

Information Spillovers in Asset Markets with Correlated Values

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Abstract

We study the effect of information spillovers and transparency in a dynamic setting with adverse selection and correlated asset values. A trade (or lack thereof) by one seller can provide information about the quality of other assets in the market. In equilibrium, the information content of this trading behavior is endogenously determined. We show that this endogeneity of information leads to multiple equilibria when the correlation between asset values is sufficiently high. That is, if buyers expect “bad” assets to trade quickly, then a seller with a bad asset has reason to be concerned about negative information being revealed, which induces her to trade quickly. Conversely, if buyers do not expect bad assets to trade quickly, then the seller has less to be concerned about and is more willing to wait. We study the implications for policies that target market transparency. We show that total welfare is higher when markets are fully transparent than when the market is fully opaque. However, both welfare and trading activity can decrease in the degree of market transparency.

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1 Introduction

If asset values are correlated and traders have some private information then trading volume of one asset can be informative about the value of other assets. Thus, transaction transparency can be potentially important for both informational efficiency and the efficiency with which assets are reallocated. Indeed, the empirical literature has documented that the degree of market transparency matters, and there is an ongoing policy debate about whether to require transactional transparency for a variety of asset classes.¹

Our goal in this paper is to develop a theoretical framework from which to understand the role of transparency and information spillovers in markets where asset values are correlated. The basic model involves two sellers (i and j), each with an indivisible asset that has a value which is either low or high. Asset values are positively correlated and each seller is privately informed about the value of her asset, but does not know the value of the other seller's asset. As in Akerlof (1970), although there is common knowledge of gains from trade, buyers are concerned about the possibility of buying a lemon. Trading takes place via a competitive decentralized market over the course of two periods. In the first period, potential buyers can approach a seller and make offers. If a seller rejects all offers in the first period, then she can entertain more offers from new buyers in the second period. In this framework, inefficiencies arise from lack or delay of trade.

The key novel ingredient of the model is that if, say, seller i trades in the first period, then with some probability this trade is observed by potential buyers of seller j 's asset, prior to them making offers in the second period. We refer to this probability as the degree of market transparency, and we consider the full range of possibilities from fully opaque to fully transparent markets.

Provided that there is some degree of transparency, a trade of one asset can provide information to buyers about the value of the other asset. Importantly, the information content of observed trading behavior is endogenous. For example, suppose that seller j 's strategy involves trading with a high probability in the first period conditional on having a low-value asset and not trading conditional on having a high-value asset. Then, if asset i and asset j are highly correlated, observing a trade (or lack thereof) has a high degree of information content about the value of seller i 's asset and the degree of market transparency will play an important role in determining i 's trading strategy. On the other hand, if seller j plans to sell the asset in the first period regardless of its value, then observing a trade by this seller is completely uninformative about the quality of seller i 's asset and the degree of market transparency is irrelevant.

¹See, for example, Asquith et al. (2013) or Goldstein et al. (2007), who study the effects of increased transparency due to the introduction of TRACE in the corporate bond market.

Since asset values are correlated and, in equilibrium, lower types trade first, a trade in one of the assets is bad news about the quality of the other asset. This introduces an interdependence in the sellers' strategies. In particular, if the seller of the low quality asset j is more likely to trade in the first period, the prospects of the seller of asset i from keeping his asset become worse. Thus, as bad news are more likely to arrive, the seller of asset i would want to trade more aggressively in the first period. Conversely, if the seller of asset j is unlikely to trade in the first period, the prospects of the seller of asset i from keeping his asset become better and he is thus less inclined to trade in the first period.

Our first main result is to show that when asset values are sufficiently correlated, a high degree of transparency leads to multiple equilibria. To provide intuition as to why this multiplicity obtains, consider the case in which the assets are perfectly correlated and the market is fully transparent. First, suppose that the low-type seller j trades with probability one in the first period and the high-type seller j trades with probability zero. If seller i delays trade in the first period, then her type will be perfectly revealed by whether seller j trades. Conditional on observing a trade by seller j in the first period, buyers will correctly infer that seller i has a low value asset and offer a low price in the second period. Therefore, a low-type seller i has no incentive to delay trade and strictly prefers to trade in the first period at a low price. Hence, there exists an equilibrium in which both low-value assets trade with probability one in the first period. We refer to this as the high-volume equilibrium.

Next, suppose that the low-type seller j trades with some intermediate probability in the first period. From seller i 's perspective, there is still positive probability that her asset will be revealed by if buyer's observe a trade by seller j , but there is also some chance that seller j does not trade, in which case buyers correctly infer that seller i is more likely to have a good asset making them willing to offer a high price in the second period. The potential for getting a high price in the second period makes seller i indifferent between trading in the first period, and hence she is willing to trade with some intermediate probability. Thus, there also exists an equilibrium in which both low-value assets trade with an intermediate probability in the first period.

In fact, we show there can exist three symmetric equilibria of the model. These equilibria are ranked both in terms of the volume of trade that takes place in the first period and the total welfare. The higher is the volume of trade in the first period, the more efficiently assets are reallocated and the higher is the total welfare. Three equilibria exist provided that asset values are sufficiently correlated and the market is sufficiently transparent. When either of these conditions breaks down, the (expected) information content revealed by seller j 's trade is insufficient to induce the moderate or high-volume equilibrium and only the low-volume equilibrium exists. Therefore, both the correlation of asset values and the degree of market

transparency can impact total welfare.

We analyze the welfare effects in more detail by conducting comparative statics on both the degree of transparency and correlation of asset values. In the high-volume equilibrium, welfare is increasing in both parameters. Yet surprisingly, welfare is *decreasing* in both parameters in the moderate-volume equilibrium and *independent* of both parameters in the low-volume equilibrium. Therefore, increasing the degree of market transparency has the potential to improve or destroy welfare.

We extend the model to a setting with an arbitrary number, $N > 1$, assets. This extension bears several interpretations. The number of assets can be interpreted literally as the number of relevant correlated assets in the marketplace. Alternatively, N can be interpreted as the degree of *market integration*: the number of different assets that traders can have information regarding (e.g., the number of assets that trade on a given platform). We show that multiple equilibria can persist in this environment and discuss implications of market integration and transparency for price dispersion and informational efficiency.

Our findings help contribute to the debate on mandatory transaction transparency, which has received significant attention from policy makers in recent years. FINRA has been a strong proponent of mandated transparency arguing that it “enhances the integrity of the corporate bond market and creates a level playing field for all investors” (NASD, 2005). In July 2002, the corporate bond market underwent a significant change when FINRA (then NASD) mandated that prices and volume completed transactions be publicly disclosed. Since then, TRACE has been expanded to include other asset classes including Agency-Backed Securities and some Asset-Backed Securities. There are also ongoing efforts by regulators to increase transparency in the markets for numerous derivatives (Title VII of Dodd-Frank) and European corporate bonds (Lerner, 2011).

Opponents have objected to mandatory transparency arguing that it is unnecessary and potentially harmful. For example, if price transparency reduces dealer margins, dealers will be less willing to commit capital to hold certain securities thereby reducing liquidity.² There is mixed empirical evidence as to whether increased transparency can reduce liquidity. Asquith et al. (2013) find that increased transparency led to a significant decline in trading activity for high-yield bonds. This is in contrast to a controlled study by Goldstein et al. (2007), who find no conclusive evidence that increased transparency causes a reduction in trading activity. Our theoretical framework helps to reconcile these findings. For example, we show that increasing transparency can increase or decrease trading activity depending on the initial degree of transparency and the correlation between assets.

²In a letter to the SEC, the Bond Market Association argued that adverse effects of mandatory transparency are likely to be exacerbated for lower-rated and less frequently traded bonds.

1.1 Related Literature

Our work is related to Daley and Green (2012, 2015), who study a setting in which information is exogenously revealed to uninformed buyers. They show that exogenous information (or news) leads to a unique equilibrium in which liquidity completely dries up (i.e., there are periods in which trade occurs with probability zero). In contrast, we show that when information is endogenously revealed by the trading behavior of other market participants, there can exist multiple equilibria all of which require trade to occur with strictly positive probability in each period.

The role of transparency in offers has been previously analyzed by Nöldeke and van Damme (1990), Swinkels (1999), Hörner and Vieille (2009) and Fuchs et al. (2015). The two most important differences with respect to these papers are that: first, we consider the transparency of transaction data while they all consider the transparency of rejected offers. Second, we explore the strategic considerations of multiple sellers whose assets have correlated values, while such considerations are absent in previous work. As we show these considerations are important since they induce complementarities in the sellers' strategies and can lead to multiple equilibria.

In our model, an increase in transparency allows agents to better learn the “common” state of nature that is affecting asset payoffs. In contemporaneous work, Duffie et al. (2014) analyze the role of published benchmarks (e.g. LIBOR), which reveal such “common” states, in facilitating trade and efficiency in OTC markets. In our setting, past trades in related assets play the role of benchmarks. As in Duffie et al. (2014), we find that more transparent markets can yield higher welfare by reducing information asymmetries among markets participants (e.g., dealers, traders). However, we also show that the effect of transparency on welfare can be non-monotonic. Our work thus suggests that the welfare effects of transparency depend on the level of pre-existing transparency in the market.

There is also a large literature within accounting and finance studying the effect of public disclosure of firm specific information. Healy and Palepu (2001) and Verrecchia (2001) provide surveys of both the theoretical and empirical work in this area. One key difference from our paper and this literature is that it takes the information to be disclosed as given and studies the effects of whether, when, how much and how frequently it is made public. Instead, our focus is on the two-way interaction between trade and information that is generated by trade itself.

The idea of a two-way feedback between trading activity and market informativeness is also present in Cespa and Vives (2015). They study a two period market in the tradition of noisy rational expectations literature and find that multiple equilibria can arise when noise-trader shocks are sufficiently persistent and informed buyers care only about their short-term returns.

While our approaches are substantially different, their model also delivers equilibria that have high trading volume and market informativeness as well as equilibria in which trading volume and informativeness are low.

The rest of the paper is organized as follows. In Section 2, we lay out the basic theoretical framework and conduct preliminary analysis. In Section 3, we analyze the equilibrium of the model and present our main results. In Section 4, we consider extensions of the basic model. We conclude in Section 5. All proofs are in the Appendix.

2 The Model

In this section, we present the basic ingredients of the model, which features two indivisible assets with correlated values that can be traded in an over-the-counter market.

There are two sellers, indexed by $i \in \{A, B\}$. Each seller owns one indivisible asset and is privately informed of her asset's type, denoted by $\theta_i \in \{L, H\}$. Seller i values an asset of type θ at c_θ , with $c_L < c_H$. Each seller has multiple potential trading partners or "buyers". The value of a type θ asset to a buyer is v_θ and there is common knowledge of gains from trade, $v_\theta > c_\theta$, which can be motivated by, for example, liquidity constraints or hedging demands.

There are two trading periods: $t = 0$ and $t = 1$. In each period, two or more buyers make simultaneous price offers to each seller. The sellers discounts payoffs between the two periods according to a discount factor $\delta \in (0, 1)$. The payoff to a seller with an asset of type θ , who agrees to trade at a price p in period t is

$$(1 - \delta^t) c_\theta + \delta^t p.$$

i.e., c_θ is the present discounted value of an asset with a flow payoff of $(1 - \delta)c_\theta$. If the seller does not trade at either date, his payoff is c_θ . Similarly, the payoff to a buyer who purchases an asset of type θ at price p is given by

$$v_\theta - p.$$

Buyers are short-lived and maximize their per-period payoff. The payoff to a buyer whose offer is rejected is normalized to zero. All players are risk neutral. Without loss of generality, we restrict offers to be in the interval $[c_L, v_H]$, since it is a dominant strategy for the seller to reject any price below c_L , and it is a weakly dominated strategy for any buyer to offer a price higher than v_H .

The correlation structure of asset values is as follows. There is an (unobservable) common

state of nature s that takes values in $\{l, h\}$ with $\mathbb{P}(s = l) = \pi \in (0, 1)$. The unconditional distribution of θ_i is also $\mathbb{P}(\theta_i = L) = \pi$, whereas the distribution of θ_i conditional on the state of nature is given by $\mathbb{P}(\theta_i = L | s = l) = \lambda \in [\pi, 1)$; asset values are assumed to be independent conditional on the state of nature. Thus, the assets are correlated but imperfectly so. We consider the case of perfect correlation in Section 4.1.

As each seller has a distinct asset, we will also refer to trade for the asset of seller i as trade in market i . Importantly, asset correlation introduces the possibility that trade in one market contains information that is relevant for pricing the asset in another. We capture information spillovers across markets as follows. We suppose that buyers who trade with seller i are distinct from those who trade with seller j and henceforth refer to market i and market j to clarify this distinction. This allows us to capture the fact that in OTC markets, a trader on one platform may not observe trades that take place on another platform. To capture the degree of transparency across markets, we assume that there is a probability $\xi \in [0, 1]$ that a transaction in market i at $t = 0$ is observed by buyers in market j prior to them making offers in period $t = 1$. We refer to the parameter ξ as the level of market *transparency*, where $\xi = 1$ stands for fully transparent markets and $\xi = 0$ for fully opaque ones.³ Finally, we focus on primitives which satisfy the following assumptions.

Assumption 1. $\pi v_L + (1 - \pi)v_H < c_H$

Assumption 2. $v_L < (1 - \delta)c_L + \delta v_H$

The first assumption asserts that the adverse selection problem is sufficiently severe so as to rule out the efficient equilibrium where all types trade at $t = 0$ in both markets. The second assumption rules out the separating equilibrium where, with probability 1, the low type trades in $t = 0$ and the high type trades in $t = 1$. Together, these two assumptions will guarantee that information spillovers across markets are relevant for equilibrium behavior.

2.1 Preliminary Analysis

The key determinant of equilibrium trade in each market will be buyer's belief about the value of the seller's asset in that market. Let $\pi_{i,t}$ denote the probability that buyers in market i assign to $\theta_i = L$ at the beginning of period t . For any belief $\tilde{\pi}$, let $\bar{V}(\tilde{\pi}) \equiv \tilde{\pi}v_L + (1 - \tilde{\pi})v_H$ denote the expected value of an asset to a buyer. The strategy of a seller is a mapping from θ and the history (inclusive of the current offers) to a probability of acceptance; the strategy of

³We assume without loss that unaccepted offers are not observed neither within nor across markets. As shown in Fuchs et al. (2014), with just two types this assumption does not change the within market equilibrium.

a buyer is a mapping from the history to a probability distribution over offers in the interval $[c_L, v_H]$. We restrict attention to Perfect Bayesian Equilibria (PBE).

The equilibrium behavior at $t = 1$ corresponds to that of the familiar static model. Our primary interest, therefore, is characterizing the equilibrium trading behavior at $t = 0$. We do so by using backward induction. Consider the equilibrium in market i .

At $t = 1$: Suppose that seller has not traded at $t = 0$; otherwise, the game in this market ends. If $\pi_{i,1} > \bar{\pi} \equiv \frac{v_H - c_H}{v_H - v_L}$, then a familiar ‘market for lemons’ arises: only low type wants to trade and the price of the asset is v_L . On the other hand, if $\pi_{i,1} < \bar{\pi}$, both types trade and the price is given by $\bar{V}(\pi_{i,1})$. In the case where $\pi_{i,1} = \bar{\pi}$, either of the two cases is possible since buyers are indifferent to whether to submit v_L and attract only the low type or submit $\bar{V}(\pi_{i,1})$ and attract both types. In this case, we will allow buyers to mix between the two offers and we will denote by $\eta_i \in [0, 1]$ the probability with which the bid is $\bar{V}(\pi_{i,1})$. We summarize these results in the following lemma:

Lemma 1 *Given that the buyers’ belief about seller is $\pi_{i,1}$, the equilibrium outcome at $t = 1$ in market i satisfies the following:*

- *If $\pi_{i,1} > \bar{\pi}$, then the bid is v_L and only low type seller accepts.*
- *If $\pi_{i,1} < \bar{\pi}$, then the bid is $\bar{V}(\pi_{i,1})$ and both types accept.*
- *If $\pi_{i,1} = \bar{\pi}$, then the bid is $\bar{V}(\pi_{i,1})$ with probability $\eta_i \in [0, 1]$ and v_L with probability $1 - \eta_i$; both types accept bid $\bar{V}(\pi_{i,1})$, while only low type accepts bid v_L .*

It follows that the structure of the seller’s payoff at $t = 1$ as it depends on θ_i , $\pi_{i,1}$, and η_i , denoted by $F_{\theta_i}(\pi_{i,1}, \eta_i)$, is given by:

$$F_L(\pi_{i,1}, \eta_i) \equiv v_L + \mathbb{1}_{\{\pi_{i,1} \leq \bar{\pi}\}} \cdot \eta_i \cdot (\bar{V}(\pi_{i,1}) - v_L) \quad \text{and} \quad F_H(\pi_{i,1}, \eta_i) = \max\{c_H, \bar{V}(\pi_{i,1})\}$$

Thus, the payoff of the low type is increasing in η_i while the payoff of the high type is independent of η_i ; both payoffs are weakly decreasing in the belief $\pi_{i,1}$.

At $t = 0$: The seller decides whether to accept the bid or wait until the next period. His payoff at $t = 1$ is stochastic because buyers’ beliefs will depend on news arriving from market j and because buyers may be mixing over offers. Let $\mathbb{E}_{\theta_i}\{F_{\theta_i}(\pi_{i,1}, \eta_i)\}$ denote the expected payoff of seller i at $t = 1$, when his type is θ_i . Note that $F_H(\pi_{i,1}, \eta_i) \geq F_L(\pi_{i,1}, \eta_i)$ for any $(\pi_{i,1}, \eta_i)$. Thus, because high type believes good news are more likely than the low type, the expected payoff to waiting is higher for the high type seller than for the low type:

$$(1 - \delta)c_H + \delta\mathbb{E}_H\{F_H(\pi_{i,1}, \eta_i)\} > (1 - \delta)c_L + \delta\mathbb{E}_L\{F_L(\pi_{i,1}, \eta_i)\}$$

Hence, in equilibrium if the high type trades with positive probability at $t = 0$ then the low type will trade with probability one, and thus the bid at $t = 0$ is bounded above by the asset's unconditional expected value $\pi v_L + (1 - \pi)v_H$. This implies that the equilibrium bid at $t = 0$ must be v_L : the high type seller will not trade at $t = 0$ because, by Assumption 1, his outside option c_H is greater than $\pi v_L + (1 - \pi)v_H$. We summarize the above results in the following lemma:

Lemma 2 *The equilibrium outcome at $t = 0$ in market i satisfies the following:*

- *Buyers' bid is given by v_L .*
- *High type seller does not accept the bid, while the low type accepts with probability σ_i .*

Thus far, we have not specified the information structure in each market. To this end, let Z_i denote the information that arrives to market i between $t = 0$ and $t = 1$. By Lemma 2, there are two values that Z_i can take on equilibrium path. Let $Z_i = \emptyset$ if either trade does not occur in market j or if it occurs but is not observed in market i due to a lack of transparency, and let $Z_i = v_L$ if trade occurs in market j at price v_L and this news arrives to market i .

For $z \in \{\emptyset, v_L\}$ and $\theta \in \{L, H\}$, define $\gamma_i(z) \equiv \mathbb{P}(Z_i = z)$ and $\rho_{i,\theta}(z) \equiv \mathbb{P}(Z_i = z | \theta_i = \theta)$ to be the unconditional and type-conditional distributions of news Z_i . The following lemma provides a characterization of these distributions:

Lemma 3 *The news Z_i from market j to market i takes values in $\Omega \equiv \{\emptyset, v_L\}$ and has the following properties:*

- *The unconditional distribution of Z_i is characterized by*

$$\gamma_i(v_L) \equiv \mathbb{P}(Z_i = v_L) = (\xi \cdot \pi) \cdot \sigma_j$$

- *The distribution of Z_i conditional on θ_i is characterized by*

$$\rho_{i,L}(v_L) \equiv \mathbb{P}(Z_i = v_L | \theta_i = L) = \phi \cdot \gamma_i(v_L)$$

where $\phi \equiv \pi \left(\frac{\lambda}{\pi}\right)^2 + (1 - \pi) \left(\frac{1-\lambda}{1-\pi}\right)^2 \in [1, \pi^{-1})$ and is increasing in λ .

Thus, the distribution of news is endogenous to the trading strategies of market participants. In particular, the larger the frequency of trade σ_j of the low type, the more frequently these news arrive to market i , and vice versa. The distribution of news in turn affects the strategies of the sellers. To see this, note that if buyers in market i believe that low type sellers trade

with probabilities σ_i and σ_j , then conditional on seller i arriving to $t = 1$ and buyers observing news $Z_i = z$, their beliefs become

$$\pi_{i,1}(z) = \frac{\rho_{i,L}(z) \cdot \pi_{\sigma_i}}{\rho_{i,L}(z) \cdot \pi_{\sigma_i} + \rho_{i,H}(z) \cdot (1 - \pi_{\sigma_i})}$$

where $\pi_{\sigma_i} = \frac{(1-\sigma_i) \cdot \pi}{(1-\sigma_i) \cdot \pi + 1 - \pi}$ is the buyers' interim belief about seller i conditional on him not having traded at $t = 0$.⁴ Importantly, note that $\pi_{i,1}(z)$ depends on σ_j through the likelihood ratio $\frac{\rho_{H,i}(z)}{\rho_{L,i}(z)}$; this feedback between trade and information is at the center of the equilibrium analysis that follows.

3 Equilibrium

3.1 Exogenous News Benchmark

In this section, we consider the case of exogenous news, which will serve as a useful benchmark. In particular, we study the equilibria in market i taking as given the trading strategy σ_j in market j . The following proposition gives a full characterization of equilibrium structure in market i :

Proposition 1 *Given $\sigma_j \in [0, 1]$, an equilibrium in market i exists, is unique, and has the following properties:*

- At $t = 0$, the low type's trading strategy satisfies $\sigma_i < 1$ and

$$v_L \leq (1 - \delta)c_L + \delta \mathbb{E}_L \{ F_L(\pi_{i,1}, \eta_i) \}$$

with strict equality if $\sigma_i > 0$, where

$$\mathbb{E}_L \{ F_L(\pi_{i,1}, \eta_i) \} = \sum_{z \in \Omega} \rho_{i,L}(z) \cdot [\eta_i(z) \bar{V}(\pi_{i,1}(z)) + (1 - \eta_i(z)) v_L]$$

- At $t = 1$, for news realization $z \in \Omega$ buyers' trading strategy satisfies

$$\eta_i(z) \begin{cases} = 1 & \text{if } \pi_{i,1}(z) < \bar{\pi} \\ \in [0, 1] & \text{if } \pi_{i,1}(z) = \bar{\pi} \\ = 0 & \text{if } \pi_{i,1}(z) > \bar{\pi} \end{cases}$$

⁴When $\sigma_j = 0$, i.e., news $Z_i = v_L$ occur with probability 0; thus, we have $\rho_{i,L}(v_L) = \rho_{i,H}(v_L) = 0$ and in this case we set without loss $\pi_{i,1}(v_L) = \pi_{\sigma_i}$.

These results follow directly from combining Lemmas 1 to 3 with the fact that the trading strategy σ_i must be optimal for the low type seller. First, that $\sigma_i < 1$ follows by Assumption 2 because otherwise we would have $\pi_{i,1}(z) = 0$ for $z \in \Omega$ and $\bar{V}(0) = v_H$; thus, the low type seller would strictly prefer to wait. Second, for the low type to mix in equilibrium, he needs to be indifferent. Hence, an equilibrium with no trade at $t = 0$ can arise only when the expected payoff $\mathbb{E}_L\{F_L(\pi_{i,1}, \eta_i)\}$ of the low type is large despite him not trading at $t = 0$.⁵ We now discuss two interesting cases that this model nests.

No News and Trade. The model with exogenous news nests the simple case where there are no news. In particular, this case can be captured by assuming that either there is no information arrival or that seller j does not trade if he is low type, i.e., $\xi = 0$ and/or $\sigma_j = 0$. In this case, we can omit the contingency of $t = 1$ variables on news. Note that in the absence of news trade must occur in $t = 0$ with positive probability. If not, buyers would not update their beliefs at $t = 1$ and, by Assumption 1, high type would not accept any offer that has non-negative profits for buyers. Hence, we must have $\sigma_i \in (0, 1)$ and the low type must be indifferent to trading at $t = 0$ or waiting to $t = 1$:

$$v_L = (1 - \delta)c_L + \delta F_L(\pi_{i,1}, \eta_i)$$

where $F_L(\pi_{i,1}, \eta_i) = \eta_i \bar{V}(\pi_{i,1}) + (1 - \eta_i)v_L$. As we discuss next and as shown in DG12, introduction of news into this setting can result in delay of trade and even collapse of trade at $t = 0$.

News and No Trade. Let the trading probability σ_j of seller j be given and let us construct an equilibrium where market i does not trade at $t = 0$, i.e., $\sigma_i = 0$. In such an equilibrium, it must be the case that the low type prefers to wait rather than trade:

$$v_L \leq (1 - \delta)c_L + \delta \mathbb{E}_L\{F_L(\pi_{i,1}, \eta_i)\}_{\sigma_i=0}$$

When there is no trade at $t = 0$ in market i , there can neither be trade at $t = 1$ following the realization of news $Z_i = v_L$; however, there must be trade following $Z_i = \emptyset$. In fact, assume further that following event $Z_i = \emptyset$ trade occurs with probability 1 in market i . Then, we can use Proposition 1 to show that

$$\mathbb{E}_L\{F_L(\pi_{i,1}, \eta_i)\}_{\sigma_i=0} = \hat{\pi}v_L + (1 - \hat{\pi})v_H$$

⁵The existence and uniqueness of equilibria follow from the fact that the continuation payoff is monotonic in σ_i and η_i and because its range is the entire interval $[v_L, v_H]$.

where $\hat{\pi} \equiv \phi\gamma_i(v_L) + (1 - \phi\gamma_i(v_L)) \cdot \frac{1 - \phi\gamma_i(v_L)}{1 - \gamma_i(v_L)} \cdot \pi$ is the probability that the low type assigns to receiving offer v_L at $t = 1$. Then, a sufficient condition for this to be an equilibrium is that

$$\hat{\pi} \leq \min \left\{ 1 - \frac{1 - \delta}{\delta} \frac{v_L - c_L}{v_H - v_L}, \bar{\pi} \right\}$$

In other words, the low type must want to wait at $t = 0$ and the high type must accept the offer at $t = 1$ following news $Z_i = \emptyset$. This condition can be shown to be satisfied for an interval of values for $\xi \cdot \sigma_j \in (0, 1)$ when δ is sufficiently large and π is sufficiently close to $\bar{\pi}$. Thus, no trade is indeed a possibility when news are exogenous.

With endogenous information, however, the trading probability σ_j is an equilibrium outcome and the condition which generates no trade is no longer guaranteed to hold. In fact, as we will show in the next section, the equilibrium imposes sufficient restrictions on σ_j so as to ensure that trade occurs with positive probability.

3.2 Endogenous News

In the previous section, we took the trading strategy of seller j as given and then studied the trading strategy of seller i . The full equilibrium of the economy, however, requires that the trading strategy in market j also be an equilibrium outcome. We will without loss of generality focus on symmetric equilibria, in which the trading strategies in the two (symmetric) markets are identical: $\sigma \equiv \sigma_A = \sigma_B$ and $\eta \equiv \eta_{A,1} = \eta_{B,1}$; we show in the Appendix that asymmetric equilibria do not exist. In what follows, we therefore drop the subscripts indicating a market's identity.

The first result of this section is that endogeneity of information implies that there is always trade at $t = 0$:

Proposition 2 *The equilibria of this economy have the properties stated in Proposition 1, except that at $t = 0$ the low type's trading strategy satisfies $\sigma \in (0, 1)$ and $v_L = (1 - \delta)c_L + \delta\mathbb{E}_L\{F_L(\pi_1, \eta)\}$. In particular, trade occurs with positive probability at $t = 0$.*

The intuition for this result is the following. If there is no trade at $t = 0$, then there will be no news. But then buyers will not update their beliefs and they will believe that the sellers are of an average type at $t = 1$. By Assumption 1, however, the high type will not trade at any price that makes non-negative profits for buyers and the bid at this date will be v_L w.p. 1. But then the low type will not want to delay trade in order to get v_L that he can get by trading at $t = 0$. Thus, $\sigma = 0$ can no longer be an equilibrium. This result highlights the importance of thinking about the source of information in financial markets. If much of the information

available to market participants is generated through trade itself, then the equilibrium puts a lower bound on the amount of trade that we should observe.

Next, we show that the endogeneity of information can introduce complementarities in the trading behavior of markets and lead to equilibrium multiplicity. To see this, consider the continuation payoff of the low type seller at $t = 1$:

$$\mathbb{E}_L\{F_L(\pi_1, \eta)\} = \sum_{z \in \Omega} \rho_L(z) \cdot [\eta(z)\bar{V}(\pi_1(z)) + (1 - \eta(z))v_L]$$

Given the distribution of news (γ, ρ_L) , this continuation value is increasing in σ because buyers become more optimistic about the seller who arrives at $t = 1$: $\pi_1(z)$ is decreasing in σ for $z \in \Omega$. This would be the only effect if news were exogenous. With endogenous news, however, the trading probability σ also affects the distribution of news (γ, ρ_L) . There are two effects to consider.

First, buyers' beliefs $\{\pi_1(z)\}_{z \in \Omega}$ depend on σ only conditional on trade not having occurred in the other market, i.e., when $Z = \emptyset$. This is because if the other market trades, the seller's type in that market is revealed fully. On the other hand, conditional on no trade in the other market, higher σ makes buyers more optimistic about the seller. Intuitively, more trade by the low type in the other market implies that this seller is more likely to be of high type if he has not traded yet. Due to correlation, this means that the seller in own market is more likely to be of high type. This increases the prices that the seller expects to receive at $t = 1$ and, therefore, his expected continuation payoff.

Second, the trading strategy σ affects how frequently bad news arrive to each market. As σ increases, the low type seller in each market trades more frequently; as a result, both $\gamma(v_L)$ and $\rho_L(v_L)$ increase. Because prices are lower following the event of trade in the other market, the low type becomes more pessimistic when $\rho_L(v_L)$ increases, and this effect is stronger the more correlated are the assets. This reduces the expected payoff of the low type at $t = 1$ and induces him to want to trade sooner. As we show next, when transparency and correlation are sufficiently high, this effect can be strong enough so as to generate equilibrium multiplicity.

Proposition 3 *The equilibria of this economy, all of which are symmetric, fall into one of the following categories:*

1. *Low Trade:* There exists $\bar{\delta} < 1$ such that for $\delta > \bar{\delta}$, there is an equilibrium in which the trading strategies of the sellers and buyers satisfy $\pi_1(\emptyset) = \bar{\pi}$, and $\eta(v_L) = 0 < \eta(\emptyset)$.

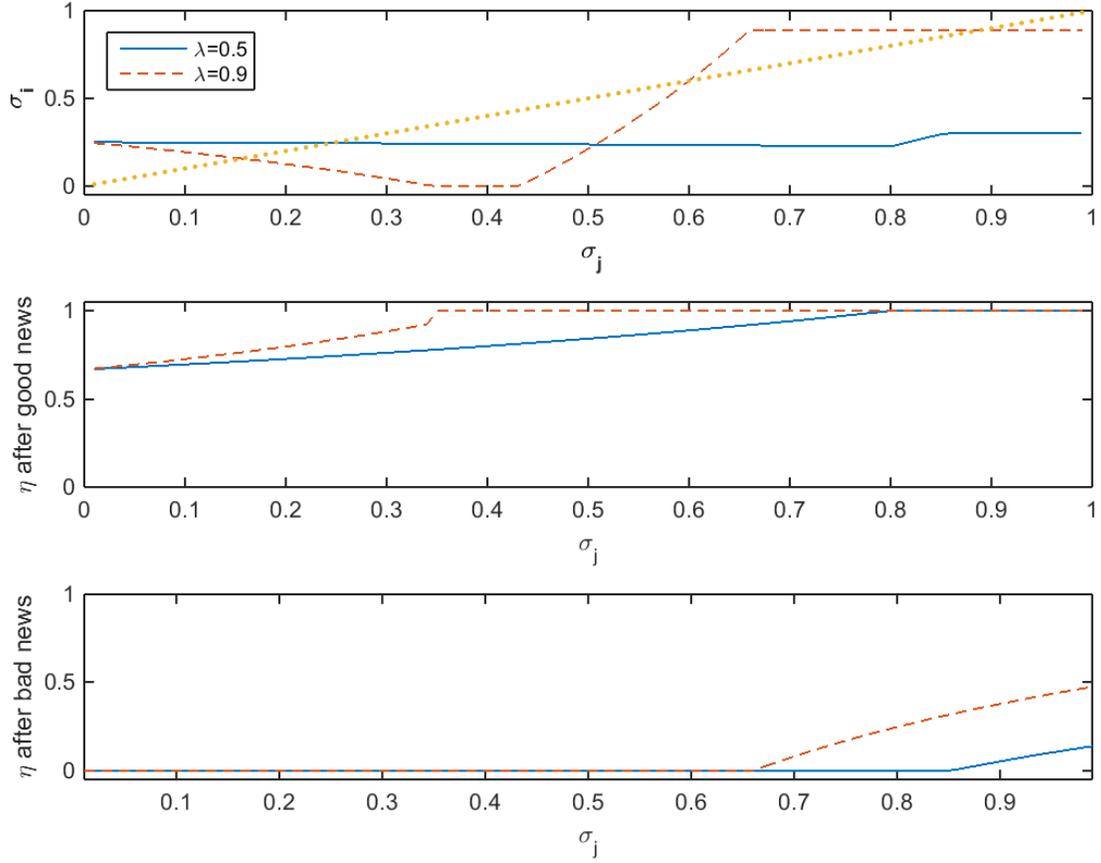


Figure 1: **Strategic Interactions in Transparent Markets**

Parameters: $c_L = 0$, $c_H = 0.2$, $v_L = 0.1$, $v_H = 0.25$, $\pi = 0.4$, $\delta = 0.6$, and $\xi = 1$.

2. *High Trade*: Fix $\delta < 1$, then there exist $\bar{\lambda}_\delta, \bar{\xi}_\delta < 1$ such that for $\lambda > \bar{\lambda}_\delta$ and $\xi > \bar{\xi}_\delta$, there is an equilibrium in which the trading strategies of the sellers and buyers satisfy $\pi_1(v_L) \leq \bar{\pi}$, and $\eta(v_L) \leq 1 = \eta(\emptyset)$.
3. *Medium Trade*: Fix $\delta > \bar{\delta}$, $\lambda > \bar{\lambda}_\delta$, and $\xi > \bar{\xi}_\delta$, then there is an equilibrium in which the trading strategies of the sellers and buyers satisfy $\pi_1(\emptyset) < \bar{\pi} < \pi_1(v_L)$, and $\eta(v_L) = 0 < 1 = \eta(\emptyset)$.

Importantly, the three types of equilibria coexist when $\delta > \bar{\delta}$, $\lambda > \bar{\lambda}_\delta$, and $\xi > \bar{\xi}_\delta$, and the equilibrium is unique when either δ , λ , or ξ are sufficiently small. Finally, let σ^q with $q = LT, MT, HT$ be the equilibrium trading probability at $t = 0$ in the low, medium, and high trade equilibria respectively, then $\sigma^{LT} < \sigma^{MT} < \sigma^{HT}$ whenever the three types of equilibria coexist.

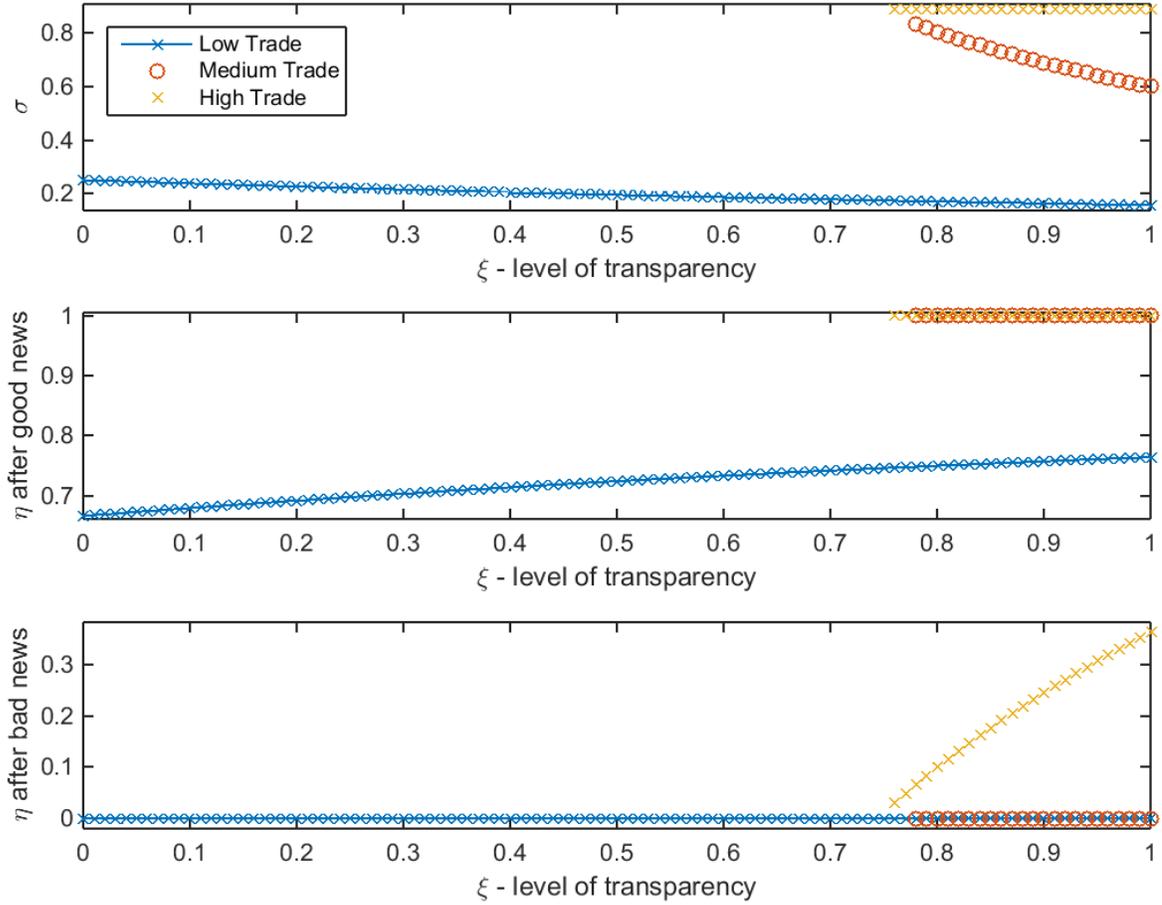


Figure 2: **Transparency and Multiple Equilibria**

Parameters: $c_L = 0$, $c_H = 0.2$, $v_L = 0.1$, $v_H = 0.25$, $\pi = 0.4$, $\delta = 0.6$, and $\lambda = 0.9$.

These results are illustrated in Figures 1 – 3. The top panel of Figure 1 plots the equilibrium trading probability in one market against an exogenously given trading probability in the other market, for two different values of the correlation parameter λ . As we can see, when correlation is high, the relationship between the two trading probabilities is positive, i.e., the trading behavior in the two markets is complementary. In fact, as the figure illustrates, the complementarity is sufficiently strong so as to introduce equilibrium multiplicity - the crossing with the 45 degree line indicates an equilibrium in both markets. The difference in the behavior of markets along different equilibria can be seen not only due to differences in σ but also due to differences in buyers' strategies η illustrated in the bottom two panels of Figure 1.

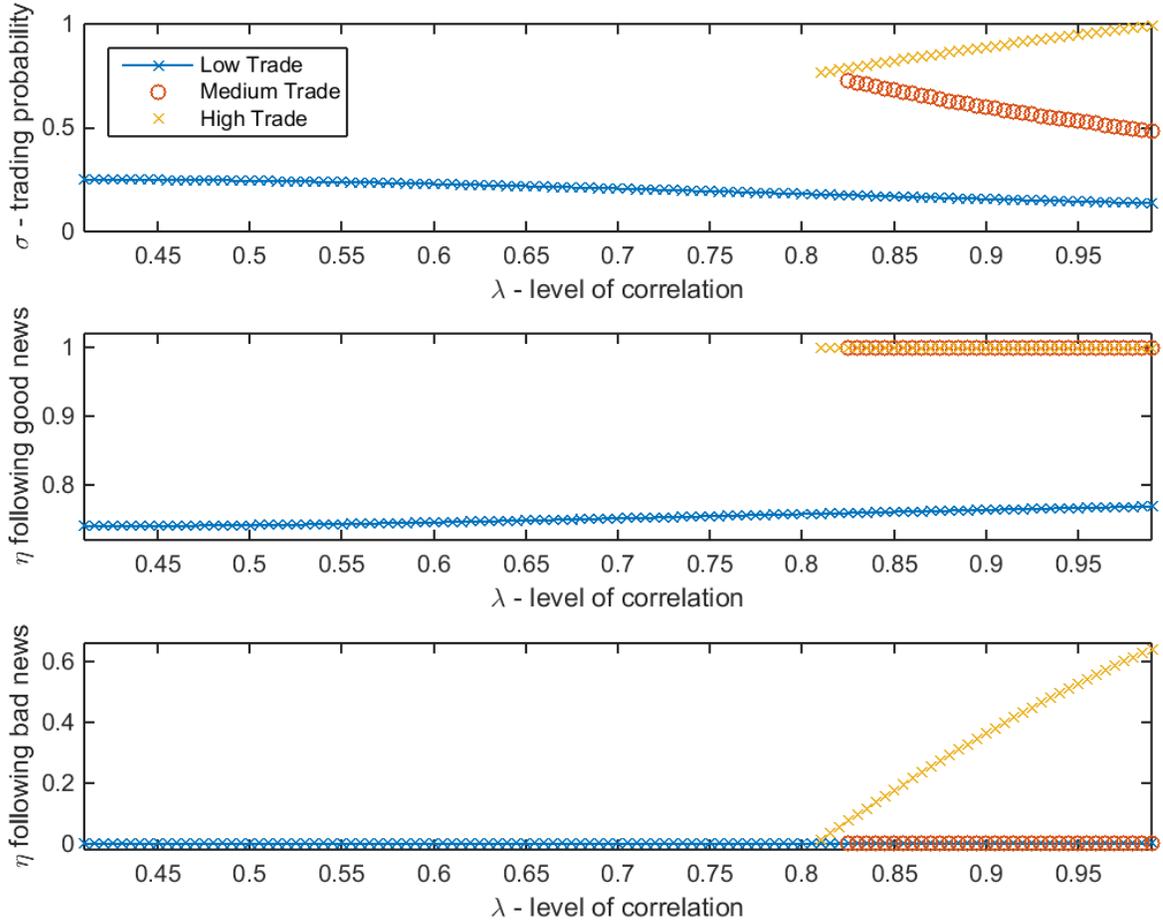


Figure 3: **Correlation and Multiple Equilibria**

Parameters: $c_L = 0$, $c_H = 0.2$, $v_L = 0.1$, $v_H = 0.25$, $\pi = 0.4$, $\delta = 0.6$, and $\xi = 1$.

Figures 2 and 3, on the other hand, impose an equilibrium in the two markets and plot the behavior of the equilibrium as a function of the level of transparency and correlation between the two markets. As we can see, equilibrium multiplicity kicks in only when the level of transparency and/or correlation is sufficiently high. Of particular interest is the dramatic difference in the trading behavior across the equilibria when such multiplicity exists. As we will show next, these differences in trading behavior across equilibria also translate into dramatic differences in welfare.

3.3 Welfare

In this section, we study how the level of market transparency and the degree of correlation among assets affects the welfare of market participants. Let $W_\theta(\xi, \lambda)$ denote the equilibrium payoff of a type θ seller when transparency is ξ and correlation is λ . Since, in equilibrium, low type sellers are always indifferent between trade at $t = 0$ (at a price of v_L) and waiting until $t = 1$, we have that $W_L(\xi, \lambda) = v_L$ for all (ξ, λ) . That is, the expected utility of the low type is independent of the level of transparency and correlation. On the other hand, the equilibrium payoff of the high type will depend on both transparency and correlation and it is given by

$$W_H(\xi, \lambda) = (1 - \delta)c_H + \delta\mathbb{E}_H\{F_H(\pi_1, \eta)\}$$

where $F_H(\pi_1, \eta) = \max\{c_H, \bar{V}(\pi_1)\}$. Thus, a higher level ξ of transparency or correlation is welfare improving for sellers if and only if $\mathbb{E}_H\{\max\{c_H, \bar{V}(\pi_1)\}\}$ is higher.

Note that because buyers always break even in equilibrium, in order to make welfare comparisons we can focus on the payoff of the high type. Thus, any improvement in the high type's welfare is also a Pareto improvement. The following proposition proves the main result of this section, stating that while welfare with transparency and correlation is always higher than in their absence, the effect of transparency and correlation on welfare can be non-monotonic.

Proposition 4 *For any $\xi, \tilde{\xi} > 0$ and $\lambda, \tilde{\lambda} > \pi$, we have $W_H(\xi, \lambda) \geq W_H(\tilde{\xi}, \pi)$ and $W_H(\xi, \lambda) \geq W_H(0, \tilde{\lambda})$ with strict inequalities if and only if in an equilibrium with transparency ξ and correlation λ the high type seller strictly prefers to trade following the event of no trade in the other asset, i.e., outside the low trade equilibrium. Furthermore, the effect of transparency and correlation on welfare can be non-monotonic.*

Finally, let $W_H^q(\xi, \lambda)$ with $q = LT, MT, HT$ be the welfare of the high type in the low, medium, and high trade equilibria respectively, then $W_H^{LT}(\xi, \lambda) < W_H^{MT}(\xi, \lambda) < W_H^{HT}(\xi, \lambda)$ when the three types of equilibria coexist.

Figure 4 illustrates these results. In the bottom panel of the figure, when transparency is low, there is a unique equilibrium. In the particular parameterization below, the equilibrium features buyers mixing following the event of no trade in the other market. Here, the high type is indifferent whether to trade or not and any increase in transparency translates into change in trading behavior keeping welfare unchanged. On the other hand, when transparency becomes sufficiently large, multiplicity kicks in and we have dramatic differences in welfare levels across equilibria. The equilibrium with the lowest welfare (low trade equilibrium) still behaves as before; however, as we can see, welfare is strictly higher in the medium and high

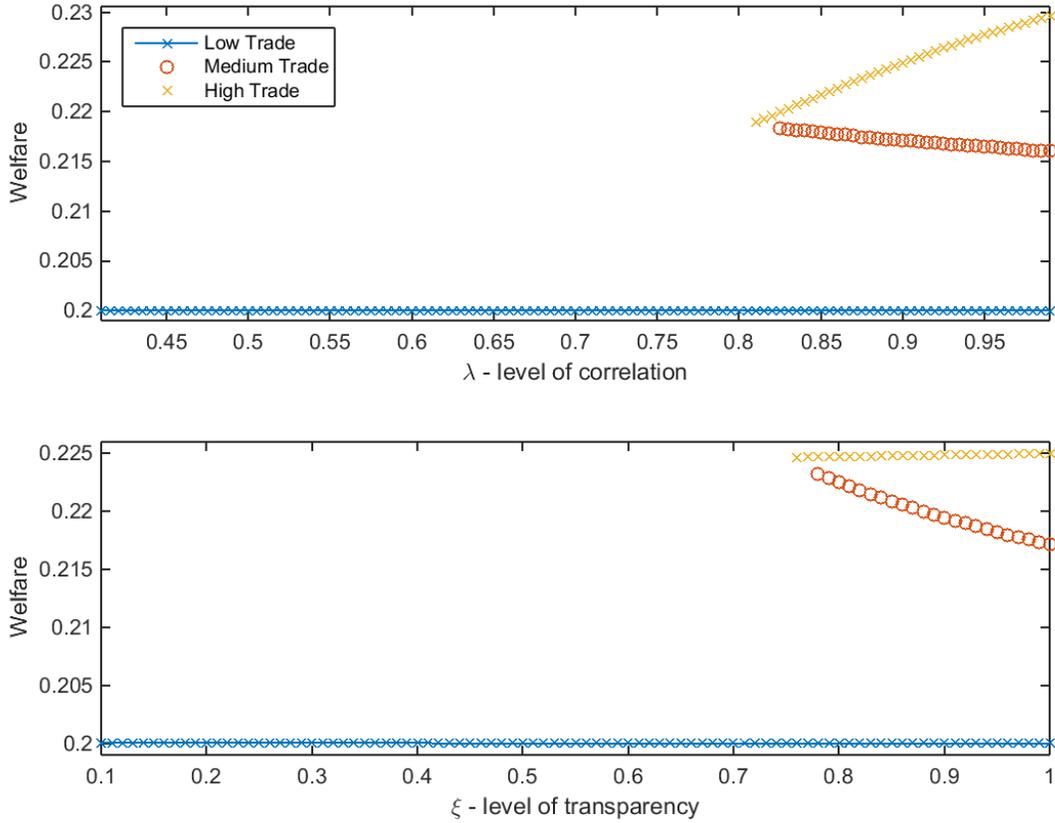


Figure 4: **Welfare**

Parameters: $c_L = 0$, $c_H = 0.2$, $v_L = 0.1$, $v_H = 0.25$, $\pi = 0.4$, and $\delta = 0.6$. In top panel, $\xi = 1$; in bottom panel, $\lambda = 0.9$.

trade equilibria. Along these equilibria, there is trade w.p. 1 in the event of no trade in the other market. However, whether trade occurs following the event of trade in the other market turns out to be crucial for whether welfare increases in transparency. Note that along the medium trade equilibrium, where trade does not occur following bad news, transparency reduces welfare. This is because in this region the negative news effect tends to dominate the positive effect that transparency has on prices. On the other hand, in the high trade equilibrium, the price effect is dominant and welfare increases in transparency. The intuition for the effect of correlation on welfare is analogous.

4 Robustness and Extensions

In this section, we investigate the robustness of our results and consider several extensions of the basic framework.

4.1 Perfect Correlation

We study the case where the traded assets are perfectly correlated, i.e., $\lambda = 1$. When correlation is perfect, we may have to worry about buyers' off-equilibrium beliefs. Suppose that the equilibrium specifies that low type trades immediately at $t = 0$, but that only one of the sellers has traded. In this case, buyers can put any probability $\pi^{off} \in [0, 1]$ to the remaining seller being low type. Then the continuation value of the low type seller can be expressed as:

$$\mathbb{E}_L\{F_L(\pi_1, \eta)\} = \sum_{z \in \Omega} \rho_L(z) \cdot [\eta(z)\bar{V}(\pi_1(z)) + (1 - \eta(z))v_L]$$

where $(\gamma, \rho_L, \eta, \pi_1)$ are as in Lemma 3 and Proposition 1, but with $\lambda = 1$ and $\sigma = 1 \implies \pi_1(v_L) = \pi^{off}$. There are two sets of equilibria to consider depending on whether the low type plays a pure strategy of trading immediately or a mixed trading strategy. By the same reasoning as before, an equilibrium with no trade is not possible.

First, as with imperfect correlation, we can have the low type mix between trade at $t = 0$ and $t = 1$. In such equilibria, the low type must be indifferent whether to trade at $t = 0$ or $t = 1$ and his continuation value is

$$\mathbb{E}_L\{F_L(\pi_1, \eta)\} = \rho_L(v_L) \cdot v_L + (1 - \rho_L(v_L)) \cdot [\eta(\emptyset)\bar{V}(\pi_1(\emptyset)) + (1 - \eta(\emptyset))v_L]$$

Since this continuation value is the limit of continuation values with imperfect correlation as λ goes to 1 and $\eta(v_L) = 0$, we conclude that these equilibria are the limits of low and medium trade equilibria of the economy with imperfect correlation.

Second, in contrast to imperfect correlation, we can have the low type seller trade immediately at $t = 0$. In that case, the low type receives a payoff v_L and the high type receives a payoff $(1 - \delta)c_H + \delta v_H$, and this equilibrium exists if with the most pessimistic off-equilibrium belief $\pi^{off} = 1$, we have

$$v_L \geq (1 - \delta)c_L + \delta \mathbb{E}_L\{F_L(\pi_1, \eta)\}|_{\sigma=1}$$

Intuitively, if the low type expects the other low type to trade and reveal their common type, then there is no incentive to delay trade to $t = 1$. Now, despite being in pure strategies, these equilibria are the limits of the high trade equilibria with imperfect correlation. To see this,

note that the latter equilibria require that the belief following trade in the other market be sufficiently high:

$$\pi_1(v_L) = \frac{\rho_L(v_L) \cdot \pi_\sigma}{\rho_L(v_L) \cdot \pi_\sigma + \rho_H(v_L) \cdot (1 - \pi_\sigma)} = \frac{1}{1 + \frac{1 - \phi\pi}{(1 - \sigma)\phi\pi}} \leq \bar{\pi}$$

Hence, along high trade equilibria we must have $\lim_{\lambda \rightarrow 1} \sigma = 1$. Thus, as with perfect correlation, the low type's payoff will be v_L and the high type's payoff will approach $(1 - \delta)c_H + \delta v_H$. The following proposition provides a summary of the above results:

Proposition 5 *The equilibria with perfect correlation are equivalent, in terms of welfare and trading probability σ , to the limits of equilibria with imperfect correlation as λ goes to 1. Thus, multiple equilibria continue to be possible when correlation is perfect as well.*

4.2 Many Assets

In this section, we extend the analysis to the case of many assets. Suppose that there are $N + 1$ sellers, each seller endowed with an asset with payoffs as described in Section 2. Let Z be the news that arrive to a market, and note that now $Z \in \{v_L, \emptyset\}^N$; this is because each market now can release news v_L or \emptyset . In any symmetric equilibrium, we only need to specify how many v_L offers have been observed by buyers. We will denote by z_k the event that exactly k transactions with offer v_L have been observed. Then the unconditional and type-conditional distributions of news are given by:

$$\begin{aligned} \gamma(z_k) &\equiv \mathbb{P}(Z = z_k) = \sum_{j=k}^N \binom{j}{k} (\xi\sigma)^k (1 - \xi\sigma)^{j-k} \mathbb{P}(c_L^j, c_H^{N-j}) \\ \rho_L(z_k) &\equiv \mathbb{P}(Z = z_k | c_L) = \sum_{j=k}^N \binom{j}{k} (\xi\sigma)^k (1 - \xi\sigma)^{j-k} \mathbb{P}(c_L^j, c_H^{N-j} | c_L) \end{aligned}$$

where $\mathbb{P}(c_L^j, c_H^{N-j})$ is the unconditional probability that only j sellers are of low type and $\mathbb{P}(c_L^j, c_H^{N-j} | c_L)$ is that probability conditional on seller $N + 1$ being of low type as well. These probabilities are in turn given by:

$$\begin{aligned} \mathbb{P}(c_L^j, c_H^{N-j}) &= \binom{N}{j} \left(\lambda^j (1 - \lambda)^{N-j} \pi + \left(\frac{1 - \lambda}{1 - \pi} \pi \right)^j \left(1 - \frac{1 - \lambda}{1 - \pi} \pi \right)^{N-j} (1 - \pi) \right) \\ \mathbb{P}(c_L^j, c_H^{N-j} | c_L) &= \binom{N}{j} \left(\lambda^j (1 - \lambda)^{N-j} \cdot \lambda + \left(\frac{1 - \lambda}{1 - \pi} \pi \right)^j \left(1 - \frac{1 - \lambda}{1 - \pi} \pi \right)^{N-j} \cdot (1 - \lambda) \right) \end{aligned}$$

We now show that the multiple equilibria can arise with many assets as well. Although there can now be many more equilibria than in the two asset case, we will focus our attention to two types of equilibria which, as before, we label low and high trade equilibria.

High Trade Equilibrium. Let us construct the counterpart of the high trade equilibrium of Proposition 3. Consider equilibria that feature $\eta(z_N) > 0$, i.e. where at $t = 1$ trade occurs with positive probability even following the event where every other seller has been observed to have traded at $t = 0$. Then, the high type must weakly prefer to trade following this event:

$$\pi_1(z_N) = \frac{\rho_L(z_N) \cdot \pi_\sigma}{\rho_L(z_N) \cdot \pi_\sigma + \rho_H(z_N) \cdot (1 - \pi_\sigma)} \leq \bar{\pi}$$

But because we have $\lim_{\lambda, \xi \rightarrow 1} \rho_H(z_N) \rightarrow 0$ and $\lim_{\lambda, \xi \rightarrow 1} \rho_L(z_N) \rightarrow \sigma^N$, if σ were bounded away from 1, we would also have that $\lim_{\lambda, \xi \rightarrow 1} \pi_1(z_N) = 1$, which is a contradiction. Thus, we conclude that $\lim_{\lambda, \xi \rightarrow 1} \sigma = 1$ and $\lim_{\lambda, \xi \rightarrow 1} \rho_L(z_N) = 1$, which implies that the range of the low type's continuation value converges to the interval $(v_L, v_H]$. This establishes the result that when λ and ξ are large enough, this type of equilibrium exists.

Low Trade Equilibrium. Let us now construct the counterpart of the low trade equilibrium of Proposition 3. Consider equilibria that feature $\eta(z_0) \leq 1$, i.e. where at $t = 1$ trade occurs with positive probability only following the event where no other seller has been observed to have traded at $t = 0$. Then, the high type must weakly prefer to trade following this event:

$$\pi_1(z_0) = \frac{\rho_L(z_0) \cdot \pi_\sigma}{\rho_L(z_0) \cdot \pi_\sigma + \rho_H(z_0) \cdot (1 - \pi_\sigma)} = \bar{\pi}$$

To show that the low trade equilibria exist for large δ , λ , and ξ we need to show that the range of the low type's continuation value does not converge to the singleton $\{v_L\}$ as λ, ξ go to 1, which is equivalent to showing that $\lim_{\lambda, \xi \rightarrow 1} \rho_L(z_0) > 0$. But because we have $\lim_{\lambda, \xi \rightarrow 1} \rho_H(z_0) = 1$ and $\lim_{\lambda, \xi \rightarrow 1} \rho_L(z_0) = (1 - \lim_{\lambda, \xi \rightarrow 1} \sigma)^N$, it suffices to show that $\lim_{\lambda, \xi \rightarrow 1} \sigma < 1$. If, to the contrary, we had $\lim_{\lambda, \xi \rightarrow 1} \sigma = 1$, then

$$\bar{\pi} = \lim_{\lambda, \xi \rightarrow 1} \pi_1(z_0) = \lim_{\lambda, \xi \rightarrow 1} \frac{\rho_L(z_0) \cdot \pi_\sigma}{\rho_L(z_0) \cdot \pi_\sigma + \rho_H(z_0) \cdot (1 - \pi_\sigma)} = 0 < \pi$$

which contradicts the fact that $\pi_1(z_0) = \bar{\pi}$ in this type of equilibrium.

The following proposition summarizes these results, and states the analogue of the welfare result for the case of many assets. Let $W_{N, \theta}(\xi, \lambda)$ denote the welfare of type θ seller when transparency is ξ and correlation is λ . Then we have the following result.

Proposition 6 *Fix N . Then (i) there exists $\bar{\delta} < 1$ such that the low trade equilibrium exists*

whenever $\delta > \bar{\delta}$, and (ii) there exist $\bar{\lambda}_{N,\delta}, \bar{\xi}_{N,\delta} < 1$ such that the high trade equilibrium exists whenever $\lambda > \bar{\lambda}_{N,\delta}$ and $\xi > \bar{\xi}_{N,\delta}$. These equilibria coexist whenever $\delta > \bar{\delta}$, $\lambda > \bar{\lambda}_{N,\delta}$, and $\xi > \bar{\xi}_{N,\delta}$.

For any $\xi, \tilde{\xi} > 0$ and $\lambda, \tilde{\lambda} > \pi$, we have $W_{N,H}(\xi, \lambda) \geq W_{N,H}(\tilde{\xi}, \pi)$ and $W_{N,H}(\xi, \lambda) \geq W_{N,H}(0, \tilde{\lambda})$ with strict inequalities if and only if in an equilibrium with transparency ξ and correlation λ the high type seller strictly prefers to trade following the event that no trade has been observed in all other assets, i.e., outside the low trade equilibrium.

5 Conclusions

In this paper, we study informational spillovers and transparency in markets where the assets have correlated values. In this setting, the information transmitted from one market to another is endogenous to trading activity. As a result, the degree of transparency of financial markets can have important implications for trading behavior and welfare in these markets. The important insights that we derived were the following. First, contrary to an economy with exogenous revealed information, when the information revealed is endogenously determined by trading behavior, there must be trade in every period with strictly positive probability. Second, the endogenous nature of information introduces complementarities in the trading behavior across markets. In particular, when correlation and transparency are sufficiently high, these complementarities can be sufficiently strong so as to lead to multiple equilibria that differ in their trading volume, prices, and welfare. In terms of overall welfare, fully transparent markets are always preferred to fully opaque markets. However, an interior level of transparency may be optimal depending on the type of equilibrium on which traders coordinate.

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6 Appendix

Proof of Proposition 1. Let $\sigma_j \in [0, 1]$ be given. From Lemmas 1 and 2, we only need to determine the low type's strategy σ_i and the buyers' strategy η_i . By Assumption 2, $\sigma_i = 1$ cannot be an equilibrium since then the price at $t = 1$ would be v_H and the low type would want to wait; hence, $\sigma_i < 1$. In that case, we must have $v_L \leq (1 - \delta)c_L + \delta \mathbb{E}_L \{F_L(\pi_{i,1}, \eta_i)\}$ with strict equality if $\sigma_i > 0$. The equilibrium strategy of the buyers follows from Lemma 1. Finally, uniqueness follows from the monotonicity of $\mathbb{E}_L \{F_L(\pi_{i,1}, \eta_i)\}$ in σ and η_i . ■

Proof of Proposition 2. If $\sigma = 0$, then there is no news and thus $v_L \leq (1 - \delta)c_L + \delta F_L(\pi_{i,1}, \eta_i)$ with strict equality if $\sigma > 0$. But, at $\sigma = 0$, we have

$$F_L(\pi_{i,1}, \eta_i) = \eta_i(\pi v_L + (1 - \pi)v_H) + (1 - \eta_i)v_L$$

and since $\pi > \bar{\pi}$ we must have $\eta_i = 0$. Hence, $v_L > (1 - \delta)c_L + \delta v_L$ and the low type strictly prefers to trade at $t = 0$, a contradiction. Thus, $\sigma \in (0, 1)$. ■

Proof of Proposition 3. It is clear the equilibrium falls into one of the three categories specified. First, I show that high trade equilibria exist when λ and ξ are sufficiently large. Let $\bar{\sigma}$ be such that $\pi_1(v_L)|_{\sigma=\bar{\sigma}} = \bar{\pi}$, and note that if $\sigma \geq \bar{\sigma}$ then

$$\begin{aligned} \mathbb{E}_L\{F_L\} &= (\xi\pi\phi) \cdot \sigma \times [\eta(v_L)(\pi_1(v_L)|_{\sigma \geq \bar{\sigma}} \cdot v_L + (1 - \pi_1(v_L)|_{\sigma \geq \bar{\sigma}}) \cdot v_H) + (1 - \eta(v_L))v_L] \\ &+ (1 - \xi\pi\phi) \cdot \sigma \times (\pi_1(\emptyset)|_{\sigma \geq \bar{\sigma}} \cdot v_L + (1 - \pi_1(\emptyset)|_{\sigma \geq \bar{\sigma}}) \cdot v_H) \end{aligned}$$

where $\eta(v_L) \in (0, 1)$ only if $\sigma = \bar{\sigma}$. Because $\lim_{\lambda, \xi \rightarrow 1} (\xi\pi\phi) \cdot \bar{\sigma} = 1$, the range of the continuation value in this type of equilibrium becomes $(v_L, v_H]$ as λ, ξ go to 1. This establishes the existence of high trade equilibria. Second, I show that the low trade equilibria exist when δ is sufficiently large. Let $\bar{\sigma}$ be such that $\pi_1(\emptyset)|_{\sigma=\bar{\sigma}} = \bar{\pi}$, and note that the continuation value of the low type is

$$\mathbb{E}_L\{F_L\} = (\xi\pi\phi) \cdot \bar{\sigma} \times v_L + (1 - (\xi\pi\phi) \cdot \bar{\sigma}) \times [\eta(\emptyset)c_H + (1 - \eta(\emptyset))v_L]$$

where $\eta(\emptyset) \in [0, 1]$. Thus, in this type of equilibrium, the continuation has the following range:

$$\mathbb{E}_L\{F_L\} \in [v_L, (\xi\pi\phi) \cdot \bar{\sigma} \times v_L + (1 - (\xi\pi\phi) \cdot \bar{\sigma}) \times c_H]$$

Because $\sup_{\xi, \lambda} (\xi\pi\phi) \cdot \bar{\sigma} < 1$ in this type of equilibrium, we have that the low trade equilibria exist whenever δ is greater than some $\bar{\delta} < 1$, independently of the values of λ and ξ . Note that the ranges of the continuation values in the two equilibria overlap whenever $\delta > \bar{\delta}$ and λ and ξ are sufficiently large. Finally, the range of the continuation value in the medium trade

equilibrium falls in between the continuation values of the high and the low trade equilibria. Hence, whenever these two equilibria exist, the medium trade equilibrium exists as well.

For uniqueness, we first show that when either ξ and/or λ are sufficiently small, then the equilibrium is unique. To this end, we just need to show that the continuation value of the low type is monotonic in σ along each of the three types of equilibria and that the ranges of these equilibria do not overlap.

- High Trade Equilibria: the continuation value of the low type is given by

$$\mathbb{E}_L \{F_L\} = \rho_L \cdot \bar{V}(\pi_{i,1}(v_L)) + (1 - \rho_L) \cdot \bar{V}(\pi_{i,1}(\emptyset))$$

Differentiation with respect to σ yields:

$$\frac{d\mathbb{E}_L \{F_L\}}{d\sigma} = \underbrace{(\xi\phi\pi) [\bar{V}(\pi_{i,1}(v_L)) - \bar{V}(\pi_{i,1}(\emptyset))]}_{\text{news effect}} + \underbrace{\rho_L \frac{d\bar{V}(\pi_{i,1}(v_L))}{d\sigma} + (1 - \rho_L) \frac{d\bar{V}(\pi_{i,1}(\emptyset))}{d\sigma}}_{\text{price effect}}$$

Where the “news effect” is negative since $\pi_{i,1}(\emptyset) < \pi_{i,1}(v_L)$ and the “price effect” is positive since $\pi_{i,1}(v_L)$ and $\pi_{i,1}(\emptyset)$ are increasing in σ . Since the “news effect” can be made arbitrarily small by having ξ and/or λ small, we have monotonicity in σ .

- Low Trade Equilibria: the continuation value of the low type is given by

$$\mathbb{E}_L \{F_L\} = \rho_L \cdot v_L + (1 - \rho_L) \cdot [\eta(\emptyset) \bar{V}(\pi_{i,1}(\emptyset)) + (1 - \eta(\emptyset)) v_L]$$

In this equilibrium, σ is fixed and thus the continuation value is trivially monotonic.

- Medium Trade Equilibria: the continuation value of the low type is given by

$$\mathbb{E}_L \{F_L\} = \rho_L \cdot v_L + (1 - \rho_L) \cdot \bar{V}(\pi_{i,1}(\emptyset))$$

Differentiation with respect to σ yields:

$$\frac{d\mathbb{E}_L \{F_L\}}{d\sigma} = \underbrace{(\xi\phi\pi) [v_L - \bar{V}(\pi_{i,1}(\emptyset))]}_{\text{news effect}} + \underbrace{(1 - \rho_L) \frac{d\bar{V}(\pi_{i,1}(\emptyset))}{d\sigma}}_{\text{price effect}}$$

It is clear that the “news effect” can again be made arbitrarily small by making ξ small. On the other hand, the medium trade equilibrium disappears as $\lambda \rightarrow \pi$ since $\pi_{i,1}(\emptyset), \pi_{i,1}(v_L) \rightarrow \pi_\sigma$.

Finally, monotonicity of the continuation value in the medium trade equilibrium also means that the ranges of the continuation values in the three types of equilibria do not overlap.

We now show that the equilibrium is also unique when δ is sufficiently small. Let $\widehat{\delta}$ be defined by $v_L = (1 - \widehat{\delta})c_L + \widehat{\delta}v_H$. Recall that we have assumed that $\delta > \widehat{\delta}$ to rule out the separating equilibrium, which would be unique if this condition were violated. We now show that even when $\delta > \widehat{\delta}$, it becomes unique as δ comes close to $\widehat{\delta}$. Note that when δ is close to $\widehat{\delta}$, only high trade equilibrium is possible; hence, we only need to consider the low type's continuation value along this type of equilibrium. But note that as δ approaches $\widehat{\delta}$, we must have σ approach 1. Hence, we can again make the “news effect” arbitrarily small because along the high trade equilibrium $\lim_{\delta \rightarrow \widehat{\delta}} \pi_{i,1}(v_L) = \lim_{\delta \rightarrow \widehat{\delta}} \pi_{i,1}(\emptyset) = 0$. We have thus established equilibrium uniqueness. To rank the trading probabilities, note that we have $\pi_1^{HT}(v_L) \leq \bar{\pi} < \pi_1^{MT}(v_L) < 1 = \pi_1^{LT}(v_L)$ and $\pi_1(v_L)$ is decreasing in σ . This is sufficient to conclude that $\sigma^{LT} < \sigma^{MT} < \sigma^{HT}$. ■

Non-Existence of Asymmetric Equilibria. Let $f(\sigma)$ denote the equilibrium trading probability of one seller when the other seller trades with probability σ ; since $f(\sigma)$ is unique for each $\sigma \in [0, 1]$, we will refer to $f(\cdot)$ as the best-response function. To rule out asymmetric equilibria, we need to show that the equation

$$f(f(x)) = x$$

has at most one solution for $x \in [0, 1]$. For contradiction, let x_1, x_2 with $x_1 < x_2$ be solutions to the above equation, then we have that

$$f(x_1) = x_2$$

and

$$f(x_2) = x_1$$

But then we must have that $f(x_2) < f(x_1)$, i.e., $f(\cdot)$ must be decreasing somewhere on the interval $[x_1, x_2]$. Recall that, for a given x , $f(x)$ is determined by one of the following conditions:

1. High type does not trade following bad news and is indifferent following good news:

$$\pi_{i,1}(\emptyset) |_{\sigma_i=f(x), \sigma_j=x} = \bar{\pi}$$

In this case, $f(\cdot)$ is clearly decreasing.

2. High type does not trade following bad news and trades following good news:

$$\frac{v_L - (1 - \delta) c_L}{\delta} \leq \rho_L v_L + (1 - \rho_L) \bar{V}(\pi_{i,1}(\emptyset)) |_{\sigma_i=f(x), \sigma_j=x}$$

with strict equality when $f(x) > 0$. In this case, $f(\cdot)$ is (weakly) increasing as shown in Lemma 3.1.

3. High type is indifferent following bad news and trades following good news:

$$\pi_{i,1}(v_L) |_{\sigma_i=f(x), \sigma_j=x} = \bar{\pi}$$

In this case, $f(\cdot)$ is clearly increasing.

4. High type trades following bad and good news:

$$\frac{v_L - (1 - \delta) c_L}{\delta} = \rho_L \bar{V}(\pi_{i,1}(v_L)) + (1 - \rho_L) \bar{V}(\pi_{i,1}(\emptyset)) |_{\sigma_i=f(x), \sigma_j=x}$$

In this case, $f(\cdot)$ is increasing as shown in Lemma 3.1.

We thus conclude that there exists an $\bar{x} \in [0, 1]$ such that, for $x \in [0, \bar{x}]$, $f(\cdot)$ is decreasing and satisfies

$$\pi_{i,1}(\emptyset) |_{\sigma_i=f(x), \sigma_j=x} = \bar{\pi}$$

and, for $x > \bar{x}$, $f(\cdot)$ is weakly increasing.

For asymmetric equilibria to exist, it must be the case that $\bar{x} > 0$. For $x \in [0, 1]$, define function $g(\cdot)$ by

$$g(x) = 1 - \frac{1 - \frac{1-\phi\pi}{1-\pi} \cdot \pi \cdot \xi \cdot x}{1 - \phi \cdot \pi \cdot \xi \cdot x} \cdot \frac{1 - \pi}{\pi} \cdot \frac{\bar{\pi}}{1 - \bar{\pi}}$$

and note that this function agrees with $f(\cdot)$ on the interval $[0, \bar{x}]$. By Lemma 3.2 below, $g(g(x)) = x$ has at most one solution in the interval $[0, 1]$. Hence, for asymmetric equilibria to exist, it must be the case that $g(g(0)) \leq 0$. However, note that

$$g(0) = 1 - \frac{1 - \pi}{\pi} \cdot \frac{\bar{\pi}}{1 - \bar{\pi}}$$

\implies

$$\begin{aligned}
g(g(0)) &= 1 - \frac{1 - \frac{1-\phi\cdot\pi}{1-\pi} \cdot \pi \cdot \xi \cdot g(0)}{1 - \phi \cdot \pi \cdot \xi \cdot g(0)} \cdot \frac{1 - \pi}{\pi} \cdot \frac{\bar{\pi}}{1 - \bar{\pi}} \\
&= 1 - \frac{1 - \frac{1-\phi\cdot\pi}{1-\pi} \cdot \pi \cdot \xi \cdot \left(1 - \frac{1-\pi}{\pi} \cdot \frac{\bar{\pi}}{1-\bar{\pi}}\right)}{1 - \phi \cdot \pi \cdot \xi \cdot \left(1 - \frac{1-\pi}{\pi} \cdot \frac{\bar{\pi}}{1-\bar{\pi}}\right)} \cdot \frac{1 - \pi}{\pi} \cdot \frac{\bar{\pi}}{1 - \bar{\pi}} \\
&> 1 - \frac{1}{1 - \left(1 - \frac{1-\pi}{\pi} \cdot \frac{\bar{\pi}}{1-\bar{\pi}}\right)} \cdot \frac{1 - \pi}{\pi} \cdot \frac{\bar{\pi}}{1 - \bar{\pi}} \\
&= 0
\end{aligned}$$

This is a contradiction, and we conclude that asymmetric equilibria cannot exist.

Next, we prove Lemma 3.1 which was used to rule out asymmetric equilibria. Here, we show that the best response function $f(x)$ is non-decreasing for x satisfying: $\pi_1(\emptyset)|_{\sigma_i=f(x), \sigma_j=x} < \bar{\pi}$. To this end, we only need to show that the continuation payoff of the low type seller i is decreasing in the trading probability σ_j in the cases 2 and 4. For $\gamma \in (0, \pi)$, define functions F_1 and F_2 by

$$F_1(\gamma) = \phi\gamma + \frac{(1 - \phi\gamma)^2}{1 - \phi\gamma + \left(1 - \frac{1-\phi\cdot\pi}{1-\pi} \cdot \gamma\right) \cdot \frac{1-\pi\sigma}{\pi\sigma}}$$

and

$$F_2(\gamma) = \frac{\phi\gamma}{1 + \frac{1-\phi\cdot\pi}{1-\pi} \cdot \frac{1-\pi\sigma}{\pi\sigma}} + \frac{(1 - \phi\gamma)^2}{1 - \phi\gamma + \left(1 - \frac{1-\phi\cdot\pi}{1-\pi} \cdot \gamma\right) \cdot \frac{1-\pi\sigma}{\pi\sigma}}$$

These functions are the probabilities that the low type assigns to receiving offer v_L in cases 2 and 4 respectively, where γ is the probability of bad news arriving.

Lemma 3.1 *The functions F_1 and F_2 are monotonically increasing in γ for $\gamma \in (0, \pi)$.*

Proof. Differentiation of $F_1(\gamma)$ w.r.t. γ yields

$$\begin{aligned}
F_1'(\gamma) &= \phi + (1 - \phi\gamma) \cdot \frac{-2\phi \left[1 - \phi\gamma + \left(1 - \frac{1-\phi\cdot\pi}{1-\pi} \gamma\right) \cdot \frac{1-\pi\sigma}{\pi\sigma}\right] + (1 - \phi\gamma) \left[\phi + \frac{1-\phi\cdot\pi}{1-\pi} \cdot \frac{1-\pi\sigma}{\pi\sigma}\right]}{\left[1 - \phi\gamma + \left(1 - \frac{1-\phi\cdot\pi}{1-\pi} \gamma\right) \cdot \frac{1-\pi\sigma}{\pi\sigma}\right]^2} \\
&= \left[\phi + \frac{1 - \phi \cdot \pi}{1 - \pi} \cdot \frac{1 - \pi\sigma}{\pi\sigma}\right] \pi_1(\emptyset)^2 - 2\phi\pi_1(\emptyset) + \phi
\end{aligned}$$

where $\pi_1(\emptyset) = \frac{1}{1 + \frac{1-\phi\cdot\pi}{1-\pi} \cdot \frac{1-\pi\sigma}{\pi\sigma}}$. This quadratic is minimized at x given by

$$x = \frac{\phi}{\phi + \frac{1-\phi\cdot\pi}{1-\pi} \cdot \frac{1-\pi\sigma}{\pi\sigma}} > \pi\sigma \geq \pi_1(\emptyset)$$

which implies that

$$F_1'(\gamma) > -\frac{\phi^2}{\phi + \frac{1-\phi\cdot\pi}{1-\pi} \cdot \frac{1-\pi_\sigma}{\pi_\sigma}} + \phi = \frac{\frac{1-\phi\cdot\pi}{1-\pi} \cdot \frac{1-\pi_\sigma}{\pi_\sigma}}{\phi + \frac{1-\phi\cdot\pi}{1-\pi} \cdot \frac{1-\pi_\sigma}{\pi_\sigma}} > 0$$

This establishes that F_1 is increasing. Differentiation of $F_2(\gamma)$ w.r.t. γ yields

$$F_2'(\gamma) = \left[\phi + \frac{1-\phi\cdot\pi}{1-\pi} \cdot \frac{1-\pi_\sigma}{\pi_\sigma} \right] \cdot \pi_1(\emptyset)^2 - 2\phi \cdot \pi_1(\emptyset) + \phi \cdot \frac{\phi}{\phi + \frac{1-\phi\cdot\pi}{1-\pi} \cdot \frac{1-\pi_\sigma}{\pi_\sigma}}$$

This quadratic has only one root at

$$x = \frac{\phi}{\phi + \frac{1-\phi\cdot\pi}{1-\pi} \cdot \frac{1-\pi_\sigma}{\pi_\sigma}} > \pi_\sigma \geq \pi_1(\emptyset)$$

which implies that $F_2'(\gamma) > 0$. ■

Next, we prove Lemma 3.2 which was also used to rule out asymmetric equilibria.

Lemma 3.2 For $x \in [0, 1]$, define function $g(\cdot)$ by

$$g(x) = 1 - \frac{1 - \frac{1-\phi\pi}{1-\pi} \cdot \pi \cdot \xi \cdot x}{1 - \phi \cdot \pi \cdot \xi \cdot x} \cdot \frac{1-\pi}{\pi} \cdot \frac{\bar{\pi}}{1-\bar{\pi}}$$

Then the equation $g(g(x)) = x$ has at most one solution in $[0, 1]$.

Proof. Note that $g'(\cdot), g''(\cdot) < 0$. Hence, $g(g(x)) = x$ has at most one solution in $[0, \bar{x}]$ if $g(0) < \bar{x}$, where recall that \bar{x} is defined by $g(\bar{x}) = 0$. Let $\alpha \equiv \frac{1-\pi}{\pi} \cdot \frac{\bar{\pi}}{1-\bar{\pi}}$, then we have $g(0) = 1 - \alpha$ and

$$\begin{aligned} (\xi\pi) \cdot \bar{x} &= \frac{1-\alpha}{\phi - \frac{1-\phi\pi}{1-\pi}} \\ &> \pi \cdot (1-\alpha) \\ &= \pi \cdot g(0) \end{aligned}$$

and the result follows. ■

Proof of Proposition 4. Let us prove the result about the effect of transparency on welfare. The proof for the effect of correlation on welfare is analogous. Fix $\lambda > \pi$ and suppose that

$\xi > 0$ and in equilibrium we have $\pi_1(\emptyset) = \bar{\pi}$. Then we must have that

$$\mathbb{E}_L \{F_L\} |_{\xi>0} = (\xi\pi\phi) \cdot \bar{\sigma} \times v_L + (1 - (\xi\pi\phi) \cdot \bar{\sigma}) \times [\eta(\emptyset) c_H + (1 - \eta(\emptyset)) v_L] < c_H$$

where $\bar{\sigma}$ satisfies $\pi_1(\emptyset) |_{\sigma=\bar{\sigma}} = \bar{\pi}$. Thus, $\pi_1(\emptyset) = \bar{\pi}$ must also be satisfied in the equilibrium with $\xi = 0$. In both cases, $W_H(\xi, \lambda) = W_H(0, \cdot) = c_H$. Now, suppose that when $\xi > 0$, in equilibrium we have $\pi_1(\emptyset) < \bar{\pi}$. If when $\xi = 0$, we have $\pi_1(\emptyset) = \bar{\pi}$, then the result is immediate; so suppose that this is not the case. Then the result follows from

$$\mathbb{E}_H \{F_H\} |_{\xi>0} = \mathbb{E}_L \{F_L\} |_{\xi>0} = \mathbb{E}_L \{F_L\} |_{\xi=0} = \mathbb{E}_H \{F_H\} |_{\xi=0}$$

For an example of non-monotonicity, see Figure 4. Finally, to rank welfares, note that the welfare of the high type in equilibrium of type q is given by

$$W_H^q(\xi, \lambda) = (1 - \delta) c_H + \delta \cdot [\rho_H^q(v_L) \max\{c_H, \bar{V}(\pi_1^q(v_L))\} + (1 - \rho_H^q(v_L)) \max\{c_H, \bar{V}(\pi_1^q(\emptyset))\}]$$

where $\rho_H^q(v_L) = \frac{1-\phi\pi}{1-\pi}\gamma$ and $\gamma = \xi\pi\sigma$. Hence, we have $W_H^{HT}(\xi, \lambda), W_H^{MT}(\xi, \lambda) > c_H = W_H^{LT}(\xi, \lambda)$ and we are left to show that $W_H^{HT}(\xi, \lambda) > W_H^{MT}(\xi, \lambda)$. Since $W_H^q(\xi, \lambda)$ is increasing in own trading probability for $q = MT, HT$, we only need to show that $W_H^q(\xi, \lambda)$ is also increasing in the probability γ of news arrival. This is shown in the following lemma:

Proof of Lemma 4.1. For $\gamma \in (0, \pi)$, define functions $G_1(\cdot)$ and $G_2(\cdot)$ by

$$G_1(\gamma) = \frac{1 - \phi\pi}{1 - \pi} \gamma \cdot \bar{\pi} + \left(1 - \frac{1 - \phi\pi}{1 - \pi} \gamma\right) \cdot \pi_1(\emptyset)$$

and

$$G_2(\gamma) = \frac{1 - \phi\pi}{1 - \pi} \gamma \cdot \pi_1(v_L) + \left(1 - \frac{1 - \phi\pi}{1 - \pi} \gamma\right) \cdot \pi_1(\emptyset)$$

Then $G_1(\cdot)$ and $G_2(\cdot)$ are decreasing in γ .

Proof. Differentiation of G_1 w.r.t. γ yields

$$\begin{aligned} G_1'(\gamma) &= \frac{1 - \phi\pi}{1 - \pi} \cdot (\bar{\pi} - \pi_1(\emptyset)) + \left(1 - \frac{1 - \phi\pi}{1 - \pi} \gamma\right) \cdot \pi_1'(\emptyset) \\ &= \frac{1 - \phi\pi}{1 - \pi} (\bar{\pi} - \pi_1(\emptyset)) - \frac{\phi - \frac{1-\phi\pi}{1-\pi}}{1 - \phi\gamma} (1 - \pi_1(\emptyset)) \pi_1(\emptyset) \\ &= \frac{\phi - \frac{1-\phi\pi}{1-\pi}}{1 - \phi\gamma} \cdot \pi_1^2(\emptyset) - \left(\frac{1 - \phi\pi}{1 - \pi} + \frac{\phi - \frac{1-\phi\pi}{1-\pi}}{1 - \phi\gamma}\right) \cdot \pi_1(\emptyset) + \frac{1 - \phi\pi}{1 - \pi} \cdot \bar{\pi} \end{aligned}$$

This is a positive quadratic in $\pi_1(\emptyset)$ and is thus maximized at the extremes: at $\pi_1(\emptyset) = \bar{\pi}$

(the highest possible value) and at $\pi_1(\emptyset) = \hat{\pi}$ such that $\pi_1(v_L) = \bar{\pi}$ (the lowest possible value). Since we have that $G'_1(\gamma)|_{\pi_1(\emptyset)=\bar{\pi}} < 0$, we need to show that $G'_1(\gamma)|_{\pi_1(\emptyset)=\hat{\pi}} < 0$, where

$$\hat{\pi} = \frac{1}{1 + \frac{1 - \frac{1-\phi\pi}{1-\pi}\gamma \phi(1-\pi)}{1-\phi\gamma} \frac{1-\bar{\pi}}{1-\phi\pi} \frac{1-\bar{\pi}}{\bar{\pi}}}$$

But this follows because:

$$\begin{aligned} \text{sgn}(G'_1(\gamma)|_{\pi_1(\emptyset)=\hat{\pi}}) &= \text{sgn}\left(\frac{1-\phi\pi}{1-\pi}(\bar{\pi}-\hat{\pi}) - \frac{\phi - \frac{1-\phi\pi}{1-\pi}}{1-\phi\gamma}(1-\hat{\pi})\hat{\pi}\right) \\ &= \text{sgn}\left(\frac{1-\phi\pi}{1-\pi} \frac{1 - \frac{1-\phi\pi}{1-\pi}\gamma \phi(1-\pi)}{1-\phi\gamma} (1-\bar{\pi})\hat{\pi} - \frac{\phi - \frac{1-\phi\pi}{1-\pi}}{1-\phi\gamma}(1-\hat{\pi})\hat{\pi}\right) \\ &= \text{sgn}\left(\left(1 - \frac{1-\phi\pi}{1-\pi}\gamma\right)(1-\bar{\pi}) - \left(1 - \frac{1-\phi\pi}{1-\pi}\phi^{-1}\right)(1-\hat{\pi})\right) \\ &= \text{sgn}\left(1 - \frac{\frac{1}{1-\phi\gamma} \frac{\phi(1-\pi)}{1-\phi\pi}}{\bar{\pi} + \frac{1 - \frac{1-\phi\pi}{1-\pi}\gamma \phi(1-\pi)}{1-\phi\gamma} (1-\bar{\pi})}\right) \\ &= \text{sgn}\left(\bar{\pi}(1-\phi\gamma)(1-\phi\pi) + \left(1 - \frac{1-\phi\pi}{1-\pi}\gamma\right)\phi(1-\pi)(1-\bar{\pi}) - \phi(1-\pi)\right) \\ &\leq \text{sgn}(\bar{\pi} - \phi\bar{\pi}) < 0 \end{aligned}$$

Differentiation of G_2 w.r.t. γ yields

$$\begin{aligned} G'_1(\gamma) &= \frac{1-\phi\pi}{1-\pi} \cdot (\pi_1(v_L) - \pi_1(\emptyset)) + \left(1 - \frac{1-\phi\pi}{1-\pi}\gamma\right) \cdot \pi'_1(\emptyset) \\ &< \pi_1(v_L) - \pi_1(\emptyset) + (1-\gamma) \cdot \pi'_1(\emptyset) = H'(\gamma) = 0 \end{aligned}$$

since $H(\gamma) \equiv \gamma \cdot \pi_1(v_L) + (1-\gamma) \cdot \pi_1(\emptyset) = \pi_\sigma$ where $\pi_\sigma \equiv \frac{(1-\sigma)\pi}{1-\sigma\pi}$ is constant in γ . ■