

# Structural changes in the US economy: Bad Luck or Bad Policy?

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## Abstract

This paper investigates the relationship between time variations in output and inflation dynamics and monetary policy in the US. There are changes in the structural coefficients and in the variance of the structural shocks. The policy rules in the 1970s and 1990s are similar as is the transmission of policy disturbances. Inflation persistence is only partly a monetary phenomena. Variations in the systematic component of policy have limited effects on the dynamics of output and inflation. Results are robust to alterations in the auxiliary assumptions.

Key words: Monetary policy, Inflation persistence, Transmission of shocks, Time varying coefficients structural VARs

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# 1 Introduction

There is considerable evidence suggesting that the US economy has fundamentally changed over the last couple of decades. In particular, several authors have noted a marked decline in the volatility of real activity and in the volatility and persistence of inflation since the early 1980s (see e.g. Blanchard and Simon (2000), McConnell and Perez Quiroz (2001) and Stock and Watson (2003)). What are the reasons behind such a decline? A stream of literature attributes these changes to alterations in the mechanisms through which exogenous shocks spread across sectors and propagate over time. Since the transmission mechanism depends on the structure of the economy, such a viewpoint implies that important characteristics, such as the behavior of consumers and firms or the preferences of policymakers, have changed over time. The recent literature has paid particular attention to monetary policy. Several studies, including Clarida, Gali and Gertler (2000), Cogley and Sargent (2001) (2005), Lubik and Schorfheide (2004), have argued that monetary policy was "loose" in fighting inflation in the 1970s but became more aggressive since the early 1980s and see in this change of attitude the reason for the observed changes in inflation and output. This view, however, is far from unanimous. For example, Bernanke and Mihov (1998), Leeper and Zha (2003), Orphanides (2004), find little evidence of significant changes in the policy rule used in the last 25-30 years while Hanson (2001) claims that the propagation of monetary shocks has been stable. Sims (2001) and Sims and Zha (2004) suggest that changes in the variance of exogenous shocks are responsible for the observed changes.

This controversy is not new. In the past rational expectations econometricians (e.g. Sargent (1984)) have argued that policy changes involving regime switches dramatically alter private agent decisions and, as a consequence, the dynamics of the macroeconomic variables, and searched for historical episodes supporting this view (see e.g. Sargent (1999)). VAR econometricians, on the other hand, often denied the empirical relevance of this argument suggesting that the systematic portion of monetary policy has rarely been altered and that policy changes are better characterized as random variations for the non-systematic part (Sims (1982)). This long standing debate now has been cast into the dual framework of "bad policy" (failure to adequately respond to inflationary pressure) vs. "bad luck" (shocks are drawn from a distribution whose moments vary over time) and new evidence has been collected thanks to the development of methods which allow to examine time variations

in the structure of the economy and in the variance of the exogenous processes. Overall, and despite recent contributions, the role that monetary policy had in shaping the observed changes in the US economy is still open.

This paper provides novel evidence on this issue. Our framework of analysis is a time varying coefficients VAR model, similar to Cogley and Sargent (2001), where the coefficients evolve according to a nonlinear transition equation which puts zero probability on paths associated with explosive roots, and we use Markov Chain Monte Carlo (MCMC) methods to estimate the posterior distributions of the quantities of interest. Cogley and Sargent (2005) and Primiceri (2005) add to this framework a stochastic volatility model for the reduced form innovations. We also allow the variance of the forecast errors to vary over time but, as in Canova (1993), we do this in a simpler and more intuitive manner, which retains conditional linearity and links changes in the variance of the coefficients to changes in the variance of the forecast errors in an economically meaningful way.

We identify structural disturbances via sign restrictions on dynamic response of certain variables to shocks. While we focus on monetary policy disturbances, the methodology is well suited to jointly identify multiple sources of structural disturbances (see e.g. Canova and De Nicolò (2002)). We choose to work with sign restrictions for two reasons. First, the contemporaneous zero restrictions conventionally used are often absent in those theoretical models one likes to use to guide the interpretation of the results. Second, while the restrictions we employ are robust to the parameterization, common to both flexible and sticky price models (see e.g. Gambetti et al. (2005)) and independent of whether the economic environment delivers determinate or indeterminate solutions (see e.g. Lubik and Schorfheide (2004)), those imposed by zero type restrictions leave the system underidentified when indeterminacies emerge. This is important since one version of the bad policy hypothesis relies on the presence of indeterminacies in the earlier part of the sample.

The resulting structural system can be used to evaluate the magnitude of structural variations produced by changes in i) the systematic component of policy, ii) the propagation of policy shocks, iii) the variance of the structural disturbances and iv) the rest of the economy. Moreover, we can do this examining short and long run features of the estimated system. Both reduced form time varying coefficient and structural but constant coefficient approaches are unable to separate the relative importance of i)-iv) in accounting for the observed changes.

Contrary to the literature up to date, we construct posterior distributions which are consistent with the information available at each  $t$ . While such an approach complicates estimation considerably, it provides a more reliable measure of time variations present in the structural system and of the timing of the changes, if they exist.

We innovate relative to the existing literature in another important dimension. Because time variations in the coefficients induce important non-linearities, standard statistics summarizing the dynamics in response to structural shocks are inappropriate. For example, since at each  $t$  the coefficient vector is perturbed by a shock, assuming that between  $t + 1$  and  $t + k$  no shocks other than the monetary policy disturbance hit the system may give misleading results. To trace out the evolution of the economy in response to structural shocks, we employ a different concept of impulse response function, which shares similarities with those used in Koop, Pesaran and Potter (1996), Koop (1996), and Gallant, Rossi and Tauchen (1993). In particular, impulse responses are defined as the difference between two conditional expectations, differing in the arguments of their conditioning sets. The combined use of a robust identification scheme, of recursive analysis and of appropriately defined responses is crucial to deliver meaningful answers to the questions at stake.

Four main conclusions emerge from our investigation. First, as in Bernanke and Mihov (1998) and Leeper and Zha (2003), we find that excluding the Volker experiment, the monetary policy rule has been quite stable over time. Interestingly, point estimates of the coefficients obtained in the end of the 1990's are similar to those obtained in the late 1970's. Second, as in Sims and Zha (2004), we find posterior evidence of a decrease in the uncertainty surrounding the structural disturbances of the system but no synchronization in the timing of the changes in the variance of the shocks hitting various equations. Third, we show that the transmission of policy shocks has been very stable: both the shape and the persistence of output and inflation responses are very similar over time and quantitative differences statistically small. Fourth, we find that structural inflation persistence has statistically changed over time, that both monetary and non-monetary factors account for its magnitude and that the relative contribution of monetary policy shocks is increasing since the early 1980s.

We investigate, by way of counterfactuals, whether a more aggressive policy response to inflation would have made a difference for the dynamics of output and inflation. Such a stance would have reduced inflationary pressures and produced significant output costs

in 1979, but produced no measurable inflation effects in the 1980s or 1990s and a perverse outcome in the 2000s. Hence, while the Fed could have had some room to improve economic performance at the end of the 1970s, altering the policy response to observable variables, it seems unlikely that such an alteration would have produced the changes observed in the US economy. Finally, we show that our results are robust to a number of changes in some auxiliary assumptions, in particular, the treatment of trends, the variables included in the VAR and to the identification procedure.

Overall, while the crudest version of the "bad policy" proposition has low posterior support, the evidence appears to be consistent both with more sophisticated versions of this proposition as well with the alternative "bad luck" hypothesis. To disentangle the two interpretations, a model in which preferences, technologies and the distributions of the shocks are allowed to change along with the preferences of the Fed is needed. While such a model is still too complex to be analyzed and estimated with existing tools, approximations of the type employed in Canova (2004), can shed important light on this issue.

The rest of the paper is organized as follows. Section 2 presents the reduced form model, describes our identification scheme and the approach used to obtain posterior distributions for the structural coefficients. Section 3 defines impulse response functions which are appropriate for our TVC-VAR model. Section 4 presents the results and Section 5 concludes. Two appendices describes the technical details involved in the computation of impulse responses and of posterior distributions.

## 2 The empirical model

Let  $y_t$  be a  $n \times 1$  vector of time series with the representation

$$y_t = A_{0,t} + A_{1,t}y_{t-1} + A_{2,t}y_{t-2} + \dots + A_{p,t}y_{t-p} + \varepsilon_t \quad (1)$$

where  $A_{0,t}$  is a  $n \times 1$  vector;  $A_{i,t}$ , are  $n \times n$  matrices,  $i = 1, \dots, p$ , and  $\varepsilon_t$  is a  $n \times 1$  Gaussian white noise process with zero mean and covariance  $\Sigma_t$ . Let  $A_t = [A_{0,t}, A_{1,t}, \dots, A_{p,t}]$ ,  $x'_t = [1_n, y'_{t-1}, \dots, y'_{t-p}]$ , where  $1_n$  is a row vector of ones of length  $n$ . Let  $vec(\cdot)$  denote the stacking column operator and let  $\theta_t = vec(A'_t)$ . Then (1) can be written as

$$y_t = X'_t \theta_t + \varepsilon_t \quad (2)$$

where  $X_t' = (I_n \otimes x_t')$  is a  $n \times (np + 1)n$  matrix,  $I_n$  is a  $n \times n$  identity matrix, and  $\theta_t$  is a  $(np + 1)n \times 1$  vector. If we treat  $\theta_t$  as a hidden state vector, equation (2) represents the observation equation of a state space model. We assume that  $\theta_t$  evolves according to

$$p(\theta_t | \theta_{t-1}, \Omega_t) \propto \mathcal{I}(\theta_t) f(\theta_t | \theta_{t-1}, \Omega_t) \quad (3)$$

where  $\mathcal{I}(\theta_t)$  is an indicator function discarding explosive paths of  $y_t$ . Such an indicator is necessary to make dynamic analysis sensible and, as we will see below, it is easy to implement numerically. We assume that  $f(\theta_t | \theta_{t-1}, \Omega_t)$  can be represented as

$$\theta_t = \theta_{t-1} + u_t \quad (4)$$

where  $u_t$  is a  $(np + 1)n \times 1$  Gaussian white noise process with zero mean and covariance  $\Omega_t$ . We select this simple specification because more general AR and/or mean reverting structures were always discarded in out-of-sample model selection exercises. We assume that  $\Sigma_t = \Sigma \quad \forall t$ ; that  $\text{corr}(u_t, \varepsilon_t) = 0$ , and that  $\Omega_t$  is diagonal. At first sight, these assumptions may appear to be restrictive, but they are not. For example, the first assumption does not imply that the forecast errors are homoschedastic. In fact, substituting (4) into (2) we have that  $y_t = X_t' \theta_{t-1} + v_t$  where  $v_t = \varepsilon_t + X_t' u_t$ . Hence, one-step ahead forecast errors have a time varying non-normal heteroschedastic structure even assuming  $\Sigma_t = \Sigma$  and  $\Omega_t = \Omega$ . The assumed structure is appealing since it is coefficient variations that impart heteroschedastic movements to the variance of the forecasts errors (see Canova (1993); Sims and Zha (2004) and Cogley and Sargent (2005) have alternative specifications). The second assumption is standard but somewhat stronger and implies that the dynamics of the model are conditionally linear. Sargent and Hansen (1998) showed how to relax this assumption by equivalently letting the innovations of the measurement equation to be serially correlated. Since in our setup  $\varepsilon_t$  is, by construction, a white noise process, the loss of information caused by imposing uncorrelation between the shocks is likely to be small. The third assumption implies that each element of  $\theta_t$  evolves independently but it is irrelevant since structural coefficients are allowed to evolve in a correlated manner.

Let  $S$  be a square root of  $\Sigma$ , i.e.,  $\Sigma = SS'$ . Let  $H_t$  be an orthonormal matrix, independent of  $\varepsilon_t$ , such that  $H_t H_t' = I$  and let  $J_t^{-1} = H_t' S^{-1}$ .  $J_t$  is a particular decomposition of  $\Sigma$  which transforms (2) in two ways: it produces uncorrelated innovations (via the matrix  $S$ ) and gives a structural interpretation to the equations of the system (via the matrix  $H_t$ ).

Premultiplying  $y_t$  by  $J_t^{-1}$  we obtain

$$J_t^{-1}y_t = J_t^{-1}A_{0,t} + \sum_j J_t^{-1}A_{j,t}y_{t-j} + e_t \quad (5)$$

where  $e_t = J_t^{-1}\varepsilon_t$  satisfies  $E(e_t) = 0$ ,  $E(e_te_t') = I_n$ . Equation (5) represents the class of "structural" representations of  $y_t$  we are interested in. For example, a standard Choleski representation can be obtained setting  $S$  to be lower triangular and  $H_t = I_n$  and more general patterns of zero restrictions result choosing  $S$  to be non-triangular and  $H_t = I_n$ . In this paper  $S$  is arbitrary and  $H_t$  implements interesting economic restrictions.

Letting  $C_t = [J_t^{-1}A_{0,t}, J_t^{-1}A_{1,t}, \dots, J_t^{-1}A_{p,t}]$ , and  $\gamma_t = \text{vec}(C_t')$ , (5) can be written as

$$J_t^{-1}y_t = X_t'\gamma_t + e_t \quad (6)$$

As in fixed coefficient VARs, there is a mapping between  $\gamma_t$  and  $\theta_t$  since  $\gamma_t = (J_t^{-1} \otimes I_{np})\theta_t$  where  $I_{np}$  is a  $(np+1) \times (np+1)$  identity matrix. Whenever  $\mathcal{I}(\theta_t) = 1$ , we have

$$\gamma_t = \gamma_{t-1} + \eta_t \quad (7)$$

where  $\eta_t = (J_t^{-1} \otimes I_{np})u_t$ , the vector of shocks to structural parameters, satisfies  $E(\eta_t) = 0$ ,  $E(\eta_t\eta_t') = E((J_t^{-1} \otimes I_{np})u_t u_t' (J_t^{-1} \otimes I_{np})')$ . Hence, the vector of structural shocks  $\xi_t' = [e_t', \eta_t']'$  is a white noise process with zero mean and covariance matrix  $E\xi_t\xi_t' = \begin{bmatrix} I_n & 0 \\ 0 & E((J_t^{-1} \otimes I_{np})u_t u_t' (J_t^{-1} \otimes I_{np})') \end{bmatrix}$ . Since each element of  $\gamma_t$  depends on several  $u_{it}$  via the matrix  $J_t$ , shocks to structural parameters are no longer independent

The structural model (6)-(7) contains two types of shocks: disturbances to the observations equations,  $e_t$ , and disturbances to structural parameters,  $\eta_t$ . While the formers have the usual interpretation, the latters are new. To understand their meaning, suppose that the  $n$ -th equation of (6) is a monetary policy equation and suppose we summarized it by  $\tilde{\gamma}_t = [\gamma_{(n-1)(np+1),t}, \dots, \gamma_{n(np+1),t}]'$ , which describes, say, how interest rates respond to the developments in the economy, and the policy shock  $e_{n,t}$ . Then, if variations in the parameters regulating preferences and technologies are of second order, an assumption commonly made in the literature,  $\tilde{\gamma}_t$  captures changes in the preferences of the monetary authorities with respect to developments in the rest of the economy.

In our setup, identifying structural shocks is equivalent to choosing  $H_t$ . As in Faust (1998), Canova and De Nicoló (2002) and Uhlig (2005), we select  $H_t$  so that the sign of the

impulse response functions at  $t + k, k = 1, 2, \dots, K_1$  matches some theoretical restriction. In particular, we assume that a contractionary monetary policy shock must generate a non-positive effects on output, inflation and nominal balances and a non-negative effect on the interest rate for two quarters. (see Gambetti et al. (2005) for a class of DSGE models which robustly generates this set of restrictions).

We choose sign restrictions to identify shocks for two reasons. First, the contemporaneous zero restrictions conventionally used are often absent in those theoretical (DSGE) models economists like to use to guide the interpretation of VAR results. Second, a set zero restrictions which satisfies the standard order condition for identification, does not deliver an identified system in the case of indeterminacy (in this case, there are  $n+1$  shocks). Sign restrictions do not suffer from this problem. Moreover, as shown by Lubik and Schorfheide (2004), a small scale version of the model used in Gambetti et al (2005) delivers the same qualitative implications we use as identifying both in determinate and in indeterminate scenarios. In the last section we report, as a robustness check, results obtained identifying policy shocks as the third element of a Choleski system, i.e. we let the interest rate to react to output and inflation but assume that it has no effects within a quarter on these variables. Since we are interested in recovering the systematic and non-systematic part of monetary policy and in analyzing how the economy responds to their changes over time, we arbitrarily orthogonalize the other disturbances without giving them any structural interpretation.

### 3 Impulse Responses

One question we would like to address is whether the transmission of monetary policy shocks has changed over time. In a fixed coefficient model, impulse response functions provide information on how the variables react to policy shocks. Impulse responses are typically computed as the difference between two realizations of  $y_{i,t+k}$  which are identical up to time  $t$ , but one assumes that between  $t+1$  and  $t+k$  a shock in  $e_j$  occurs only at time  $t+1$  and the other that no shocks take place at all dates between  $t+1$  and  $t+k, k = 1, 2, \dots$ .

In a TVC model, responses computed this way are inadequate since they disregard the fact that between  $t+1$  and  $t+k$  the coefficients of the system may also change. Hence, meaningful impulse response functions ought to measure the effects of a shock in  $e_{jt+1}$  on  $y_{it+k}$ , allowing future shocks to the coefficients to be non-zero. For this reason, our impulse

responses are obtained as the difference between two conditional expectations of  $y_{t+k}$ . In both cases we condition on the history of the data  $(y_1, \dots, y_t)$ , of the states  $(\theta_1, \dots, \theta_t)$ , on the structural parameters of the transition equation (which are function of  $J_t$ ) and all future shocks. However, in the first case we condition on a draw for the current shock, while in the second we condition on current shock being zero.

Formally speaking, let  $y^t = [y'_1, \dots, y'_t]'$  be a history for  $y_t$ ;  $\theta^t = [\theta'_1, \dots, \theta'_t]'$  be a trajectory for the states up to  $t$ ,  $y_{t+1}^{t+k} = [y'_{t+1}, \dots, y'_{t+k}]'$  be a collection of future observations and  $\theta_{t+1}^{t+k} = [\theta'_{t+1}, \dots, \theta'_{t+k}]'$  be a future trajectory of states. Let  $V_t = (\Sigma, \Omega_t)$ ; recall that  $\xi'_t = [e'_t, \eta'_t]'$  and let  $\zeta'_t = [u'_t, \epsilon'_t]'$ . Let  $\xi_{i,t+1}^\delta$  be a particular realization of size  $\delta$  in  $\xi_{i,t+1}$  and let  $\mathcal{I}_t^1 = \{y^t, \theta^t, V_{t+1}, J_{t+1}, \xi_{j,t+1}^\delta, \xi_{-j,t+1}, \zeta_{t+2}^{t+\tau}\}$  and  $\mathcal{I}_t^2 = \{y^t, \theta^t, V_{t+1}, J_{t+1}, \xi_{t+1}, \zeta_{t+2}^{t+\tau}\}$  be conditioning sets, where  $\xi_{-j,t+1}$  indicates all shocks excluding the one in the  $j$ -th component. Then an impulse response function to a shock  $\xi_{j,t+1}^\delta$ ,  $j = 1, \dots, n$  is defined as:<sup>1</sup>

$$IR_y(t, k) = E(y_{t+k} | \mathcal{I}_t^1) - E(y_{t+k} | \mathcal{I}_t^2) \quad k = 1, 2, \dots \quad (8)$$

(8) resembles the impulse response function suggested by Gallant et al. (1996), Koop et al. (1996) and Koop (1996). Three important differences need to be noted. First, rather than treating histories as random variables, we condition on a particular realization of  $y^t$ . Since we want to analyze how responses vary over time, history dependence is a must. Second, responses to shocks to the measurement equation are independent of the sign and the size of the shocks (as it is in a fixed coefficient case). This is not the case for shocks to the transition equation. Third, we do not condition on a particular realization of  $\theta_{t+1}$  (for example, its conditional mean) but instead treat  $\theta_{t+1}$  as a random variable and integrate it out when calculating impulse responses. Integrating  $\theta_t$  out of impulse responses allows us to concentrate attention on time differences which depend on the history of  $y_t$  but not on the size of the sample. Finally, note that  $IR_y(t, k)$  can be made state dependent, if we condition on a particular stretch of history (a boom or a recession), and that (8) coincides with standard impulse responses when coefficients are constant.

Since there are two types of shocks, we describe how to trace out the dynamics effects

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<sup>1</sup>An alternative definition of impulse responses is obtained averaging out future shocks. Our definition is preferable for two reasons: it is easier to compute and produces numerically more stable distributions; it generates impulse responses which are similar to those produced by constant coefficient impulse responses when shocks to the measurement equation are considered. Since future shocks are not averaged out, the impulse responses we present will tend to display an "excess" variability.

of each of them separately. Let  $\xi_{j,t+1} = e_{j,t+1}$ . Then

$$\begin{aligned} IR_y(t, 1) &= J_t e_{j,t+1}^\delta \\ IR_y(t, k) &= \Psi_{t+k,k-1}^j e_{j,t+1}^\delta \quad k = 2, 3, \dots \end{aligned} \quad (9)$$

where  $\Psi_{t+k,k-1} = \mathcal{S}_{n,n}[(\prod_{h=0}^{k-1} \mathbf{A}_{t+k-h}) \times J_{t+k-(k-1)}]$ ,  $\mathbf{A}_t$  is the companion matrix of the VAR at time  $t$ ;  $\mathcal{S}_{n,n}$  is a selection matrix which extracts the first  $n \times n$  block of  $[(\prod_{h=0}^{k-1} \mathbf{A}_{t+k-h}) \times J_{t+k-(k-1)}]$  and  $\Psi_{t+k,k-1}^j$  is the column of  $\Psi_{t+k,k-1}$  corresponding to the  $j$ -th shock.

When the coefficients are constant,  $\prod_h \mathbf{A}_{t+k-h} = \mathbf{A}^k$  and  $\Psi_{t+k,k-1} = \mathcal{S}_{n,n}(\mathbf{A}^k \times J_t)$  for all  $k$ . Hence (9) collapses to traditional impulse response function to unitary structural shocks. Clearly,  $IR_y(t, k)$  depends on the identifying matrix  $J_t$  and is non-explosive, since  $\Psi_{t+k,k-1}$  is the product of matrices whose eigenvalues are non-explosive.

When  $\xi_{j,t+1} = \eta_{j,t+1}$  for  $j = (n-1)(np+1), \dots, n(np+1)$ , appendix A shows that

$$\begin{aligned} IR_y(t, 1) &= [E(\mathbf{A}_{t+1,1}|\mathcal{I}_t^1) - E(\mathbf{A}_{t+1,1}|\mathcal{I}_t^2)] y_t \\ IR_y(t, k) &= E(\tilde{A}_{0,t+k}|\mathcal{I}_t^1) - E(\tilde{A}_{0,t+k}|\mathcal{I}_t^2) + E(\tilde{\Phi}_{t+k,k}|\mathcal{I}_t^1) y_t - E(\tilde{\Phi}_{t+k,k}|\mathcal{I}_t^2) y_t \\ &\quad + E\left(\sum_{j=0}^{k-1} \Psi_{t+k,j}|\mathcal{I}_t^1\right) e_{t+k-j} - E\left(\sum_{j=0}^{k-1} \Psi_{t+k,j}|\mathcal{I}_t^2\right) e_{t+k-j} \quad k = 2, 3, \dots \end{aligned} \quad (10)$$

where  $\tilde{\Phi}_{t+k,k}$  and  $\tilde{A}_{0,t+k}$  are defined in appendix A. There are three terms in (10): the first two show how shocks spread in the system through the intercept; the next two how they spread through the lags of  $y_t$  and the last two how they spread through future shocks to the structural equations. Note that when  $\xi_{j,t+1} = \eta_{j,t+1}$ ,  $IR_y(t, k)$  depends on  $J_{t+1}$  only because  $\eta_{t+1} = (J_{t+1}^{-1} \otimes I_{np}) u_{t+1}$ . Also in this case,  $IR_y$  are non-explosive.

To illustrate these definitions take an AR(1) process  $y_t = A_t y_{t-1} + e_t$  and let  $A_t = A_{t-1} + \eta_t$ . Then for a shock of size  $\delta$  to the observation equation  $IR_y(t, 1) = e_{t+1}^\delta$ ;  $IR_y(t, 2) = E_t(A_{t+2} e_{t+1}^\delta)$  and so on, while for a shock of size  $\delta$  to the measurement equation,  $IR_y(t, 1) = \eta_{t+1}^\delta y_t$ ;  $IR_y(t, 2) = E_t(\eta_{t+1}^\delta (y_t + \eta_{t+1}^\delta e_{t+1}))$ , and so on.

## 4 Estimation

The model (6)-(7) is estimated using Bayesian methods. That is, we specify prior distributions for  $\theta_0, \Sigma, \Omega_0$ , and  $H_t$  and use data up to  $t$  to compute posterior distributions of the structural parameters and of continuous functions of them. Since our sample goes from

1960:1 to 2003:2, we initially estimate the model for the sample 1960:1-1977:3 and then reestimate it moving the terminal date by one quarter up to 2003:2.

Posterior distributions for the structural parameters are not available in a closed form. MCMC methods are used to simulate posterior sequences consistent with the information available up to time  $t$ . Estimation of reduced form TVC-VAR models with or without time variations in the variance of the shocks to the transition equation is now standard (see e.g. Cogley and Sargent (2001)): it requires treating the parameters which are time varying as a block in a Gibbs sampler algorithm. Therefore, at each  $t$  and in each Gibbs sampler cycle, one runs the Kalman filter and the Kalman smoother, conditional on the draw of the other time invariant parameters. In our setup the calculations are complicated by the fact that at each cycle, we need to obtain structural estimates of the time varying features of the model. This means that, in each cycle, we need to apply the identification scheme, discarding paths which are explosive and paths which do not satisfy the restrictions we impose. The computational costs are compounded because we need to run the Gibbs sampler more than a 100 times, one per sample we analyze. Convergence was checked using a CUMSUM statistic. The results we present are based on 10,000 draws for each  $t$ <sup>2</sup>.

Because of the heavy notation involved in the construction of posterior distributions and the technicalities needed to produce draws from these posteriors, we present the details of the estimation approach in appendix B.

## 5 The Results

The data we use is taken from the FREDII data base of the Federal Reserve Bank of San Louis. In our basic exercise we use the log of (linearly) detrended real GDP, the log of first difference of GDP deflator, the log of (linearly) detrended M1 and the federal funds rate in that order. Systems containing other variables are analyzed in the next section.

We organize the presentation of the results around four general themes: (i) Do reduced form coefficients display significant variations? (ii) Are there synchronized changes in the structural coefficients and/or in the structural variances of the model? (iii) Are there changes in the propagation of monetary policy disturbances in the short and the long run? (iv) Would it have made a difference for macroeconomic performance if monetary policy

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<sup>2</sup>Total computational time for each specification on a Pentium IV machine was about 100 hours.

were more aggressive in fighting inflation, in particular, at the end of the 1970's?

## 5.1 The evolution of reduced form coefficients

The first panel of figure 1 plots the evolution of the mean of the posterior distribution of the change in reduced form coefficients in each of the four equations. The first date corresponds to estimates obtained with the information available up to time 1977:3, the last one to estimates obtained with the information up to time 2003:2.

Several interesting aspects of the figure deserve some comments. First, consistent with the evidence of Sargent and Cogley (2001) and (2005) all equations display some coefficient variation. In terms of size, the money (third) and interest rate (fourth) equations are those with the largest changes, while variations in the coefficients of the inflation (second) equation are the smallest of all. Second, while changes appear to be stationary in nature, there are few coefficients which display a clear trend over time. For example, in the output (first) equation, the coefficient on the first lag of money is drifting downward from 0.6 in 1977 to essentially zero at the end of the sample; while in the money equation, the first lagged money coefficient is drifting upward from roughly zero in 1977 to about 0.9 in 2002. Perhaps more importantly, there is little evidence of a once-and-for-all structural break in the coefficients of the output and inflation equation (i.e. coefficients do not jump at some date and stays there afterward). Third, the majority of the changes appear to be concentrated at the beginning of the sample. The 1979-1982 period is the one which displays the most radical variations; there is some coefficient drift up to 1986, and after that date variations appear to be random and small. Finally, centered 68% posterior bands for the coefficients at the beginning (1977:3) and at the end of the sample (2003:2) overlap in many cases. Therefore, barring few relevant exceptions, instabilities appear to be associated with the Volker (1979-1982) experiment and the adjustments following it. Furthermore, they are temporary and mean reverting in nature.

Figure 2 reports the posterior mean drift of inflation and a posterior mean of inflation persistence obtained in the system. The mean drift of inflation tracks well the ups and downs of inflation over the period and the posterior mean of inflation persistence shows a dramatic decline at the beginning of the 1980's. Both of these patterns agree with those presented by Cogley and Sargent (2005), despite the fact that the VAR system differ in the number and kind of variables used. To go beyond the documentation of patterns of time

variations in reduced form statistics and study whether monetary policy is responsible for the changes, we next examine the dynamics of structural coefficients.

## 5.2 Structural time variations

The second panel of figure 1 presents the evolution of the posterior mean of the changes in the lagged structural coefficients of each equation at each date in the sample. The first date corresponds again to estimates obtained with the information up to time 1977:3, the last one to estimates obtained with the information up to time 2003:2.

It is immediate to notice that changes in the structural coefficients are typically larger and more generalized than those in the reduced form coefficients. The output and the monetary policy equations are those displaying the largest absolute coefficient changes - these are up to 4 times as large as the largest absolute changes present in the other two equations - while the coefficients of the structural inflation equation are still the most stable ones. Furthermore, except for the money (demand) equation, most the variations are concentrated in the first part of the sample, are large in size, statistically and often economically significant. More interestingly from our point of view, there is a pattern in the structure of time variations. The output equation displays two regimes of coefficient variations (one with high variations up to 1986 and one with low variations thereafter) and, within the high volatility regime, the largest coefficient variations occur in 1986. The inflation equation shows the largest coefficient changes up to 1982 and, barring few exceptions, a more stable pattern resulted since then. Finally, our identified monetary policy equation displays large and erratic coefficient changes up to 1986 and coefficients variation is considerably reduced after that. Since the timing of the variations in the structural coefficients of the output and inflation equations are somewhat asynchronous with those of the monetary policy equation, figure 1 casts some doubts on a causal interpretation of the observed changes running from changes in the policy equation to changes in the dynamics of output and inflation.

Figure 3 zooms in on the evolution of the coefficients of the monetary policy equation (which is normalized to be the last one of the system). Three facts stand out. First, posterior mean estimates of all contemporaneous coefficients are humped shaped: they significantly increase from 1979 to 1982 and smoothly decline afterwards. Second, although all contemporaneous coefficients are higher at the end than at the beginning of the sample, they are typically lower than the conventional wisdom would suggest. In particular, the

contemporaneous inflation coefficient peaks at about 1.2 in 1982 and then declines to a low 0.3, on average, in the 1990s and this pattern is also shared by the two lagged inflation coefficients. In this sense, Alan Greenspan's regime was only marginally more effective than Arthur Burns's in insuring inflation stability: interest rate responses to inflation movements were barely more aggressive in the 1990s than they were in the 1970s. Note also that, again excluding the beginning of the 1980's, the estimated monetary policy rule displayed considerable stability, in line with the subsample evidence presented, e.g. by Bernanke and Mihov (1998). Since macroeconomic performance was considerably different in the two time periods, the size and characteristics of the shocks hitting the US economy in the two periods must have been different. We will elaborate on this issue later on.

Our estimated policy rule displays a six fold-increase in all contemporaneous and first lagged coefficients from 1979 to 1982. Interestingly, this increase is not limited to the inflation coefficients, but also involves output and the money coefficients. The high responsiveness of interest rates to economic conditions is consistent with the idea that by targeting monetary aggregates the Fed forced interest rates to jump to equilibrate a "fixed" money supply with a largely varying money demand - the period was characterized by a number of important financial innovations. The pervasive instability characterizing this period and the subsequent three years adjustments contrasts with the substantial stability of the coefficients of the monetary policy rule in the rest of the sample. Hence, excluding the "Volker experiment", the systematic component of monetary policy has hardly changed over time and if, any change must be noted, it is more toward a decline in the responsiveness of interest rates to economic conditions. This outcome is consistent with the "business as usual" characterization of monetary policy put forward by Leeper and Zha (2003) and with the time profile of the policy rule recursively estimated in a DSGE model (see Canova (2004)).

The evidence we have so far collected seems to give little credence to the crudest version of the "bad policy" hypothesis: there is no permanent increase in the inflation coefficient of the policy rule, nor clear evidence that the Taylor's principle was violated in the 1970's and satisfied afterwards. Both more sophisticated versions of the "bad policy" and the "back luck" hypotheses suggest that alterations in the distribution of the shocks hitting the economy are responsible for the improved macroeconomic outcome. In the former case, changes in the variance of policy shocks "caused" the observed changes; in the latter case, policy has little or nothing to do with the dynamics of output and inflation which are simply

driven variations in the distributions of the shocks hitting the two equations.

Figure 4 presents some evidence on this issue. In the top panel we report the evolution of the posterior mean estimate of the variance of the structural forecast errors and, in the bottom panel, the variations produced by its heteroschedastic component, i.e the variations induced by product of the estimated innovations in the coefficient and the regressors of the model. Three features are of interest. First, the forecast error variance in three of the four equations is humped shaped: it shows a significant increase from 1979 to 1982 followed by a smooth decline. As it happened with structural coefficients, the posterior mean estimate of the variance of the shocks in the end of the sample is roughly similar in magnitude to the posterior mean estimate obtained in 1977. Second, the time profile of the changes in the forecast error variances of the output and the inflation equations are not synchronized with the variations in the forecast error variance of our estimated policy equation, which starts declining significantly after 1986. Third, the contribution of changes in the coefficients to the forecast error variance is much larger in the output and inflation equations than in the other two equations up to 1982 but similar after that date. Shocks to the model contribute most to the variability of the forecast error between 1979 and 1982 - they account for about 50% of the variance in the output and inflation equations - but their importance declined after 1982 and the decline is stronger in the inflation equation.

### 5.3 Changes in the propagation of monetary policy disturbances?

Figure 5 reports the posterior mean responses of output and inflation to identified monetary policy shocks in each date of the sample, for horizons running from 1 to 12 quarters. We do not report interest rate responses because they are similar over time and quite standard in shape and magnitude: after the initial impulse, the increase dissipates rather quickly and becomes insignificantly different from zero after the 3th quarter for each date in the sample.

The shape of both output and inflation responses is roughly unchanged over time. Output responses are U-shaped; a through response occurs after about 3 quarters and there is a smooth convergence to zero after that date. Inflation responses are also slightly U-shaped; the effect at the one quarter horizon is typically the largest, and responses smoothly converge toward zero afterwards.

There is a small quantitative difference in the mean responses over time. For output, the posterior mean of the instantaneous response is always centered around -0.15 and the

size of the through responses at lag 3 varies in the range  $(-0.20, -0.05)$ . For inflation, minor differences occur at lag one (posterior mean varies between  $-0.07$  to  $-0.16$ ) while in 1978 responses are more persistent than at all the other dates at horizons ranging from 3 to 8.

Differences in inflation responses are both statistically and economically small. The posterior 68% confidence band for the largest discrepancy (the one at lag 1) includes zero at almost all horizons and, if we exclude the initial three years, the time path of inflation responses is unchanged over time. The posterior 68% confidence band for the largest discrepancy in output responses (the one at lag 3) does at times exclude zero - the trough response in 1982 appear to be significantly deeper than the trough response in 1978 and 1979 and at some dates after 1992 - but differences are economically small: the maximum discrepancy in the cumulative output multiplier twelve quarters ahead is only 0.5%. In other words, a one percent increase in interest rates produced output responses which differ over time on average by 0.04% points at each horizon.

Overall, the dynamics induced by monetary policy shocks are remarkably stable over time and, in agreement with the results of section 5.2, responses in the end of the 1990's look similar, in shape and size, to those in the end of the 1970's.

## 5.4 Inflation Dynamics and Monetary Policy

Cogley and Sargent (2001) and (2005) have examined measures of core inflation to establish their claim that monetary policy is responsible for the observed changes in inflation dynamics. They define core inflation as the persistent component of inflation, statistically measured by the zero frequency of the spectrum (that is, by the sum of all autocovariances of the estimated inflation process), and show i) persistence has substantially declined over time and ii) there is synchronicity between the changes in persistence and a narrative account of monetary policy changes. Pivetta and Reis (2004), using univariate conventional classical methods, dispute the first claim showing that differences over time in two measures of inflation persistence are statistically insignificant. Since our study has so far concentrated on short/medium run frequencies, we turn to investigate the longer run relationship between inflation and monetary policy. In particular, we are curious as to whether different frequencies of the spectrum carry different information and whether our basic conclusions on the role of monetary policy are altered.

Our analysis differs from existing ones in two important respects: we use output in

place of unemployment in the estimated system; we measure persistence using the estimated structural model. While the first difference is minor, the second is not. In fact, thanks the orthogonality of the structural shocks and of the ordinates of the spectrum, we can not only to describe the evolution of the spectrum of inflation over time, but also directly measure of proportion of the spectral power at frequency zero due to monetary policy shocks and describe its evolution over time. From the structural MA representation of the system we have that  $\pi_t = \sum_{i=1}^n \phi_{it}(\ell)e_{it}$ , where  $e_{it}$  is orthogonal to  $e_{jt}$ . Hence the spectrum of inflation at Fourier frequencies  $\omega$  is  $S_\pi(\omega) = \frac{1}{2\pi} \sum_{i=1}^n |\phi_{it}(\omega)|^2 \sigma_i^2$  and the component at frequency zero due to monetary policy shocks is  $S_\pi^*(\omega = 0) = \frac{1}{2\pi} |\phi_{nt}(\omega = 0)|^2 \sigma_n^2$ .

The top panel of figure 6 shows the time evolution of the posterior mean of the spectrum of inflation at the zero frequency and the contribution that monetary policy shocks had in shaping its changes. The estimate of the zero frequency displays an initial increase in 1978-1980 followed by a sharp decline the year after; since 1981 the estimated posterior mean of the zero frequency of the spectrum has been relatively stable (with the exclusion of 1991). The initial four fold jump and the following ten fold decrease are visually large and statistically significant. In fact, the bottom panel of figure 6 indicates that the 68% posterior band for the differences between the log spectrum in 1979 and 1996 (the date with the lowest estimates) does not include zero at the zero frequency. At all other frequencies, differences over time are negligible both in terms of size and shape. Hence, except for the zero frequency, the posterior distribution of the spectrum of inflation has also been relatively stable. What is the role of monetary policy shocks? The top panel of figure 6 indicates that the two graphs track each other reasonably well suggesting that, at least in terms of timing, monetary policy shocks are important in determining inflation persistence dynamics. Second, the contribution of monetary policy to inflation persistence varies over time: fluctuations are large and the percentage explained ranges from about 20 to about 75 percent. Interestingly, there is a significant trend increase since 1981. Third, there is a substantial portion of inflation persistence (roughly, 50 percent on average) which has nothing to do with monetary policy shocks. While the determination of the forces behind this large percentage is beyond the scope of this paper, one can conjecture that real and financial factors could account for these variations. As mentioned, the years between 1978 and 1982 were characterized by financial innovations and high nominal interest rate variability. The pattern present at the zero frequency over this period is consistent with

these two features while the subsequent decline is consistent with the reduction of the volatility of interest rate disturbances shown in figure 4.

In conclusions, there is visual and statistical evidence of instabilities in the posterior mean of inflation persistence. Changes in the posterior mean of inflation persistence go hand in hand with changes in the contribution of monetary policy shocks. Perhaps more importantly, we find that the contribution of monetary policy shocks to variations in the posterior means of inflation persistence is smaller than expected, that factors other than monetary policy are crucial to understand its evolution over time, and that the relative contribution of monetary policy has increased since the early 1980s.

### **5.5 What if monetary policy would have been more aggressive?**

It is common in the literature to argue, by mean of counterfactuals, that monetary policy failed to perform an inflation stabilization role in the 1970s (see e.g Clarida, Gali and Gertler (2000) or Boivin and Giannoni (2002)) and that, had it followed a more aggressive stance against inflation, dramatic changes in the economic performance would have resulted. While exercises of this type are meaningful only in dynamic models with clearly stated microfoundations, our structural setup allows us to approximate the ideal type of exercise without falling into standard Lucas-critique type of traps. In fact, to the extent that the monetary policy equation we have identified is structural, and given that we estimate posterior distributions which are consistent with the information available at each  $t$ , we can examine what would have happened if the policy response to inflation was significantly stronger, where by this we mean a (permanent) two standard deviations increased in the inflation coefficients above the estimated posterior mean. Figure 7 plots the percentage output and inflation changes from the value of the baseline year which would have been produced at selected dates in the sample. To interpret the numbers note, e.g., that the maximum inflation response in 1979 (-5 percent) correspond to a 1.0 point absolute decline in the annual inflation recorded at that date (which was around 19 percent) and that a 15 percent decline in 2003, at the annual rate of 2.5 percent, corresponds to an absolute fall of less than a 0.4 points.

A permanent more aggressive stance would have had important inflation effects in 1979, primarily in the medium run. However, at all dates in the 1980s and 1990s, the effect would have been statistically negligible. Interestingly, if such a policy were used in 2003, it would

have produced a small but significant medium run increase in inflation. A tougher stance on inflation, however, is not painless: important output effects would have been generated. In 1979, the fall would have lasted about four years while the 7 percent fall recorded in 2003 would have lasted for quite a long time. The Phillips curve trade-off, measured here by the conditional correlation between output and inflation in response to the change, displays an interesting pattern: it is positive and significant in 1979, it is zero in 1983, and it is negative in 1992 and 2003, and at the last date it is statistically significant. While there are many reasons which can explain the change in the sign of the trade-off, a better control of inflation expectations and an improved credibility in the policy environment are clearly consistent with this pattern.

Overall, while there was room for stabilizing inflation in the end of the 1970, it is not clear that a tougher inflation stance would have been costless in terms of output. There is a sense in which the conventional view is right: being tough on inflation in the end of the 1970s would have produced a different macroeconomic outcome than in the end of the 1990s. However, the reasoning seems to be wrong: being tough on inflation is dangerous when the slope of the Phillips curve trade-off is different from the conventional one.

## 6 Robustness analysis

There is a number of specification choices we have made which may affect the results. In this section we analyze the sensitivity of our conclusions to variations in the identification method, in the treatment of trends, and in the variables included in the VAR.

All the results we have presented so far have been produced identifying monetary policy shocks using sign restrictions on the dynamics of money, inflation and output. Would the pattern of time variations, the estimated policy rule and the time profile of impulse responses be altered if an alternative identification scheme was used? Figure 8 shows the evolution of the variance of the forecast errors and of output and inflation responses obtained identifying policy shocks with a Choleski decomposition. Since here contemporaneous coefficients are time invariant, the evolution of structural coefficients reproduces the pattern of time variations present in the reduced form coefficients (they are simply multiplied by a constant). Therefore, the discussion of subsection 5.1 apply here without a change. Overall, the main conclusions we have derived are robust to this change: there are time variations

in the coefficients but they are not synchronized across equations; the sum of the inflation coefficients in the policy equation is roughly the same in the end of the 1970s and of the 1990s; the evolution of the estimated forecast error variances reproduces the one present in figure 4; impulse responses are broadly similar across time. Clearly, there are changes in pattern of responses relative to our baseline case - inflation increases for at least a year after an interest rate shock. However, it is still true, that differences over time in the posterior mean of output and inflation responses are small and insignificant.

Some feel uncomfortable with dynamic exercises conducted in a system where linearly detrended output and linearly detrended money are used. One argument against this choice is that after these transformations these two variables are still close to be integrated and are not necessarily cointegrated. Hence, the dynamics we trace out may be spurious. A second argument, put forward in Orphanides (2004), has to do with the fact that measures of the output gap obtained linearly filtering the data are plagued by measurement error. This measurement error is presumably reduced when output growth is employed. To verify whether arguments of this type alter our conclusions we have repeated estimation using the growth rate of output and of M1 in place of the detrended values of output and M1. A sample of the results appears in figure 9, where we plot the evolution of the posterior of the contemporaneous policy coefficients, of the variances of the forecasts errors and of the time profile of output and inflation in response to a policy shock, identified using sign restrictions. Once again, our basic conclusions remain unchanged. In particular, the variability of GDP and inflation forecast errors in the 1990's is about half what it was in the 1980's and 1970's; policy coefficients are stable; the transmission of policy shocks is stable and numerical difference emerge only in the response of inflation in the medium run, which is stronger at the beginning of the sample than at the end.

We have also examined the sensitivity of our conclusions also to changes in the variables of the VAR. It is well known that small scale VAR models are appropriate only to the extent that omitted variables exert no influence on the dynamics of the included ones. A-priori it is hard to know what variables are more important and to check if our system effectively marginalized the influence of all relevant variables. We have therefore repeated our exercise substituting the unemployment rate to detrended output. Figure 10 reports the evolution of the posterior mean of the contemporaneous policy coefficients, of the variances of the forecast errors and of the responses of unemployment and inflation to a policy shock,

identified via sign restrictions. Also in this case, our conclusions appear to be robust.

Finally, it is now common to examine monetary policy in empirical and theoretical models in which money play no role. We believe that such a practice is dangerous in a system like ours for two reasons. First, omission of money may cause identification problems (demand and supply of currency can not be disentangled). Second, money was a crucial ingredient in the considerations that shaped monetary policy decisions, at least up to the end of the 1980's. Commentators have argued that the inclusions of money in the policy rule may lead to an improper characterization of the policy decision of the Fed, especially during Greenspan's tenure. Figure 11 presents a sample of the results obtained with a trivariate system which excludes money. Our basic conclusions are robust also to this change. Interestingly, the posterior mean of the contemporaneous output coefficient in policy rule is counterintuitively negative and significant, suggesting that such a system could be misspecified.

## 7 Conclusions

This paper provides novel evidence on the contribution of monetary policy to the structural changes observed in the US economy over the last 30 years. We use a time varying structural VAR model to analyze the issues. Our exercise is truly recursive and, methodologically, we innovate on the existing literature in two important respects: we provide a sign scheme to identify structural shocks in a TVC model and a way to calculate impulse responses, which is coherent with the assumptions of the model. These three feature together allows us to assess how much time variation there is in the propagation of policy shocks, both in the short and in the long run, and to run counterfactuals to understand whether permanent changes in the systematic component of policy would have significantly altered macroeconomic performance.

We would like to emphasize four main conclusions of our investigation. First, excluding the 1979-1982 period, the posterior distribution of the policy coefficients has been relatively stable over time. Second, there is a clear trend decline in the posterior mean of the variability of the shocks hitting the economy but the changes observed in the output and inflation equations are unsynchronized with those present in the policy equation. Third, the posterior distribution of responses of output and inflation to policy shocks has been relatively stable

over time, while changes in posterior distribution of inflation persistence appear to be partially related to changes in the contribution of policy shocks. Fourth, a more aggressive policy would have decreased inflation in the medium run in 1979 but not later. If this policy would have been implemented, output costs would have been large.

Since our results go against several preconceived notions present in the literature, it is important to highlight what are the features of our analysis which may be responsible for the differences. As repeatedly emphasized, our analysis uses a structural model, it is recursive and employs a definition of impulse responses which is consistent with the nature of the model we use. Previous studies which used the same level of econometric sophistication (such as Cogley and Sargent (2001), (2005)) have concentrated on reduced form estimates and were forced to use the timing of the observed changes to infer the contribution of monetary policy to changes in output and inflation. Our approach allows not only informal tests but also to quantify a-posteriori the relationship between monetary policy, output and inflation dynamics. In studies where a semi-structural Choleski based model is used, as in Primiceri (2005), the analysis is not recursive and the impulse responses are computed in the traditional way. Relative to earlier studies such as Bernanke and Mihov (1998), Hanson (2001) or Leeper and Zha (2003), which use subsample analyses to characterize the changes over time in structural VARs, we are able to precisely track the evolution of the coefficients over time and produce a more complete and reliable picture of the relatively minor variations present in the monetary policy stance in the US.

Our results agree with those obtained recursively estimating a small scale DSGE model with Bayesian methods (see Canova (2004)) and contrast with those of Boivin and Giannoni (2002) who use an indirect inference principle to estimate the parameters of a DSGE model over two subsamples. We conjecture that identification problems could be responsible for the difference since the latter method has problems exploring flat objective functions. Finally, our results are consistent with those of Sims and Zha (2004), despite the fact that, in that paper, variations in both the coefficients and the variance are accounted for with a Markov switching methodology. Relative to their work, our analysis emphasizes that factors other than monetary policy could be more important in explaining the structural changes witnessed in the US economy and provides recursive impulse response analysis.

While the decline in the variance of the shocks hitting both the economy and the coefficients of its structural representation seems to suggest that exogenous reasons are responsi-

ble for the changes in the US economy, it is important to emphasize that our conclusions are consistent both with the analysis of McConnell and Perez Quiroz (2001) and with the idea that a more transparent policy process has reduced the volatility of agent's expectations over time. It is therefore important to extend the current study, enlarging the number of variables included in the structural model, identifying other sources of shocks and disentangling possible factors which may be behind the decline in the volatility of structural shocks. Also, we have repeatedly mentioned that the monetary policy rule is similar in the 1970s and in the end of the 1990s. Why is it that inflation in the 1990s did not follow the same pattern as in the 1970s? What is the contribution of technological changes to this improved macroeconomic framework? We plan to study these and related issues in future work.

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## Appendix

### A. Impulse Responses

The structural model is

$$y_t = A_{0,t} + A_{1,t}y_{t-1} + A_{2,t}y_{t-2} + \dots + A_{p,t}y_{t-p} + S \times H_t \times e_t \quad (11)$$

and its companion form is

$$\mathbf{y}_t = \mathbf{A}_{0,t} + \mathbf{A}_t \mathbf{y}_{t-1} + \mathbf{E}_t \quad (12)$$

where  $\mathbf{A}_{0,t} = [A_{0,t}, 0, \dots, 0]'$ ,  $\mathbf{y}_t = [y_t, y_{t-1}', \dots, y_{t-p}']'$ ,  $\mathbf{E}_t = [(S \times H_t \times e_t)', 0, \dots, 0]'$  are  $n(np+1) \times 1$  vectors and

$$\mathbf{A}_t = \begin{bmatrix} A_{1t} & A_{2t} & \dots & A_{p-1t} & A_{pt} \\ I_k & 0 & \dots & 0 & 0 \\ 0 & I_k & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & I_k & 0 \end{bmatrix}$$

is an  $n(np+1) \times n(np+1)$  matrix. Recursively substituting into (12) we obtain

$$\mathbf{y}_{t+k} = \mathbf{A}_{0,t+k} + \sum_{j=1}^{k-1} \Phi_{t+k,j} \mathbf{A}_{0,t+k-j} + \Phi_{t+k,k} \mathbf{y}_t + \sum_{j=0}^{k-1} \Phi_{t+k,j} \mathbf{E}_{t+k-j}$$

where  $\Phi_{t+k,j} = \prod_{i=0}^{j-1} \mathbf{A}_{t+k-i}$ ,  $j = 1, 2, \dots$ ,  $\Phi_{t+k,0} = I_{np}$ . Let  $\mathcal{S}_{(h,h')}(X)$  be a selection matrix extracting  $h$ -rows and  $h'$ -columns of the matrix  $X$ . Since  $y_t = \mathcal{S}_{(n,1)}(\mathbf{y}_t)$ , setting  $\tilde{\Phi}_{t+k,k} = \mathcal{S}_{(n,n^2p)}(\Phi_{t+k,k})$  and  $\Phi_{t+k,j} = \mathcal{S}_{(n,n)}(\Phi_{t+k,j})$  we have

$$y_{t+k} = \tilde{A}_{0,t+k} + \tilde{\Phi}_{t+k,k} \mathbf{y}_t + \sum_{j=0}^{k-1} \Psi_{t+k,j} e_{t+k-j} \quad (13)$$

where  $\tilde{A}_{0,t+1} = A_{0,t+1}$ ;  $\tilde{A}_{0,t+k} = A_{0,t+k} + \sum_{j=1}^{k-1} \Phi_{t+k,j} A_{0,t+k-j}$  for  $k > 1$ ;  $\Psi_{t+k,j} = \Phi_{t+k,j} \times S \times H_{t+k-j}$  and  $\Psi_{t+k,0} = \Phi_{t+k,0} \times S \times H_{t+k} \equiv S \times H_{t+k} = J_{t+k}$ .

Partition  $e_t = (e_{i,t}, e_{-i,t})$ , where  $e_{i,t}$  is an element of  $e_t$  and  $e_{-i,t}$  is the vector containing the other  $n-1$  elements of  $e_t$ , and  $H_t = (h_{it}, h_{-it})$ , where  $h_{it}$  is a column of  $H_t$  corresponding to  $e_{i,t}$  and  $h_{-it}$  is the matrix formed by the other  $n-1$  columns. Then equation (13) is

$$y_{t+k} = \tilde{A}_{0,t+k} + \tilde{\Phi}_{t+k,k} \mathbf{y}_t + \sum_{j=0}^{k-1} \Psi_{t+k,j}^i e_{i,t+k-j} + \sum_{j=0}^{k-1} \Psi_{t+k,j}^{-i} e_{-i,t+k-j} \quad (14)$$

where  $\Psi_{t+k,j}^i = \Phi_{t+k,j} \times S \times h_{it+k-j}$  and  $\Psi_{t+k,j}^{-i} = \Phi_{t+k,j} \times S \times h_{-it+k-j}$ .

### Shocks to the Non-Systematic Component

Let  $\mathcal{I}_t^1 = \{y^t, \theta^t, V_{t+1}, J_{t+1}, \xi_{i,t+1}^\delta, \xi_{-i,t+1}, \zeta_{t+2}^{t+k}\}$  and  $\mathcal{I}_t^2 = \{y^t, \theta^t, V_{t+1}, J_{t+1}, \xi_{-i,t+1}, \zeta_{t+2}^{t+k}\}$ .

Taking conditional expectations of (14) we obtain

$$\begin{aligned} E(y_{t+k}|\mathcal{I}_t^l) &= E(\tilde{A}_{0,t+k}|\mathcal{I}_t^1) + E(\tilde{\Phi}_{t+k,k}|\mathcal{I}_t^l) \mathbf{y}_t + \\ &+ E\left(\sum_{j=0}^{k-1} \Psi_{t+k,j}^i e_{i,t+k-j}|\mathcal{I}_t^l\right) + E\left(\sum_{j=0}^{k-1} \Psi_{t+k,j}^{-i} e_{-i,t+k-j}|\mathcal{I}_t^l\right) \end{aligned} \quad (15)$$

$l = 1, 2$ . Since  $e_t$  and  $\eta_t$  are orthogonal:

$$\begin{aligned} E(y_{t+k}|\mathcal{I}_t^1) - E(y_{t+k}|\mathcal{I}_t^2) &= E(\Psi_{t+k,k-1}^i e_{i,t+k-k+1}|\mathcal{I}_t^1) - E(\Psi_{t+k,k-1}^i e_{i,t+k-k+1}|\mathcal{I}_t^2) \\ &= \Psi_{t+k,k-1}^i e_{i,t+k-k+1} - \Psi_{t+k,k-1}^i E(e_{i,t+k-k+1}|\mathcal{I}_t^2) \\ &= \Psi_{t+k,k-1}^i e_{i,t+1} \end{aligned} \quad (16)$$

### Shocks to the Systematic Component

Let  $\eta_{i,t+1}$  be a shock to the systematic component of the  $i$ -th equation and set  $\eta_{-i,t+1} = 0$ .

Taking expectations with respect to  $\mathcal{I}_t^l$ ,  $l = 1, 2$  we have

$$E(y_{t+k}|\mathcal{I}_t^l) = E(\tilde{A}_{0,t+k}|\mathcal{I}_t^l) + E(\tilde{\Phi}_{t+k,k}|\mathcal{I}_t^l) \mathbf{y}_t + E\left(\sum_{j=0}^{k-1} \Psi_{t+k,j}|\mathcal{I}_t^l\right) e_{t+k-j} \quad (17)$$

Taking the difference between conditional expectations we obtain equation (10).

## B. Estimation

### Priors

Let  $T$  be the end of the estimation sample and let  $K_1$  be the number of periods for which the identifying restrictions must be satisfied. Let  $H_T = \rho(\omega_T)$  be a rotation matrix whose columns represents orthogonal points in the hypersphere and let  $\omega_T$  be a vector in  $R^6$  whose elements are  $U[0, 1]$  random variables. Let  $\mathcal{M}_T$  be the set of impulse response functions at  $T$  satisfying the restrictions and let  $F(\mathcal{M}_T)$  be an indicator function which is one if the identifying restrictions are satisfied, that is, if  $(\Psi_{T+1,1}^i, \dots, \Psi_{T+K_1,K_1}^i) \in \mathcal{M}_T$ , and zero otherwise. Let the prior for  $\theta^{T+K_1}$ ,  $\Sigma_T$ ,  $\Omega_T$ ,  $H_T$  be

$$p(\theta^{T+K_1}, \Sigma_T, \Omega_T, \omega_T) = p(\theta^{T+K}|\Sigma_T, \Omega_T)p(\Sigma_T, \Omega_T)F(\mathcal{M}_T)p(H_T) \quad (18)$$

Here  $p(\theta^{T+K}|\Sigma_T, \Omega_T) \propto I(\theta^{T+K})f(\theta^{T+K}|\Sigma_T, \Omega_T)$  where  $I(\theta^{T+K}) = \prod_{t=0}^{T+K} I(\theta_t)$  and  $f(\theta^{T+K}|\Sigma_T, \Omega_T) = f(\theta_0) \prod_{t=0}^{T+K} f(\theta_t|\theta_{t-1}, \Sigma_t, \Omega_t)$ . Hence,  $p(\theta^{T+K}|\Sigma_T, \Omega_T)$  is truncated normal.

We assume that  $\Sigma_t = \Sigma$ ,  $\Omega_t = \kappa_1 \Omega_{t-1} + \kappa_2 \Omega$  and that  $\Sigma$  and  $\Omega$  have independent inverse Wishart priors with scale matrices  $\Sigma_0^{-1}$ ,  $\Omega_0^{-1}$  and degrees of freedom  $\nu_{01}$  and  $\nu_{02}$ . We also assume that the prior for  $\theta_0$  is truncated Gaussian independent of  $\Sigma_T$  and  $\Omega_T$ , i.e.  $f(\theta_0) \propto I(\theta_0)N(\bar{\theta}, \bar{P})$ . Finally we assume a uniform prior  $p(H_T)$  since all rotation matrices are a-priori equally likely. Collecting the pieces, the joint prior is:

$$p(\theta^{T+K}, \Sigma_T, \Omega_T, \omega_T) \propto I(\theta^{T+K})F(\mathcal{M}_T)[f(\theta_0) \prod_{t=0}^{T+K} f(\theta_t|\theta_{t-1}, \Sigma_t, \Omega_t)]p(\Sigma_T)p(\Omega_T) \quad (19)$$

Note that when  $H_t = I_n$ ,  $F(\mathcal{M}_T) \equiv 1$ .

We "calibrate" the prior by estimating a fixed coefficients VAR using data from 1960:1 up to 1969:1. We set  $\bar{\theta}$  equal to the point estimates of the coefficients and  $\bar{P}$  to the estimated covariance matrix;  $\Sigma_0$  is equal to the estimated covariance matrix of VAR innovations;  $\Omega_0 = \varrho \bar{P}$  and  $\nu_{10}$  and  $\nu_{20}$  are set to small numbers (so as to make the prior close to non-informative). The parameter  $\varrho$  measures how much the time variation is allowed in coefficients. Although as  $T$  grows the likelihood dominates, the choice of  $\varrho$  matters in finite samples. We choose  $\varrho$  as a function of  $T$  i.e. for the sample 1969:1-1981:2,  $\varrho = 0.0025$ ; for 1969:1-1983:2,  $\varrho = 0.003$ ; for 1969:1-1987:2,  $\varrho = 0.0035$ ; for 1969:1-1989:2,  $\varrho = 0.004$ ; for 1969:1-1995:4,  $\varrho = 0.007$ ; for 1969:1-1999:1,  $\varrho = 0.008$ , and for 1969:1-2003:2,  $\varrho = 0.01$ . This range of values implies a quiet conservative prior coefficient variations: in fact, time variation accounts between 0.35 and a 1 percent of the total coefficients standard deviation. After some specification searches, we set  $\kappa_1 = 0.8$ ,  $\kappa_2 = 0.1$ .

## Posteriors

To draw posterior sequences we need  $p(H_T, \theta_{T+1}^{T+K}, \theta^T, \Sigma_T, \Omega_T|y^T)$ , which is analytically intractable. Luckily we can decompose it into simpler components. First, note that

$$\begin{aligned} p(H_T, \theta_{T+1}^{T+K}, \theta^T, \Sigma_T, \Omega_T|y^T) &\equiv p(H_T, \theta^{T+K}, \Sigma_T, \Omega_T|y^T) \\ &\propto p(y^T|H_T, \theta^{T+K}, \Sigma_T, \Omega_T)p(H_T, \theta^{T+K}, \Sigma_T, \Omega_T) \end{aligned} \quad (20)$$

Second, since the likelihood is invariant to any orthogonal rotation  $p(y^T|H_T, \theta^{T+K}, \Sigma_T, \Omega_T) = p(y^T|\theta^{T+K}, \Sigma_T, \Omega_T)$ . Third,  $p(H_T, \theta^{T+K}, \Sigma_T, \Omega_T) = p(\theta^{T+K}, \Sigma_T, \Omega_T)F(\mathcal{M}_T)p(H_T)$ . Thus

$$p(H_T, \theta^{T+K}, \Sigma_T, \Omega_T|y^T) \propto p(\theta^{T+K}, \Sigma_T, \Omega_T|y^T)F(\mathcal{M}_T)p(H_T) \quad (21)$$

where  $p(\theta^{T+K}, \Sigma_T, \Omega_T | y^T)$  is the posterior distribution for the reduced form parameters, which, in turn can be factored as

$$p(\theta^{T+K}, \Sigma_T, \Omega_T | y^T) = p(\theta_{T+1}^{T+K} | y^T, \theta^T, \Sigma_T, \Omega_T) p(\theta^T, \Sigma_T, \Omega_T | y^T)$$

The first term on the right hand side of (22) represents beliefs about the future and the second term the posterior density for states and hyperparameters. Here  $p(\theta_{T+1}^{T+K} | y^T, \theta^T, \Sigma_T, \Omega_T) = p(\theta_{T+1}^{T+K} | \theta^T, \Sigma_T, \Omega_T) = \prod_{k=1}^K p(\theta_{T+k} | \theta_{T+k-1}, \Sigma_T, \Omega_T)$  because the states are Markov. Finally, since  $\theta_{T+k}$  is conditionally truncated normal with mean  $\theta_{T+k-1}$  and variance  $\Omega_T$

$$\begin{aligned} p(\theta_{T+1}^{T+K} | \theta^T, \Sigma_T, \Omega_T) &= I(\theta_{T+1}^{T+K}) \prod_{k=1}^K f(\theta_{T+k} | \theta_{T+k-1}, \Sigma, \Omega_T) \\ &= I(\theta_{T+1}^{T+K}) f(\theta_{T+1}^{T+K} | \theta^T, \Sigma_T, \Omega_T) \end{aligned} \quad (22)$$

The second term in (22) can be factored as

$$p(\theta^T, \Sigma_T, \Omega_T | y^T) \propto p(y^T | \theta^T, \Sigma_T, \Omega_T) p(\theta^T, \Sigma_T, \Omega_T) \quad (23)$$

The first term is the likelihood function which, given the states, has a Gaussian shape so that  $p(y^T | \theta^T, \Sigma_T, \Omega_T) = f(y^T | \theta^T, \Sigma_T, \Omega_T)$ . The second term is the joint posterior for states and hyperparameters. Hence:

$$p(\theta^T, \Sigma_T, \Omega_T | y^T) \propto f(y^T | \theta^T, \Sigma_T, \Omega_T) p(\theta^T | \Sigma_T, \Omega_T) p(\Sigma_T, \Omega_T) \quad (24)$$

Since  $p(\theta^T | \Sigma_T, \Omega_T) \propto I(\theta^T) f(\theta^T | \Sigma_T, \Omega_T)$  where  $f(\theta^T | \Sigma, \Omega) = f(\theta_0 | \Sigma, \Omega) \prod_{t=1}^T f(\theta_t | \theta_{t-1}, \Sigma_t, \Omega_t)$  and  $I(\theta^T) = \prod_{t=0}^T I(\theta_t)$ , we have

$$p(\theta^T, \Sigma_T, \Omega_T | y^T) \propto I(\theta^T) f(y^T | \theta^T, \Sigma_T, \Omega_T) f(\theta^T | \Sigma_T, \Omega_T) p(\Sigma_T, \Omega_T) = I(\theta^T) p_u(\theta^T, \Sigma_T, \Omega_T | y^T) \quad (25)$$

where  $p_u(\theta_T, \Sigma_T, \Omega_T | y^T) \equiv f(y^T | \theta^T, \Sigma_T, \Omega_T) f(\theta^T | \Sigma_T, \Omega_T) p(\Sigma_T, \Omega_T)$  is the posterior density obtained if no restrictions are imposed. Collecting pieces we finally have

$$\begin{aligned} p(H_T, \theta_{T+1}^{T+K}, \theta^T, \Sigma_T, \Omega_T | y^T) &\propto I(\theta_{T+1}^{T+K}) f(\theta_{T+1}^{T+K} | \theta^T, \Sigma_T, \Omega_T) I(\theta^T) p_u(\theta^T, \Sigma_T, \Omega_T | y^T) \\ &\quad F(\mathcal{M}_T) p(H_T) \end{aligned} \quad (26)$$

Note that for  $H_t = I$ ,  $F(\mathcal{M}_T) = p(H_T) = 1$ .

### Drawing structural parameters

Given (26) draws for the structural parameters can be obtained as follows

1. Draw  $(\theta^T, \Sigma_T, \Omega_T)$  from the unrestricted posterior  $p_u(\theta^T, \Sigma_T, \Omega_T | y^T)$  via the Gibbs sampler (see below). Apply the filter  $I(\theta^T)$ .
2. Given  $(\theta^T, \Sigma_T, \Omega_T)$ , draw a sequence for  $\theta_{T+1}^{T+K}$ , i.e. draw  $u_{T+k}$  from  $N(0, \Omega_T)$  and iterate in  $\theta_{T+k} = \theta_{T+k-1} + u_{T+k}$ ,  $K$  times. Apply the filter  $I(\theta^{T+K})$ .
3. Draw  $\omega_{i,T}$  for  $i = 1, \dots, 6$  from a  $U[0, 1]$ . Draw  $H_T = \rho(\omega_T)$ .
4. Given  $\Sigma_T$ , find the matrix  $S_T$ , such that  $\Sigma_T = S_T S_T'$ . Construct  $J_T^{-1}$ .
5. Compute  $(\Psi_{T+1,1}^{i,\ell}, \dots, \Psi_{T+K,K}^{i,\ell})$  for each replication  $\ell$ . Apply the filter  $F(\mathcal{M}_T)^\ell$ .

### Drawing reduced form parameters

The Gibbs Sampler we use to compute the posterior for the reduced form parameters iterates on two steps. The implementation is identical to Cogley and Sargent (2001).

- Step 1: States given hyperparameters

Conditional on  $(y^T, \Sigma_T, \Omega_T)$ ,  $p_u(\theta^T | y^T, \Sigma_T, \Omega_T) = f(\theta_T | y^T, \Sigma_T, \Omega_T) \prod_{t=1}^{T-1} f(\theta_t | \theta_{t+1}, y^t, \Sigma_t, \Omega_t)$ .

All densities on the right end side are Gaussian they their conditional means and variances can be computed using the Kalman smoother. Let  $\theta_{t|t} \equiv E(\theta_t | y^t, \Sigma_t, \Omega_t)$ ;  $P_{t|t-1} \equiv \text{Var}(\theta_t | y^{t-1}, \Sigma_t, \Omega_t)$ ;  $P_{t|t} \equiv \text{Var}(\theta_t | y^t, \Sigma_t, \Omega_t)$ . Given  $P_{0|0}$ ,  $\theta_{0|0}$ ,  $\Omega$  and  $\Sigma$

$$\begin{aligned}
P_{t|t-1} &= P_{t-1|t-1} + \Sigma \\
\mathcal{K}_t &= (P_{t-1|t-1} X_t) (X_t' P_{t-1|t-1} X_t + \Omega_t)^{-1} \\
\theta_{t|t} &= \theta_{t-1|t-1} + \mathcal{K}_t (y_t - X_t' \theta_{t-1|t-1}) \\
P_{t|t} &= P_{t-1|t-1} - \mathcal{K}_t (X_t' P_{t-1|t-1}) \\
\Omega_t &= \kappa_1 \Omega_{t-1} + \kappa_2 \Omega
\end{aligned} \tag{27}$$

The last iteration gives  $\theta_{T|T}$  and  $P_{T|T}$  which are the conditional means and variance of  $f(\theta_T | y^T, \Sigma_T, \Omega_T)$ . Hence  $f(\theta_T | y^T, \Sigma_T, \Omega_T) = N(\theta_{T|T}, P_{T|T})$ . The other  $T - 1$  densities can be computed using the backward recursions

$$\begin{aligned}
\theta_{t|t+1} &= \theta_{t|t} + P_{t|t} P_{t+1|t}^{-1} (\theta_{t+1} - \theta_{t|t+1}) \\
P_{t|t+1} &= P_{t|t} - P_{t|t} P_{t+1|t}^{-1} P_{t|t}
\end{aligned} \tag{28}$$

where  $\theta_{t|t+1} \equiv E(\theta_t|\theta_{t+1}, y^t, \Sigma_t, \Omega_t)$  and  $P_{t|t+1} \equiv Var(\theta_t|\theta_{t+1}, y^t, \Sigma_t, \Omega_t)$  are the conditional means and variances of the remaining terms in  $p_u(\theta^T|y^T, \Sigma_t, \Omega_t)$ . Thus  $f(\theta_t|\theta_{t+1}, y^t, \Sigma_t, \Omega_t) = N(\theta_{t|t+1}, P_{t|t+1})$ . Therefore, to sample  $\theta^T$  from the conditional posterior we proceed backward, sampling  $\theta^T$  from  $N(\theta_{T|T}, P_{T|T})$  and  $\theta^t$  from  $N(\theta_{t|t+1}, P_{t|t+1})$  for all  $t < T$ .

• Step 2: Hyperparameters given states

Conditional on the states and the data,  $\varepsilon_t$  and  $u_t$  are observable and Gaussian. Hence their posterior is inverse-Wishart with parameters  $\Sigma_{1T} = \Sigma_0 + \Sigma_T^*$ ,  $\Omega_{1T} = \Omega_0 + \Omega_T^*$ ,  $\nu_{11} = \nu_{01} + T$ ,  $\nu_{12} = \nu_{02} + T$  and  $\Sigma_T^*$  and  $\Omega_T^*$  are proportional to the covariance estimators  $\frac{1}{T} \sum_{t=1}^T \varepsilon_t \varepsilon_t'$ ;  $\frac{1}{T} \sum_{t=1}^T u_t u_t'$ , respectively

Under regularity conditions and after a burn-in period, iterations on these two steps produce draws from  $p_u(\theta^T, \Sigma_T, \Omega_T|y^T)$ . In our exercises  $T$  varies from 1977:3 to 2003:2. For each of these  $T$ , 10000 iterations of the Gibbs sampler are made. CUMSUM graphs are used to check for convergence and we found that the chain had converged, roughly, after 2000 draws for each date. The densities for the parameters are typically well behaved and none is multimodal. We keep one every four of the remaining 8000 draws. After discarding all the draws generating explosive paths and that fail to satisfy the identification restrictions, we are left with about 250-300 draws for each date to conduct structural inference.

### Drawing structural impulse responses

Given a draw from the posterior of the structural parameters, the calculation of impulse responses to shocks to the non-systematic component is straightforward. In fact, given the an draw for  $(\theta^{T+K}, \Sigma_T, \Omega_T, H_{T+1})$  we calculate  $\Psi_{T+k,k}$  at each draw, compute the posterior median and the 68% central credible set at each horizon  $k$ . To compute responses to shocks to the systematic component we proceed as follows.

1. Draw  $\theta_T, \Sigma_T, \Omega_T$  and  $H_{T+1}$  from their posterior distribution.
2. Compute  $S_T$  and draw sequences for  $\epsilon_{T+1}^{T+K}$  and  $u_{T+2}^{T+K}$ .
3. Fix  $\eta_{i,T+1}^\delta$ , draw  $\eta_{-i,T+1}$  from the conditional distribution  $(\eta_{-i,T+1}|\eta_{i,T+1} = \delta)$  and form  $\eta_{T+1}^\delta$ . Compute  $u_{T+1}^\delta = (J_{T+1}^{-1} \otimes I_{np+1})^{-1} \eta_{T+1}^\delta$  and let  $u_{T+1}^{\delta, T+k} = \{u_{T+1}^\delta, u_{T+2}^{T+K}\}$
4. Using  $u_{T+1}^{\delta, T+k}$ ,  $\epsilon_{T+1}^{T+K}$ ,  $\theta_T$ . Compute  $\theta_{T+1}^{T+K} \tilde{\Phi}_{T+k,k}$ ,  $\tilde{A}_{0,T+k}$ ,  $\sum_{j=0}^{k-1} \Phi_{T+k,j} \epsilon_{T+k-j}$  for  $k = 1, \dots, K$ . This is a draw for  $E(y_{T+k}|I_t^1)$ .

5. Fix  $\eta_{i,T+1}^0$ , draw  $\eta_{-i,T+1}$  from the conditional distribution  $(\eta_{-i,T+1}|\eta_{i,T+1} = 0)$  and form  $\eta_{T+1}^0$ . Compute  $u_{T+1}^0 = (J_{T+1}^{-1} \otimes I_{np+1})^{-1} \eta_{T+1}^0$  and let  $u_{T+1}^{0,T+k} = \{u_{T+1}^0, u_{T+2}^{T+K}\}$
6. Using  $u_{T+1}^{0,T+k}$ ,  $\epsilon_{T+1}^{T+K}$ ,  $\theta_T$  compute  $\theta_{T+1}^{T+K}$ ,  $\tilde{\Phi}_{T+k,k}$ ,  $\tilde{A}_{0,T+k} \sum_{j=0}^{k-1} \Phi_{T+k,j} \epsilon_{T+k-j}$  for  $k = 1, \dots, K$  and  $y_{T+k}^0$ . This is a draw for  $E(y_{T+k}|I_t^2)$ .
7. Take the difference of the realizations in 4. and 6. for each draw.

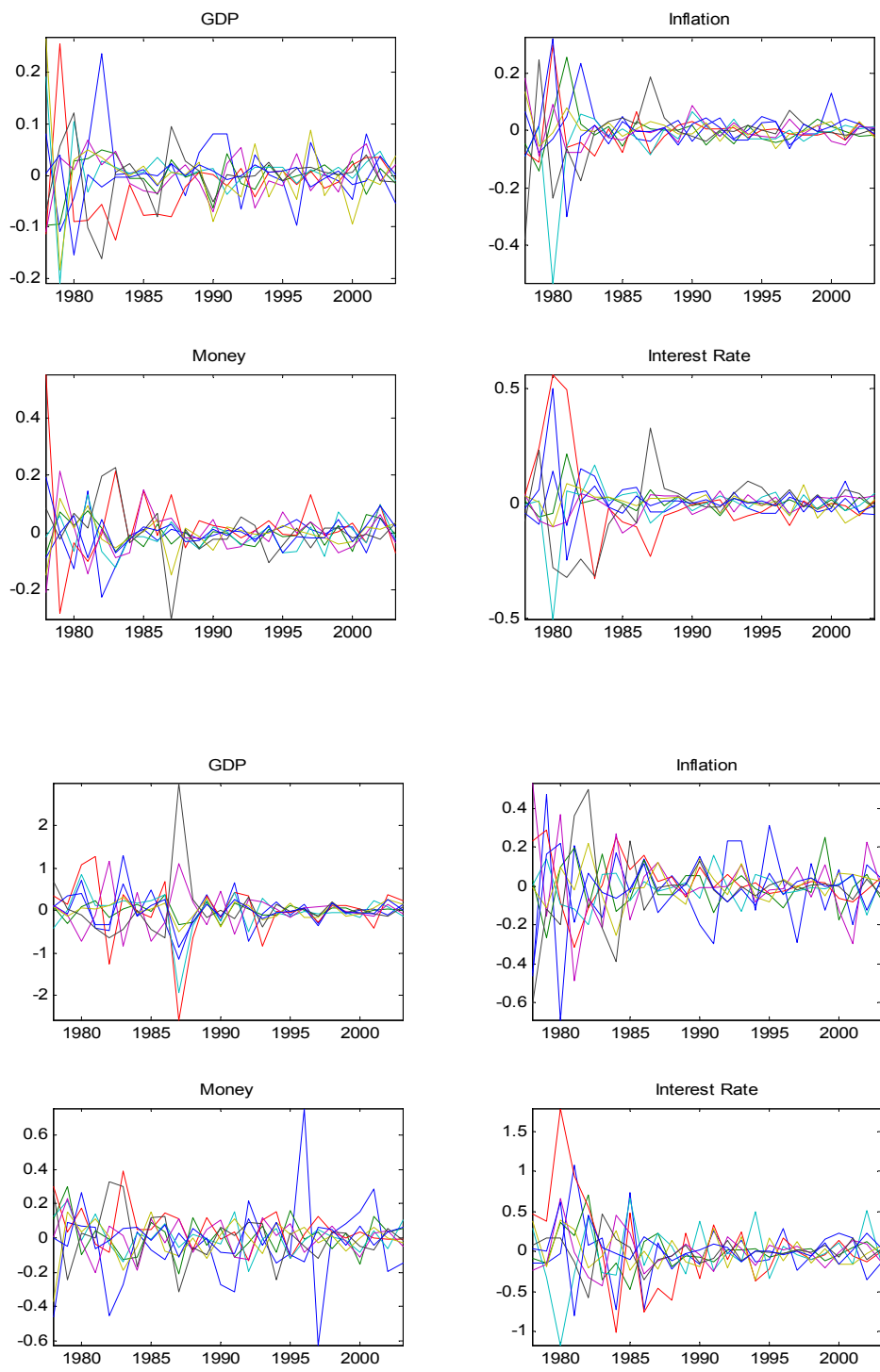


Figure 1: Mean changes: reduced form coefficients (top), structural coefficients (bottom)

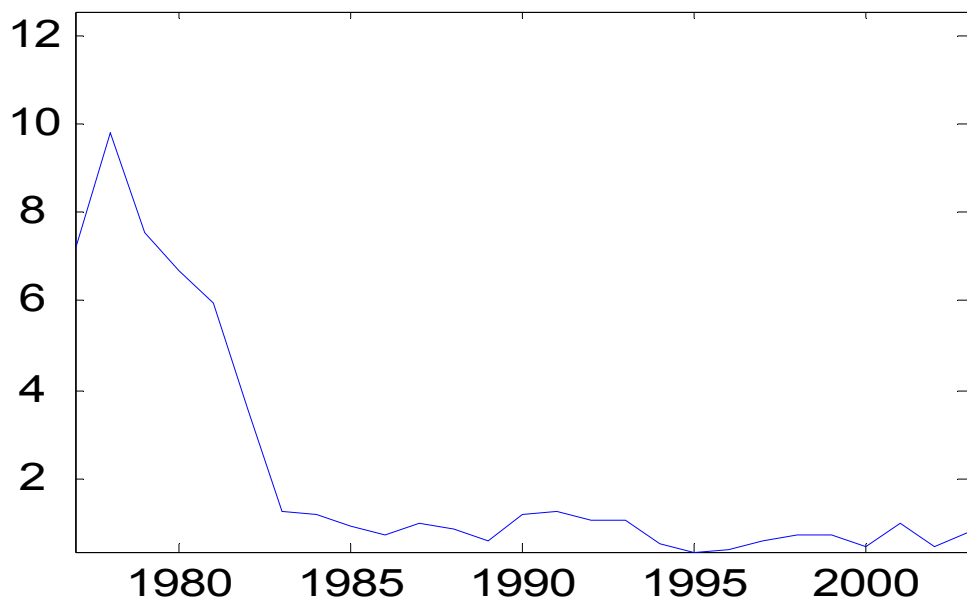
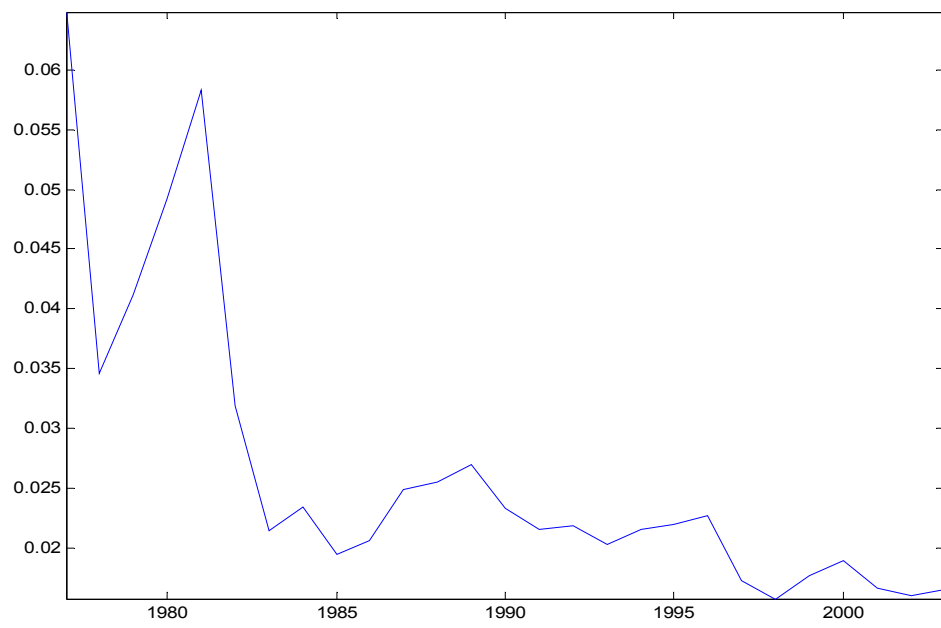


Figure 2: Reduced form mean inflation drift and mean inflation persistence.

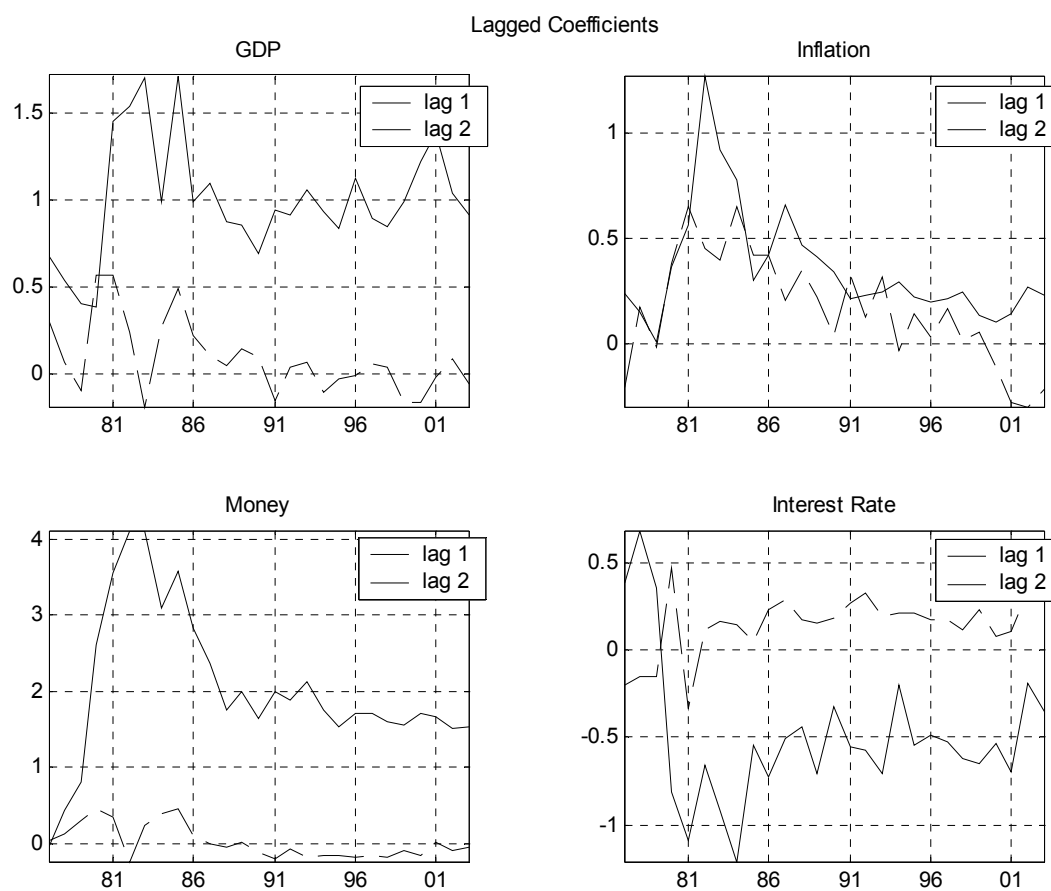
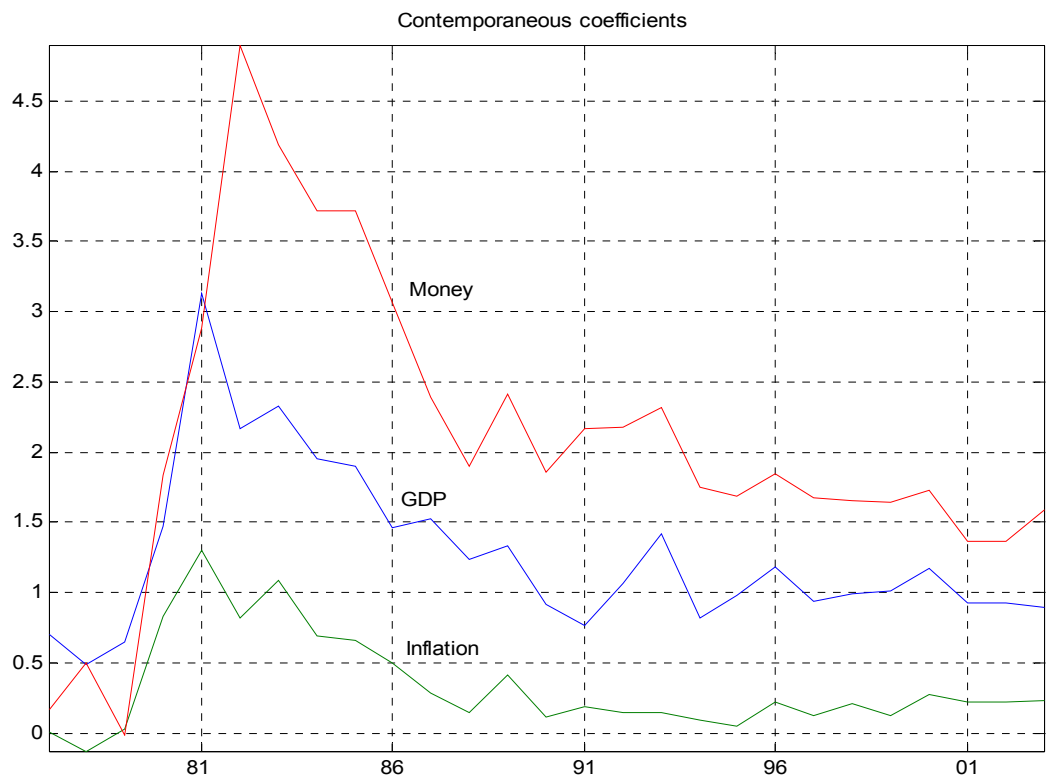


Figure 3: Structural coefficients, monetary policy equation

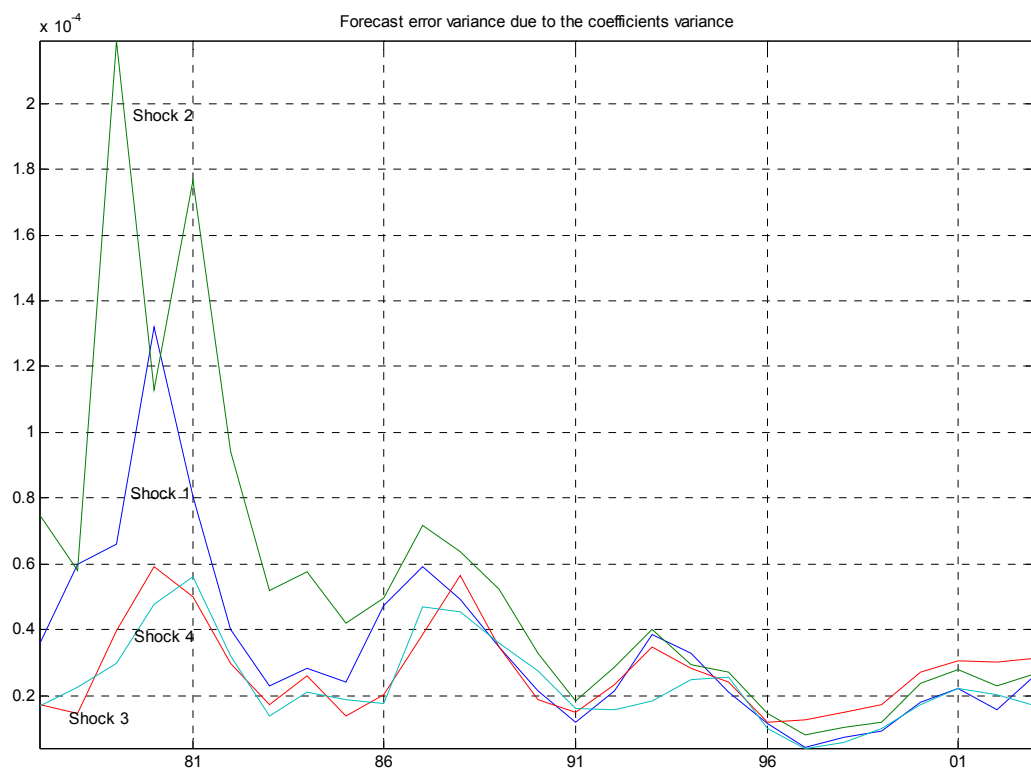
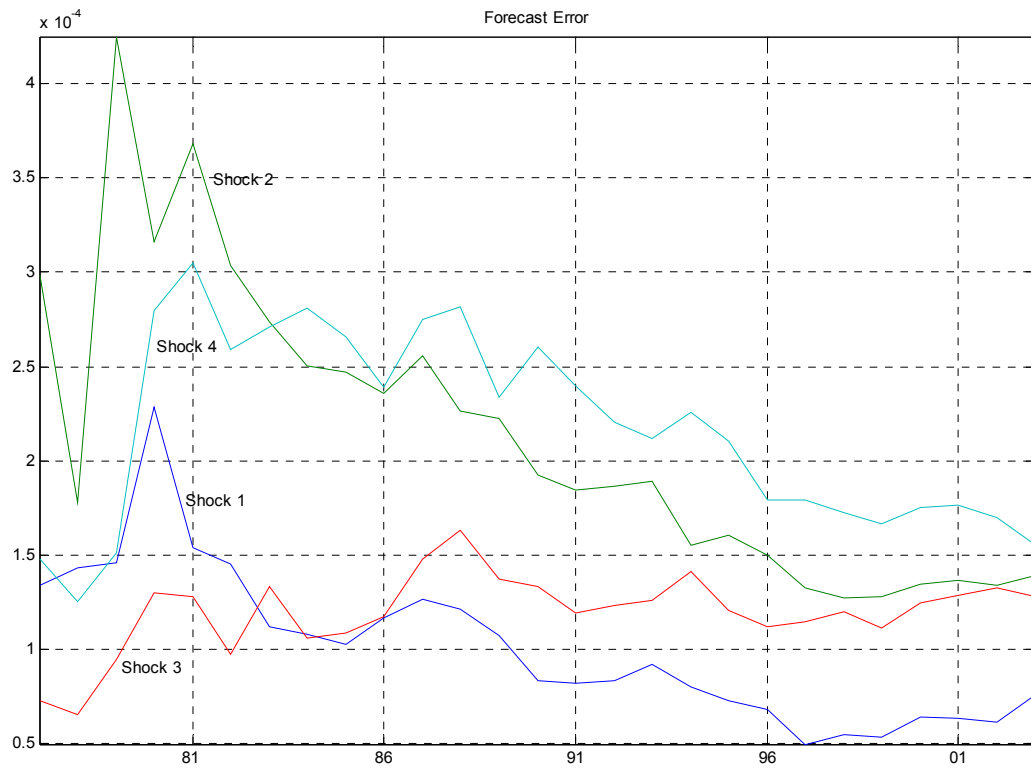


Figure 4: Forecast error variance

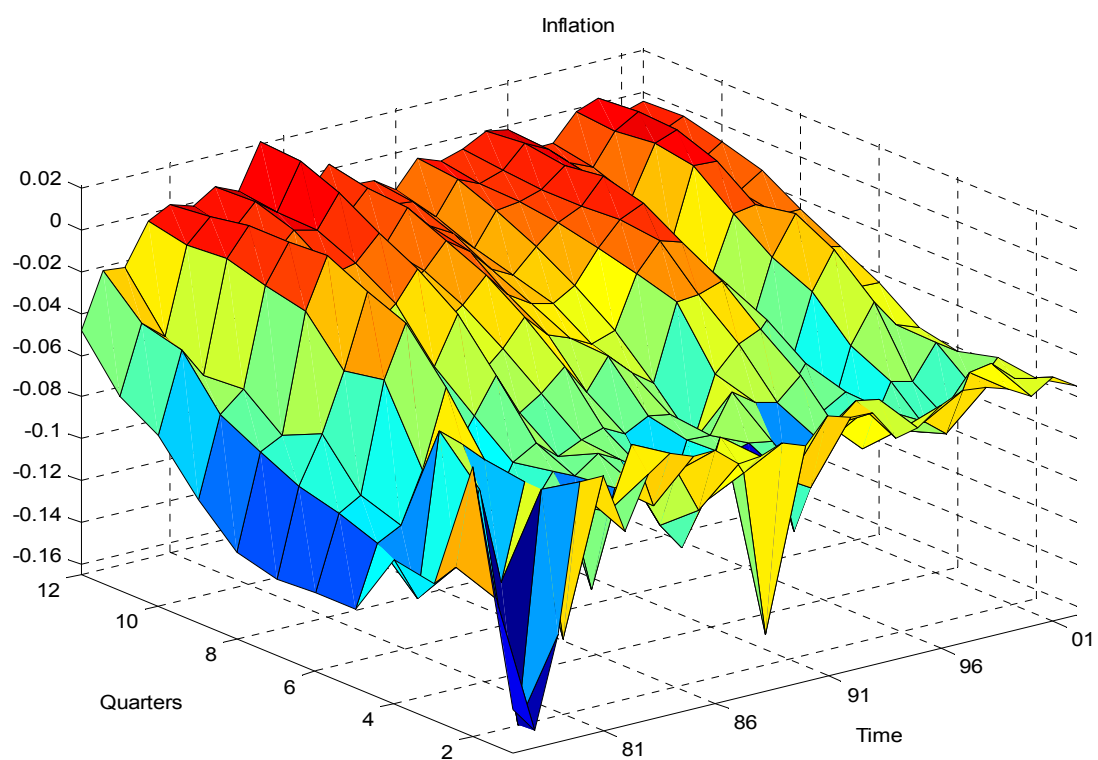
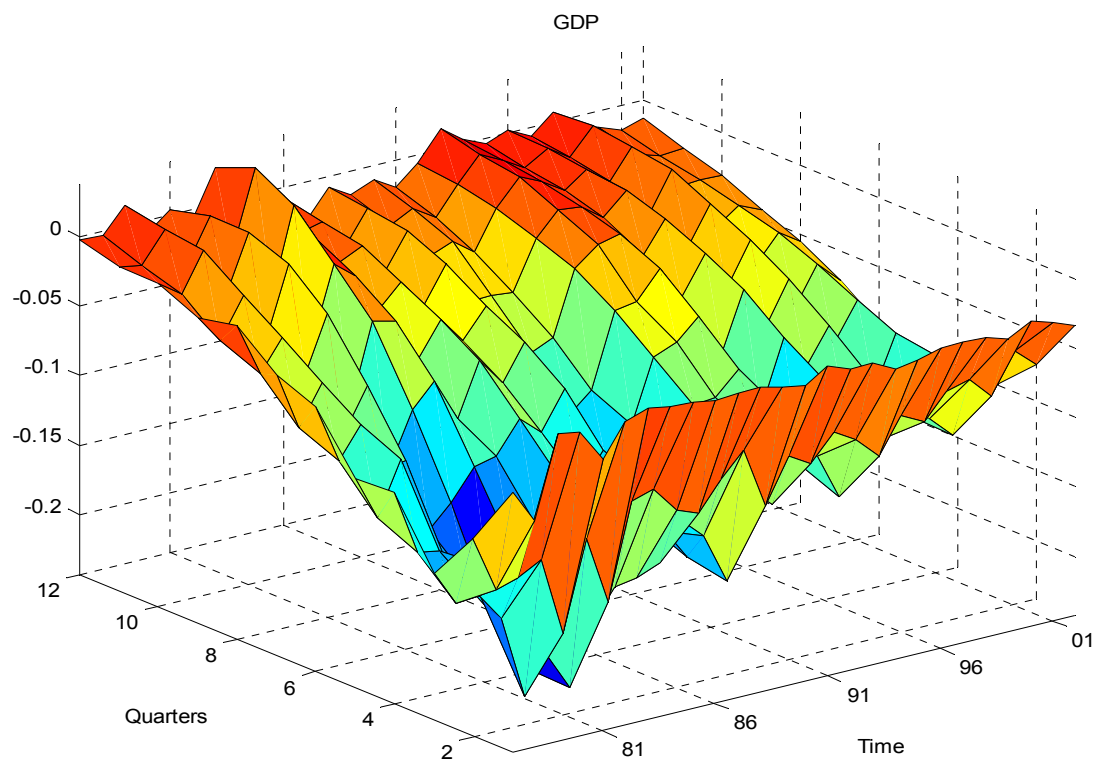


Figure 5: Structural impulse response to monetary policy shocks

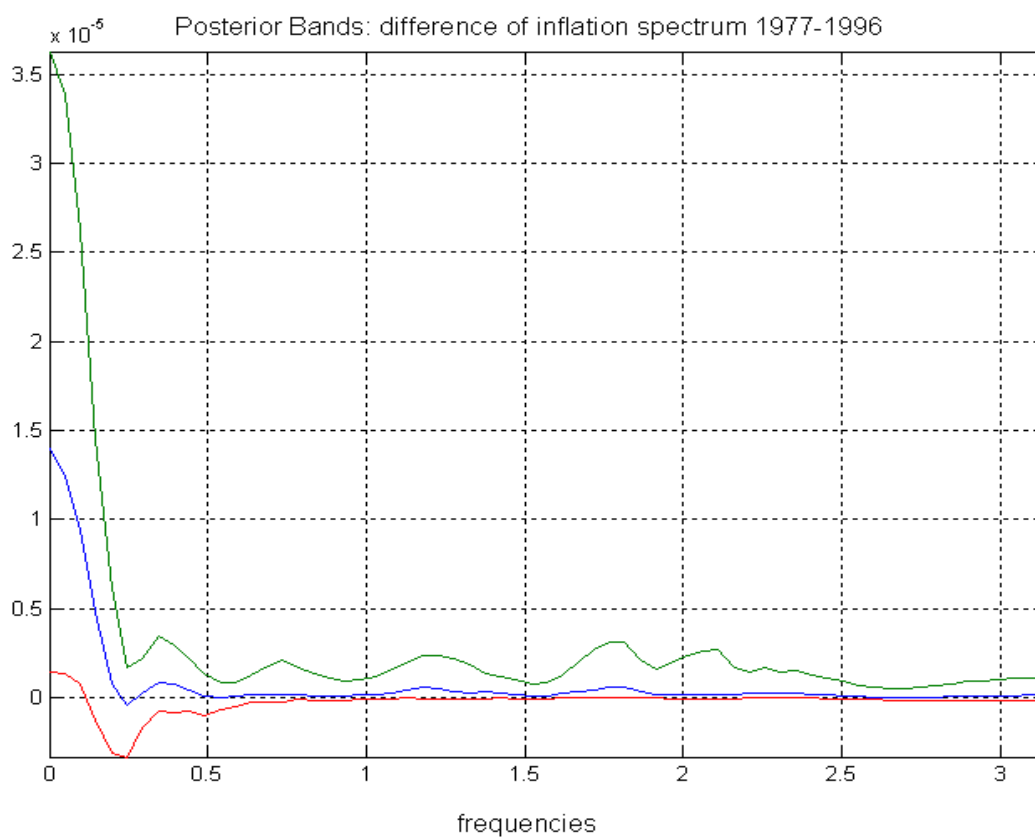
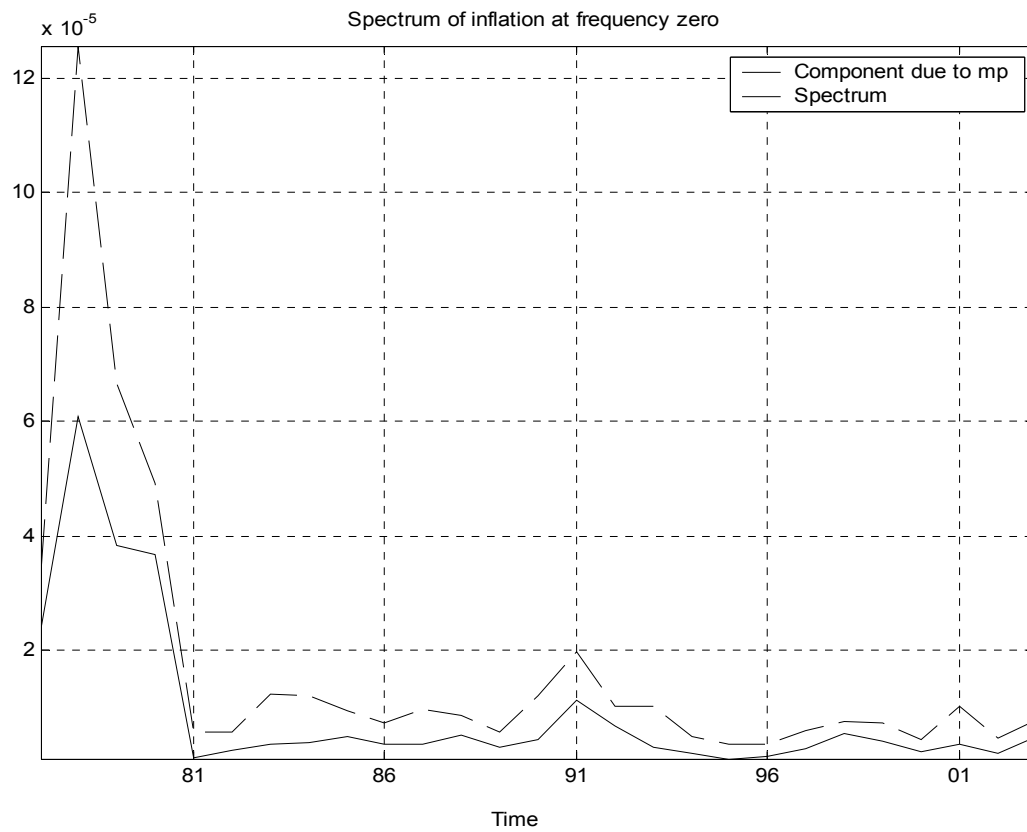


Figure 6: Inflation: persistence and spectrum

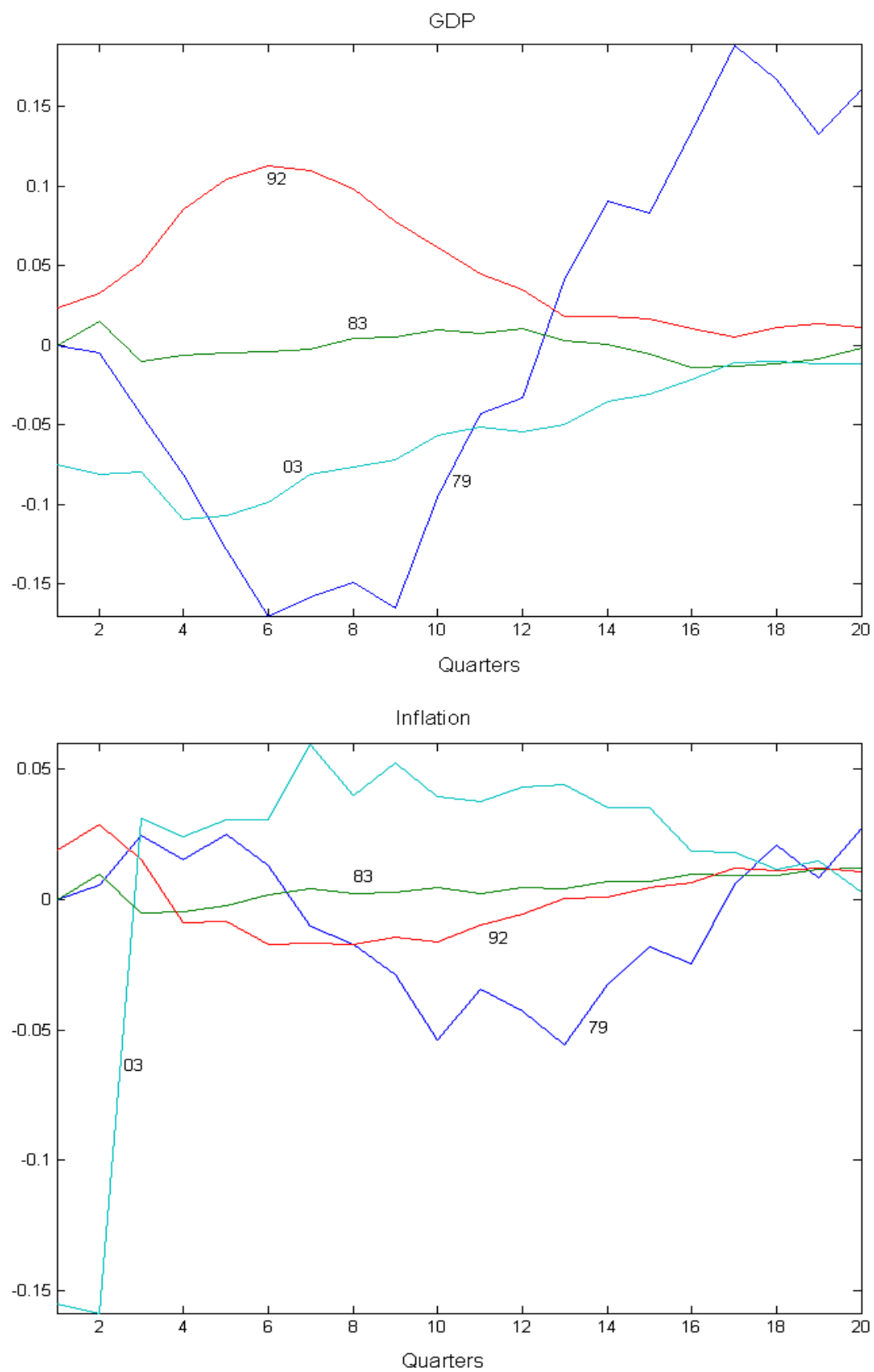


Figure 7: Impulse responses: more aggressive stance on inflation.

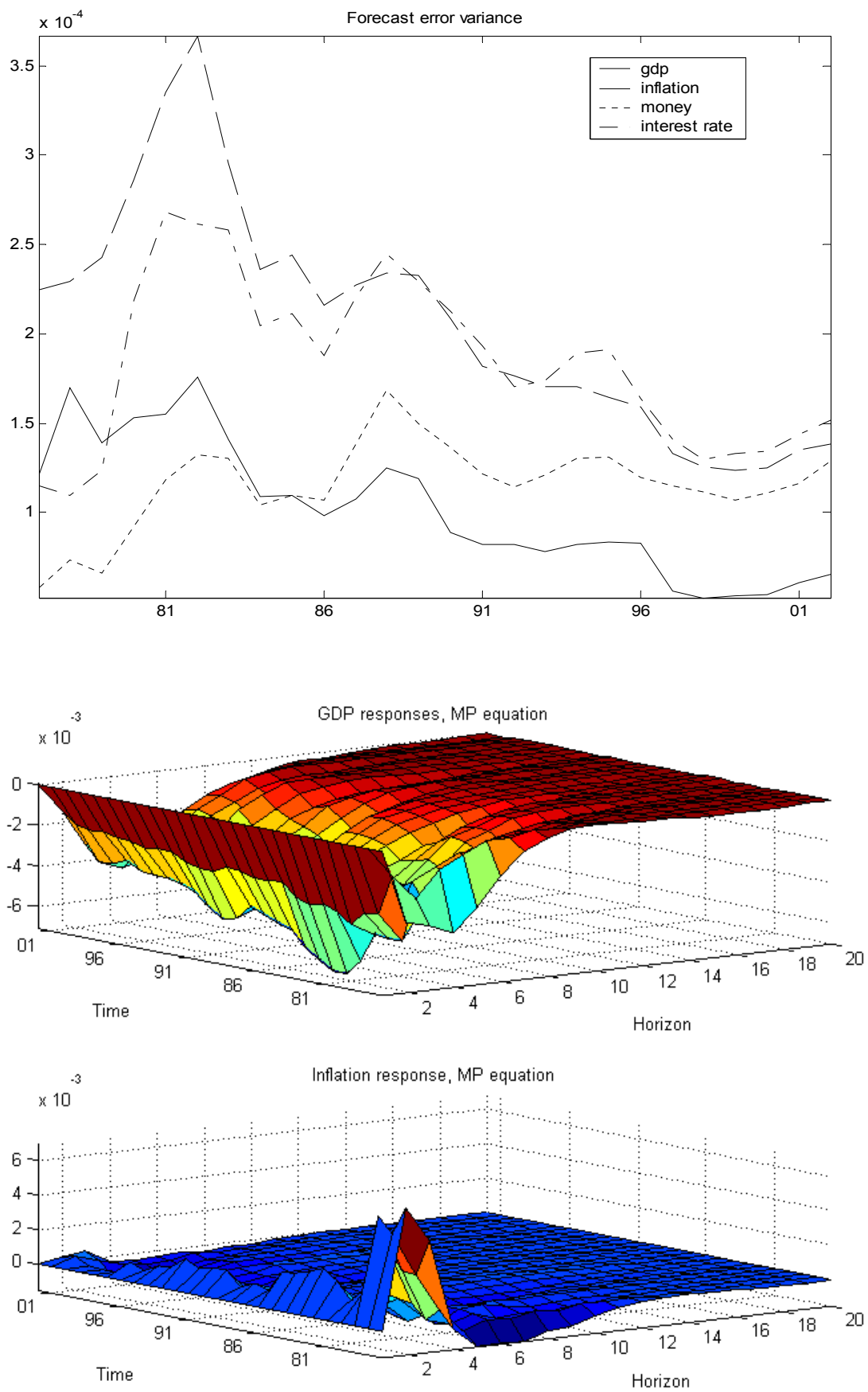


Figure 8: Forecast error variance and impulse response, Choleski identification.

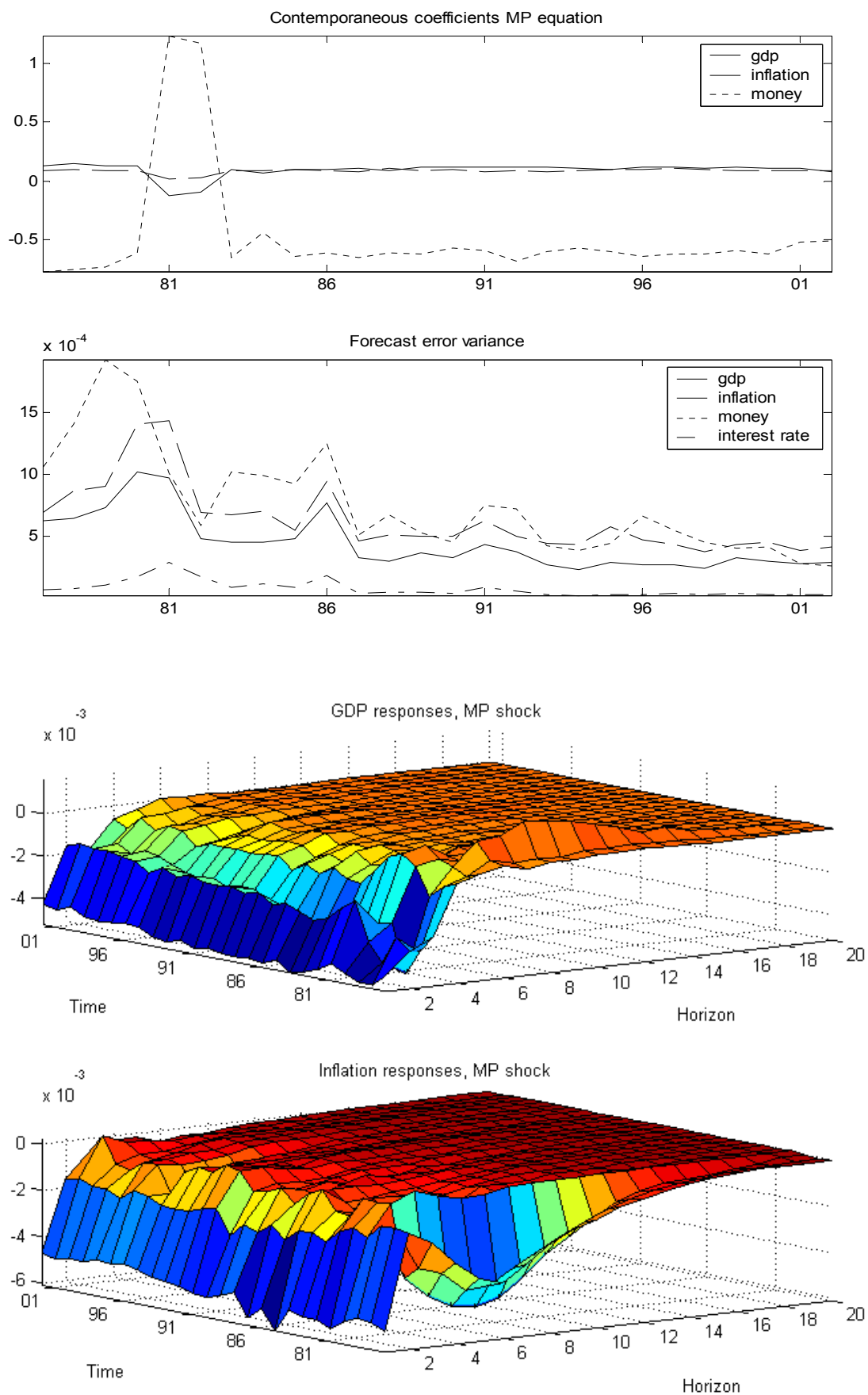


Figure 9: Contemporaneous coefficients, forecast error variance and impulse response functions, output and money in growth rates.

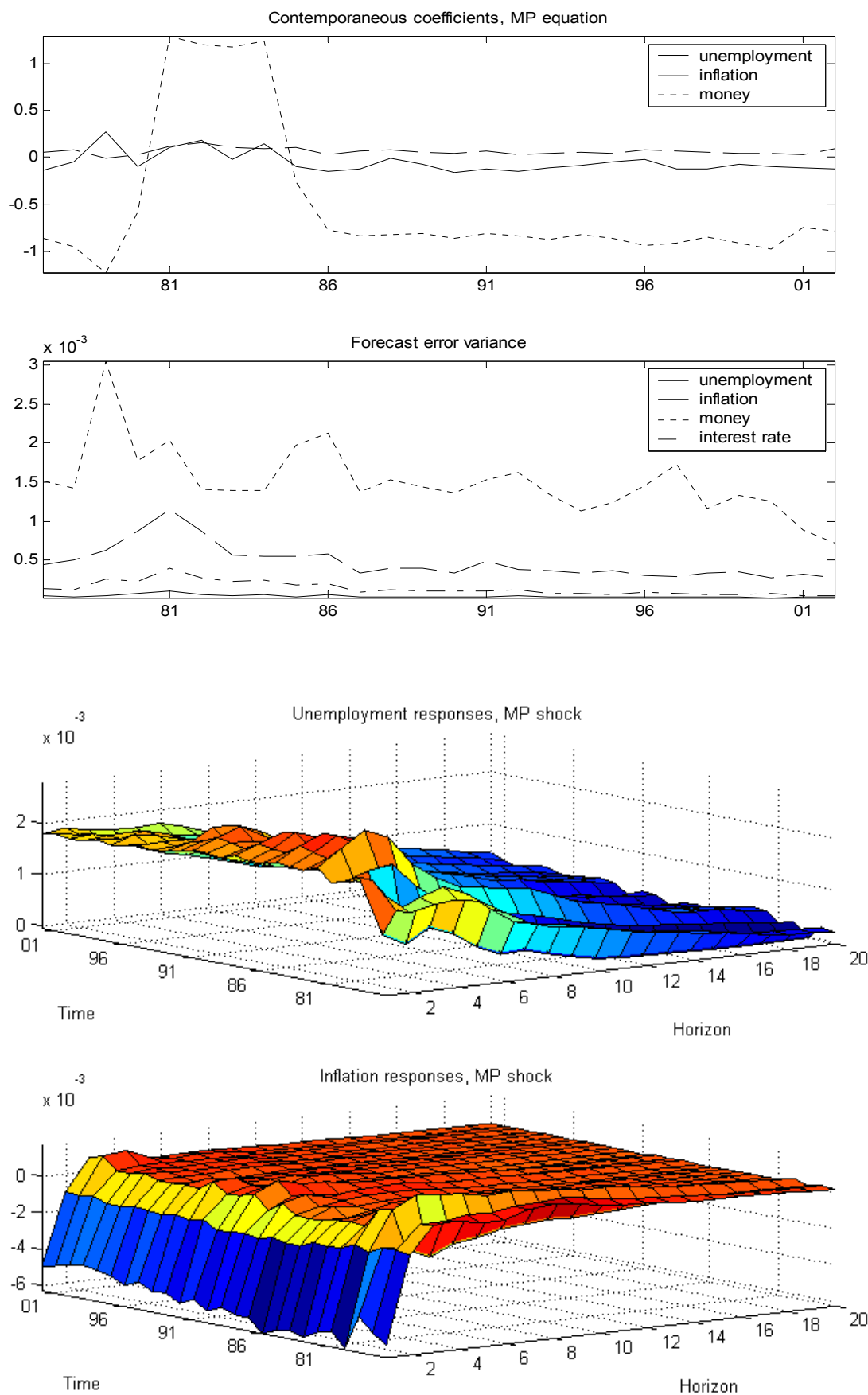


Figure 10: Contemporaneous coefficients, forecast error variance and impulse response functions, system with unemployment

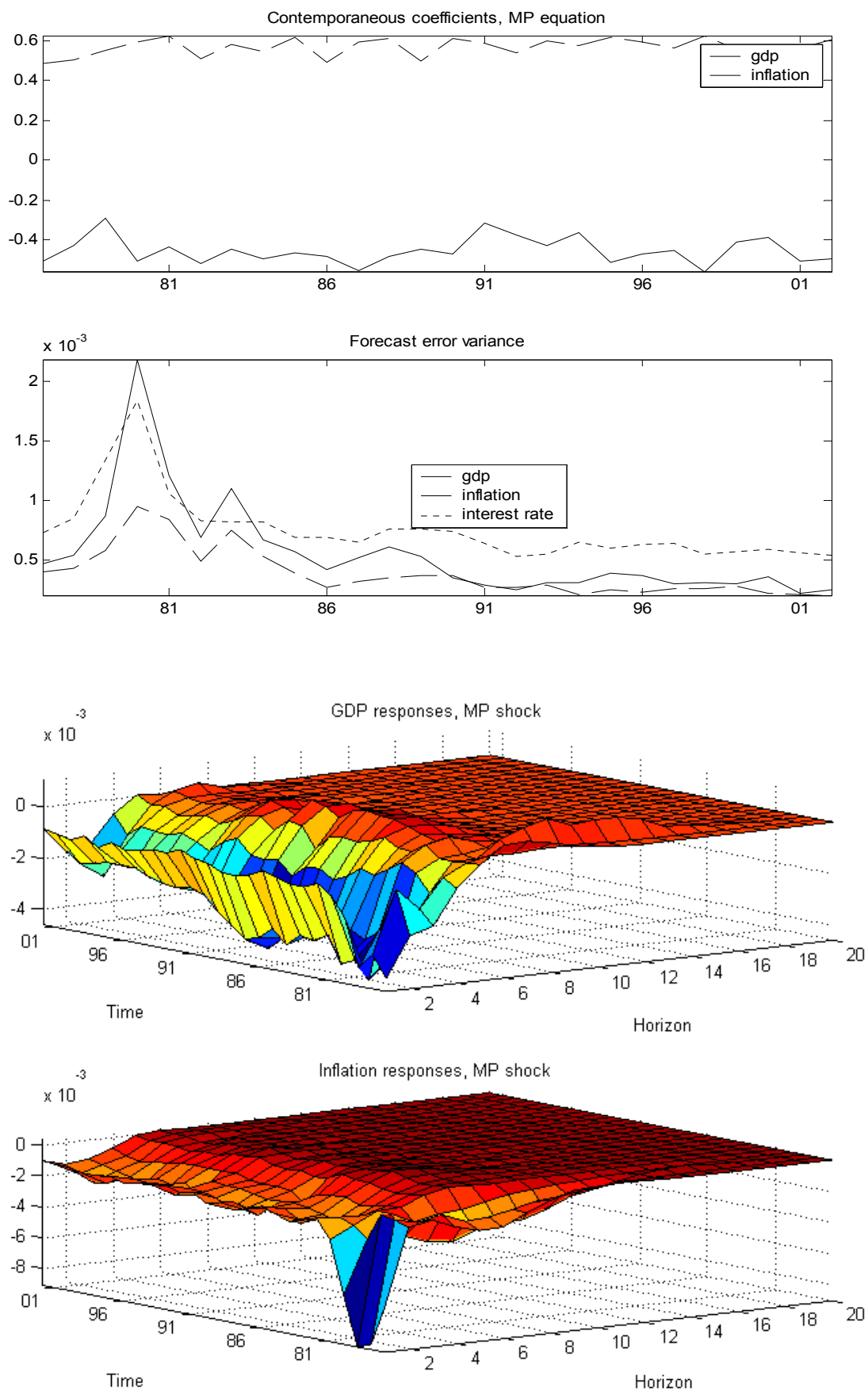


Figure 11: Contemporaneous coefficients, forecast error variance and impulse response functions, trivariate system without money.