formula that we are seeking. Note that we can approximate this derivative by parameterizing the conditional expectation as a polynomial, and that we can obtain an approximation to this derivative by running one non-linear regression after solving the model with autarky [10] .

[10] Another approach would have been the following two-step procedure: first approximate the *value*

Appendix 2

In forming Program 4 we have used some special features of the model at hand. Similar ideas could be applied to a number of different models where expectations appear in the restrictions in different ways. For example, if the expectations enter in a non-linear way, so that (3) is replaced by

$$\phi[E_t[\sum_{i=0}^{\infty}\delta^i u(c_{t+i})], k_t, s_t] \geq 0 \tag{39}$$

for a known function ϕ, then it is not posible to rearrange the objective function of Program 3 in a direct way in order to obtain Program 4. In this case, one should write

$$\max\ E_0\sum_{t=0}^{\infty}\delta^t[\lambda u(c_t) - \tau_t + \mu_t\phi[U_t, k_t, s_t]] \tag{40}$$

subject to technology constraints, $\mu_t \geq 0$ and

$$u_t = E_t[\sum_{i=0}^{\infty}\delta^i u(c_{t+i})] \tag{41}$$

Then, (41) can be put in the Lagrangian as we did with equation (3) above and the accumulation of the multipliers of equation (41) will play the role of M_t.

Another possibility is that a given restriction involves conditional expectations in t and t-1, for example, for a given α, we could have the constraint

$$E_t[\sum_{i=0}^{\infty}\delta^i u(c_{t+i})]$$

$$\geq \alpha E_{t-1}[\sum_{i=0}^{\infty}\delta^i u(c_{t-1+i})] \tag{42}$$

then, if we tried to go from Program 3 to Program 4 directly we could not apply the law of iterated expectations and eliminate the symbol E_{t-1}. Nevertheless, we can rewrite this as

$$U_t \geq \alpha U_{t-1} \tag{43}$$

$$U_t = E_t[\sum_{i=0}^{\infty}\delta^i u(c_{t+i})] \tag{44}$$

and proceed as before.

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function as a conditional expectation of future discounted utilities, and then take the derivative of this approximated value function. The second step here is problematic: if we use a polynomial to approximate the value function there is no reason to believe that the *derivative* of the polynomial will be close to the derivative of the value function. This procedure would be justified only if we used cubic or quadratic splines to approximate the value function.

TABLE 1

Model	Mean of Growth Rate of Output	Utility of the Manager	Mean of Capital in Steady State	Increase in Consumption
Autarky	2.88%	-7.72	2.57	
PO	3.90%	-7.16	2.53	3.84%
PC	2.91%	-7.66	2.53	0.39%
PI	3.90%	-7.20	2.53	3.55%

Note:"Mean of the growth rate..." refers to the mean during the first fifteen periods across independent realizations. The utility of the manager is measured at time zero and using many independent replications of the model, conditioning on $k_0 = 1$ but drawing the initial shock θ_0 from the steady state distribution. The "Increase in consumption" refers to the permanent increase in consumption that would equal the present value achieved in the autarkic environment with the present values achieved in the other environments.

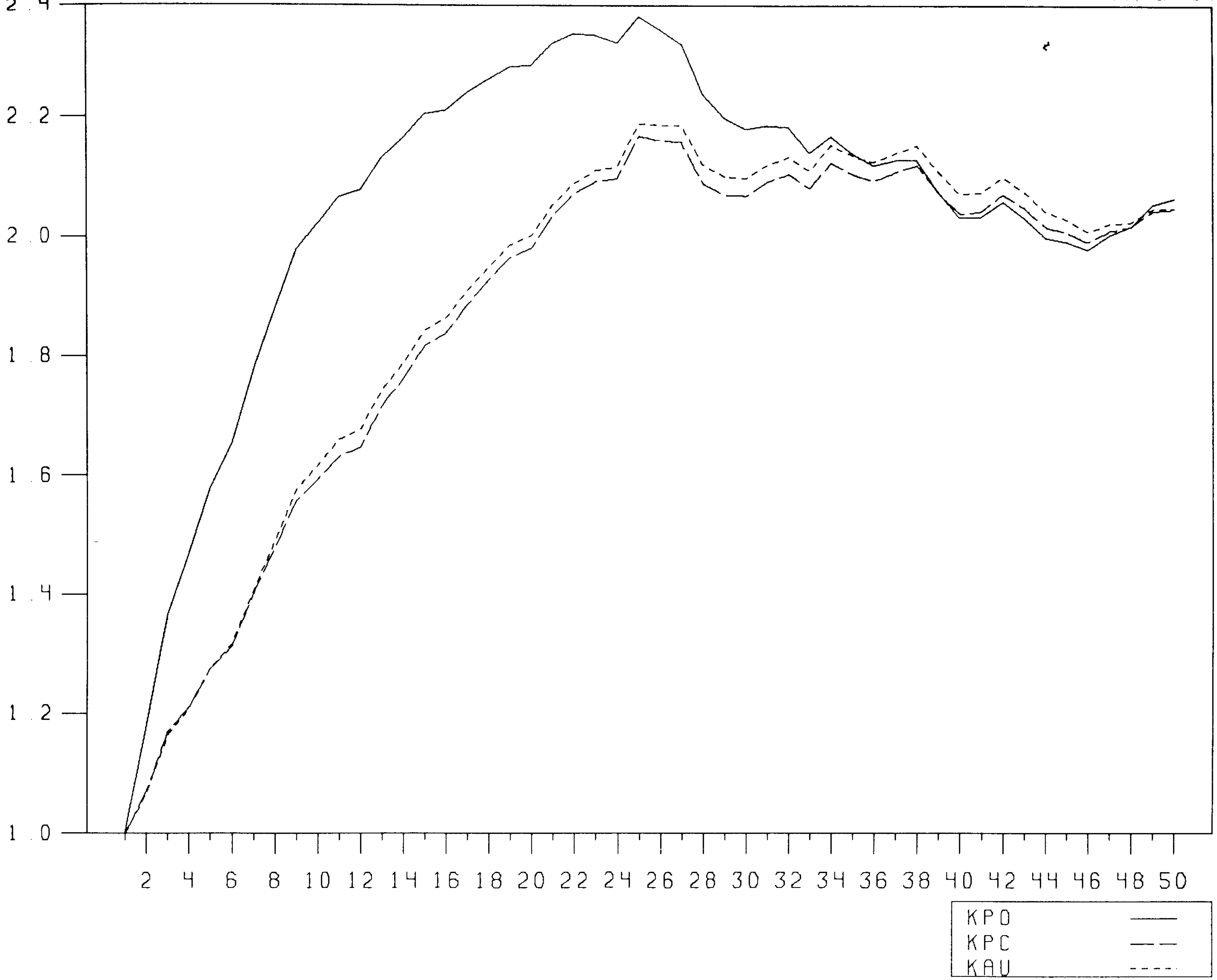

2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
KPO
KPC
KAU

Figure 2

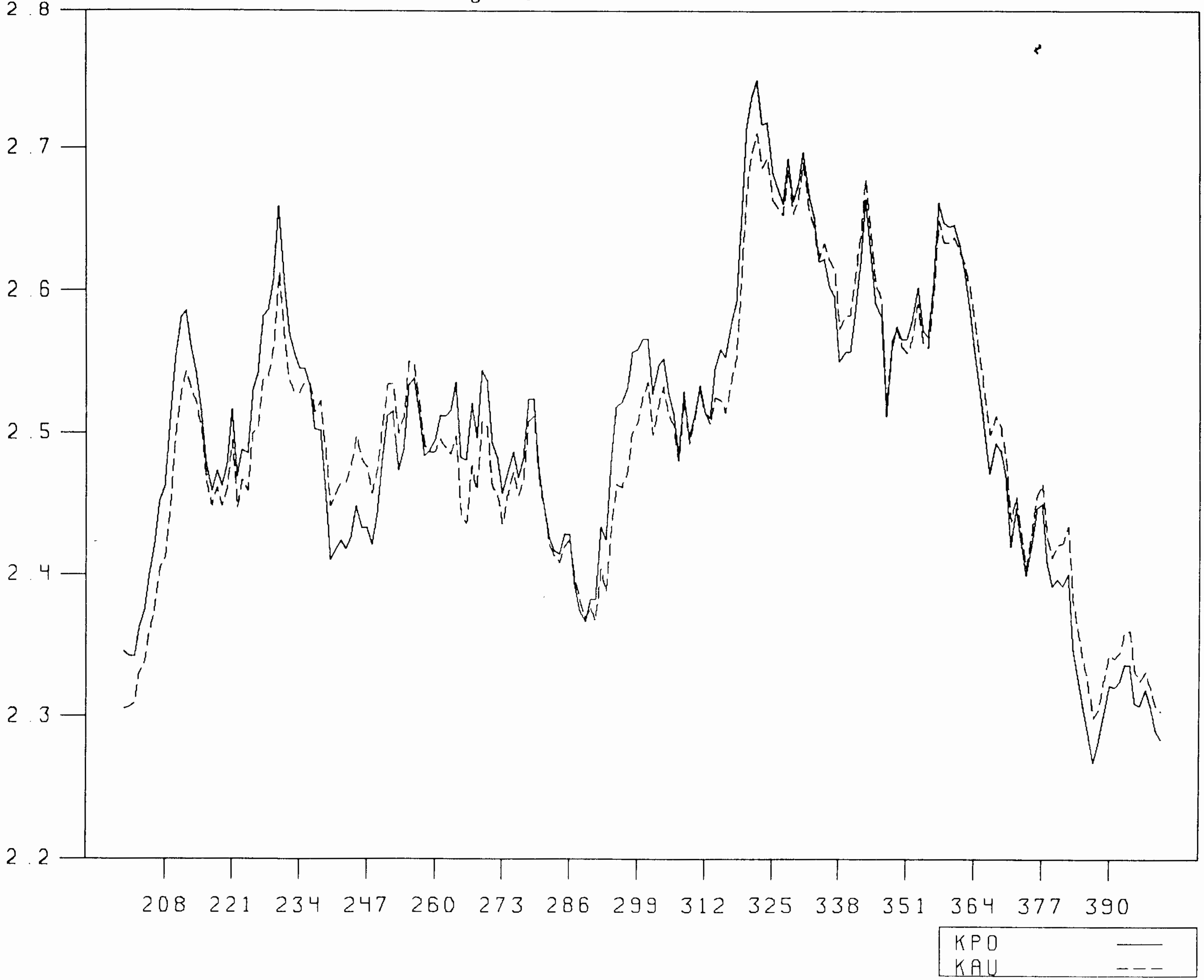
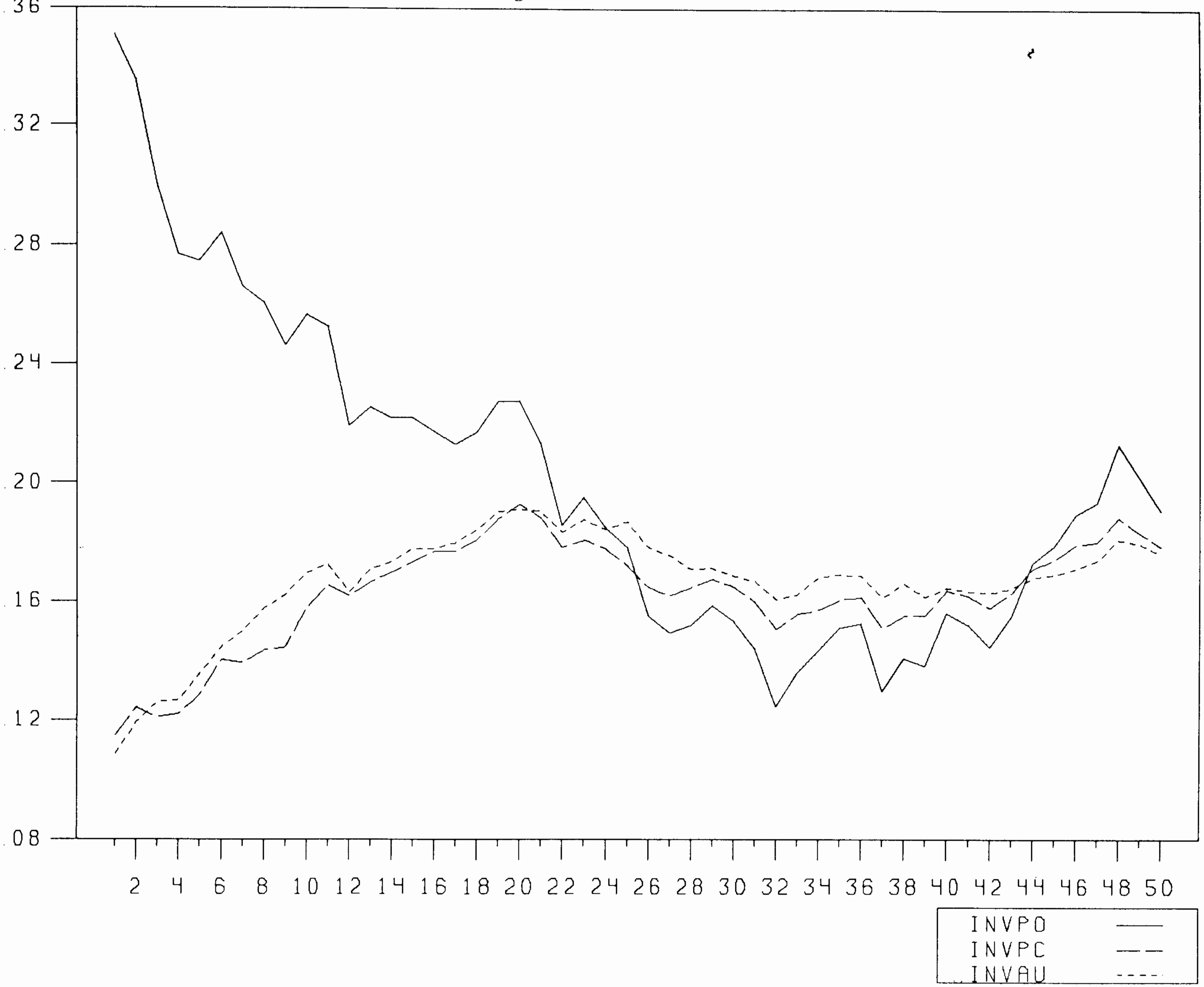
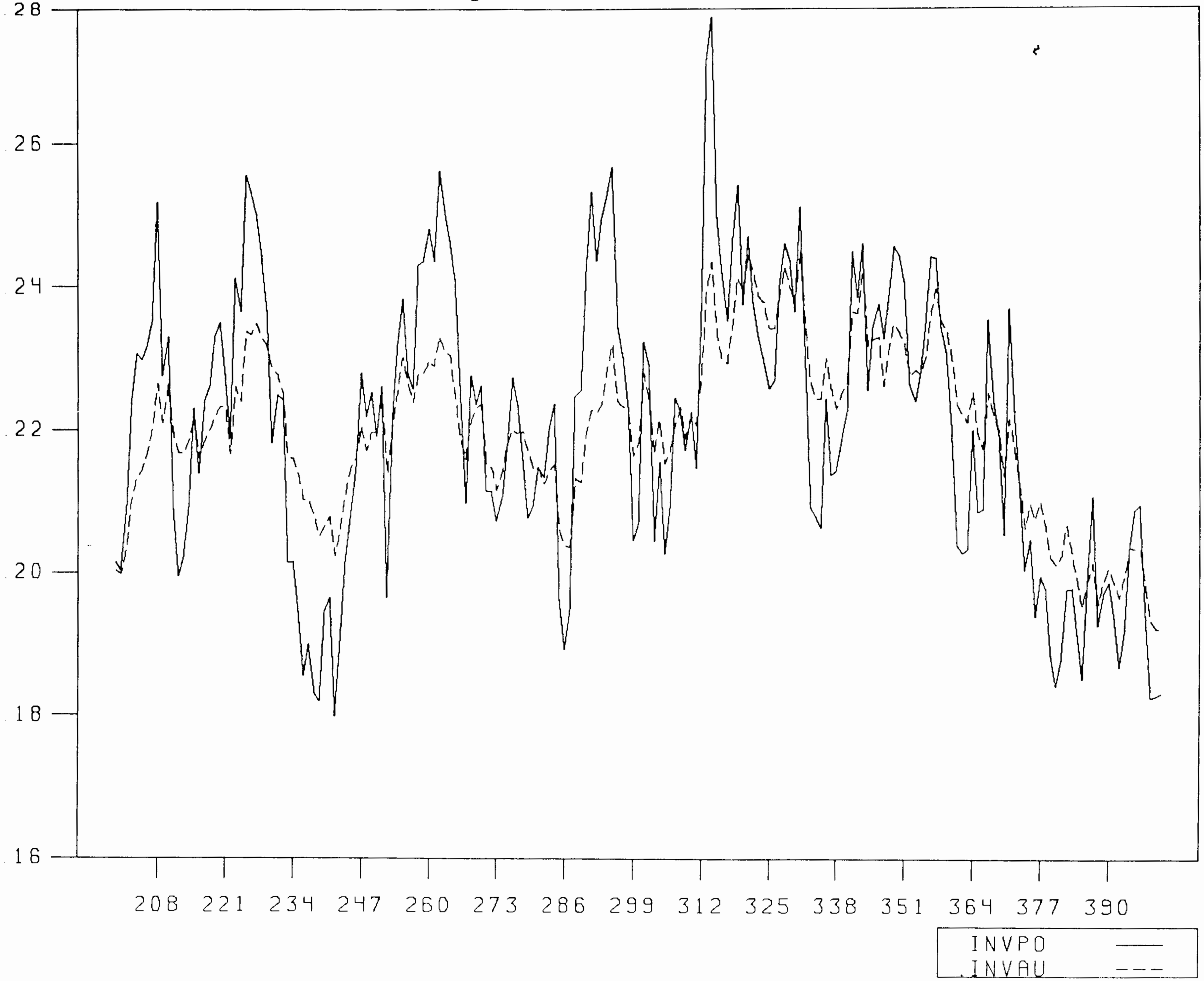

2.8
2.7
2.6
2.5
2.4
2.3
2.2
208
221
234
247
260
273
286
299
312
325
338
351
364
377
390
KPO
KAU

Figure 3



.36
.32
.28
.24
.20
.16
.12
.08
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
INVPO
INVPC
INVAU

Figure 4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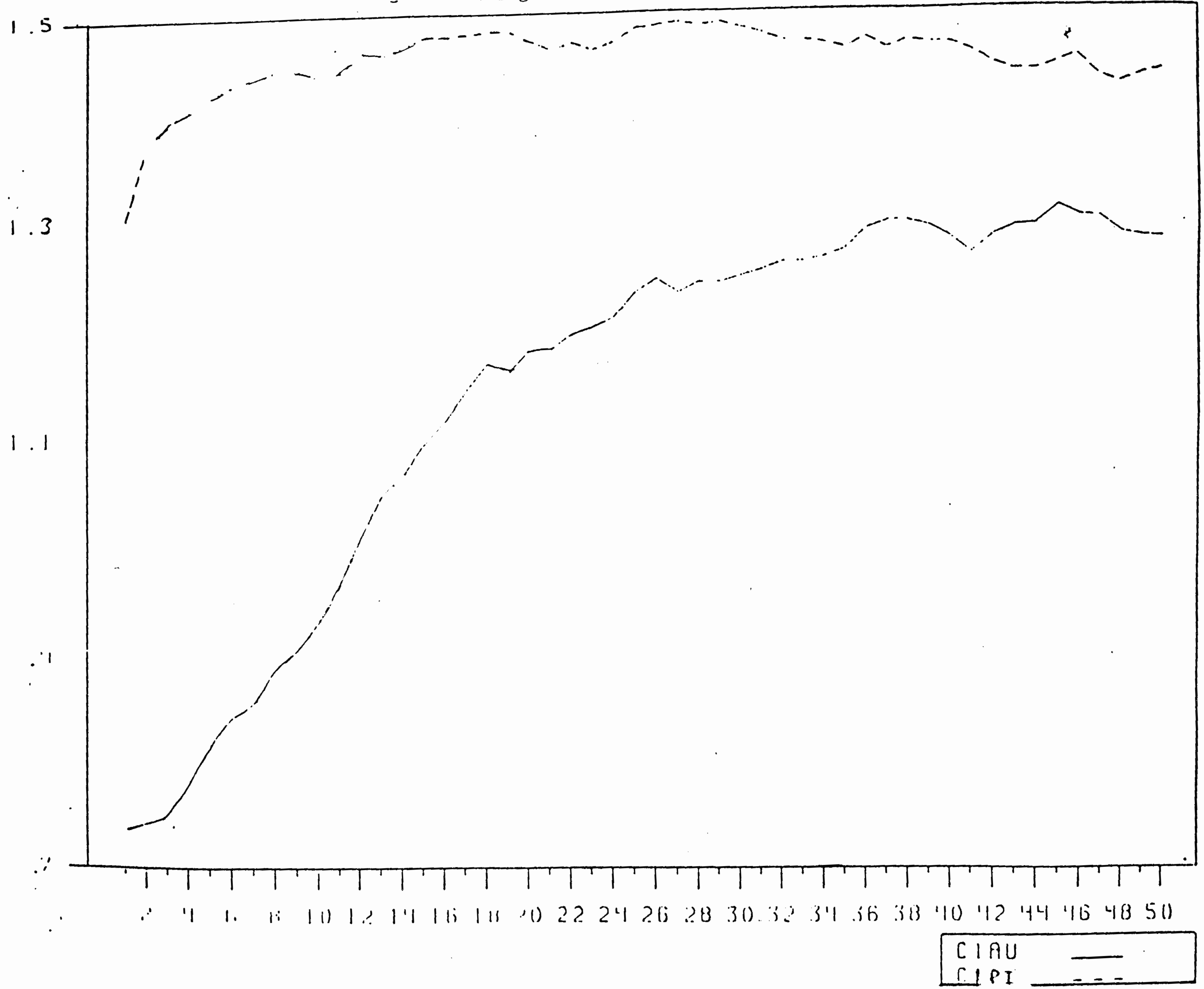
26
24
22
20
18
16
208
221
234
247
260
273
286
299
312
325
338
351
364
377
390
INVPO
INVAU

Figure 5



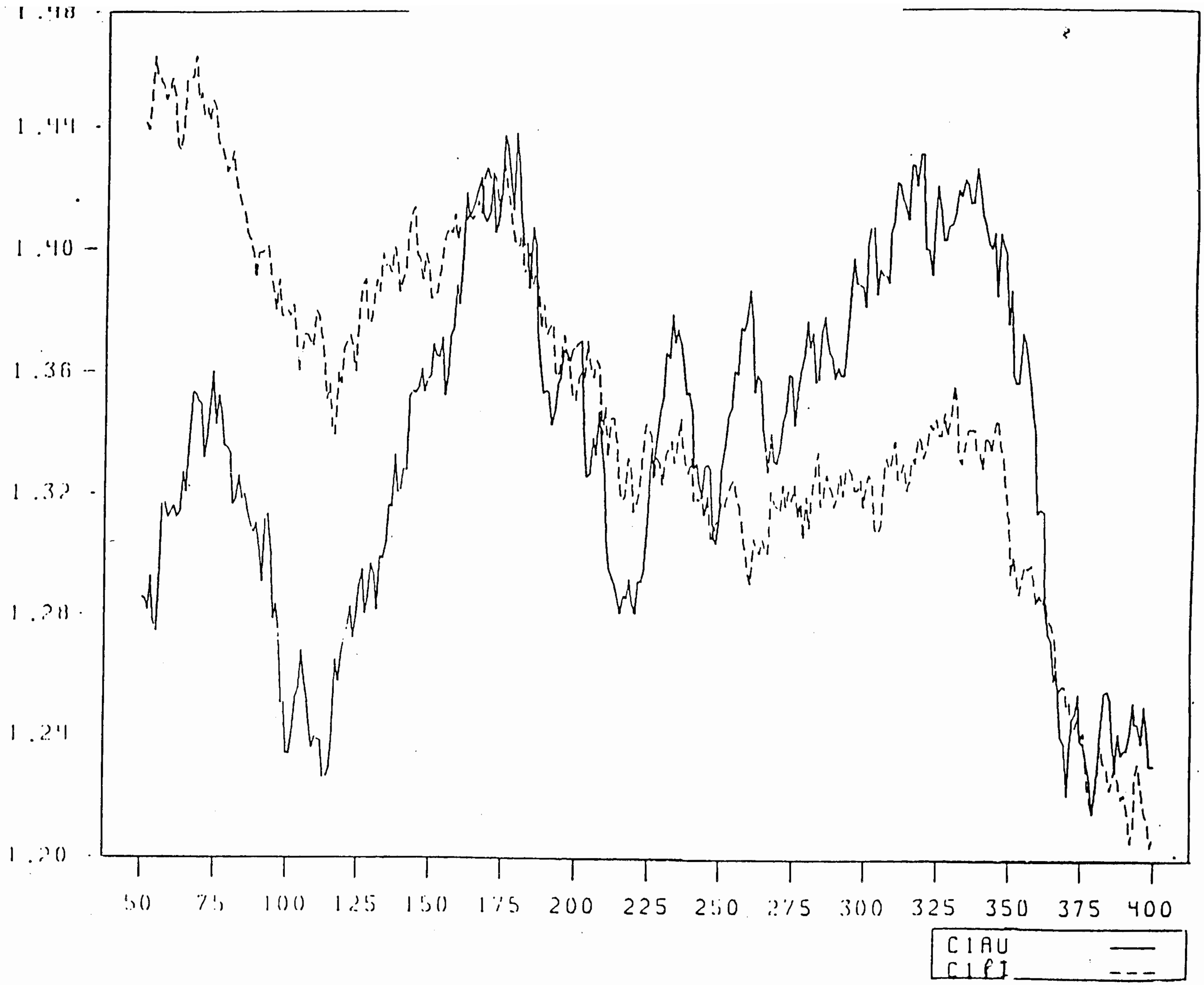

1.48
1.44
1.40
1.36
1.32
1.28
1.24
1.20
50
75
100
125
150
175
200
225
250
275
300
325
350
375
400
CIAU
CIAI

Figure 7

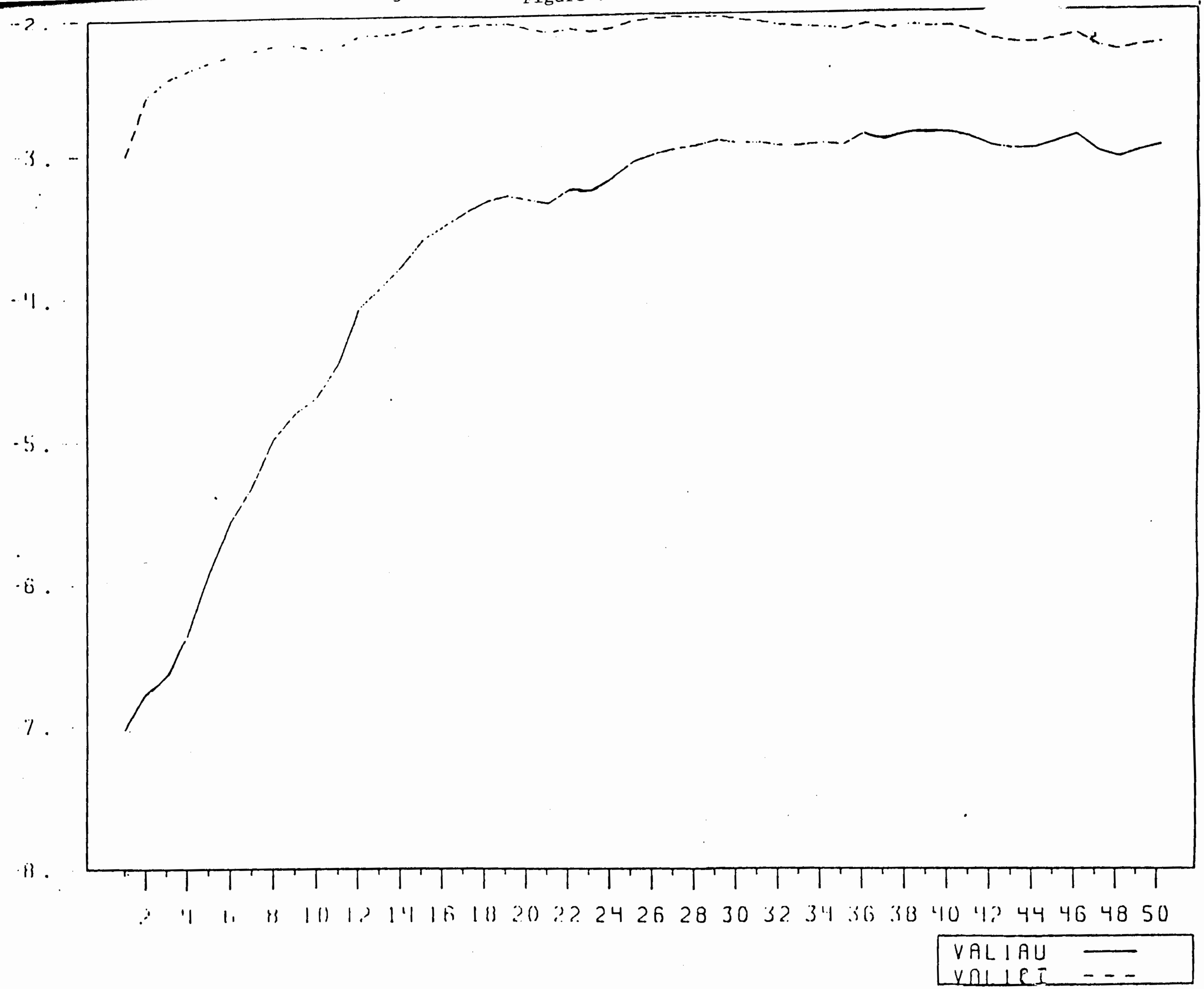
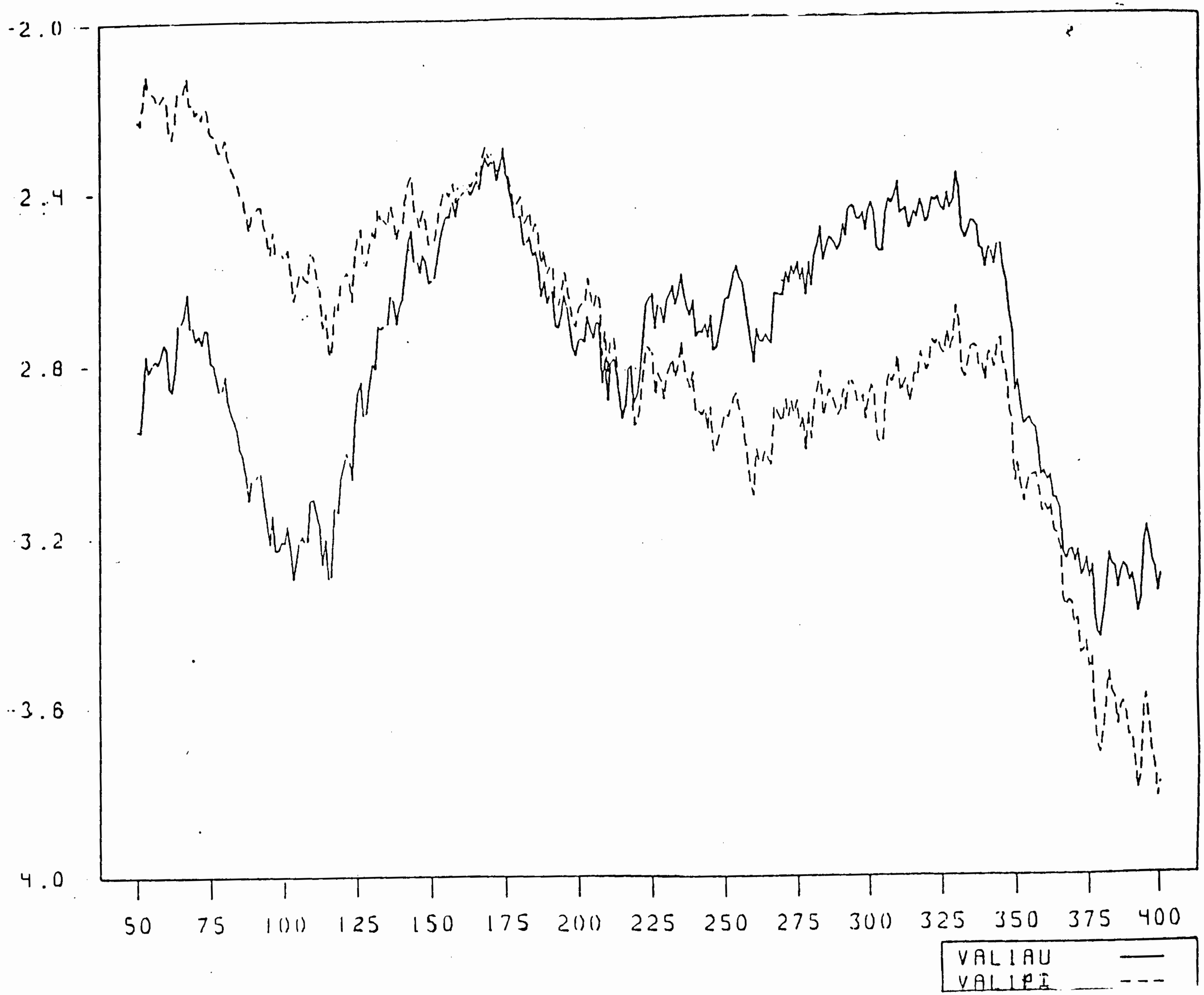
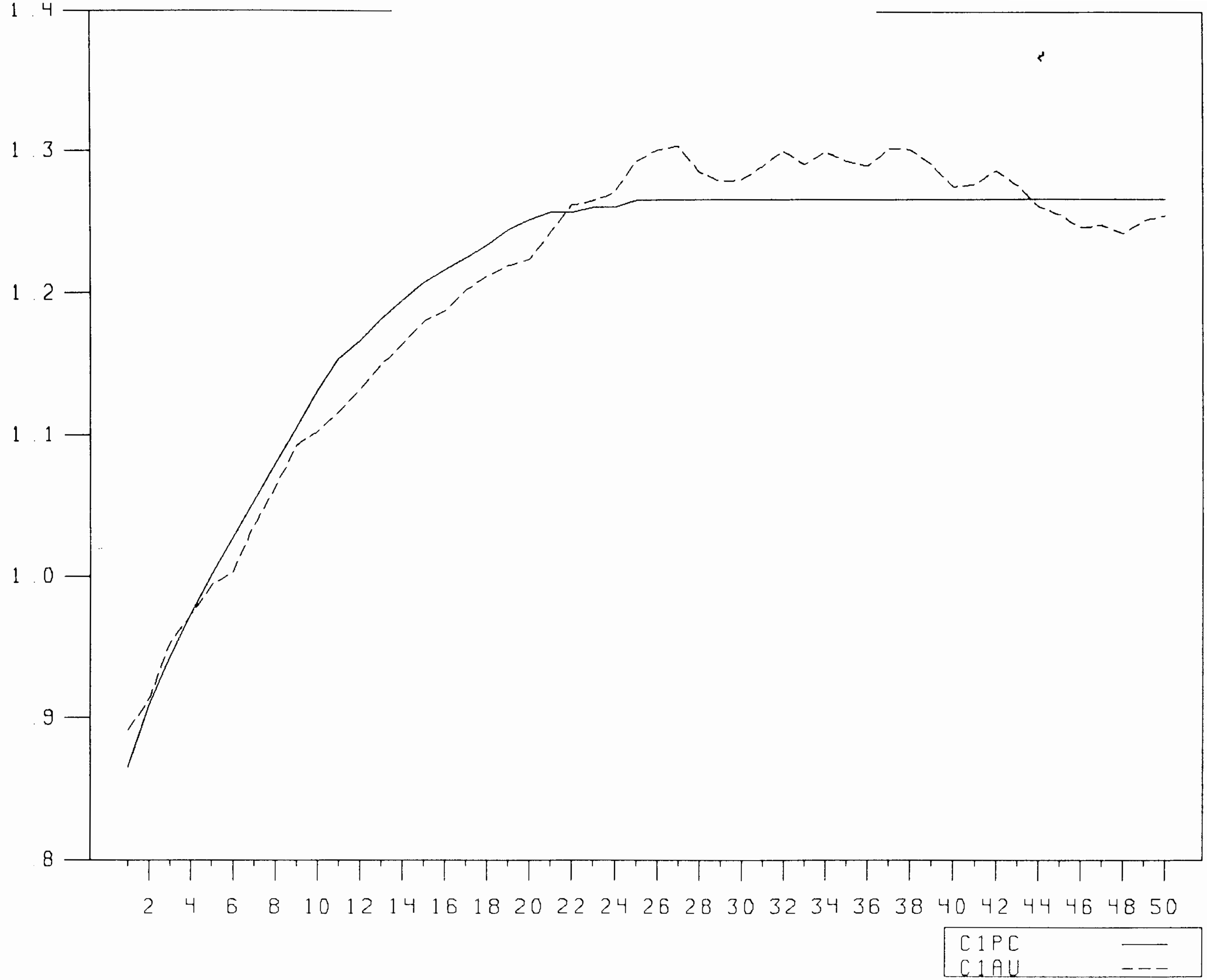

-2.
-3.
-4.
-5.
-6.
-7.
-8.
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
VALIAU
VALIET

Figure 8



-2.0
2.4
2.8
3.2
3.6
4.0
50
75
100
125
150
175
200
225
250
275
300
325
350
375
400
VALIAU
VALIPI


1.4
1.3
1.2
1.1
1.0
.9
.8
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
C1PC
C1AU

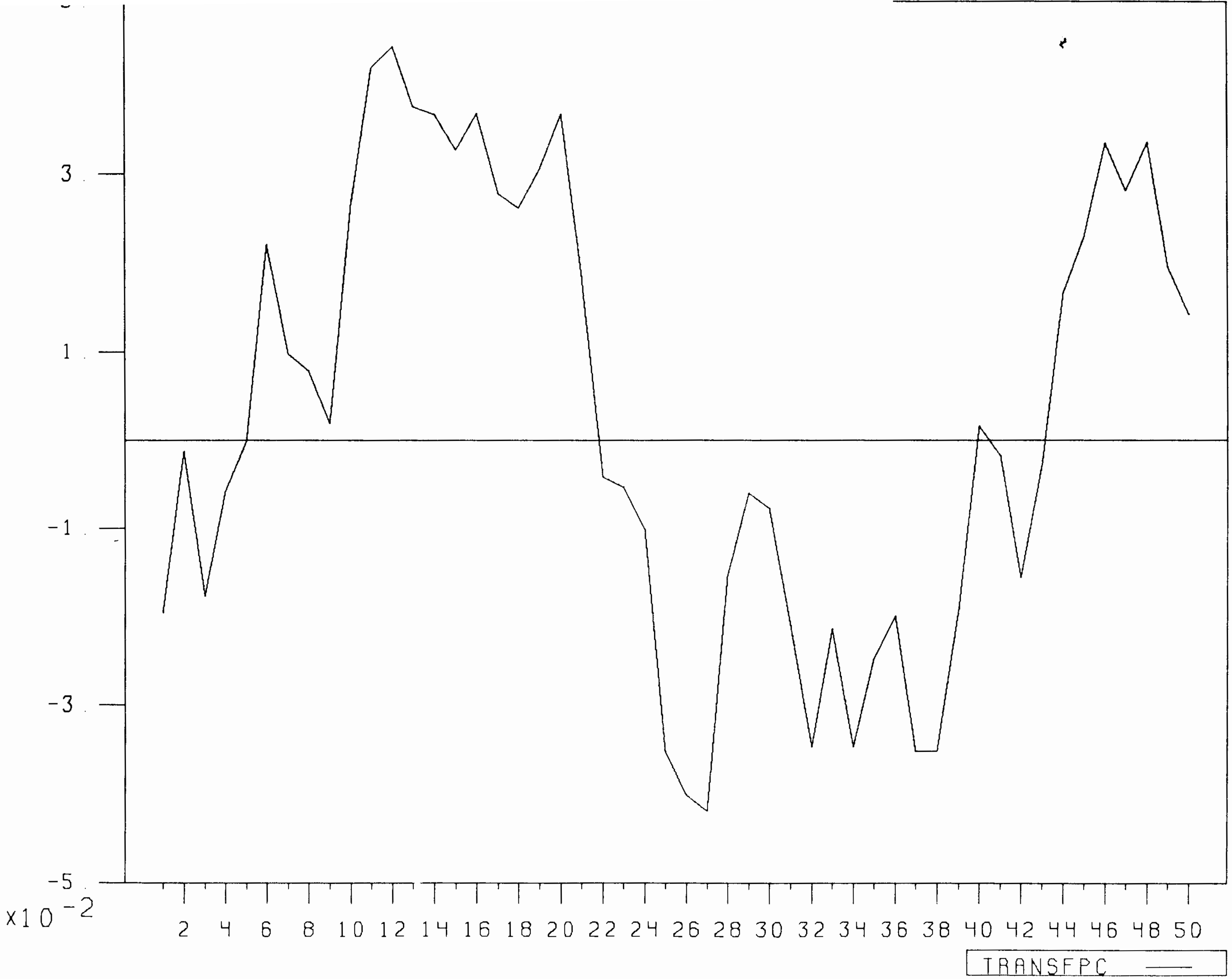
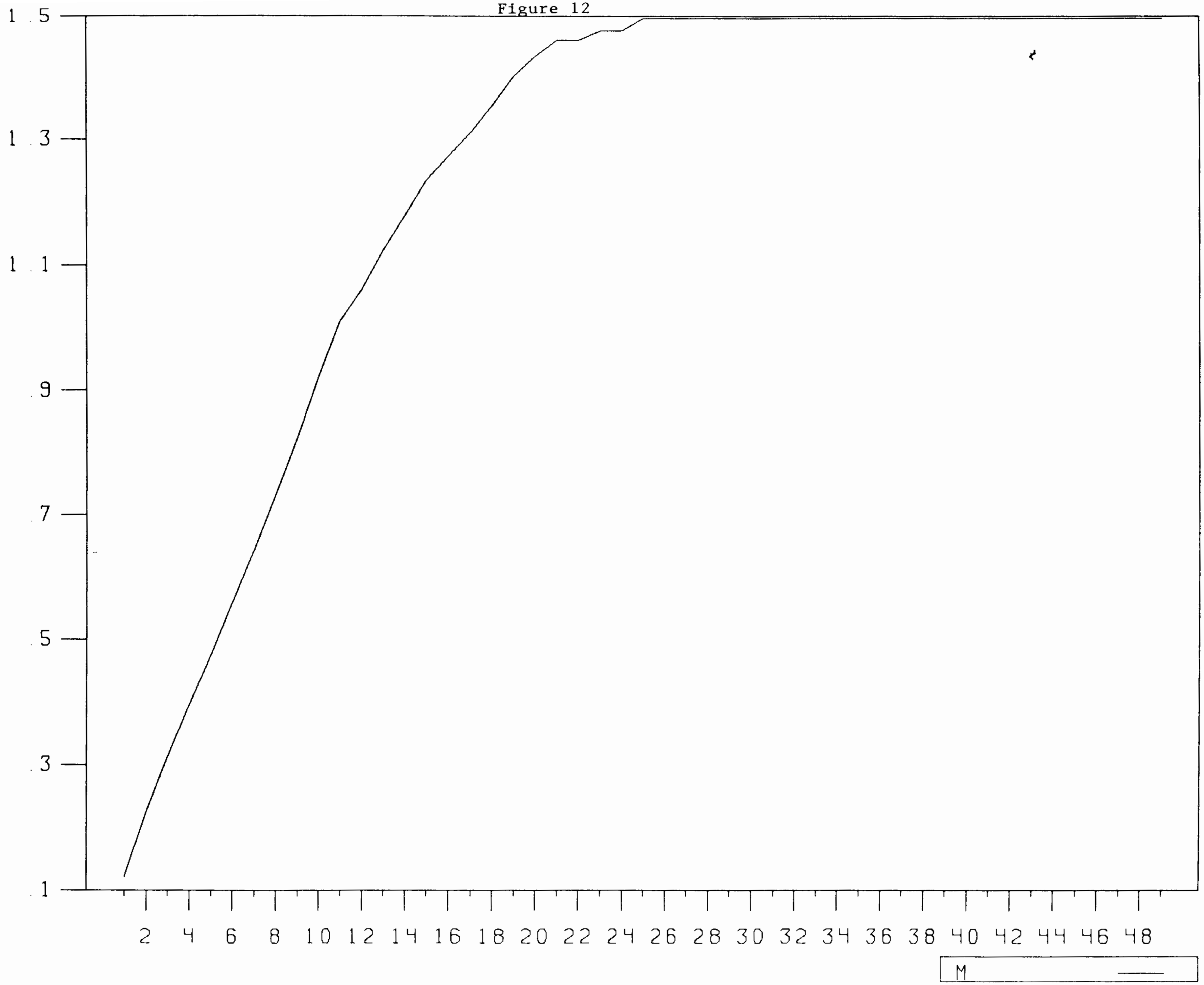

3
1
-1
-3
-5
x10^-2
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
TRANSFPC

Figure 12



1.5
1.3
1.1
.9
.7
.5
.3
.1
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48
M

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