Capacity to Within One Bit of a Class of Gaussian Multicast Channels with Interference

Sriram Sridharan
Electrical and Computer Engineering
University of Texas at Austin
Austin TX 78712, USA
Email: sridhara@ece.utexas.edu

Angel Lozano
Bell Labs (Alcatel-Lucent)
791 Holmdel-Keyport Road, Holmdel
NJ 07733, USA
Email: aloz@alcatel-lucent.com

Harish Viswanathan
Bell Labs (Alcatel-Lucent)
600 Mountain Ave., New Providence
NJ 07974, USA
Email: harishv@alcatel-lucent.com

Sriram Vishwanath
Electrical and Computer Engineering
University of Texas Austin
Austin TX 78712, USA
Email: sriram@ece.utexas.edu

***Abstract*—This paper studies the fundamental operational limits of a class of Gaussian multicast channels with an interference setting. In particular, the paper considers two base stations multicasting separate messages to distinct sets of users. In the presence of channel state information at the transmitters and at the respective receivers, the capacity region of the Gaussian multicast channel with interference is characterized to within one bit. At the crux of this result is an extension to the multicast channel with interference of the Han-Kobayashi or the Chong-Motani-Garg achievable region for the interference channel.**

I. Introduction

In a multicast channel, a transmitter communicates a common message to a plurality of receivers. In the wireless arena in particular, multicasting is poised to become a central feature of emerging wireless systems with enticing applications such as mobile TV, newscasting, etc. Relevant results on multicasting include [1]–[6].

In a cellular system, each base station can act as a separate multicast transmitter. If either different messages are communicated or the same message is communicated asynchronously, what results is a multicast channel with interference (MCI). This setting is clearly related to the classic interference channel, a well studied problem [7]–[14], but whose capacity region remains unknown except for some special cases [9]–[11]. In [14] specifically, the capacity region of the Gaussian interference channel has recently been computed to within one bit irrespective of the channel parameters.

Here, we consider a basic embodiment of the MCI where two base stations communicate distinct messages to respective groups of users (cf. Fig. 1) and we develop an achievable scheme that represents a natural extension of the Han-Kobayashi scheme [8], [13]. In the Han-Kobayashi coding scheme, each transmitter splits its message in two parts and each receiver decodes a portion of the message intended for the other user, thereby enabling partial interference cancelation.

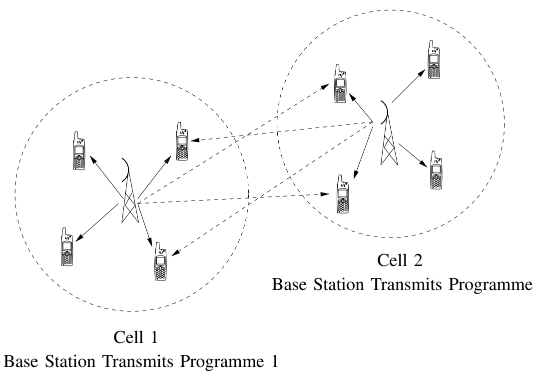

Fig. 1. Basic MCI setting.

We extend this coding scheme by allowing the transmitters to split the message into a number of small sub-messages (the actual number depends on the number of receivers associated with each transmitter). We describe a novel decoding strategy for the proposed coding scheme that enables us to demonstrate that the achievable region can be obtained straightforwardly from the achievable regions of some relevant two-user interference channels. Using the outer bounds developed in [14], we then show that the achievable region is within one bit of the capacity region. We also show that the achievable region equals the entire capacity region for some special channel parameter instances.

Notation: capital letters denote random variables, lower-case letters denote their realizations, and calligraphic letters denote alphabets. We denote vectors of length n with boldface letters (e.g. $\mathbf{x}^n$), and the i^{th} element of a vector $\mathbf{x}^n$ by x_i.

II. System Model

Let k_1 and k_2 denote the number of receivers associated with transmitters 1 and 2, respectively (cf. Fig. 2). The MCI with

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two transmitters, k_1 receivers corresponding to transmitter 1, and k_2 receivers corresponding to transmitter 2, consists of two input alphabets, $\mathcal{X}_1$ and $\mathcal{X}_2$, $k_1 + k_2$ output alphabets, $\mathcal{Y}_{1,1}, \ldots, \mathcal{Y}_{1,k_1}, \mathcal{Y}_{2,1}, \ldots, \mathcal{Y}_{2,k_2}$, and a probability transition function given by

$$p(y_{1,1}, \ldots y_{2,k_2} | x_1, x_2) = p(y_{1,1}|x_1, x_2) \ldots p(y_{2,k_2}|x_1, x_2). \tag{1}$$

For the special case of additive interference and Gaussian

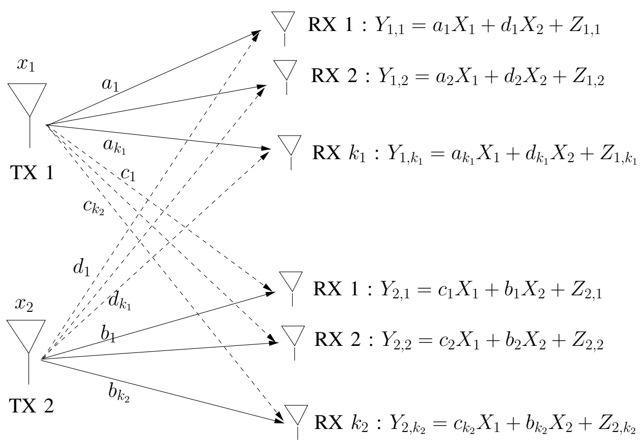

Fig. 2. System model for MCI.

noise MCI, the channel gain from each transmitter to each receiver is denoted as in Fig. 2, which also presents the input-output relationships. These gains are assumed constant over a transmission block and known by both transmitters. The receivers, in turn, are assumed to know only their own gains from each of the transmitters. Transmitter 1 communicates message M_1 to its k_1 receivers while transmitter 2 communicates message M_2 to its k_2 receivers. The codewords must satisfy the power constraints

$$\begin{aligned} &\tfrac{1}{n}\textstyle\sum_{i=1}^{n} E\left[X_{1,i}^2\right] \le P_1 \\ &\tfrac{1}{n}\textstyle\sum_{i=1}^{n} E\left[X_{2,i}^2\right] \le P_2. \end{aligned} \tag{2}$$

The additive Gaussian noise at the receivers, denoted by $Z_{1,1}, \ldots, Z_{2,k_2}$, is temporally i.i.d. and $\mathcal{CN}(0,1)$. The correlation between the noise at different receivers can be arbitrary and does not affect the capacity region of the MCI since the receivers do not co-operate with each other.

Let $\mathsf{SNR}_{i,j}$ and $\mathsf{INR}_{i,j}$ denote the signal-to-noise and interference-to-noise ratios at receiver j associated with transmitter i. Then,

$$\begin{aligned} \mathsf{SNR}_{1,j} &= |a_j|^2 P_1, \quad \mathsf{INR}_{1,j} = |d_j|^2 P_2 \\ \mathsf{SNR}_{2,j} &= |b_j|^2 P_2, \quad \mathsf{INR}_{2,j} = |c_j|^2 P_1. \end{aligned} \tag{3}$$

A $(2^{nR_1}, 2^{nR_2}, n)$ code for the MCI with independent information consists of two message sets, $M_1 \in \{1, \ldots, 2^{nR_1}\}$ and $M_2 \in \{1, \ldots, 2^{nR_2}\}$, two encoding functions

$$f_1 : M_1 \to \mathcal{X}_1^n, \qquad f_2 : M_2 \to \mathcal{X}_2^n. \tag{4}$$

such that the codewords satisfy the power constraints in (2), and $k_1 + k_2$ decoding functions

$$g_{1,1} : \mathcal{Y}_{1,1}^n \to M_1 \quad \ldots \quad g_{2,k_2} : \mathcal{Y}_{2,k_2}^n \to M_2. \tag{5}$$

Let $E_{i,j}(m_i)$ denote the event that $g_{i,j}(Y_{i,j}^n) \neq m_i$. Here, j takes values in $\{1, \ldots k_1\}$ or $\{1, \ldots, k_2\}$ depending on whether i is 1 or 2, respectively. The average probability of error, $P_e^{(n)}$, equals

$$\frac{1}{2^{n(R_1+R_2)}} \sum_{m_1, m_2} \Pr\{E_{1,1}(m_1) \cup \ldots \cup E_{2,k_2}(m_2) | (m_1, m_2)\}. \tag{6}$$

A rate pair (R_1, R_2) is achievable on the MCI if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes such that $P_e^{(n)} \to 0$. The capacity region of the MCI, denoted by $\mathcal{C}_{\mathrm{MCI}}^G$, is equal to the set of all achievable rate pairs.

Let IC_{j_1,j_2} denote the Gaussian interference channel between user j_1 associated with transmitter 1 and user j_2 associated with transmitter 2, with power constraints P_1 and P_2 and input-output relationships

$$\begin{aligned} Y_{1,j_1} &= a_{j_1} X_1 + d_{j_1} X_2 + Z_{1,j_1} \\ Y_{2,j_2} &= c_{j_2} X_1 + b_{j_2} X_2 + Z_{2,j_2}. \end{aligned} \tag{7}$$

An interference channel can be classified into three types depending on the gains. It is a strong interference channel if $c_{j_2} \ge a_{j_1}$ and $d_{j_1} \ge b_{j_2}$, it is a weak interference channel if $c_{j_2} < a_{j_1}$ and $d_{j_1} < b_{j_2}$, and otherwise it is a mixed interference channel. Let $\mathcal{C}_{\mathrm{IC}}^{(j_1,j_2)}$ denote the capacity region of IC_{j_1,j_2}. An immediate observation is that, for $1 \le j_1 \le k_1$ and $1 \le j_2 \le k_2$, we have $\mathcal{C}_{\mathrm{MCI}}^G \subseteq \mathcal{C}_{\mathrm{IC}}^{(j_1,j_2)}$.

III. Main Results

The Chong-Motani-Garg region for the interference channel uses 3 auxiliary random variables [13], [15]. In our achievable region, we will use $2k_1k_2 + 1$ auxiliary random variables. Let $\mathcal{Q}$, $\mathcal{U}_1, \ldots, \mathcal{U}_{k_1k_2}$ and $\mathcal{V}_1, \ldots, \mathcal{V}_{k_1k_2}$ be those auxiliary random variables defined on arbitrary sets with $\mathcal{Q}$ corresponding to the time-sharing parameter. Let $\mathbf{U}^k$ and $\mathbf{V}^k$ denote, respectively, the vectors $(U_1, \ldots, U_k)$ and $(V_1, \ldots, V_k)$. Let $\mathcal{P}$ be the set of probability distributions $P(q, u_1, \ldots, u_{k_1k_2}, v_1, \ldots, v_{k_1k_2}, x_1, x_2)$ that factor as

$$\begin{aligned} P(q) \times \left(\Pi_{i=1}^{k_1k_2} P(u_i|q)\right) \times P(x_1|\mathbf{u}^{k_1k_2}, q) \\ \times \left(\Pi_{i=1}^{k_1k_2} P(v_i|q)\right) \times P(x_2|\mathbf{v}^{k_1k_2}, q). \end{aligned} \tag{8}$$

For each $(j_1, j_2) \in \{1, \ldots, k_1\} \times \{1, \ldots, k_2\}$, let $r(j_1, j_2)$ and $s(j_1, j_2)$ be integers such that $r(j_1, j_2)$, $s(j_1, j_2) \in \{0, \ldots, k_1k_2\}$. Let $\mathbf{r}$ and $\mathbf{s}$ denote the vectors $(r(1,1), r(1,2), \ldots, r(k_1,k_2))$ and $(s(1,1), s(1,2), \ldots, s(k_1,k_2))$ respectively. Define $\hat{U} = U^{r(j_1,j_2)}$ and $\hat{V} = V^{s(j_1,j_2)}$. We take any $P \in \mathcal{P}$ and fix $\mathbf{r}$ and $\mathbf{s}$.

Let $\mathcal{R}^P_{j_1,j_2}$ be the set of rate pairs (R_1, R_2) that satisfy

$$\begin{aligned}
R_1 &\le I(X_1; Y_{1,j_1}|\hat{V}Q) \\
R_2 &\le I(X_2; Y_{2,j_2}|\hat{U}Q) \\
R_1 + R_2 &\le I(X_1\hat{V}; Y_{1,j_1}|Q) + I(X_2; Y_{2,j_2}|\hat{U}\hat{V}Q) \\
R_1 + R_2 &\le I(X_1; Y_{1,j_1}|\hat{U}\hat{V}Q) + I(X_2\hat{U}; Y_{2,j_2}|Q) \\
R_1 + R_2 &\le I(X_1\hat{V}; Y_{1,j_1}|\hat{U}Q) + I(X_2\hat{U}; Y_{2,j_2}|\hat{V}Q) \\
2R_1 + R_2 &\le I(X_1\hat{V}; Y_{1,j_1}|Q) + I(X_1; Y_{1,j_1}|\hat{U}\hat{V}Q) \\
&\quad + I(X_2\hat{U}; Y_{2,j_2}|\hat{V}Q) \\
R_1 + 2R_2 &\le I(X_2; Y_{2,j_2}|\hat{U}\hat{V}Q) + I(X_2; \hat{U}; Y_{2,j_2}|Q) \\
&\quad + I(X_1\hat{V}; Y_{1,j_1}|\hat{U}Q).
\end{aligned} \tag{9}$$

Let $\mathcal{R}_{\text{in}}$ denote the closure of the convex hull of the set of rate pairs (R_1, R_2) described by

$$\bigcup_{P\in\mathcal{P}} \bigcup_{\mathbf{r},\mathbf{s}} \bigcap_{(j_1,j_2)} \mathcal{R}^P_{j_1,j_2}. \tag{10}$$

Then, we have the following theorem.

Theorem 3.1: The capacity region of the MCI satisfies

$$\mathcal{R}_{\text{in}} \subseteq \mathcal{C}^G_{\text{MCI}}. \tag{11}$$

Proof: See Section IV.

The region described by $\mathcal{R}_{\text{in}}$ is in fact an achievable region for any memoryless MCI. Next, we describe an achievable region for the Gaussian case in particular.

Let us fix $(j_1, j_2) \in \{1, \ldots, k_1\} \times \{1, \ldots, k_2\}$. Let $\mathcal{R}^G_{j_1,j_2}$ be the achievable region of the Gaussian IC_{j_1,j_2} as described in [14] (cf. Appendix). We define $\mathcal{R}^G_{\text{in}}$ to be the set of rate pairs given by

$$\bigcap_{1\le j_1\le k_1} \bigcap_{1\le j_2\le k_2} \mathcal{R}^G_{j_1,j_2}. \tag{12}$$

Then, the following theorem describes an achievable region for the Gaussian MCI.

Theorem 3.2: The capacity region of the MCI satisfies

$$\mathcal{R}^G_{\text{in}} \subseteq \mathcal{C}^G_{\text{MCI}}. \tag{13}$$

Proof: See Section IV.

The following lemma establishes that the achievable region given by $\mathcal{R}^G_{\text{in}}$ is within one bit of the capacity region of the Gaussian MCI. That is, if (R_1, R_2) lies on the boundary of $\mathcal{R}^G_{\text{in}}$, then (R_1+1, R_2+1) lies outside the capacity region of the MCI.

Lemma 3.1: The achievable region $\mathcal{R}^G_{\text{in}}$ is within one bit of the capacity region of MCI, $\mathcal{C}^G_{\text{MCI}}$.

Proof: For any $1 \le j_1 \le k_1$ and $1 \le j_2 \le k_2$, the capacity region of IC_{j_1,j_2} is a superset of $\mathcal{C}^G_{\text{MCI}}$. Let (R_1, R_2) lie on the boundary of $\mathcal{R}^G_{\text{in}}$. Then, (R_1, R_2) lies on the boundary of $\mathcal{R}^G_{j_1,j_2}$ for some $(j_1, j_2) \in \{1, \ldots, k_1\} \times \{1, \ldots, k_2\}$. From [14], it follows that $(R_1 + 1, R_2 + 1)$ lies outside the boundary of $\mathcal{C}^{(j_1,j_2)}_{\text{IC}}$, the capacity region of IC_{j_1,j_2}. Hence, $(R_1 + 1, R_2 + 1)$ lies outside the boundary of $\mathcal{C}_{\text{MCI}}$. ∎

The following lemma looks at a special case of the MCI where the achievable region described by $\mathcal{R}^G_{\text{in}}$ is equal to the capacity region of the MCI.

Lemma 3.2: Suppose $\min(c_1, \ldots, c_{k_2}) \ge \max(a_1, \ldots, a_{k_1})$ and $\min(d_1, \ldots, d_{k_1}) \ge \max(b_1, \ldots, b_{k_2})$, then $\mathcal{R}^G_{\text{in}} = \mathcal{C}_{\text{MCI}}$.

Proof: All the k_1k_2 interference channels are strong interference channels. Hence, $\mathcal{R}^G_{j_1,j_2}$ equals the capacity region of IC_{j_1,j_2} for each j_1 and j_2 and thus $\mathcal{R}^G_{\text{in}} = \mathcal{C}^G_{\text{MCI}}$. ∎

IV. Proofs of Theorems 3.1 and 3.2

Proof of Theorem 3.1: We fix a probability distribution $P \in \mathcal{P}$. For each $(j_1, j_2) \in \{1, \ldots, k_1\} \times \{1, \ldots, k_2\}$, choose $0 \le r(j_1, j_2) \le k_1k_2$ and $0 \le s(j_1, j_2) \le k_1k_2$. We describe encoding and decoding schemes to show that $\bigcap_{j_1,j_2} \mathcal{R}^P_{j_1,j_2}$ is achievable, where $\mathcal{R}^P_{j_1,j_2}$ is described in (9).

Encoding is as follows: A codeword $\mathbf{Q}^n$ is generated according to $\Pi^n_{i=1}P(q)$. For each $1 \le i \le k_1k_2$, transmitter 1 generates $2^{nR_{1,i}}$ independent codewords $\mathbf{U}^n_i$ according to $\Pi^n_{j=1}P(u_{ij}|q_j)$. For each $(i_1, \ldots i_{k_1k_2}) \in \{1, \ldots, 2^{nR_{1,1}}\} \times \ldots \times \{1, \ldots, 2^{nR_{1,k_1k_2}}\}$, generate $2^{nR_{1,k_1k_2+1}}$ independent codewords $\mathbf{X}^n_1$ according to the distribution $\Pi^n_{j=1}P(x_{1j}|u_{1j}(i_1), \ldots, u_{k_1k_2\ j}(i_{k_1k_2}), q_j)$, where $u_{1j}(i_1)$ denotes the j^{th} sample of the i^{th}_1 codeword of $\mathbf{U}^n_1$ and so on. Hence, if the message of transmitter 1 corresponds to index $(i_1, \ldots, i_{k_1k_2+1})$, transmitter 1 sends the codeword $\mathbf{X}^n_1(i_1, \ldots, i_{k_1k_2+1})$.

Similarly, for each $1 \le i \le k_1k_2$, transmitter 2 generates $2^{nR_{2,i}}$ independent codewords $\mathbf{V}^n_i$ according to $\Pi^n_{j=1}P(v_{ij}|q_j)$. For each $(l_1, \ldots, l_{k_1k_2}) \in \{1, \ldots, 2^{nR_{2,1}}\} \times \ldots \times \{1, \ldots, 2^{nR_{2,k_1k_2}}\}$, generate $2^{nR_{2,k_1k_2+1}}$ independent codewords $\mathbf{X}^n_2$ according to the distribution $\Pi^n_{j=1}P(x_{2j}|v_{1j}(l_1), \ldots, v_{k_1k_2j}(l_{k_1k_2}), q_j)$, where $v_{1j}(l_1)$ denotes the j^{th} sample of the l^{th}_1 codeword of $\mathbf{V}^n_1$ and so on. Hence, if the message of transmitter 2 corresponds to index $(l_1, \ldots, l_{k_1k_2+1})$, transmitter 2 sends the codeword $\mathbf{X}^n_2(l_1, \ldots, l_{k_1k_2+1})$.

Decoding: Each receiver corresponding to transmitter 1 splits into k_2 virtual receivers. Similarly, each receiver associated with transmitter 2 splits into k_1 virtual receivers. Let $1 \le j_1 \le k_1$ and $1 \le j_2 \le k_2$. Consider virtual receiver j_2 within receiver j_1 associated with transmitter 1. This virtual receiver attempts to find unique indices $(i_1, \ldots, i_{k_1k_2+1}, l_1, \ldots, l_{r(j_1,j_2)})$ such that

$$\Big(\mathbf{Q}^n, \mathbf{U}^n_1(i_1), \ldots, \mathbf{U}^n_{k_1k_2}(i_{k_1k_2}), \mathbf{X}^n_1(i_1, \ldots, i_{k_1k_2+1}), \mathbf{V}^n_1(l_1), \ldots \mathbf{V}^n_{r(j_1,j_2)}(l_{r(j_1,j_2)}), \mathbf{Y}^n_1\Big) \tag{14}$$

is jointly typical.

Similarly, virtual receiver j_1 within receiver j_2 receiver associated with transmitter 2 attempts to find unique indices $(l_1, \ldots, l_{k_1k_2+1}, i_1, \ldots, i_{s(j_1,j_2)})$ such that

$$\Big(\mathbf{Q}^n, \mathbf{V}_1^n(l_1), \ldots, \mathbf{V}_{k_1k_2}^n(l_{k_1k_2}), \mathbf{X}_2^n(l_1, \ldots, l_{k_1k_2+1}), \mathbf{U}_1^n(i_1), \ldots \mathbf{U}_{s(j_1,j_2)}^n(i_{r(j_1,j_2)}), \mathbf{Y}_2^n\Big) \quad (15)$$

is jointly typical.

Using steps similar to those used in [12, Lemma 3], the above decoding process can be shown to be successful if, for each $(j_1, j_2) \in \{1, \ldots, k_1\} \times \{1, \ldots, k_2\}$, the rate pair (R_1, R_2) lies in the set $\mathcal{R}_{j_1,j_2}^{P*}$ described by:

$$\begin{aligned}
R_1 &\leq I(X_1; Y_{1,j_1}|\hat{V}Q) \\
R_1 &\leq I(X_1; Y_{1,j_1}|\hat{U}\hat{V}Q) + I(X_2\hat{U}; Y_{2,j_2}|\hat{V}Q) \\
R_2 &\leq I(X_2; Y_{2,j_2}|\hat{U}Q) \\
R_2 &\leq I(X_2; Y_{2,j_2}|\hat{U}\hat{V}Q) + I(X_1\hat{V}; Y_{1,j_1}|\hat{U}Q) \\
R_1 + R_2 &\leq I(X_1\hat{V}; Y_{1,j_1}|Q) + I(X_2; Y_{2,j_2}|\hat{U}\hat{V}Q) \\
R_1 + R_2 &\leq I(X_1; Y_{1,j_1}|\hat{U}\hat{V}Q) + I(X_2\hat{U}; Y_{2,j_2}|Q) \\
R_1 + R_2 &\leq I(X_1\hat{V}; Y_{1,j_1}|\hat{U}Q) + I(X_2\hat{U}; Y_{2,j_2}|\hat{V}Q) \\
2R_1 + R_2 &\leq I(X_1\hat{V}; Y_{1,j_1}|Q) + I(X_1; Y_{1,j_1}|\hat{U}\hat{V}Q) \\
&\quad + I(X_2\hat{U}; Y_{2,j_2}|\hat{V}Q) \\
R_1 + 2R_2 &\leq I(X_2; Y_{2,j_2}|\hat{U}\hat{V}Q) + I(X_2; \hat{U}; Y_{2,j_2}|Q) \\
&\quad + I(X_1\hat{V}; Y_{1,j_1}|\hat{U}Q).
\end{aligned} \quad (16)$$

The set $\mathcal{R}_{j_1,j_2}^{P*}$ has two additional inequalities (one for R_1 and one for R_2) when compared with $\mathcal{R}_{j_1,j_2}^{P}$. To achieve a rate pair (R_1, R_2), an element of $\mathcal{R}_{j_1,j_2}^{P}$ but not that of $\mathcal{R}_{j_1,j_2}^{P*}$, we use steps similar to those used in [12, Lemma 2]. Precisely, the receivers choose $r(j_1, j_2)$ or $s(j_1, j_2)$ as zero and successful decoding takes place if the rate pair $(R_1, R_2) \in \mathcal{R}_{j_1,j_2}^{P}$ for the original choice of $r(j_1, j_2)$ and $s(j_1, j_2)$ for each (j_1, j_2). Hence, successful decoding is possible if (R_1, R_2) lies in the intersection of $\mathcal{R}_{j_1,j_2}^{P}$. ■

Proof of Theorem 3.2: This result follows from Theorem 3.1 by choosing the auxiliary random variables appropriately. For each $(R_1, R_2) \in \mathcal{R}_{\text{in}}^{G}$, we will demonstrate that there exist auxiliary variables $U_i, V_i, \hat{U}, \hat{V}$ and integers $r(j_1, j_2), s(j_1, j_2)$ for each pair (j_1, j_2) such that the inequalities in (9) are satisfied.

To this end, we set the time sharing auxiliary random variable $\mathcal{Q} = \{\phi\}$. The auxiliary random variables $U_1, \ldots, U_{k_1k_2}$ and $V_1, \ldots, V_{k_1k_2}$ are chosen to be independent and Gaussian with zero mean. The channel inputs X_1 and X_2 are related to the auxiliary random variables as

$$\begin{aligned}
X_1 &= U_1 + \ldots + U_{k_1k_2} + W_1 \\
X_2 &= V_1 + \ldots + V_{k_1k_2} + W_2
\end{aligned} \quad (17)$$

where W_1 and W_2 are Gaussian with zero mean and independent of the U_i's and V_i's.

To determine the power levels for the auxiliary variables defined above we rely on the results in [14], where the authors compute an achievable region for a two-user Gaussian interference channel. Each transmitter splits its message into two parts (a private part and a common part) and allocates certain amount of power to each part. The common part of the message is decoded by the other receiver to perform partial interference cancelation. The power splitting scheme in [14] is described in the Appendix.

Consider the interference channel between user j_1 associated with transmitter 1 and user j_2 associated with transmitter 2. Using the approach described in the Appendix, we can compute the power splitting required for this interference channel since, by definition, (R_1, R_2) is in the achievable region of this two-user interference channel. We let $\alpha(j_1, j_2)$ and $\beta(j_1, j_2)$ represent the power allocated to the common part of the message by each transmitter. Let A denote the set $\{\alpha(j_1, j_2) : 1 \leq j_1 \leq k_1, 1 \leq j_2 \leq k_2\}$. Similarly, $B = \{\beta(j_1, j_2) : 1 \leq j_1 \leq k_1, 1 \leq j_2 \leq k_2\}$. Note that the sets A and B may have values that repeat themselves. Let A_{inc} and B_{inc} be the set of elements of A and B arranged in increasing order. The powers allocated to the auxiliary random variables are denoted by P_{u_i} and P_{v_i} for $1 \leq i \leq k_1k_2$. The power allocation is given by

$$\begin{aligned}
P_{u_i} &= A_{\text{inc}}(i) - A_{\text{inc}}(i-1) \\
P_{v_i} &= B_{\text{inc}}(i) - B_{\text{inc}}(i-1)
\end{aligned}, \quad 1 \leq i \leq k_1k_2. \quad (18)$$

where $A_{\text{inc}}(0) = B_{\text{inc}}(0) = 0$. The powers allocated to W_1 and W_2 are

$$\begin{aligned}
P_{W_1} &= P_1 - \textstyle\sum_{i=1}^{k_1k_2} P_{u_i} \\
P_{W_2} &= P_2 - \textstyle\sum_{i=1}^{k_1k_2} P_{v_i}.
\end{aligned} \quad (19)$$

Finally, for any j_1 and j_2, we choose $r(j_1, j_2)$ and $s(j_1, j_2)$ such that

$$\begin{aligned}
\textstyle\sum_{i=1}^{r(j_1,j_2)} P_{u_i} &= \alpha(j_1, j_2)P_1 \\
\textstyle\sum_{i=1}^{s(j_1,j_2)} P_{v_i} &= \beta(j_1, j_2)P_2.
\end{aligned} \quad (20)$$

Defining $\hat{U} = U^{r(j_1,j_2)}$ and $\hat{V} = V^{s(j_1,j_2)}$ and letting them play the role of S_1 and S_2, respectively (as defined in the Appendix), we can show that the intersection of the achievable region given by [14] of all the k_1k_2 interference channels is achievable. ■

V. Numerical Results

The achievable region derived in this paper is guaranteed to be within one bit of the capacity region. Hence, in the high-power regime the achievable region $\mathcal{R}_{\text{in}}^{G}$ in (13) can be expected to be far larger than that of other strategies like TDM. However, in the low-power regime it is possible that TDM outperforms the strategy underlying $\mathcal{R}_{\text{in}}^{G}$.

In Fig. 3, $\mathcal{R}_{\text{in}}^{G}$ is compared with the outer bound of the MCI and with the achievable region using TDM. We posit a system with 10 receivers associated to each of the two transmitters.

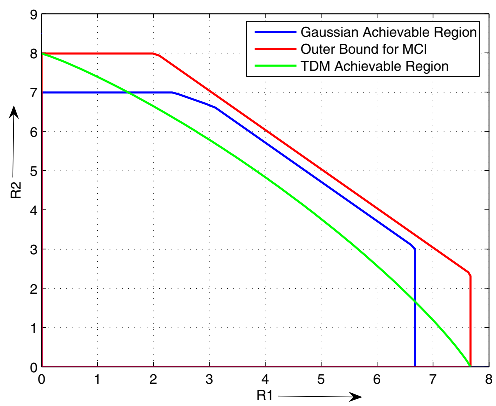

Fig. 3. Comparison of $\mathcal{R}_{\text{in}}^G$ with outer bound and with TDM.

The channels are modeled as Rayleigh-faded with the same average SNR and INR of 30dB and are generated randomly and independently. As the number of users increases, the users with lower SNR's will tend to be a burden on $\mathcal{R}_{\text{in}}^G$. In that regime of large numbers of users, it may be better to revert to TDM, whose performance itself will be within 1 bit of optimality. It should also be noted that $\mathcal{R}_{\text{in}}^G$ is obtained with a single realization of auxiliary random variables. It is possible that the limiting interference channel is a weak interference channel, in which case, $\mathcal{R}_{\text{in}}^G$ will not achieve the points corresponding to the maximum individual rates (as is the case in Fig. 3). By taking the union over all possible auxiliary random variables, a region containing the TDM region can be obtained.

Appendix

Consider the following two-user Gaussian interference channel

$$\begin{aligned} Y_1 &= aX_1 + dX_2 + Z_1 \\ Y_2 &= cX_1 + bX_2 + Z_2. \end{aligned} \tag{21}$$

where the transmitters have power constraints P_1 and P_2, respectively, and the noise at the receivers is complex Gaussian with zero mean and unit variance. Then,

$$\begin{aligned} \mathsf{SNR}_1 &= |a|^2 P_1, \qquad \mathsf{INR}_1 = |d|^2 P_2 \\ \mathsf{SNR}_2 &= |b|^2 P_2, \qquad \mathsf{INR}_2 = |c|^2 P_1. \end{aligned} \tag{22}$$

The coding strategy [14] is to choose

$$X_1 = T_1 + S_1 \quad \text{and} \quad X_2 = T_2 + S_2. \tag{23}$$

where T_1, T_2, S_1, S_2 are independent Gaussian auxiliary random variables. T_1 and T_2 represent the private parts of codewords, whereas S_1 and S_2 represent the common parts, which will also be decoded by the unintended receiver. The powers allocated to T_1, T_2, S_1 and S_2 depend on a, b, c and d. The achievable rate region is the same as that given in (9), except that we replace $\hat{U}, \hat{V}, Y_{1,j_1}, Y_{2,j_2}$ with S_1, S_2, Y_1, Y_2. We describe the power allocation to the auxiliary random variables, denoting such powers by $P_{t_1}, P_{s_1}, P_{t_2}$ and P_{s_2}.

Case 1: In a weak interference channel ($|c| < |a|$, $|d| < |b|$),

$$\begin{aligned} P_{t_1} &= \tfrac{\min(1,\mathsf{INR}_2)}{|c|^2}, \qquad P_{t_2} = \tfrac{\min(1,\mathsf{INR}_1)}{|d|^2} \\ P_{s_1} &= P_1 - P_{t_1}, \qquad P_{s_2} = P_2 - P_{t_2}. \end{aligned} \tag{24}$$

Case 2: In a mixed interference channel ($|d| \geq |b|$, $|c| < |a|$),

$$\begin{aligned} P_{t_1} &= \tfrac{\min(1,\mathsf{INR}_2)}{|c|^2}, \qquad P_{t_2} = 0 \\ P_{s_1} &= P_1 - P_{t_1}, \qquad P_{s_2} = P_2. \end{aligned} \tag{25}$$

Case 3: In a mixed interference channel ($|c| \geq |a|$, $|d| < |b|$),

$$\begin{aligned} P_{t_2} &= \tfrac{\min(1,\mathsf{INR}_1)}{|d|^2}, \qquad P_{t_1} = 0 \\ P_{s_2} &= P_2 - P_{t_2}, \qquad P_{s_1} = P_1. \end{aligned} \tag{26}$$

Case 4: In a strong interference channel ($|c| \geq |a|$, $|d| \geq |b|$), assign all the power to S_1 and S_2, i.e., $P_{t_1} = P_{t_2} = 0$. The achievable region in this case is the capacity region of the strong interference channel.

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