Endogenous Credit Cycles and Financial Dampening in an Adverse Selection Economy*

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January 2005

Abstract

We propose an adverse selection framework in which the financial sector has a dual role. It amplifies or dampens exogenous shocks and also generates endogenous fluctuations. We fully characterize constrained optimal contracts in a setting in which entrepreneurs need to borrow and are privately informed about the quality of their projects. Our characterization is novel in analyzing pooling and separating allocations in a context of multi-dimensional screening: specifically, the amounts of investment undertaken and of entrepreneurial net worth are used to screen projects.

We then embed these results in a dynamic competitive economy. First, we show how endogenous regime switches in financial contracts may generate fluctuations in an economy that exhibits no dynamics under full information. Unlike previous models of endogenous cycles, our result does not rely on entrepreneurial net worth being counter-cyclical or inconsequential for determining investment. Secondly, the model shows the different implications of adverse selection as opposed to pure moral hazard. In particular, and contrary to standard results in the macroeconomic literature, the financial system may dampen exogenous shocks in the presence of adverse selection.

*I am deeply indebted to Paolo Siconolfi for constant encouragement. I am particularly grateful to John Geanakoplos, John Donaldson, Pietro Reichlin and Tano Santos for discussions and important insights. I also thank Max Amarante, Dirk Bergemann, Alex Citanna, Bruce Preston, Bernard Salanie, Guido Sandleris, Filippo Taddei, Wouter Vergote and seminar participants at Columbia, Yale and at the IV Villa Mondragone Workshop in Economic Theory and Econometrics.

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1 Introduction

An important research agenda in macroeconomics has been to analyze the relationship between the business cycle and the condition of firms' and households' balance sheets. The focus of this research has been to study the way in which exogenous shocks may be amplified through the financial system, whereas an old idea among macroeconomists - that the financial system may itself be a source of endogenous volatility¹ - has received significantly less attention.

In this paper, we propose a unified framework in which the financial system has a dual role, as an amplifier or dampener of exogenous shocks and as a generator of fluctuations. A crucial and novel feature of our framework is that both of these roles arise in a context in which borrower net worth is pro-cyclical and plays an important role in determining investment. We show that in the presence of adverse selection, the financial system may act as a dampener of exogenous shocks and, even in absence of such shocks, as a source of economic fluctuations.

We consider a setting in which borrowers are privately informed about the returns of their investment projects. Financial intermediaries seek to mitigate the asymmetry of information by offering a menu of contracts. Unlike traditional models that analyze this phenomenon by assuming that the size of investment projects is exogenously fixed (Stiglitz and Weiss [38] or Bester [11],[12]) or that the return to the latter is linear (Bencivenga and Smith [7]), we allow for a concave investment function so that the size of projects is determined endogenously in equilibrium. This feature allows entrepreneurs to be screened through the amounts of collateral that they provide and of investment that they undertake.

As is well known, adverse selection settings like the one we consider are problematic when it comes to existence of equilibria. In particular, models of screening like Rothschild-Stiglitz [30] do not have an equilibrium whenever the pooling allocation Pareto dominates its separating counterpart. We avoid this problem by following Hellwig [23] in modeling competition in the credit market as a three-stage game. As we explain below, this specification always has an equilibrium, which in particular is constrained optimal. It is this feature that ultimately allows us to fully characterize pooling and separating equilibria in a setting of multi-dimensional screening, a feature that is - to our knowledge - completely novel. We show that entrepreneurial wealth is crucial in determining whether the credit market equilibrium will entail pooling or separation of projects, thus leading to different levels of aggregate investment.

This framework is then embedded in a dynamic competitive economy. We first show that, contrary to the standard result in the literature, exogenous shocks may be dampened by the presence of adverse selection. This may happen because, under this form of asymmetric information, the amount that each type of entrepreneur can borrow depends on what other types are borrowing. When our economy faces a positive shock, entrepreneurial wealth increases but so does the total

¹Different but well-known versions of this view can be found in Friedman [20] and Keynes [24].

amount of savings in the economy. This implies that loan contracts must change in order to restore market clearing while at the same time satisfying incentive compatibility. The way in which these two requirements interact ultimately determines how the proportion of funds allocated to each type of project varies and, consequently, whether shocks are dampened or amplified by the presence of asymmetric information. If we abstract from the market clearing role of contracts - for example, by assuming that we are dealing with a small, open economy in which the interest rate is set exogenously - the traditional result of financial amplification is restored.

Secondly, we show that an economy that exhibits no dynamics under full information and that is not subject to exogenous perturbations may nonetheless display aggregate fluctuations in the presence of adverse selection. This result stems from the fact that changes in entrepreneurial net worth may generate regime switches between pooling and separating equilibria in the credit market. It must be stressed that, unlike previous models that analyze endogenous cycles in settings of asymmetric information, the latter arise in our framework despite the pro-cyclicality of net worth and its importance in determining investment. To facilitate the comparison of this mechanism with a well understood benchmark, we analyze it in a dynamic setting that is nearly identical to that in Bernanke and Gertler [8].

It may be helpful to elaborate further on the mechanism behind the previous result. When entrepreneurial wealth is low, screening is relatively costly in our economy since it is done predominantly by restricting the amount of investment undertaken by good entrepreneurs. Hence, there is a strong tendency to pool all projects and have good entrepreneurs cross-subsidize their bad counterparts. If the average quality of investment in the economy is above a certain threshold, pooling contracts will yield a relatively high level of investment, future output and - more specifically - future entrepreneurial wealth.

As entrepreneurial wealth increases, though, the screening possibilities of intermediaries are enhanced, since they can increasingly screen through collateralization requirements. Consequently, intermediaries can eventually design profitable contracts tailored to attract the most productive entrepreneurs from the pool. These contracts induce lower levels of investment relative to the pooling equilibrium but entail no cross-subsidization, and hence provide funds at a lower cost. In this way, there is a “flight of quality” phenomenon by which the best entrepreneurs are lured away from the pooling equilibrium, which unravels into a separating regime and in so doing leads to a decrease in investment, future output and future entrepreneurial net worth. If this effect is sufficiently large, the economy may revert to a pooling equilibrium and the process starts again.

Although the main objective of this paper is conceptual in nature, the mechanism behind endogenous fluctuations in our model is consistent with different strands of stylized evidence. In the first place, booms may revert into recessions in our framework even though entrepreneurial wealth is pro-cyclical and plays a crucial role in determining equilibrium levels of investment, both features

that seem consistent with empirical evidence.² This is in contrast with previous models dealing with endogenous reversion mechanisms in the presence of asymmetric information, in which net worth is either counter-cyclical or it is inconsequential for the allocation of credit. An additional feature of our model that seems to be consistent with stylized evidence is that our endogenous reversion mechanism is driven by changes in lending standards. In a study that analyzes contract terms of commercial and industrial loans over a sixteen year period, Asea and Blomberg [4] find that bank lending standards do in fact change over the business cycle and seem have a significant effect on the dynamics of the latter. More interestingly, their conclusions seem to support the view that “during booms asymmetric information in credit markets may cause good projects to draw in bad ones”, generating “the opposite of Akerlof’s celebrated Lemon’s principle.” This is consistent with the mechanism in our model, by which lending booms are possible insofar good projects cross-subsidize their bad counterparts.

It is worthwhile at this stage to highlight some additional features of our framework and relate it to the existing literature. We first comment further on our static environment of adverse selection. Our scenario is similar to that of Besanko and Thakor [10] in that they also allow for the inclusion of investment as a dimension in the optimal contracts. The main differences between our setup and theirs lie in our technological assumptions and in their use of the reactive equilibrium as a solution concept. By using Hellwig’s characterization of competition in markets with adverse selection, we are instead able to analyze sequential Nash equilibria. This characterization, which essentially reduces to adding a third stage to games of screening, allows constrained optimal pooling allocations to survive as equilibria of the model. This is so because, when analyzing whether to deviate from the pooling equilibrium, contract-designers anticipate that they will eventually attract all agents with their deviation, making the latter unprofitable.

In terms of financial dampening, our result has an interesting conceptual underpinning. In particular, the reason for which the role of the financial system as an amplification mechanism differs in the presence of adverse selection as opposed to pure moral hazard or private information is closely related to the work of Rustichini and Siconolfi. They note that - in economies with moral hazard or private information - the incentive compatibility constraint operates on individual allocations. The presence of adverse selection, on the other hand, is essentially different, since incentive compatibility then restricts allocations across types. In such an environment, as Rustichini and Siconolfi [35] state, “the notion of incentive compatible individual allocations is then meaningless and must be replaced by the notion of an incentive compatible set of joint allocations”. As we previously mentioned, this is precisely the origin of the dampening effect in our model, in which joint allocations must be

²For evidence of procyclicality on profit margins and cash flows in UK manufacturing see Small ([36],[37]). There is ample evidence on the procyclicality of corporate cash flows, corporate earnings and asset prices in the United States. A brief discussion on evidence in this regard is provided by Longstaff and Piazzesi [28]. For a survey on the evidence regarding the impact of net worth on investment, see Hubbard [22].

incentive compatible while at the same time clearing the credit market.

Regarding the way in which endogenous cycles arise in our framework, we believe it to be natural in a fairly standard adverse selection setting. Besides the aforementioned considerations regarding net worth, it is also the case that our mechanism does not rely on particular relative price changes³ or on assumptions regarding the mix of adverse selection and moral hazard. There are many similarities between our model and that of Reichlin and Siconolfi [31], for example: the latter, however, relies on a particular mix between these two types of asymmetric information, which ultimately makes it difficult to identify the underlying assumptions that generate different effects. Moreover, the source of our reversion mechanism, regime switches between pooling and separating contracts in a context of multi-dimensional screening, is novel and interesting in its own right. In this sense, our framework complements the existing literature by highlighting a new reversion mechanism that arises in the presence of adverse selection.

Finally, cycles in our setting are deterministic and arise in a fully rational environment. Evidently, this is in contrast to views that attribute volatility to some form of irrational behavior, as in the Keynesian concept of “animal spirits” or Kindleberger’s [25] account of financial crises. It is interesting to note, however, that due to the pro-cyclicality of net worth, fluctuations in our economy could be observationally equivalent to some form of irrationality: an outside observer would see investment decrease in a context of increasing output and net worth and in the absence of any exogenous shock. In our model, though, regime switches between pooling and separating contracts may induce rational entrepreneurs to contract investment even as net worth increases.

Our paper is akin to the substantial body of work that has analyzed the macroeconomic significance of borrowers’ balance sheets in determining consumption and investment. Prominent examples of this are the papers by Bernanke and Gertler [8] and Kiyotaki and Moore [26] which - as a common feature with much of this literature - rest on the notion that, in the presence of asymmetric information regarding borrowers, loan contracts will entail the use of collateral and credit rationing. Thus, to the extent that net worth is pro-cyclical, shocks to the economy have a direct impact on borrowers’ balance sheets thereby affecting the degree to which borrowing is constrained and, consequently, the level of investment. This literature emphasizes different macroeconomic implications of credit market imperfections but, importantly, the general conclusion that emerges is that the business cycle is amplified by the presence of informational frictions in the latter.⁴

Recent work has tried to complement this literature by analyzing mechanisms through which

³In Aghion et. al. ([1],[2]) and Suarez and Susmann ([39]), relative price changes decrease entrepreneurial net worth.

⁴Bernanke, Gertler and Gilchrist [9] provide a short review of this literature. Different but related approaches can be found in the work of Angeletos and Calvet [3], who focus on the impact of incomplete markets on entrepreneurs’ willingness to invest as opposed to its effect on their ability to borrow, and of Buera [15] and Cagetti and De Nardi [17], who focus on the interaction between borrowing constraints and occupational choices.

asymmetric information in credit markets may not just lead to the amplification of exogenous shocks, but may itself be a source of endogenous fluctuations. The papers by Suarez and Sussman [39], Azariadis and Smith [6], Reichlin and Siconolfi [30] and, although somewhat different in its objective, Aghion, Bacchetta and Banerjee [1] fall within this category.⁵ Although they all succeed in their purpose and are interesting at different levels, their fluctuations rely either on generating counter-cyclical dynamics for entrepreneurial net worth or on assuming that the latter plays no role in determining the equilibrium level of investment.

In [39], net worth is assumed to be counter-cyclical: when output is high, prices fall and - along with them - so does the net worth of entrepreneurs. It is this feature that underlies the endogenous reversion mechanism. As Reichlin [33] notes, however, net worth is generally thought to be pro-cyclical and empirical evidence on asset prices, profit margins and cash flows seems to support this notion. The mechanism in [1] is somewhat similar in spirit, although the price change that brings about fluctuations is not that of the final good but that of the input used to produce it: when output expands, the price of the input increases and - in some scenarios - may contract profits and hence future production.⁶ Finally, in [6] and [30] entrepreneurs either have no endowment or it plays no role in determining the allocation of credit. Once again, this is at odds with empirical findings by which investment seems to be significantly affected by firms' net worth.

The paper is structured as follows. Section 2 discusses the baseline model of the credit market. It contains a complete characterization of separating and pooling equilibria and of some properties of regime switches. Section 3 then embeds the model in an OLG framework, analyzes the dynamics of the economy under asymmetric information, and proves that shocks may be dampened or amplified by the financial system. Section 4 introduces a simplified version of the dynamic model and shows how the presence of asymmetric information may generate fluctuations in a setting that is otherwise characterized by a complete lack of dynamics. Finally, Section 5 presents a discussion on the main assumptions of the model and Section 6 concludes.

2 The Baseline Model

2.1 Setup

Assume a two period economy with entrepreneurs, households and financial intermediaries which we refer to as banks. There is a continuum of households, which are evenly distributed over the interval $[0, M]$. At time 0, households are endowed with an invariant amount A of the economy's

⁵Of course, the work dealing with endogenous cycles extends beyond these papers, which focus on the role of asymmetric information. For a more general review of the theoretical work on endogenous cycles, see Reichlin [32].

⁶The model of Aghion, Banerjee and Piketty [2] presents a mechanism similar to that in [1]. The former, however, focuses on the general equilibrium effects of interest changes. During booms, investment expands until the consequent rise in interest rates increases the debt burden of entrepreneurs and in so doing decreases investment.

only consumption good and they have monotonically increasing preferences over second-period consumption. Given this assumption, their preferences need not be specified.

2.1.1 Entrepreneurs

There also exists a continuum of entrepreneurs distributed in $[0, 1]$. Entrepreneurs are endowed with a decreasing returns to scale technology for transforming period 0 consumption goods into period 1 consumption goods. Like consumers, entrepreneurs value only period 1 consumption and are risk neutral, so that they maximize the expected value of profits.

We assume that there are only two types of entrepreneurs, which we refer to as bad (B) and good (G). Entrepreneurs of each type are distributed over intervals of length λ^j , $j = B, G$ where $\lambda^G + \lambda^B = 1$. The production technology of j -type ($j = B, G$) entrepreneurs who invest an amount I of the good at time zero yields a gross return of $\alpha^j f(I)$ with probability p^j and zero otherwise. It is assumed that,

- (A.1.) $f'(I) > 0$, $f''(I) < 0$ for all $I > 0$, and $f'(0) = \infty$ while $\lim_{I \rightarrow \infty} f'(I) = 0$. Regarding the difference in technology between different types of entrepreneurs, we assume that $\alpha^B > \alpha^G$ and $p^G \alpha^G > p^B \alpha^B$.

The technological assumptions in (A.1.) are similar to those commonly used in the credit rationing literature, namely second-order stochastic dominance (SOSD). The only difference is that, in our setup, the B technology is not just a mean-preserving spread of its G counterpart but actually has a lower expected return. This latter assumption allows us to unambiguously rank technologies and will play a crucial role when we analyze dynamics.

Entrepreneurs are endowed with an amount W of the consumption good at time 0, which they cannot use to finance their production.⁷ Investment in projects is hence fully funded by bank borrowing, making investment and loan sizes equivalent in our framework and terminology. More specifically, the relationship between banks and entrepreneurs is assumed to be the following:

- (A.2.) A contract between banks and entrepreneurs is an array (I, R, c) , where I is the amount borrowed, R is the gross interest factor on the loan and c is the percentage of the loan that entrepreneurs must collateralize by using their own wealth.⁸ If the investment is successful, entrepreneurs pay back the amount borrowed adjusted by the interest factor: otherwise, they

⁷This assumption is introduced to simplify the exposition, but it is not restrictive. In fact, the contracts we study - with collateral - could alternatively be rewritten as contracts in which entrepreneurs invest in their projects and banks provide additional funds. We adopt the former interpretation in the belief that our contracts are easier to analyze, deliver a more tractable framework and encompass the cases in which entrepreneurial wealth can or cannot be directly invested.

⁸Contracts could be easily modified to include a probability of rationing ε . In the Appendix we show, however, that our Inada assumption on technology would immediately imply that ε is always 0 at equilibrium.

default and the bank keeps the goods put up as collateral and the interest borne by them. Finally, and since they cannot invest it directly in the project, entrepreneurs deposit their endowment in the bank for a gross interest factor of r .

Thus, (A.2.) tells us that the expected profit for any entrepreneur j ($j = B, G$) of obtaining a loan according to contract (I, R, c) is given by⁹

$$\pi^j(I, R, c) = p^j[\alpha^j f(I) - RI] - (1 - p^j)rcI \quad (1)$$

Different types of entrepreneurs differ in their willingness to accept a higher interest factor in exchange for a lower collateral requirement. It can be easily verified that - for any level of I and r - the marginal rate of substitution between the interest factor R and the collateral requirement c is $\frac{r(1-p^j)}{p^j}$ for an entrepreneur of type j .

In terms of informational asymmetry, we will follow [10] and [31] by assuming that borrowers' types cannot be observed either directly or through realized project returns. Hence, all agents other than the owner of the project can only verify whether the latter was successful or not. In such a scenario it is known that the optimal contractual form is that of debt as assumed in (A.2.).¹⁰

2.1.2 Banks

There exists a finite number of banks that collect deposits from individuals and entrepreneurs and offer loan contracts to entrepreneurs. Banks are assumed to be risk neutral and competitive: on the deposit side, they take r as given (the interest factor on deposits) and they Nash compete on the loan market by designing contracts. As was mentioned earlier, each contract is a triple (I, R, c) that specifies the amount of the loan, the interest factor and the percentage of collateral that must be put up by the entrepreneur.

We assume that each bank gets the same share of total deposits and, if they design the same contract, they get the same share and composition of loan applications. Given (A.2.), a bank's expected profit of accepting an application for a contract (I, R, c) from a type j entrepreneur are given by

$$p^j(RI) + (1 - p^j)r(cI) - rI \quad (2)$$

⁹Of course, entrepreneurs also obtain a return of Ar on their bank deposits. This, however, is independent of the type of loan that they obtain.

¹⁰In a more general environment, Boyd and Smith ([13]) show that debt can arise as the optimal contractual form under adverse selection and costly state verification provided that verification costs are sufficiently high.

2.2 Main Properties of Loan Contracts

In the present section, we discuss the main properties of loan contracts taking the interest rate as given, i.e. assuming an infinitely elastic supply of funds. In the basic model presented in the previous subsection, households can save only by depositing their wealth in the bank. Since entrepreneurs also deposit all their wealth in banks, the role of the latter is just to allocate the total wealth of the economy among the pool of applicants.

Under full information, it can be easily shown that the model has a unique equilibrium in which banks offer contracts $(I^{G*}, \alpha^G f'(I^{G*}), 0)$ and $(I^{B*}, \alpha^B f'(I^{B*}), 0)$ to G - and B -type entrepreneurs, respectively. In this case, I^{G*} and I^{B*} are determined so that $r = p^G \alpha^G f'(I^{G*}) = p^B \alpha^B f'(I^{B*})$, and banks receive zero profits in expectations. There is no need for the use of rationing or collateral, and the aforementioned contracts imply $I^{G*} > I^{B*}$ and a higher interest factor ($\alpha^j f'(I^j)$) for bad entrepreneurs than for their good counterparts.¹¹

Now consider the case of asymmetric information, in which banks are not able to distinguish among different types of borrowers. There is a direct analogy between such a scenario and the Rothschild-Stiglitz insurance model. In the latter, an equilibrium does not always exist: in particular, it fails to do so when the pooling allocation is Pareto superior to the separating allocation. To avoid this problem, we follow Hellwig [23] and model competition in the credit market as having three stages. In the first stage, banks design contracts: in the second stage, entrepreneurs apply for these contracts and, in the third stage, banks decide whether to accept or reject these applications.

Hellwig applies the concept of sequential equilibrium to this game and shows that an equilibrium always exists: in particular, the specification allows for the existence of pooling contracts as equilibria when the values of the parameters prevent the existence of separating equilibrium contracts in Rothschild-Stiglitz games.¹² More interestingly, an application of the Kohlberg-Mertens stability criterion selects only the allocation most preferred by G -type entrepreneurs as equilibria of the model. In other words, the most robust outcome of the aforementioned game form will be the separating contract insofar as the latter provides G -type entrepreneurs with higher profits than any pooling contracts. On the contrary, if there are pooling contracts that are Pareto superior to the separating contract, the one mostly preferred by G -types will emerge as the most robust equilibrium of the model.¹³ It is important to keep this in mind because it implies that bank competition will ultimately yield contracts which are constrained efficient (among the set of contracts yielding zero profits and no cross subsidization). This feature by which contracts are constrained

¹¹In the remainder of the paper, we use the superscript $*$ to denote the level of variables at equilibrium in the presence of symmetric information.

¹²Alternatively, the existence problem could be avoided by explicitly modelling competitive contract markets in which banks and entrepreneurs interact: such a setting always has an equilibrium (see Gale [21] and Geanakoplos and Dubey [19]) which in particular may entail pooling (see Gale [21] and Martin [29]).

¹³Note the similarity in spirit between Hellwig's specification and Wilson's [41] concept of anticipatory equilibrium.

optimal, which is standard in the macroeconomic literature, will be central to our characterization of the equilibria.¹⁴

In what follows, we will analyze the equilibrium contracts for an economy indexed by an interest rate-entrepreneurial wealth pair (r, W) . We first characterize the separating equilibrium: as we will see, the interesting feature of these contracts is that the size of loans and the rate of collateralization are both used as screening devices. When there is no wealth to be used as collateral, the whole weight of screening is borne by the size of loans, and the investment undertaken by G entrepreneurs is constrained with respect to the full information benchmark. This constraint is relaxed as the relative wealth of entrepreneurs increases, making it possible to screen more through collateral and less through loan size. We then characterize pooling contracts and show that collateral also plays an important role in determining them. Finally, we solve for the full-equilibrium of the economy, in which the interest rate r is determined endogenously, and discuss its properties.

2.2.1 Separating Equilibria

Under the assumptions of exclusivity and no cross-subsidization, a separating equilibrium is defined as follows.

Definition 1 *For a given interest-rate entrepreneurial wealth pair (r, W) , a separating equilibrium is a set of contracts $C^{SEP}(r, W) = \{(I^B, R^B, c^B), (I^G, R^G, c^G)\}$ satisfying the following conditions:*

1. *Feasibility: contracts must respect the collateralization constraint.*

$$c^j \in [0, \frac{W}{I^j}] \text{ for } j = B, G \quad (3)$$

2. *Incentive Compatibility: each entrepreneur applies to the contract designed for his type.*

$$\begin{aligned} p^i[\alpha^i f(I^j) - R^j I^j] - (1 - p^i)rc^j I^j &\leq p^i[\alpha^i f(I^i) - R^i I^i] - r(1 - p^i)c^i I^i r \\ &\text{for } i \neq j, i, j = G, B \end{aligned} \quad (4)$$

3. *Zero profit condition for banks: each contract must yield banks zero profits in expectation.*

$$r = p^j R^j + (1 - p^j)rc^j \text{ for } j = G, B \quad (5)$$

4. *No bank can profit by offering alternative contracts.*

¹⁴For a discussion of Hellwig's characterization, see Riley [34].

Conditions (1)-(3) merit no further comment: note simply that (3) stems from bank competition together with the no cross-subsidization condition. As for condition (4), and for the reasons outlined above, it implies that a separating equilibrium will not exist when such an allocation is Pareto dominated by a pooling contract, since in such a case banks will have an incentive to deviate and offer the latter. The resulting contracts are characterized in the following proposition.

Proposition 1 *Given (r, W) , the separating equilibrium is a set of contracts $C^{SEP}(r, W) = \{(I^B, R^B, c^B), (I^G, R^G, c^G)\}$ satisfying,*

$$p^B \alpha^B f'(I^B) = r \quad (6)$$

$$R^B = \frac{r}{p^B} \quad (7)$$

$$c^B = 0 \quad (8)$$

$$p^G \alpha^G f'(I^G) = r + \psi [\alpha^B p^B f'(I^G) - \frac{p^B}{p^G} r] \quad (9)$$

$$\text{where } \psi = \frac{\eta}{[1 - \frac{p^B}{p^G}] I^G r} \quad (10)$$

$$\text{and } \eta > 0 \text{ when } c^G = \frac{W}{I^G}$$

$$c^G = \frac{[\alpha^B p^B f(I^G) - \frac{p^B}{p^G} I^G r] - p^B \alpha^B f(I^B) - r I^B}{[(1 - p^B) - p^B \frac{(1-p^G)}{p^G}] r I^G} \leq 1 \quad (11)$$

Proof. See Appendix. ■

Conditions (6)-(8) imply that - at equilibrium - contracts taken by B -type entrepreneurs entail no distortions. Thus, they are lent the efficient amount at the given interest rate and they have no need to provide collateral. As is usually the case in adverse selection models, only one of the incentive compatibility constraints binds at equilibrium: in particular, it is the B type constraint.

It is therefore on the contracts taken by G -type entrepreneurs that the interest of the equilibrium lies. Condition (9) implies that - in the presence of a binding incentive compatibility constraint induced by a restricted amount of collateral - G entrepreneurs will be rationed with respect to the full information allocation, in the sense that they will receive smaller loans than they would desire at the prevailing interest rate.¹⁵ Essentially, the design of the separating contracts reduces to a problem of multidimensional screening, in which the loan size and the rate of collateralization are used in order to induce separation between different technologies.¹⁶

¹⁵Once again, it must be recalled that the optimal contracts will entail no rationing in the Stiglitz-Weiss sense, since Inada conditions make the use of this instrument excessively costly.

¹⁶Note that the remaining dimension of the contract, R , is immediately pinned down by the zero-profit condition once c is determined.

In order to provide a simple graphical interpretation of Proposition 1, we define a “no mimicry constraint” (NMC) as the set of G -type contracts that satisfy the incentive compatibility constraint (4) and banks’ zero profit condition (5). The following figure depicts the NMC in the (I^G, c^G) space.

No Mimicry Constraint (for given r)

For low levels of c^G , it must be the case that $I^G \neq I^B$: this is so because when collateral is relatively scarce, screening must be done through investment. In such a scenario, the only way to discourage B -type entrepreneurs from applying to G contracts is by restricting or expanding the amount of investment that they must undertake relative to their efficient amount of investment.

For higher rates of collateralization, though, there is less of a need to screen through investment and the incentive compatible levels of I^G therefore draw closer to I^B . When both loan sizes are equal, I^G is no longer used for screening and the full weight of the separation must fall on the rate of collateralization. Hence, the latter is maximized at this point, at which it reaches one.

Of all the contracts on the NMC, only those satisfying the collateralization constraint can be implemented at equilibrium. Graphically, this means that the equilibrium G contract must lie at the intersection of the NMC and the collateralization constraint. As the following figure shows, there will be a pair of such contracts for any given level of wealth W . Between these contracts, competition among banks will select the one that maximizes G -type profits: this is the contract entailing a lower loan size and a higher rate of collateralization.¹⁷ Therefore, point S in the figure below represents the equilibrium G -type contract when entrepreneurs are endowed with wealth

¹⁷In the Appendix we prove why this is the case. Intuitively, both contracts entail the same total level of collateral and therefore B -types can only be indifferent between the two if the difference in output between them equals the difference in repayments in case of success. However, if this is the case, G -type entrepreneurs prefer the one with the lower loan size since their MRS between loan size and payments in case of success is lower than that of B -types.

W .

Equilibrium G contract under binding wealth constraint

This result, by which G – type entrepreneurs are screened in equilibrium by restricting their loan size, closely resembles the traditional Rothschild-Stiglitz result. Nonetheless, we believe that it is particularly surprising in the context of our production economy. In the complete absence of collateral, it implies that *the safer, more productive technology will be constrained not only with respect to the efficient allocation, but even with respect to its riskier, less productive counterpart*.¹⁸

Let (I^*, R^*, c^*) denote the incentive compatible contract that would deliver the full information loan size to G – type entrepreneurs. Trivially, if entrepreneurial wealth W exceeds I^*c^* , the separating contracts will be able to recover the first-best allocation. Thus, efficiency is achieved in our economy in the presence of sufficient collateral. The following remark describes, for a given level of the interest rate r , the separating contracts as a function of W .¹⁹

Remark 1 *For a given interest rate r , the separating loan contracts are characterized by the following features:*

1. *In the absence of collateralizable wealth, $c^G = 0$ and $I^G < I^B = I^{B*}$.*
2. *As collateralizable wealth increases, I^G increases and I^B is constant.*
3. *The rate of collateralization c^G is increasing in the level of collateralizable wealth until $I^G = I^B$: at this point, c^G reaches one and it decreases when W increases further.*

¹⁸The separating equilibrium with no collateral is fully derived in the Appendix.

¹⁹The Appendix characterizes comparative statics of the separating contracts with respect to changes in r .

2.2.2 Pooling Equilibria

As was mentioned earlier, Hellwig's three-stage characterization of competition in the credit market allows for the existence of a pooling equilibrium when the latter Pareto dominates the separating allocation. A pooling equilibrium of our economy is defined as follows,

Definition 2 *For a given interest rate-entrepreneurial wealth pair (r, W) , a pooling equilibrium is a contract $C^{POOL}(r, W) = \{(\bar{I}, \bar{R}, \bar{c})\}$ satisfying the following conditions:*

1. *Feasibility: the pooling contract must respect the collateralization constraint.*
2. *Zero profit condition for banks: when offered to a pool of applicants representative of the population, the contract must yield banks zero profits in expectation.*
3. *No bank can profit by offering alternative contracts.*

In traditional models of signaling, the existence of a pooling equilibrium entails cross-subsidies from good types to bad types: this is also true in our framework, although the extent of such transfers ultimately depends on the wealth of entrepreneurs and on the way in which payments are divided between the interest factor and the rate of collateralization. As the following proposition shows, any pooling equilibrium will entail a binding collateralization constraint, although the amount of investment will be independent of entrepreneurial wealth.

Proposition 2 *Given (r, W) , a pooling equilibrium is given by a contract $C^{POOL} = \{(\bar{I}, \bar{R}, \bar{c})\}$ satisfying,*

$$p^G \alpha^G f'(\bar{I}) = \frac{p^G}{\bar{p}} r \quad (12)$$

$$\bar{R} = r \left[\frac{1 - (1 - \bar{p})\bar{c}}{\bar{p}} \right] \quad (13)$$

$$\bar{c} = \frac{W}{\bar{I}} \quad (14)$$

where $\bar{p} = \lambda^G p^G + \lambda^B p^B$ denotes the average probability of success of all projects in the economy.

Proof. See Appendix. ■

Condition (14) says that a pooling equilibrium must entail a binding collateralization constraint and, consequently, that the degree of cross-subsidization it exhibits depends on entrepreneurial wealth. Higher rates of collateralization decrease the average cost of funds for G -type entrepreneurs and hence increase their profits. Since the equilibrium pooling contract will be the one that maximizes the profits of G entrepreneurs while yielding zero profits to banks, condition (14) must hold at equilibrium.

The same reasoning applies for the level of investment in a pooling equilibrium as defined implicitly in (12). When $W = 0$, the condition must clearly be satisfied since it simply equates marginal productivity of investment in the G sector to the marginal cost of funds. As entrepreneurial wealth increases, however, so does the rate of collateralization of the pooling contract: in doing so, the average cost of funds decreases and profits increase for G -types. The size of the loan, however, remains fixed, since the cost of the marginal dollar borrowed by G entrepreneurs is always $\frac{p^G}{\bar{p}}r$ and therefore (12) must hold.

A final consideration will prove useful in order to better understand the pooling equilibrium: both the pooling loan size ($\bar{I}$) and the B -type loan size in the separating equilibrium (I^B) are independent of entrepreneurial wealth. In fact, their relative magnitude depends only on the value of $\bar{p}$, and simple arithmetic yields the following remark.

Remark 2 *For any interest rate r , it is the case that*

$$\bar{I}(r, \bar{p}) \lesseqgtr I^B(r) \Leftrightarrow \bar{p} \lesseqgtr \frac{\alpha^B p^B}{\alpha^G} \quad (15)$$

where $\bar{I}$, I^B and $\bar{p}$ are defined as above.

Thus, if the ratio of G - to B -types in the economy is high enough to make $\bar{p}$ surpass the threshold in (15), the pooling allocation will entail a higher loan size than B -type contracts in the separating equilibrium. This feature will be invoked later on in order to analyze the conditions under which the economy displays a pooling or a separating equilibrium.

2.2.3 Characterization of Switching Regimes

We now analyze the conditions under which the equilibrium of an economy indexed by (r, W) will be pooling or separating. As discussed before, the equilibrium will be pooling or separating depending on the profits that the optimal pooling contract yields G -type entrepreneurs vis-à-vis the optimal separating contract.²⁰ We define switching points as the levels of entrepreneurial wealth $W^*(r)$ that determine - for a given interest rate - a change in regime from pooling to separating or vice-versa. In the remainder of the paper, we use $C^{EQ}(r, W)$ to denote the equilibrium contracts for given levels of wealth and the interest rate.

Since the existence of a pooling equilibrium depends on the profits it yields G -type entrepreneurs, it is most likely to happen when separating contracts are least efficient with respect to the full information allocation. This occurs when entrepreneurial wealth and - consequently - collateral are low.

²⁰We show in the Appendix (Section 7.1.5) that, whenever G -types prefer the optimal pooling to the optimal separating contract, so do B -types.

We proceed as follows: first, we restrict parameter values for which all economies indexed by $(r, 0)$ display a pooling equilibrium, i.e., for which the latter arises in the absence of collateralizable wealth. We then prove that the mapping $W^*(r)$ is a function under such restrictions, so that the switching point is uniquely determined by the interest rate.

Lemma 3 *If $\bar{p} \geq \frac{\alpha^B p^B}{\alpha^G}$ then $C^{EQ}(r, 0) = C^{POOL}(r, 0)$ for all values of r .*

Proof. See Appendix. ■

Lemma 3 determines a threshold value of $\bar{p}$ above which the equilibrium is always pooling when $W = 0$. From (15), this benchmark allows us to relate loan sizes in the pooling and separating equilibria: in particular, when the pooling loan size is weakly larger than the B -type loan size in the separating contracts, an economy with no collateral will pool all loans regardless of the interest rate.

Having established this benchmark, we now prove that the aforementioned restriction on $\bar{p}$ also guarantees the existence of a unique switching point for each level of r .

Lemma 4 *If $\bar{p} \geq \frac{\alpha^B p^B}{\alpha^G}$, the mapping $W^*(r)$ is a function, i.e., there is a unique switching point for each value of the interest rate.*

Proof. See Appendix. ■

Therefore, whenever $\bar{p} \geq \frac{\alpha^B p^B}{\alpha^G}$, there exists a unique switching level of wealth $W^*(r)$ for each value of r : for $W < W^*(r)$, $C^{EQ}(r, W) = C^{POOL}(r, W)$, whereas $C^{EQ}(r, W) = C^{SEP}(r, W)$ otherwise.

2.2.4 Behavior of Investment at the Switching Points

We now analyze the behavior of aggregate investment at the switching points. When $\bar{p} = \frac{\alpha^B p^B}{\alpha^G}$, we have shown that $\bar{I}(r) = I^B(r)$. Additionally, we prove in the Appendix that the following also holds

$$W^*(r) = I^B(r)$$

so that the switching point will be given by the level of entrepreneurial wealth that is equal to B -type loans. That is

$$I^G(r, W^*(r)) = W^*(r) = I^B(r)$$

Hence, when $\bar{p} = \frac{\alpha^B p^B}{\alpha^G}$, the transition from pooling to separating is smooth. For $W < W^*(r)$, the economy displays a pooling equilibrium. At $W^*(r)$, there is a switch in regime and a separating

equilibrium emerges: aggregate investment, however, remains constant under the new regime, since the change to separating contracts does not affect loan sizes.

When $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$, however, there is a discontinuity in investment once the regime switches from pooling to separating. In this case - as we have shown - $\bar{I}(r) > I^B(r)$, so that the switch from pooling to separating must entail a contraction in the amount invested by B -type entrepreneurs. The same will be true of their G -type counterparts, for whom $I^G(r, W^*(r)) < \bar{I}(r)$: this must necessarily be the case, since at the switching point these entrepreneurs are by definition indifferent between both kinds of contracts. The pooling contract, however, entails some degree of cross-subsidization, whereas the separating one does not. Thus, in order for G -type entrepreneurs to be indifferent between them, the latter one must entail a lower loan size. Therefore, when the economy switches from a pooling to a separating equilibrium under the assumption that $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$, there is a contraction in the amount invested by all entrepreneurs. Lemma 5 summarizes this discussion.

Lemma 5 1. If $\bar{p} = \frac{\alpha^B p^B}{\alpha^G}$, then

$$I^G(r, W^*(r)) = W^*(r) = I^B(r) = \bar{I}(r)$$

2. If $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$, then

$$\bar{I}(r) > I^G(r, W^*(r)) > W^*(r) > I^B(r)$$

Proof. See Appendix. ■

2.3 Equilibria of the Economy for Endogenous r

In this section, we consider the case in which the aggregate amount of deposits in the economy is exogenously determined²¹ and so the interest rate must adjust in order to achieve an equilibrium in the credit market.²² Formally, an equilibrium of the economy is defined as follows,

Definition 3 For given levels of W and A , an equilibrium of the economy is an interest rate r and a set of contracts $C^{EQ}(r, W)$ such that,

1. Given r and W , the contracts in $C^{EQ}(r, W)$ are equilibrium contracts, i.e., they are feasible, incentive compatible, satisfy the zero-profit condition and there are no profitable deviations for banks.

²¹ Although the assumption of inelastic supply of funds is admittedly restrictive, it allows us to concentrate exclusively on the equilibrium contracts. Our results would not be affected by assuming that total savings are increasing in r .

²² Note that from the banks' perspective, the problem of designing optimal contracts is exactly the same under any environment, since even in the present scenario each bank will take the interest rate as given and will perceive the supply of deposits as being completely elastic at that rate. Thus, the only thing we need to analyze is the implications of the aforementioned contracts for the interest rate and the sizes of loans.

2. Given r , total investment as implied by $C^{EQ}(r, W)$ equals savings ($MA + W$).

Although our qualitative results do not depend on it, we simplify the exposition by avoiding the possibility of having multiple equilibria. Since the latter may arise as a consequence of the non-monotonicity of I^G with respect to the interest rate (see Section 7.1.4 in the Appendix), we rule it out by assuming that - within the separating regime - total investment is inversely related to the interest rate for all values of W . The following Lemma provides a sufficient condition in order for this to be the case.

Lemma 6 *Let $\vartheta(r, W)$ be the total amount invested by entrepreneurs as a function of the interest rate and collateralizable wealth. If for all pairs $\{I^G(r, W), I^B(r)\}$ of loan sizes corresponding to the separating contracts the following condition is satisfied,*

$$\left| \frac{1}{\alpha^B p^B f''(I^B(r, W))} \right| \lambda^B > \frac{[I^G(r, W) \frac{p^B}{p^G} + (1 - \frac{p^B}{p^G})W - I^B(r, W)]}{\alpha^B p^B f'(I^G(r, W)) - \frac{p^B}{p^G} r} \lambda^G \quad (16)$$

then $\vartheta'_r(r, W) < 0$ for all values of W .

Proof. See Appendix. ■

We invoke (16) throughout the remainder of the paper. Proposition 7 proves existence of equilibrium under (16) and $\bar{p} \geq \frac{\alpha^B p^B}{\alpha^G}$. When the latter inequality is strict, Lemma 5 shows that regime switches generate discontinuities in the level of investment, thereby generating problems for the existence of equilibrium. We address this issue by introducing a convexifying device. At the switching point, note that optimal pooling and separating contracts yield G entrepreneurs the same amount of profits. Convexity may then be restored by the introduction of random contracts, which are defined as a probability $\omega \in [0, 1]$ together with a pair of pooling and separating contracts. To be more precise, we follow [31] and introduce random contracts as follows:

1. Firms apply to a lottery that randomizes between a pooling contract and pair of separating contracts.
2. The lottery realization determines whether the applicant is entitled to a pooling or a separating contract.
3. If the lottery realization determines that the applicant is entitled to a separating contract, the applicant decides whether to receive the G or the B contract.

In the Appendix, we explain why this restores existence. We also explain the reason for which random contracts emerge as a Nash equilibrium of the three-stage game when the strategy space of banks is enriched to include randomization as specified above.

Proposition 7 *In the presence of positive collateral and binding wealth constraints, an economy indexed by W and satisfying $\bar{p} \geq \frac{\alpha^B p^B}{\alpha^G}$ always has an equilibrium which:*

1. *May fail to be unique if (16) is not satisfied.*
2. *May entail randomization between pooling and separating contracts when $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$.*

Proof. See Appendix. ■

3 Asymmetric Information Dynamics: Financial “Dampening”

Most of the macroeconomic literature regarding informational frictions in the credit market tends to conclude that the presence of the latter amplifies the effects of exogenous shocks. These results stem mainly from the existence of private information in the form of costly state verification (as in Bernanke and Gertler [8], who draw on Townsend [40]) or moral hazard issues in the form of “hold-up” problems (as is implicit in Kiyotaki and Moore [26]), which constrain the amount of borrowing that firms may undertake. Either by directly increasing firms’ endowment or by raising the market value of their assets, positive shocks help align incentives, expand firms’ borrowing capabilities and investment, and are thus amplified through the credit market.

In the present section we show how, in our adverse selection setting, the effects of exogenous shocks may actually be dampened by the financial sector even though borrowers’ net worth is pro-cyclical. In order to do so, we focus on the dynamics of the separating contracts: this decision is motivated by the fact that entrepreneurial wealth is inconsequential for determining investment under the pooling regime, so that the dynamics of the economy under the latter are qualitatively similar to those that arise under full information.²³ It is in the separating regime, on the other hand, where the changes in entrepreneurial wealth affect the composition of investment. We leave the possibility of switches between pooling and separating regimes for the next section, in which we analyze how endogenous fluctuations may arise as a consequence of adverse selection.

3.1 Dynamic Setup

Based on our previous static framework, we now draw on Diamond [18] and develop a stylized OLG model to analyze contract dynamics and to characterize the way in which shocks are amplified or dampened through the financial sector. More specifically, we embed the contracting problem analyzed in the previous section into the simplest possible OLG framework, with zero population growth, inelastic supply of savings and full depreciation of the capital stock.

²³In terms of the model, our decision to focus on the dynamics of the separating contracts can be interpreted as applying to an economy with a relatively low proportion of G – *type* entrepreneurs.

At any point in time, a new generation of measure one is born: of this generation, a measure λ^G is assumed to be composed of G – *type* entrepreneurs, a measure λ^B is composed of B – *type* entrepreneurs and the remaining measure is composed of households. Thus, $\lambda^G + \lambda^B < 1$. Both households and entrepreneurs maximize consumption in their old age, where the latter are risk neutral and the preferences of the former are inessential in exactly the same way as was assumed for the static analysis. All members of each generation supply inelastically one unit of labor when they are young, which is combined with capital from the old to produce a consumption good according to a constant returns to scale technology denoted by

$$y_t = \theta g(1, k_{t-1}) \quad (17)$$

where g is a continuously differentiable and concave constant returns to scale function, k is per-capita stock of capital (and aggregate stock, due to the normalization on population size) and its subscript denotes the date of birth of the generation that owns it. Both old and young are paid a competitive price for supplying their factors of production, so that the young receive,

$$w_t(k_{t-1}) = \theta[g(k_{t-1}) - k_{t-1}g'(k_{t-1})] \quad (18)$$

for their labor while the old receive

$$q_t(k_{t-1}) = \theta g'(k_{t-1}) \quad (19)$$

per unit of capital, where q_t denotes the marginal productivity of capital in the production of the final good at time t .

Since everyone is interested in consuming only during old age, the income of the young is saved in its entirety, while that of the old is fully consumed. Thus, at any point in time, total savings of this economy will be equal to w_t . This amount is deposited in banks, which serve as intermediaries and channel it to entrepreneurs by means of the separating contracts derived in the previous section. Entrepreneurs then borrow in order to produce capital according to their respective technologies, and their payments to banks are ultimately distributed among depositors.²⁴

With this basic setup in mind, we will now formalize the discussion by defining an intertemporal equilibrium. For expositional convenience, we express loans and investment in terms of the final good whereas repayment and collateral are expressed in terms of capital. Abusing notation, we use

$$\rho_t^e = \frac{r_t}{q_{t+1}^e}$$

to denote the ratio between the interest rate at time t and the expected price of capital at time $t + 1$. The latter expectation is formed according to perfect foresight.

²⁴Note the similarity with [8] and [14] in that the source of wealth for entrepreneurs is the wage they receive while young.

Definition 4 For a given initial value w_0 , an intertemporal equilibrium of the asymmetric information economy is defined as a trajectory

$$\{k_t, w_t, q_{t+1}^e, r_t, C^{EQ}(w_t, \rho_t^e) : t \geq 0\}$$

such that, for all t

1. $C^{EQ}(w_t, \rho_t^e) = C^{SEP}(w_t, \rho_t^e)$ as characterized in Proposition 1
2. There is perfect foresight in forming expectations regarding the price of capital
3. Labor and capital market clearing conditions (18) and (19) are satisfied
4. $\{I_t^B, I_t^G\} \in C^{SEP}(w_t, \rho_t^e)$ satisfy

$$\lambda^G I_t^G(w_t, \rho_t^e) + \lambda^B I_t^B(\rho_t^e) = w_t \quad (20)$$

In the case of full information, the definition of equilibrium is analogous to the previous one, the only difference being that the equilibrium contracts equalize marginal productivities across technologies.

Since ρ_t^e in this economy essentially depends on the total amount of savings and on the way in which it is allocated across technologies, we can rewrite the equilibrium conditions of this economy at time t as the following system of equations

$$k_t = K(\beta_t, w_t) \quad (21)$$

$$w_t = \varpi(k_{t-1}, \theta) \quad (22)$$

$$F(\beta_t, w_t) = 0 \quad (23)$$

where β_t denotes the share of total savings allocated to the G technology. Hence, (21), (22) and (23) represent, respectively, the production of capital given investment, the equilibrium wage given the capital stock and the relationship between savings and its allocation under the separating regime as implied by equilibrium contracts.

By considering the relationships implied by (22) and (23), we can specify (21) as

$$k_t = K(\beta_t(w_t(k_{t-1})), w_t(k_{t-1})) \quad (24)$$

where β_t is defined as before and K denotes the gross return of investment as expressed by,

$$\lambda^G \alpha^G p^G f\left(\frac{\beta_t(w_t(k_{t-1})) \cdot w_t(k_{t-1})}{\lambda^G}\right) + \lambda^B \alpha^B p^B f\left(\frac{(1 - \beta_t(w_t(k_{t-1}))) \cdot w_t(k_{t-1})}{\lambda^B}\right) \quad (25)$$

Equation (25) shows how the amount of capital produced depends on the total amount of savings - $w_t(k_{t-1})$ - both directly and indirectly. The latter effect arises from the impact of wages

on equilibrium contracts and, consequently, on the way in which credit is allocated among different technologies. The accumulation path of capital in the economy is then given by,

$$\frac{dk_t}{dk_{t-1}} = \left(\frac{\partial K}{\partial \beta_t} \frac{d\beta_t}{dw_t} + \frac{\partial K}{\partial w_t} \right) \frac{\partial w_t}{\partial k_{t-1}} \quad (26)$$

With this basic setup in mind, we analyze the dynamics of the model in the presence of full information. After that, we will turn our attention the dynamics of the asymmetric information economy when equilibrium contracts are separating.

3.2 Full Information Dynamics

The present subsection briefly analyzes some aspects of the (trivial) dynamics of the model under full information. In the latter setting, loan sizes will satisfy

$$\alpha^G p^G f'(I_t^G) = \alpha^B p^B f'(I_t^B)$$

in all periods. Thus, not surprisingly,

$$\frac{\partial K}{\partial \beta_t} = \alpha^G p^G f'(I_t^G) w_t(k_{t-1}) - \alpha^B p^B f'(I_t^B) w_t(k_{t-1}) = 0 \quad (27)$$

meaning that, since the marginal productivity of investment is equalized across different technologies, changes in the allocation of deposits have no marginal effects on the gross return to investment. By replacing (27) into (26), we can express the rate of capital accumulation as

$$\frac{dk_t}{dk_{t-1}} = [\beta_t \alpha^G p^G f'(I_t^G) + (1 - \beta_t) \alpha^B p^B f'(I_t^B)] w_t' = \alpha^G p^G f'(I_t^G) w_t' > 0 \quad (28)$$

so that the current capital stock affects the future stock by the increase in wages adjusted by the marginal productivity of investment. In such a scenario, a steady state is given by a level of capital k^* such that,

$$k^* = \lambda^G \alpha^G p^G f\left(\frac{\beta(w(k^*)) \cdot w(k^*)}{\lambda^G}\right) + \lambda^B \alpha^B p^B f\left(\frac{(1 - \beta(w(k^*))) \cdot w(k^*)}{\lambda^B}\right) \quad (29)$$

Stability of such a steady state in turn requires,

$$\frac{dk_t}{dk_{t-1}} = \alpha^G p^G f'(I_t^G) w_t' < 1 \quad (30)$$

Since we are interested in analyzing the dynamics of the asymmetric information scenario, we assume that the baseline full information economy has a unique, stable and nonoscillatory steady state.

3.3 Asymmetric Information Dynamics

We now analyze the case of asymmetric information. We focus on the case with binding collateralization constraints, which is evidently the one of economic interest.

The main difference that arises with the full information economy is that now, the allocation of deposits among different technologies (β) will be constrained by the presence of collateral. In our setting, in which the only source of wealth for young entrepreneurs are wages, this means that β will be a function of the latter not only as determinants of deposits but also in their role as entrepreneurial wealth. Thus, capital accumulation in the asymmetric information economy will be given by,

$$\begin{aligned} \frac{dk_t}{dk_{t-1}} = & [\beta_t \alpha^G p^G f'(I_t^G) + (1 - \beta_t) \alpha^B p^B f'(I_t^B)] w_t' \\ & + [\alpha^G p^G f'(I_t^G) - \alpha^B p^B f'(I_t^B)] w_t' \frac{d\beta_t}{dw_t} w_t \end{aligned} \quad (31)$$

where the first and second terms on the right hand side represent, respectively, the productivity of savings generated by additional capital and the change in allocative efficiency stemming from the variation in wealth. The first term, which reflects the weighted marginal productivity of investment, will by definition be lower in the presence of asymmetric information and binding wealth constraints (since in such a scenario the allocation of funds will be inefficient). The second term, on the other hand, describes the change in the proportion of credit allocated to the G – type sector: not surprisingly, this term may in principle be positive or negative depending on conditions which will be thoroughly analyzed below. An obvious consideration is that, if contracts eventually achieve the efficient allocation, the growth rate of the asymmetric information economy will be equal to that of the benchmark economy. In such a situation, the marginal productivity of investment will be equalized across sectors and growth will thus depend only on the latter.

For any given level of capital, then, the productivity of investment will be lower in the presence of asymmetric information due to the inefficiency induced by the equilibrium contracts.²⁵ Moreover, the proportion of funds allocated to the B technology may itself decrease or increase with economic growth, depending on the way in which the latter affects the incentive compatibility constraint. Let us note that the asymmetric information economy has at least one stable steady state:

Lemma 8 *If the full information economy has a unique, stable steady state with a capital stock of k^* , the asymmetric information economy under the separating regime will display at least one stable steady state with $\hat{k} \leq k^*$.*

Proof. See Appendix. ■

²⁵In what remains of this application, we refer somewhat loosely to the “efficiency” of contracts in terms of β , the proportion of funds allocated to the S technology.

3.3.1 Evolution of Investment under the Separating Contracts

Thus far we have analyzed the evolution of capital as a function of w and β , without specifying the relationship between these two variables. We now analyze this relationship in order to characterize contract dynamics, paying particular attention at the case in which $I^G < I^B$: as we saw in the static framework, this case corresponds to an economy in which $\frac{w}{I^B} < 1$.

We differentiate the equilibrium mapping (23) and obtain the following relationship between total savings and the share invested in the G technology,

$$\frac{d\beta_t}{dw_t} = \frac{[(1 - \frac{p^B}{p^G}) - (\frac{\beta_t}{\lambda^G})[\alpha^B p^B f'(\frac{\beta_t w_t}{\lambda^G}) - \frac{p^B}{p^G} \rho_t^e] - p^B \alpha^B f''(I_t^B)(\frac{1-\beta_t}{\lambda^B})[I_t^B - \frac{p^B}{p^G} I_t^G - w_t[1 - \frac{p^B}{p^G}]]}{(\frac{w_t}{\lambda^G})[\alpha^B p^B f'(\frac{\beta_t w_t}{\lambda^G}) - \frac{p^B}{p^G} \rho_t^e] - p^B \alpha^B f''(I_t^B)(\frac{w_t}{\lambda^B})[I_t^B - \frac{p^B}{p^G} I_t^G - w_t[1 - \frac{p^B}{p^G}]]} \quad (32)$$

Remark 3 *The terms in (32) have the following signs:*

1. $[\alpha^B p^B f'(\frac{\beta_t w_t}{\lambda^G}) - \frac{p^B}{p^G} \rho_t^e] > 0$ always, regardless of relative loan sizes;
2. $[(1 - \frac{p^B}{p^G}) - (\frac{\beta_t}{\lambda^G})[\alpha^B p^B f'(\frac{\beta_t w_t}{\lambda^G}) - \frac{p^B}{p^G} \rho_t^e]]$, is the slope of the NMC with the sign changed. Thus, when $I^G < I^B$ this term is negative, and it is positive when the opposite inequality holds;
3. $-p^B \alpha^B f''(I_t^B)[I_t^B - \frac{p^B}{p^G} I_t^G - w_t(1 - \frac{p^B}{p^G})]$, which is found both in the numerator and the denominator, will be negative or positive depending on the relative size of loans. When $I^B > I^G$, the term will be positive, whereas it will become negative if the inequality is reversed.

The first two expressions in the previous remark are the first-order effects on contracts, while the third one is a second-order effect induced by changes in the interest rate. If we consider an economy in which $I^B > I^G$ and analyze (32) in light of the remark, we can say the following:

- a) The denominator is positive. An increase in β_t has no first order effects on B – type profits while it raises the profits associated to G – type contracts, therefore exerting pressure on the incentive compatibility constraint. Additionally, there is a second order effect associated to the increase in interest rates that accompanies a reallocation of funds towards the G -sector. When $I^B > I^G$, this effect reinforces the previous one by increasing the relative cost of B – type contracts.
- b) Regarding the numerator, the first order effects (given by the first two terms) are negative. This stems, once again, from the fact that changes in the overall level of credit (while keeping β_t constant) have first-order implications only for the profitability of G – type contracts: in particular, this effect makes the latter more attractive whenever $I^B > I^G$. Additionally,

increases in the volume of credit have a second order effect through the changes in interest rate that they induce: this effect tends to make B -type loans relatively more (less) attractive whenever $I^B > I^G (<)$.

(a) and (b) together imply that, in the presence of sufficiently small second-order effects (i.e., if the marginal productivity of investment does not fall too rapidly), an economy with loan sizes $I^G < I^B$ will allocate an increasing proportion of its resources to the B -type technology as it expands. This implies that such an economy would never revert to the “right” loan sizes as it grows, thereby displaying a steady state with less capital than its full information counterpart and in which the B -type technology invests more than G -type entrepreneurs.

3.4 Financial Dampening

From our previous analysis, we identified two effects of an increase in wages. On one hand, higher wages increase the amount of savings in the economy and - consequently - the amount of investment and the marginal productivity of labor. This effect, which reinforces the original increase in wages, is at work both in the full information benchmark and in our adverse selection economy. On the other hand, there is a second effect which is consequential exclusively in the latter: an increase in wages affects the allocation of savings between technologies. If entrepreneurial wealth is low relative to investment, that this effect may work against the original increase in wages by expanding the proportion of funds allocated to the less productive technology.

We analyze these effects more closely by evaluating the long-run effects of a productivity shock in the full and asymmetric information economies. To do so, we work with the wage mapping (22), which we rewrite to account for (21) and (23) to deliver

$$w_t = \theta[g(k_{t-1}(\beta_t(w_{t-1}), w_{t-1})) - k_{t-1}(\beta_t(w_{t-1}), w_{t-1})g'(k_{t-1}(\beta_t(w_{t-1}), w_{t-1}))]$$

By Lemma 8, we know that the wage mapping has at least one stable steady state under asymmetric information. If we denote $\hat{w}$ to be such a steady-state in our economy, we can evaluate the long-run effect of a productivity shock in the presence of asymmetric information by considering the following elasticity,

$$\hat{\xi}_{\hat{w}, \theta} = \frac{\partial \hat{w} / \partial \theta}{\hat{w} / \theta} = \frac{1}{1 + \hat{k} \theta g''(\hat{k}) \frac{d\hat{k}}{d\hat{w}}} \quad (33)$$

When the economy faces a productivity shock, the steady state level of wages is affected directly through the increase in labor productivity but also indirectly through the increase in the stock of capital. This increase in the capital stock is itself affected by the increase in wages, which expands savings and hence capital accumulation. These effects are captured by (33) where the impact of wages on the capital stock is given by

$$\begin{aligned} \frac{d\hat{k}}{d\hat{w}} = & [\hat{\beta}\alpha^G p^G f'(\hat{I}^G) + (1 - \hat{\beta})\alpha^B p^B f'(\hat{I}^B)] + \\ & [\alpha^G p^G f'(\hat{I}^G) - \alpha^B p^B f'(\hat{I}^B)] \frac{d\hat{\beta}}{d\hat{w}} \hat{w} \end{aligned} \quad (34)$$

A first effect, positive and captured by the first line of (34), represents the increase in investment generated by the greater availability of funds. A second effect represents the reallocation of funds that takes place as a consequence of contract dynamics. This aspect, which is captured by the second line in (34), is positive or negative depending on the way in which $\hat{\beta}$ is affected by wages.

In the case of the full information economy, for which we denote steady state levels by the superscript $*$, the analogous elasticity is,

$$\xi_{w^*,\theta}^* = \frac{\partial w^*/\partial \theta}{w^*/\theta} = \frac{1}{1 + \theta k^* g''(k^*) \frac{dk^*}{dw^*}} \quad (35)$$

Proposition 9 $\hat{\xi}_{\hat{w},\theta} < \xi_{w^*,\theta}^*$ iff

$$\hat{k} \cdot |g''(\hat{k})| \frac{d\hat{k}}{d\hat{w}} < k^* \cdot |g''(k^*)| \frac{dk^*}{dw^*} \quad (36)$$

When comparing the elasticities $\xi_{w^*,\theta}^*$ and $\hat{\xi}_{\hat{w},\theta}$, there are different effects which act in opposite direction under standard assumptions. Since under asymmetric information the steady state entails a relatively low level of capital and output, the productivity of capital and - for a given allocation of credit - investment is higher, tending to amplify the effects of shocks. On the other hand, though, the relatively inefficient allocation of investment across technologies, plus the fact that this inefficiency may even increase as a consequence of the shock, tend to dampen the effects of the latter under asymmetric information. If this effect is sufficiently strong, the adverse selection economy will dampen the impact of exogenous shocks, and this will be the case even though entrepreneurs' wealth will be positively correlated with the latter.

In terms of (36), under the standard assumption by which $k \cdot |g''(k)|$ is decreasing in k , the increase in wages induced by an increase in the capital stock is greater in the adverse selection economy. In such a scenario, then, a necessary condition for (36) to be satisfied is that the increase in the capital stock directly induced by an expansion in savings is higher under full than under asymmetric information as captured in (34). In other words, the inefficiency in the allocation of investment and the evolution of this inefficiency as dictated by contract dynamics in (32) must be substantial enough to offset the concavity effects.

Ultimately, it is the endogeneity of the interest rate that potentially dampens shocks in the presence of adverse selection. It is essentially this feature, which forces contracts to clear the market

while being incentive compatible, that may lead to a decrease in the efficiency of investment as a consequence of growth. In an economy in which the interest rate is fixed at r , there is no direct impact of wages on investment under full information. In the adverse selection economy under the separating regime, though, an expansion in wages increases the amount that G entrepreneurs can invest and, consequently, leads directly to an increase in the stock of capital. In such a scenario, exogenous perturbations will necessarily be amplified by the presence of asymmetric information.

To summarize, in the full information economy a positive shock increases the amount of period t savings, raising investment while maintaining marginal productivities equalized across sectors. With adverse selection, however, there is an additional effect that must be taken into account. First of all, investment is allocated inefficiently across technologies. Moreover, as (31) illustrates, an increase in wages will affect not just the total amount of investment but also its allocation between sectors, since β will also change as credit expands. When an increase in collateralizable wealth cannot be fully absorbed by G - type entrepreneurs without violating incentive compatibility, the interest rate must fall and B - type investment must expand in order to restore market clearing. It is thereby the feature inherent to adverse selection, by which the notion of incentive compatibility restricts joint as opposed to individual allocation, what generates the possibility of financial dampening in the presence of an exogenous shock.

4 Regime Switching and Endogenous Cycles

We now analyze the properties of regime switches between pooling and separating contracts by using a slightly modified version the OLG framework described in the previous section. In particular, and as a way to simplify the analysis, we introduce the following assumptions.

1. The economy may borrow or lend any amount of the consumption good at a fixed rate given by r .²⁶

This assumption, which delivers a framework nearly identical to that used in [8] for analyzing the financial accelerator, is useful in simplifying the analysis. As we will explain shortly, it implies that both the full information economy and the asymmetric information economy under the pooling regime exhibit no dynamics. The former implication is useful because it provides a clear benchmark against which to compare our results, whereas the latter greatly simplifies the analysis of regime switches and allows for a clearer presentation of the mechanism at work.

²⁶ Alternatively, we could have directly followed [8] and assumed that; a) the economy has a storage technology with a return of r and; b) that the proportion of entrepreneurs in the total population is sufficiently small so as to guarantee that - for any level of q - aggregate supply of funds exceeds aggregate demand. This would also deliver a cost of funds fixed at r .

2. It is also assumed that $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$, so that the optimal pooling contract Pareto dominates the optimal separating one in the absence of entrepreneurial wealth. As can be appreciated from Lemma 5, this assumption also implies that a switch of regime is accompanied by a discontinuous change in investment.

The present section proceeds as follows. First we show that neither the full information benchmark or the asymmetric information economy under the pooling regime exhibit dynamics. We then characterize the dynamics of the asymmetric information economy under the separating regime. Finally, we analyze the properties of regime switches and show how the adverse selection economy may fluctuate in the absence of exogenous perturbations.

4.1 Full Information Benchmark and Pooling Regime Dynamics

Under full information, the equilibrium of the economy is trivial. At any point in time t , investment in both sectors must be such that their marginal productivities are equalized, thus satisfying,

$$\alpha^G p^G f'(I_t^{G*}) = \alpha^B p^B f'(I_t^{B*}) = \frac{r}{q_{t+1}^e}$$

where q_{t+1}^e denotes the expected rental price of capital at $t + 1$ which, by perfect foresight, must satisfy

$$q_{t+1}^e = \theta g'(k_t^*(q_{t+1}^e))$$

while $k_t^*(q_{t+1}^e)$ is defined as

$$\begin{aligned} k_t^*(q_{t+1}^e) = & \lambda^G \alpha^G p^G f(I_t^{G*}(q_{t+1}^e)) \\ & + \lambda^B \alpha^B p^B f(I_t^{B*}(q_{t+1}^e)) \end{aligned}$$

The previous conditions show that the amount of investment at time t and the resulting stock of capital are independent of state variables. Consequently, our assumption regarding the existence of a unique, stable steady state in the full information economy implies that the latter converges in one period to the said steady state regardless of initial conditions. We denote the steady state values of capital and factor prices in the full information economy by $\{k^*, w^*, q^*\}$.

This consideration also applies to the asymmetric information economy under the pooling regime. In fact, time t wages do not play any role in determining the size of the pooling loan at time t , which from (12) is defined implicitly by

$$\alpha^G p^G f'(\bar{I}_t) = \frac{r}{q_{t+1}^e} \frac{p^G}{\bar{p}}$$

where q_{t+1}^e must now satisfy

$$q_{t+1}^e = \theta g'(k_{t,POOL}(q_{t+1}^e))$$

and $k_{t,POOL}(q_{t+1}^e)$ is defined as

$$k_{t,POOL}(q_{t+1}^e) = [\lambda^G \alpha^G p^G + \lambda^B \alpha^B p^B] f(\bar{L}_t(q_{t+1}^e))$$

Since the size of the pooling loan depends only on parameters of the economy and on q_{t+1}^e , once again, the capital stock of an economy in the pooling regime will be immediately pinned down. Thus, as in the case of the full information economy, there will be a unique, stable steady state to which the economy will converge in one period: we denote the steady state values of capital and factor prices in the asymmetric information economy under the pooling regime by $\{k_{POOL}, w_{POOL}, q_{POOL}\}$. The following figure displays the pooling dynamics in the (w_{t-1}, w_t) space: for any value of w_{t-1} , $w_t = w_{POOL}$ and the economy therefore “jumps” to the steady state regardless of initial conditions.

Wage Dynamics under Pooling Contracts

4.2 Separating Regime Dynamics

In both the full information economy and the asymmetric information economy under the pooling regime, investment is independent of wages. Under a separating regime, current investment in the G technology is increasing in current wages, and future wages are in turn increasing in the former. This generates a direct relationship between current and future wages.

In this scenario, loan sizes

$$\{I_{t,SEP}^B(q_{t+1}^e), I_{t,SEP}^G(q_{t+1}^e, w_t)\}$$

will be such that $I_{t,SEP}^B(q_{t+1}^e)$ is implicitly defined by

$$\alpha^B p^B f'(I_{t,SEP}^B) = \frac{r}{q_{t+1}^e}$$

while $I_{t,SEP}^G(q_{t+1}^e, w_t)$ is determined by the incentive compatibility and zero profit conditions as in Proposition 1. Once again, q_{t+1}^e must satisfy

$$q_{t+1}^e = \theta g'_k(1, k_{t,SEP}(q_{t+1}^e, w_t))$$

where $k_{t,SEP}(q_{t+1}^e, w_t)$ is defined as

$$\begin{aligned} k_{t,SEP}(q_{t+1}^e, w_t) = & [\lambda^G \alpha^G p^G f(I_{t,SEP}^G(q_{t+1}^e, w_t)) \\ & + \lambda^B \alpha^B p^B f(I_{t,SEP}^B(q_{t+1}^e))] \end{aligned}$$

In the separating regime, then, the asymmetric information economy will display dynamics since the equilibrium level of investment depends on wages and hence on the capital stock. The economy under the separating regime might display a unique, stable steady state or multiple steady states: we focus on the former case for simplicity, although our results are not restricted to this scenario. We denote the steady state values of capital and factor prices in the asymmetric information economy under the separating regime by $\{k_{SEP}, w_{SEP}, q_{SEP}\}$.

The following figure illustrates the dynamics of the asymmetric information economy under the separating regime. Throughout the analysis, it is assumed that

$$w_{SEP} < w_{POOL} \tag{37}$$

meaning that the steady state of the economy under the separating regime is lower than that under the pooling regime. Such an economy always exists, since (37) basically implies that the latter regime is highly productive with respect to the former, a natural outcome in an environment with a high average quality of entrepreneurs and a low proportion of entrepreneurs with respect to the total population. This last feature makes the separating allocation relatively unproductive since it minimizes the importance of wages as collateralizable wealth.

Wage Dynamics under Separating Contracts

4.2.1 Switch of Regimes and Cycles

We now address the issue of regime switches. The following proposition identifies their existence. In particular, it establishes that there is a partition of the wage space into three distinct regions and that the economy either pools, separates or mixes between both regimes depending on the region at which it finds itself. It is of interest to remark that the proposition is independent of the steady-state considerations mentioned in the previous subsection, and it hinges only on the assumption by which $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$.

Proposition 10 *Assume an economy in which $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$. Given the wage interval $[0, \bar{w}]$, where $\bar{w}$ is assumed to be arbitrarily large, there exists a unique pair of switching wages (w_1, w_2) such that:*

- *If $w_t \leq w_1$ then equilibrium loan contracts at time t are pooling*
- *If $w_t \geq w_2$ then equilibrium loan contracts at time t are separating*
- *If $w_1 < w_t < w_2$ then equilibrium loan contracts at time t involve randomization between pooling and separating contracts*

Proof. The proof is constructive and is divided into four steps. We first evaluate the pairs (w_t, q_{t+1}^e) that constitute an equilibrium under the separating regime. We then analyze the combinations of w_t and q_{t+1}^e that make G entrepreneurs indifferent between the pooling and separating

contracts. Finally, we characterize the pairs (w_t, q_{t+1}^e) that constitute an equilibrium under the pooling regime and we use our construction to prove the proposition.

Step 1: Suppose first that at time t equilibrium loan contracts are separating. From Proposition 1, separating contracts define a monotonically decreasing function:

$$q_{t+1}^e = S^{EQ}(w_t)$$

which determines the equilibrium price of capital tomorrow as a function of today's wage. Let

$$\begin{aligned}\bar{q} &= S^{EQ}(0) \\ q^* &= S^{EQ}(\bar{w})\end{aligned}$$

so that $\bar{q}$ denotes the equilibrium price of capital at time $t+1$ under the separating contracts without any entrepreneurial wealth and q^e denotes the equilibrium price of capital under full information.

Step 2: A second relationship between (w_t, q_{t+1}^e) , which we denote by

$$q_{t+1}^e = S^{SW}(w_t)$$

determines, for each wage level w_t , the expected price of capital at which G -type entrepreneurs are indifferent between the pooling and the separating regimes. From Lemma 4, this mapping is a function. As is shown in the Appendix (Section 7.5.2), moreover, it is an increasing function since at the switching points; a) higher values of wages increase the relative profitability of separating contracts, while; b) from Lemma 5, $\bar{I} > I_{SEP}^G$ so that a higher expected price of capital increases the relative profitability of pooling contracts. Given w_t , G -type entrepreneurs prefer pooling contracts whenever $q_{t+1}^e > S^{SW}(w_t)$ and prefer separating contracts otherwise. The following figure summarizes the discussion so far.

Step 3: We characterize the pooling equilibrium in terms of the previous figure. From our previous discussion, the price of capital under the pooling equilibrium is constant and equal to q_{POOL} . Suppose that the expected price of capital under the separating regime is also q_{POOL} : we argue that, in such a case, the optimal contracts must be separating. This stems from the observation that, if the supply of capital is the same under the pooling and the separating regimes, the latter must entail greater (lower) investment by G -types (B -types) and no cross subsidization: hence, the separating regime must yield G -types higher profits and must therefore be an equilibrium. Graphically, then, q_{POOL} must lie below the intersection of $S^{EQ}(w_t)$ and $S^{SW}(w_t)$, as the following figure shows:

Determination of Switching Wages (w_1, w_2)

Step 4: We define w_1 and w_2 implicitly by

$$\begin{aligned} q_{POOL} &= S^{SW}(w_1) \\ S^{EQ}(w_2) &= S^{SW}(w_2) \end{aligned}$$

and prove the proposition by using our construction.

For any wage below w_1 the equilibrium contracts can only be pooling. For any such wage level, the equilibrium price of capital lies above S^{EQ} under both the pooling and separating regimes, and hence pooling contracts constitute in fact an equilibrium.

For any wage above w_2 , on the other hand, the equilibrium contracts can only be separating. Once again, for any such wage level, the equilibrium price of capital lies below S^{EQ} under both the pooling and separating regimes, and hence separating contracts constitute in fact an equilibrium.

Consider now wage levels between w_1 and w_2 . In such a scenario, equilibrium contracts can be neither pooling nor separating, and will involve randomization between the two. To see this, note that if contracts are separating, the equilibrium price of capital is such that there are incentives to pool. On the other hand, if contracts are pooling, the equilibrium price of capital is such that there are incentives to separate. Thus, the only possible equilibrium involves randomization between both contracts so that, at the expected price of capital, G – type entrepreneurs are indifferent between the two. ■

We now characterize the dynamic behavior of the asymmetric information economy in terms of the relative ordering between the switching wages w_1 and w_2 and the steady state wages under the pooling and separating regimes. We identify three distinct cases:

1. Case 1: $w_{POOL} \leq w_1$. The economy has a unique, stable steady state at w_{POOL} and convergence is monotonic. Depending on initial conditions, the economy may start in a pooling, separating or mixing regime and will eventually converge to w_{POOL} . This case is illustrated in the figure below, where the solid dark line represents the equilibrium wage mapping.

Wage Dynamics under Case 1: $w_{POOL} \leq w_1$

2. Case 2: $w_{SEP} \geq w_2$. The economy has a unique, stable steady state at w_{SEP} and convergence may be oscillatory. For all initial wage levels above w_2 , the economy is always in a separating regime and monotonically converges to the steady state. For wage levels below w_2 , the economy may converge monotonically to w_{SEP} from below or may "overshoot", reaching wages above the steady state and then converging to the latter from above. This case is illustrated in the figure below, where - once more - the solid dark line represents the equilibrium wage mapping.

mapping.

Wage Dynamics under Case 2: $w_{SEP} \geq w_2$

3. Case 3: $w_{SEP} < w_2$ and $w_{POOL} > w_1$. At least one of the switching wages lies between the steady state wages under the separating and pooling regimes. In this case, the economy will display a unique steady state, which may be stable or unstable: in both cases, though, the economy will display fluctuations. If the steady state is unstable, the economy will fluctuate permanently, whereas in the case of stability it will do so while converging to the steady state. This case is illustrated in the figure below, where the equilibrium wage mapping is represented by the solid line.

Wage Dynamics under Case 3: $w_{SEP} < w_2$ and $w_{POOL} > w_1$

Case 3 is the one that is of interest to us. In such a scenario, an economy that has no dynamics under full information displays fluctuations in the presence of adverse selection. The main intuition behind this case is fairly simple: for low levels of wages, the separating contracts yield relatively low profits to G entrepreneurs and hence the economy will pool loans. By pooling, investment and hence future output and wages expand, increasing the future level of entrepreneurial wealth: if this increase is sufficiently large with respect to the switching wages of the economy, the resulting equilibrium will entail partial or total separation in the loan contracts and a consequent fall in output. The latter, in turn, will decrease entrepreneurial wealth thereby increasing the degree of pooling, which in turn expands output and so on.

In order to better characterize our result, we must prove that an economy satisfying Case 3 does in fact exist. We do so by showing that such an economy can be constructed from any economy originally satisfying Cases 1 or 2. This is possible because the mapping $S^{SW}(w_t)$, which determines the price of capital at which G -type entrepreneurs are indifferent between the pooling and separating contracts for a given wage level, is independent of the production function of the final good and of the absolute measure of the population. The steady state levels of wages under the pooling and separating regimes, on the other hand, are not. By exploiting this feature it is then possible to transform any original economy into an alternative one satisfying $w_{SEP} < w_2$ and $w_{POOL} > w_1$.

Proposition 11 *There is a nonempty set of economies for which $w^S < w_2$ and $w^P > w_1$.*

Proof. See Appendix. ■

We provide an economic intuition for the way in which the proof works. Suppose first that we start in an economy in Case 1: loosely speaking, wage levels are relatively low in terms of equilibrium investment, so that it is optimal to pool loans for a wide range of wages. This feature can be altered by perturbing the production function of the final good so as to make labor more productive with respect to capital. Such a modification tends to increase the level of entrepreneurial wealth with respect to the optimal level of investment and, in so doing, increases the relative appeal of separating contracts. Consequently w_1 diminishes relative to the steady state levels of wages.

An opposite exercise can be done in the case of an economy satisfying Case 2: in such a scenario, steady state wages are high relative to the equilibrium level of investment and hence separating contracts are particularly appealing. A possible perturbation of this economy consists in expanding the labor supply by increasing the measure of households in the economy while preserving that of entrepreneurs. Such a modification induces an increase in the equilibrium price of capital and a decrease in equilibrium wages or - in terms of our contracts - entrepreneurial wealth tends to decrease with respect to the optimal level of investment. Consequently, w_1 expands as pooling contracts become relatively more attractive while the steady state levels of wages contract.

We have thus constructed an environment in which the full information economy exhibits no dynamics. Once we introduce asymmetric information, though, there is a class of economies that will exhibit fluctuations: some will converge to the steady state in an oscillatory manner, whereas others will fluctuate permanently.

An important feature of our reversion mechanism is that it arises naturally in a fairly standard setting of adverse selection, in which entrepreneurial wealth is pro-cyclical and provides a channel through which entrepreneurs can be screened. In this sense, we believe that credit cycles in our framework arise through a natural mechanism which does not rest on the fact that net worth is inconsequential for determining equilibrium levels of investment (as in [6] or [31]) or on assuming that it behaves in a counter-cyclical manner (as in [39]). In our model, booms are investment-driven and are made possible because good projects cross-subsidize their less productive counterparts, a feature which also seems consistent with evidence regarding bank lending standards along the business cycle (see [4]). Finally, cycles in our model arise in a fully rational and deterministic environment.

5 Discussion

Our main results have been derived in a stylized model, thereby making it natural to inquire on their robustness to alternative settings. Here we comment on some natural directions in which the assumptions of the model could be relaxed and on their effect on our basic results.

1. In the present version of the model, debt contracts arise as the optimal arrangement due to the binary distribution of investment project outcomes. In a more general setting in which project returns were characterized by a distribution over a continuum of outcomes, Boyd and Smith [13] have shown that debt contracts can still arise as the optimal contractual arrangements in the presence of sufficiently high verification costs.
2. For simplicity of exposition and in order to compare our results with a well-known benchmark, we have analyzed the possibility of endogenous cycles in an economy in which the interest rate is fixed. Allowing for a variable interest rate would not invalidate our qualitative results, although it would be necessary for the supply of savings to display a positive elasticity with respect to the interest rate.
3. We have assumed that total savings in our economy are inelastic with respect to the interest rate. Allowing for some sensitivity in this front would not affect our results, which - in the particular case of financial dampening - could be strengthened by this feature. This is due to the fact that the inefficient allocation of credit in the separating regime, plus the fact that this inefficiency might increase as a consequence of a positive shock, are both features that exert downward pressure on the equilibrium interest rate. Under our baseline assumptions, this does not affect savings, but they would be affected if they were increasing in the interest rate. This negative effect on savings would work in favor of financial dampening.
4. We have assumed that agents are risk neutral. Introducing risk averse consumers would not affect our results. Risk aversion in the preferences of entrepreneurs, on the other hand, would generate an additional cost of pledging their wealth as collateral since doing so increases consumption volatility. Hence, this effect could restrain the amount of collateral pledged and of investment undertaken in the economy under the separating regime. On the other hand, and precisely because of this reason, collateral would be more effective as a screening device. The net impact on the level of collateralization and investment would depend on the relative importance of both these effects.
5. In order to analyze both financial dampening and endogenous fluctuations in a simple, unified framework, we have assumed that agents live only for two periods. If they were to be infinitely lived, the results would greatly depend on the assumption regarding the persistence of entrepreneurial types. If entrepreneurial types were assumed to be persistent across time, constrained optimal contracts would be substantially more complicated than in our setting and it is not clear which of our results could still be valid. We could, however, adopt a more conventional approach in which entrepreneurs are randomly assigned a new type each time they undertake a new project or, as in Bernanke, Gertler and Gilchrist [9], that the level of anonymity in the credit market is sufficiently high as to preclude long-term contracts. In such

a specification of the model, our results could be valid although some additional issues would have to be considered.

- (a) First, an economy populated by infinitely lived entrepreneurs would endogenously generate a distribution of entrepreneurial wealth. In terms of our contracting problem, this would not pose a great problem as it could be dealt with by expanding the set of equilibrium contracts to a continuum indexed by levels of collateral. Hence, it would be the case at equilibrium that some wealth levels entailed pooling whereas others entailed separation. The equilibrium would require some modification in order to assign bad entrepreneurs to these different pooling contracts.
- (b) Possibly the most substantial qualification regarding our results in the case of infinitely lived agents is related to the possibility of endogenous cycles. In the OLG specification, there is a pecuniary externality at work, by which today's entrepreneurs do not take into account the effect of their investment decisions in tomorrow's net worth. In a model with infinitely lived agents, a switch from pooling to separating would have an ambiguous impact on net worth, increasing the net revenues stemming from current production but decreasing those derived from tomorrow's labor. In order for our results to go through without any modifications, the second effect should outweigh the first. A different approach would consist in eliminating all labor income so that, at each point in time, entrepreneurial net worth is given by past profits. Such a specification would, in principle, still allow for the possibility of endogenous cycles, since a contraction in investment at the switching regime would increase the wealth of good entrepreneurs but decrease that of their bad counterparts, thus having an ambiguous effect on total entrepreneurial wealth and hence on investment.

6 Concluding Remarks

The main objective of this paper has been to propose a unified framework in which to analyze the dual role of the financial system as an amplifier of exogenous shocks and as a generator of endogenous fluctuations. To do so, we developed a model of adverse selection in the credit market with concave investment projects and intermediation and analyzed an enriched contract space in which the size of loans, the interest payments and the amount of collateral required of entrepreneurs are determined at equilibrium.

Regarding the role of the financial system as an amplifier or dampener of shocks, we believe that our framework highlights clearly the essential difference between the introduction of asymmetric information as adverse selection, on one hand, and as pure moral hazard or private information, on the other. Conceptually, whereas in the latter case the incentive compatibility constraint operates

on individual allocations, the notion of incentive compatibility in the presence of adverse selection refers to a set of joint allocations.

In our framework, the presence of adverse selection generates a nonlinear relationship between entrepreneurial net worth and investment under the separating regime. In particular, there is a positive relationship between entrepreneurial wealth and its impact on the amount of investment undertaken in high productivity projects. This feature of equilibrium investment becomes particularly relevant in the context of a closed economy, in which contracts must not only be incentive compatible but must also clear the market for loans. Consequently, it is possible that when the economy experiences a positive shock that expands entrepreneurial wealth and savings in the same proportion, the need to achieve market clearing in an incentive compatible way leads to a decrease in the proportion of funds allocated to high productivity investment. If this effect is sufficiently large, shocks are “dampened” by the presence of asymmetric information despite the fact that entrepreneurial wealth is pro-cyclical.

An additional point of interest regarding this feature of our model is that the traditional result, by which shocks are amplified through the financial system, is restored in the context of a small, open economy that faces an exogenous rate of interest. Thus, it is possible in our framework that the financial system might change from a dampener to an amplifier of shocks when a small economy opens its capital market. Although the approach we adopt in this paper is conceptual, we believe this feature of the model to be potentially appealing in light of the recent debate regarding financial liberalization and volatility (see [27]).

A second aspect we have highlighted refers to the role of regime switching between pooling and separating contracts in the generation of endogenous cycles. We have done this in an environment that deliberately eliminates dynamics under full information. When adverse selection is introduced, however, endogenous cycles may arise as a consequence of changes in equilibrium contracts. More specifically, in a context where the ratio of good to bad investment projects is relatively high, low levels of entrepreneurial wealth increase the cost of screening and hence provide incentives for the pooling of loans. Pooling, however, induces an increase in investment and - consequently - in future output and entrepreneurial wealth, decreasing the future cost of screening and therefore increasing banks’ incentives to design separating contracts. In this way, there is a “flight of quality” phenomenon by which the best entrepreneurs are lured away from the pooling equilibrium, which unravels into a separating regime and in so doing contracts investment, future output and future entrepreneurial net worth. If this effect is sufficiently large, the economy reverts to a pooling equilibrium and the process starts again.

Differently from previous papers that are similar in spirit ([1],[2],[6],[31],[39]), we believe that our reversion mechanism is novel in the sense that it emerges naturally within a fairly standard framework of adverse selection. It does not rely on a particular mix of moral hazard and adverse

selection nor does it hinge on assuming that net worth is counter-cyclical or inconsequential in determining equilibrium levels of investment.

Finally, we believe that the baseline model of multidimensional screening developed in the present paper is interesting in its own right, and that it can be extended to analyze other relevant issues.

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7 Appendix

7.1 Characterization of Contracts

As explained in the main body of the paper, we follow Hellwig ([23]) and model competition as a three-stage game in which banks design contracts, firms apply to at most one of them and banks accept or reject applications. Hellwig shows that such a game always has a sequential Nash equilibrium. In particular, pooling contracts are Nash equilibria whenever they Pareto dominate the separating pairs. Hence, Nash equilibrium contracts are always efficient and it can be shown that they arise from maximizing a welfare function subject to the incentive compatibility constraints under the assumptions of exclusivity and no cross-subsidization.

Throughout the appendix, hence, we derive the family of (constrained) optimal contracts for a given interest rate-entrepreneurial wealth pair (r, W) . We seek for an optimum within the set of contracts that satisfy exclusivity and no cross-subsidization. These contracts are obtained by maximizing the borrowers' profits subject to the lenders' participation constraints. Although there are different possible interpretations of the planner's objective function, we make it equal to the profits of G type entrepreneurs.

In the case of optimal separating contracts, these profits are maximized subject to the incentive compatibility constraints, the no-cross subsidization condition (i.e., banks' zero profit conditions) and the collateralization constraints. The optimization problem is then as follows:

$$\max_{I^G, I^B, R^G, R^B, c^G, c^B} p^G [\alpha^G f(I^G) - R^G I^G] - (1 - p^G) r c^G I^G + W r$$

subject to

$$c^j \in [0, \frac{W}{I^j}] \text{ for } j = B, G$$

$$p^i [\alpha^i f(I^j) - R^j I^j] - (1 - p^i) r c^j I^j \leq p^i [\alpha^i f(I^i) - R^i I^i] - r (1 - p^i) c^i I^i r$$

for $i \neq j, i, j = G, B$

$$r = p^j R^j + (1 - p^j) r c^j \text{ for } j = G, B$$

In the case of the optimal pooling contract, the optimization problem is analogous with the difference that it is not subject to incentive compatibility constraint. More specifically, optimal pooling contracts are obtained as the result of the following problem:

$$\max_{I, c, R} \pi^G$$

subject to

$$r = \bar{p}R + (1 - \bar{p})rc \quad (38)$$

$$c \in [0, \frac{W}{I}] \quad (39)$$

where (38) and (39) represent, respectively, the zero profit conditions for banks (noting that $\bar{p} = \lambda^B p^B + \lambda^G p^G$) and the collateralization constraint.

Throughout the Appendix, then, we use the terms “optimal separating” and “optimal pooling” contracts to denote the respective solutions of these optimization problems. Of the two, the one that yields G type entrepreneurs the highest level of profits will be referred to as the “optimal contract”.

7.1.1 Decentralization of the Optimal Contracts

From Hellwig, it follows that the solutions to the previously posed optimization problems do in fact constitute equilibrium outcomes of the three-stage game. We nonetheless provide a brief review of the argument for the convenience of the reader.

Suppose, first, that the optimal contract as defined above is pooling. Trivially, the zero profit condition and collateralization constraint are satisfied. It remains to be shown that there are no profitable deviations by banks. We argue by contradiction. If a bank deviates, it must do so by attracting either G - or B - type entrepreneurs.

If it attracts G - type entrepreneurs, then all other banks will make negative profits by offering the pooling contracts: hence, all applications to these contracts must be rejected. Consequently, the deviating bank will attract all borrowers, in which case the deviation can only be profitable if it makes positive profits when all entrepreneurs apply to it. But this contradicts the original pooling contract being optimal.

On the other hand, suppose that deviating bank seeks to attract only B - *type* entrepreneurs. Then, it must be the case that the latter make higher profits under the new contract than under the pooling allocation. But, in order for this deviation to be profitable, it must yield the bank an expected return above r for each dollar lent, a condition which would immediately violate (??) and hence the optimality of the original pooling contract.

Suppose, on the other hand, that the optimal contracts are separating. As before, it needs to be shown that there are no profitable deviations by banks. It is immediate to verify that banks cannot profitably deviate by designing separating contracts or by attracting one type of borrowers only. Hence, the only potential deviation that could be profitable must consist in the design of a pooling contract.

Any profitable pooling deviation must attract G - *type* borrowers, and must therefore imply that there exists a pooling contract that - while being profitable for the deviating bank - makes

G – *type* entrepreneurs weakly better off than their separating contract. But this implies the existence of a pooling contract in which banks make zero profits while G – *type* entrepreneurs make strictly higher profits than under the original separating contracts, which contradicts the optimality of the latter.

Having shown then that the aforementioned programming problems do in fact deliver equilibrium contracts, we now characterize these.

7.1.2 Separating Contracts without Collateral

In this subsection and the next, we prove Proposition 1 and characterize optimal separating contracts. We first do so in the absence of collateral. Additionally, we allow for random rationing (i.e., being denied a loan with probability ε^j) and show that it will never take place at an optimal solution. These contracts are the solution to the following optimization problem:

$$\begin{aligned}
& \max_{I^G, I^B, \varepsilon^G, \varepsilon^B} \pi^G \\
& \text{st.} \\
& \pi^G = (1 - \varepsilon^G) \{p^G [\alpha^G f(I^G) - R^G I^B]\} \\
& r = p^G R^G \\
& r = p^B R^B \\
& (1 - \varepsilon^G) [\alpha^B p^B f(I^G) - \frac{p^B}{p^G} I^G r] \leq \\
& \quad (1 - \varepsilon^B) \{p^B [\alpha^B f(I^B) - R^B I^B]\} \\
& \varepsilon^B, \varepsilon^G \in [0, 1]
\end{aligned}$$

We solve the problem by using only one incentive compatibility constraint and conjecturing that the other one will be slack. Once we derive the optimal contracts, we will prove this conjecture to be correct. Replacing the constraints in the objective function, the problem becomes,

$$\begin{aligned}
& \max_{I^G, I^B, \varepsilon^G, \varepsilon^B} (1 - \varepsilon^G) [p^G \alpha^G f(I^G) - I^G r] \\
& \text{st.}
\end{aligned}$$

$$\begin{aligned}
\psi &: (1 - \varepsilon^G)[\alpha^B p^B f(I^G) - \frac{p^B}{p^G} I^G r] \leq (1 - \varepsilon^B)[p^B \alpha^B f(I^B) - I^B r] \\
\zeta &: \varepsilon^G \leq 1 \\
\theta &: \varepsilon^G \geq 0 \\
\tau &: \varepsilon^B \leq 1 \\
\omega &: \varepsilon^B \geq 0
\end{aligned}$$

The FOC's of this problem, plus the incentive compatibility constraint, are given by,

$$\begin{aligned}
I^B &: \psi(1 - \varepsilon^B)[p^B \alpha^B f'(I^B) - r] = 0 \\
I^G &: \{p^G \alpha^G f'(I^G) - r\} - \psi[\alpha^B p^B f'(I^G) - \frac{p^B}{p^G} r] = 0 \\
\varepsilon^G &: -[p^G \alpha^G f(I^G) - r I^G] + \psi[\alpha^B p^B f(I^G) - \frac{p^B}{p^G} I^G r] - \zeta + \theta = 0 \\
\varepsilon^B &: (-\psi)[p^B \alpha^B f(I^B) - I^B r] + \omega - \tau < 0 \\
IC &: \frac{(1 - \varepsilon^G)[\alpha^B p^B f(I^G) - \frac{p^B}{p^G} I^G r]}{(1 - \varepsilon^B)[p^B \alpha^B f(I^B) - I^B r]} = 1
\end{aligned}$$

It is straightforward to observe that $\varepsilon^B = 0$ from the corresponding first order condition. Therefore, the FOC with respect to I^B tells us that,

$$p^B \alpha^B f'(I^B) = r$$

which fully characterizes the contract offered to bad entrepreneurs. On the other hand, we know that

$$\psi = \frac{\{p^G \alpha^G f'(I^G) - r\}}{[\alpha^B p^B f'(I^G) - \frac{p^B}{p^G} r]}$$

which, replaced in the first order condition for ε^G , tells us that the optimal contracts will never entail rationing in the sense of Stiglitz and Weiss. This is an immediate consequence of Inada conditions. Using now the first order condition for loan size to the good entrepreneur, and replacing for ψ ,

$$p^G \alpha^G f'(I^G) = r + \frac{[p^G \alpha^G f(I^G) - r I^G] - \theta}{[\alpha^B p^G f(I^G) - r I^G]} [\alpha^B p^G f'(I^G) - r]$$

The latter implies that in the absence of collateral there is a distortion by which the marginal productivity of investment is higher in the good than in the bad sector. In order to see this, note from incentive compatibility that,

$$[\alpha^B p^B f(I^G) - \frac{p^B}{p^G} I^G r] = [p^B \alpha^B f(I^B) - I^B r]$$

meaning that $I^B > I^G$. First note that if $I^B = I^G$ the incentive compatibility constraint will be violated. Thus, we know $I^B \neq I^G$, where the latter could in principle be larger or smaller than the former. Since profits are concave in I , for given I^B and r there exists pair (I_1^G, I_2^G) that satisfy the incentive compatibility constraint, where $I_2^G > I^B > I_1^G$. Since both loan sizes are incentive compatible,

$$[\alpha^B p^G f(I_1^G) - I_1^G r] = [\alpha^B p^G f(I_2^G) - I_2^G r]$$

which, noting that $\alpha^G < \alpha^B$, implies

$$[\alpha^G p^G f(I_1^G) - I_1^G r] > [\alpha^G p^G f(I_2^G) - I_2^G r]$$

Thus, of the two possible values of I^G that are incentive compatible, the one that is lower than I^B yields higher profits for the good entrepreneurs. Thus, $I^G = I_1^G < I^B$.

Finally, the single crossing property, by which

$$\frac{dR}{dI}_G = \frac{\alpha^G f'(I) - R}{I} < \frac{\alpha^B f'(I) - R}{I} = \frac{dR}{dI}_B$$

guarantees that the incentive compatibility constraint for G -type entrepreneurs is slack. The following figure illustrates the separating contracts in the (I, R) space.

Separating Contracts in the Absence of Collateral

7.1.3 Separating Contracts with Collateral

As before, we solve the problem using only one of the incentive compatibility constraints and then show that the other one is slack at the optimal solution. Additionally, we exclude the probability of loan approval from the problem in order to simplify the exposition: in the same way as we did in the previous section, it is trivial to show that there is no rationing of this form at the optimal solution due to Inada conditions. Optimal separating contracts are then the solution to the following optimization problem, where c^j $j = G, B$ is used to denote the rate of collateralization.

$$\begin{aligned} & \max_{I^G, I^B, c^G, c^B, R^G, R^B} \pi^G \\ & \text{st.} \end{aligned}$$

$$\begin{aligned} r &= p^G R^G + (1 - p^G) c^G r \\ r &= p^B R^B + (1 - p^B) c^B r \end{aligned}$$

$$\begin{aligned} p^B [\alpha^B f(I^G) - R^G I^G] - (1 - p^B) r c^G I^G &= \\ p^B [\alpha^B f(I^B) - R^B I^B] - (1 - p^B) r c^B I^B \\ c^G, c^B &\in [0, \frac{W}{I^j}] \end{aligned}$$

By rewriting the objective function, the problem becomes

$$\begin{aligned} & \max_{I^G, I^B, c^G} p^G \alpha^G f(I^G) - r I^G \\ & \text{st.} \end{aligned}$$

$$\psi : [\alpha^B p^B f(I^G) - \frac{p^B}{p^G} I^G r] + [\frac{p^B}{p^G} - 1] r c^G I^G \geq p^B \alpha^B f(I^B) - r B I^B$$

$$\begin{aligned} \eta & : c^G \leq \frac{W}{I^G} \\ \mu & : c^G \geq 0 \end{aligned}$$

The FOC's of this problem are,

$$\begin{aligned}
I^B &: \psi[p^B \alpha^B f'(I^B) - r] = 0 \\
I^G &: \lambda^G \{p^G \alpha^G f'(I^G) - r\} - \eta \left(\frac{W}{(I^G)^2} \right) \\
&\quad - \psi \left\{ [\alpha^B p^B f'(I^G) - \frac{p^B}{p^G} r] + [\frac{p^B}{p^G} - 1] r c^G \right\} \\
c^G &: -\psi [\frac{p^B}{p^G} - 1] r I^G - \eta + \mu = 0 \\
IC &: \frac{[\alpha^B p^B f(I^G) - \frac{p^B}{p^G} I^G r] + [\frac{p^B}{p^G} - 1] r c^G I^G}{p^B \alpha^B f(I^B) - r I^B} = 1
\end{aligned}$$

We let $c^B = 0$ without loss of generality. Observe that I^B is implicitly defined by

$$p^B \alpha^B f'(I^B) = r$$

If the wealth constraint is not binding, the incentive compatibility constraint is itself not binding and

$$\psi = 0$$

which implies

$$p^G \alpha^G f'(I^G) = r$$

Hence, when collateral is sufficiently abundant so that collateralization constraints need not bind, the separating contracts can replicate the full information allocation. The economically interesting case is the one with a binding collateralization constraint, in which case the first order condition for I^G tells us that,

$$p^G \alpha^G f'(I^G) = r + \frac{\eta}{[1 - \frac{p^B}{p^G}] r I^G} [\alpha^B p^B f'(I^G) - \frac{p^B}{p^G} r]$$

In order to analyze this condition, we reinterpret the maximization problem in the following way. From the first order conditions, I^B will satisfy $p^B \alpha^B f'(I^B) = r$. Given the loan contract for B -type entrepreneurs then, the problem becomes to maximize G -type profits subject to a “no mimicry” (henceforth, NMC) constraint by which,

$$p^B \alpha^B f(I^G) - (\frac{p^B}{p^G} r + [1 - \frac{p^B}{p^G}] r c^G) I^G = p^B \alpha^B f(I^B) - r I^B = K$$

The previous condition specifies c^G in terms of I^G , a mapping which gives us all combinations of both variables that satisfy the zero profit condition and the incentive compatibility constraint. This mapping is given by

$$c^G = \frac{p^B \alpha^B f(I^G) - \frac{p^B}{p^G} r I^G - K}{[1 - \frac{p^B}{p^G}] r I^G}$$

Along the NMC, c^G is maximized at the value which guarantees $\frac{p^B}{p^G} r + [1 - \frac{p^B}{p^G}] r c^G = r$. The maximum value of c^G is achieved exactly when $I^G = I^B$, at which point

$$c^G = 1 \tag{40}$$

Thus, the no mimicry constraint has c^G increasing in I^G whenever the latter is smaller than I^B and decreasing whenever $I^G > I^B$. Hence, the fully efficient G -type loan size I^{G*} will be on the downward sloping section of the NMC. When wealth constraints are binding, we need to take into account an additional constraint given by,

$$c^G I^G \leq W$$

This constraint determines a hyperbola in the c^G, I^G space and - when the constraint is binding - the optimal G -type contract must lie on the intersection of the latter with the NMC, as depicted in Figure 2 in the main body of the paper.

That is, there are two pairs of values $[(c_1^G, I_1^G), (c_2^G, I_2^G)]$ satisfying the following conditions,

$$\begin{aligned} c_1^G &> c_2^G \\ I_1^G &< I_2^G \\ c_1^G I_1^G &= c_2^G I_2^G = W \\ p^B \alpha^B f(I_1^G) - \frac{p^B}{p^G} r I_1^G &= p^B \alpha^B f(I_2^G) - \frac{p^B}{p^G} r I_2^G \end{aligned}$$

where the last equation represents the fact that the no mimicry condition must be satisfied by both pairs (note that the rate of collateralization cancels out since the total amount of collateral is the same in both possible contracts). Of these two potential G -type loan contracts, we identify the one that provides G -type entrepreneurs with the highest profits by comparing,

$$p^G \alpha^G f(I_1^G) - r I_1^G \text{ with } p^G \alpha^G f(I_2^G) - r I_2^G$$

But, by the NMC, we know that,

$$p^G \alpha^B [f(I_2^G) - f(I_1^G)] = r[I_2^G - I_1^G]$$

which in turn implies,

$$p^G \alpha^G f(I_1^G) - r I_1^G > p^G \alpha^G f(I_2^G) - r I_2^G$$

meaning that of the two potential G -type contracts that satisfy both the wealth constraint and the no mimicry condition, the one with lower loan size and higher rate of collateralization is the optimal one.

Finally, we have solved the problem under the assumption that the incentive compatibility constraint for G -type entrepreneurs hold, and we now prove that this is the case. Formally, we must prove that,

$$p^G \alpha^G f(I^G) - r I^G > p^G \alpha^G f(I^B) - r \frac{p^G}{p^B} I^B$$

for any allocation that at the same time satisfies the incentive compatibility constraint for B -type entrepreneurs,

$$p^B \alpha^B f(I^G) - \left(\frac{p^B}{p^G} r + \left[1 - \frac{p^B}{p^G}\right] r c^G\right) I^G = p^B \alpha^B f(I^B) - r I^B$$

Multiplying the first expression by $\frac{p^B \alpha^B}{p^G \alpha^G}$ and subtracting the second one from it, we obtain that,

$$\begin{aligned} & \frac{p^B \alpha^B}{p^G \alpha^G} (-r I^G) + \left(\frac{p^B}{p^G} r + \left[1 - \frac{p^B}{p^G}\right] r c^G\right) I^G \\ & > \\ & \frac{\alpha^B}{\alpha^G} (-r I^B) + r I^B \end{aligned}$$

which reduces to

$$\begin{aligned} \frac{p^B \alpha^B}{p^G \alpha^G} (-I^G) + \left(\frac{p^B}{p^G} + \left[1 - \frac{p^B}{p^G}\right] c^G\right) I^G & > \left[1 - \frac{\alpha^B}{\alpha^G}\right] I^B \\ \left(1 - \frac{\alpha^B}{\alpha^G}\right) \frac{p^B}{p^G} I^G + \left[1 - \frac{p^B}{p^G}\right] W & > I^B \left(1 - \frac{\alpha^B}{\alpha^G}\right) \end{aligned} \tag{41}$$

From our previous analysis we know that condition (41) is satisfied for $W = 0$. Additionally, the derivative of the RHS of (41) with respect to W is:

$$(1 - \frac{\alpha^B}{\alpha^G}) \frac{p^B}{p^G} \left[\frac{[1 - \frac{p^B}{p^G}]r}{\alpha^B p^B f'(I^G) - \frac{p^B}{p^G} r} \right] + [1 - \frac{p^B}{p^G}]$$

which can be expressed as

$$\frac{(\frac{f'(I^G)}{f''(I^B)} - \frac{\alpha^B p^B}{\alpha^G p^G})}{\frac{\alpha^B p^B f'(I^G)}{r} - \frac{p^B}{p^G}}$$

which is strictly positive as long as I^G is constrained below its optimal level. Hence, (41) holds for all levels of W .

7.1.4 Comparative Statics of Separating Contracts

It is of interest to analyze how the separating contracts change with the interest rate. It is immediate from (6) that - as B - type contracts entail no distortions - increases (decreases) in r will induce decreases (increases) in I^B . The behavior of I^G , though, is ambiguous. This can be appreciated by differentiating the incentive compatibility constraint, which yields,

$$\frac{dI^G}{dr} = \frac{I^G \frac{p^B}{p^G} + W[1 - \frac{p^B}{p^G}] - I^B}{\alpha^B p^B f'(I^G) - \frac{p^B}{p^G} r} \quad (42)$$

Whenever r changes, both loan sizes change as well. However, changes in I^B have no first order effects on the profits of B - type entrepreneurs when they choose B - type contracts. Changes on I^G , on the other hand, have a direct effect on the profit of G - type contracts in the eyes of B - type entrepreneurs: these are always positive, as captured by the denominator in (42).

As for changes in r , they have a direct effect on the profits of both contracts, although the direction of this effect depends on the relative sizes of loans. In particular, whenever $I^G < I^B$, increases (decreases) in r make B - type contracts relatively more expensive (cheaper) than their G - type counterparts. If $I^G > I^B$, though, the effect is the opposite, and B - type contracts become relatively more attractive for B - type entrepreneurs.

Therefore, when $I^G < I^B$ an increase in the interest rate must necessarily have a negative impact on the size of I^G : it could not be otherwise, since a reduction in the latter is needed to counter the adverse impact on incentives induced by r . On the other hand, when $I^G > I^B$, an increase in r actually slackens the incentive compatibility constraint and - consequently - generates room for an increase in the size of loans to the good technology: in principle, it is even possible that the increase in interest rate generates an increase in total output, if the productivities of both technologies are sufficiently different.²⁷

²⁷ Although in a different context, this effect of interest rate increases is somewhat reminiscent of the “cleansing effect of recessions” mentioned in [16].

7.1.5 Proof of Proposition 2

Proof. As we have already discussed, optimal pooling contracts are the solution to the following optimization problem

$$\begin{aligned}
& \max_{I,c,R} \pi^G \\
& \text{st.} \\
\pi^G &= p^G \alpha^G f(I) - p^G R I - (1 - p^G) r c I + W r \\
r &= R + (1 - \bar{p}) r c \\
c &\in [0, \frac{W}{I}]
\end{aligned}$$

By replacing everything in the objective function, the problem becomes,

$$\begin{aligned}
& \max_{I,c} p^G \alpha^G f(I) - \frac{p^G}{\bar{p}} r I - (1 - \frac{p^G}{\bar{p}}) r c I + W r \\
& \text{st.} \\
\nu &: c \leq \frac{W}{I} \\
\varphi &: c \geq 0
\end{aligned}$$

The FOC's of this problem are,

$$I : p^G \alpha^G f'(I) - \frac{p^G}{\bar{p}} r - (1 - \frac{p^G}{\bar{p}}) r c - \nu (\frac{W}{I^2}) = 0 \quad (43)$$

$$c : -(1 - \frac{p^G}{\bar{p}}) r I - \nu + \varphi = 0 \quad (44)$$

From (44), the collateralization constraint, must bind, so that

$$c = \frac{W}{I}$$

Replacing (44) in (43), we obtain

$$p^G \alpha^G f'(\bar{I}) = \frac{p^G}{\bar{p}} r$$

which proves the result. ■

It is worthwhile to point out that, whenever G types prefer the optimal pooling to the optimal separating contract, so do B types. To see this, suppose B types prefer their separating contract to a pooling one, so that

$$p^B \alpha^B f(I) - \frac{p^B}{\bar{p}} r I - (1 - \frac{p^B}{\bar{p}}) r c I < p^B [\alpha^B f(I^G)] - \frac{p^B}{p^G} r I^G - (1 - \frac{p^B}{p^G}) r c^G I^G$$

where (I^G, c^G) correspond to the separating contract offered to G -types for the given interest rate and entrepreneurial wealth. However, the previous condition implies,

$$p^G \alpha^G [f(I) - f(I^G)] < \frac{\alpha^G}{\alpha^B} [\frac{p^G}{\bar{p}} (I - W) - (I^G - W)]$$

whereas, in order for the pooling allocation to be preferred by G -types with respect to the separating, the following must hold,

$$p^G \alpha^G [f(I) - f(I^G)] > [\frac{p^G}{\bar{p}} (I - W) - (I^G - W)]$$

Thus, if B -type entrepreneurs are indifferent between the pooling allocation and their corresponding separating contract, it must be the case that the latter yields lower profits to G -type entrepreneurs.

7.1.6 Characterization of Regime Switches

Proof. [Lemma 3] The proof is by contradiction. If $\bar{p} = \frac{\alpha^B p^B}{\alpha^G}$ then the profits obtained by G -types at the contract $C^P(r, 0)$ are given by,

$$\alpha^G p^G f(\bar{I}) - \frac{\alpha^G p^G}{\alpha^B p^B} r \bar{I}$$

where, from (15), $\bar{I} = I^B$ and the latter represents the size of B -type loans under contracts $C^{SEP}(r, 0)$. Suppose that G -types prefer the separating contract to the pooling, so that,

$$\alpha^G p^G f(I^G) - r I^G > \alpha^G p^G f(\bar{I}) - \frac{\alpha^G p^G}{\alpha^B p^B} r \bar{I}$$

However, that would mean that,

$$\alpha^B p^B f(I^G) - \frac{\alpha^B p^B}{\alpha^G p^G} r I^G > \alpha^B p^B f(\bar{I}) - r \bar{I}$$

which is impossible since it would imply a violation of the B -type incentive compatibility constraint which $C^{SEP}(r, 0)$ must satisfy. Suppose instead the extreme case in which $\bar{p} \approx p^G$. Then, trivially, G -types obtain higher profits from the pooling than from the separating contract. As for B -types, their profits would be given by,

$$\alpha^B p^B f(\bar{I}) - \frac{p^B}{p^G} r \bar{I}$$

which must be higher than what they obtain under the separating, since $I^G > I^{B*}$ and for $\bar{I} = I^G$,

$$\alpha^B p^B f'(\bar{I}) > \frac{p^B}{p^G} r$$

■

Proof. [Lemma 4] The proof compares the profits of G – type entrepreneurs under pooling and separating contracts for a given value of r . From the previous Lemma, we know that the restriction on $\bar{p}$ implies that

$$\pi^G(C^{POOL}(r, 0)) > \pi^G(C^{SEP}(r, 0))$$

for all values of r . Additionally, it is easy to verify that whenever $W = \bar{I}(r)$, the following also holds:

$$\pi^G(C^{POOL}(r, \bar{I}(r))) \leq \pi^G(C^{SEP}(r, \bar{I}(r)))$$

since the profits that the pooling contract yields to G – types are bounded from above by $\alpha^G p^G f(\bar{I})$ which are attained under full collateralization. G – type profits under the separating contracts, on the other hand, are bounded from below by $\alpha^G p^G f(W)$, since the rate of collateralization is weakly smaller than one.

Thus, our economy will display a pooling equilibrium when $W = 0$ regardless of the interest rate, but will display a separating equilibrium when $W = \bar{I}(r)$. By taking derivatives of $\pi^G(C^{POOL}(r, W))$

$$\frac{\partial \pi^G(C^{POOL}(r, W))}{\partial W} = \frac{p^G}{\bar{p}} r \quad (45)$$

we see that it is linear and increasing in W . On the other hand, as shown below, $\pi^G(C^{SEP}(r, W))$ is concave and increasing in W .

$$\frac{\partial \pi^G(C^{SEP}(r, W))}{\partial W} = \left(1 - \frac{p^B}{p^G}\right) r \frac{p^G \alpha^G f'(I^G(r, W)) - r}{\alpha^B p^B f'(I^G(r, W)) - r \frac{p^B}{p^G}} \quad (46)$$

$$\frac{\partial^2 \pi^G(C^{SEP}(r, W))}{\partial W^2} = \left(1 - \frac{p^B}{p^G}\right) r^2 \frac{p^B f''(I^G(r, W))(\alpha^B - \alpha^G)}{(\alpha^B p^B f'(I^G(r, W)) - r \frac{p^B}{p^G})^2} < 0 \quad (47)$$

Hence, both profits loci can only intersect once for a given value of r , thus proving that $W^*(r)$ is a function. ■

7.2 Investment at the Switching Point (Lemma 5)

The present subsection characterizes the relative sizes of pooling and separating loans at the switching point. We recall that the switching point is defined as a level of wealth satisfying the following equality:

$$\alpha^G p^G f(I^G(r, W^*(r))) - r I^G(r, W^*(r)) = \alpha^G p^G f(\bar{I}(r)) - r \bar{I}(r) \frac{p^G}{\bar{p}} - [1 - \frac{p^G}{\bar{p}}] r W^*(r) \quad (48)$$

A few aspects of the problem should be noted, always under the assumption that $\bar{p} \geq \frac{p^B \alpha^B}{\alpha^G}$:

1. $\bar{I}(r) \geq W$. Otherwise, B -types would subsidize G -types in the pooling allocation. This can never happen, since that would yield B -types a strictly lower payoff than what they receive from their separating allocation.
2. $I^G(r, W^*(r)) \leq \bar{I}(r)$, since the separating allocation provides G types with funds at a weakly lower average cost than the pooling contract.
3. $I^G(r, W^*(r)) \geq W^*(r)$ whenever the wealth constraint is binding.

We start by analyzing the case in which $\bar{p} = \frac{p^B \alpha^B}{\alpha^G}$. In such a scenario, $\bar{I}(r) = I^B(r)$ and (48) reduces to

$$\alpha^B p^B f(I^G(r, W^*(r))) - \frac{p^B \alpha^B}{p^G \alpha^G} r [I^G(r, W^*(r)) - W^*(r)] = \alpha^B p^B f(I^B(r)) - r [I^B(r) - W^*(r)] \quad (49)$$

Incentive compatibility, on the other hand, implies that

$$\alpha^B p^B f(I^G(r, W^*(r))) - \frac{p^B}{p^G} r [I^G(r, W^*(r)) - W^*(r)] = \alpha^B p^B f(I^B(r)) - r [I^B(r) - W^*(r)]$$

so that both conditions can hold if and only if

$$I^G(r, W^*(r)) = W^*(r)$$

From this equality and (49), the following equality follows

$$I^G(r, W^*(r)) = W^*(r) = I^B(r)$$

ultimately delivering that

$$I^G(r, W^*(r)) = W^*(r) = I^B(r) = \bar{I}(r)$$

If, on the other hand, $\bar{p} > \frac{p^B \alpha^B}{\alpha^G}$, $\bar{I}(r) > I^B(r)$ together with (48) imply that

$$\alpha^G \bar{p} f(I^G(r, W^*(r))) - \frac{\bar{p}}{p^G} r [I^G(r, W^*(r)) - W^*(r)] = \alpha^G \bar{p} f(\bar{I}(r)) - r [\bar{I}(r) - W^*(r)]$$

We claim that $I^G(r, W^*(r)) > I^B(r)$ and argue by contradiction. Since the opposite strict inequality can never hold, we assume that they are equal. By incentive compatibility,

$$I^G(r, W^*(r)) = W^*(r)$$

in which case the switching point is defined by the following equality

$$\alpha^G \bar{p} [f(\bar{I}(r)) - f(I^G)] = r [\bar{I}(r) - W^*(r)]$$

which can never hold due to the concavity of the investment function. By the latter, in fact, and invoking the characterization of $\bar{I}(r)$

$$r [\bar{I}(r) - W^*(r)] = \alpha^G \bar{p} [f(\bar{I}(r)) - f(I^G)] > [\bar{I}(r) - I^G(r, W^*(r))]r$$

which cannot be true if $I^G(r, W^*(r)) = W^*(r)$.

Thus, it must be the case that $I^G(r, W^*(r)) > I^B(r)$. Together with incentive compatibility, this implies that

$$I^G(r, W^*(r)) > W^*(r) > I^B(r)$$

Hence, whenever $\bar{p} > \frac{p^B \alpha^B}{\alpha^G}$, the separating allocation at the switching point will have G -type contracts with larger loans than their B -type counterparts.

7.3 Behavior of $W^*(r)$

In the present subsection we show that there is a positive relationship between the levels of entrepreneurial wealth and the interest rate r . We start from this indifference condition,

$$\alpha^G p^G f(I^G(r, W^*(r))) - r I^G(r, W^*(r)) = \alpha^G p^G f(\bar{I}(r)) - r \bar{I}(r) \frac{p^G}{\bar{p}} - [1 - \frac{p^G}{\bar{p}}] r W^*(r)$$

which allows us to obtain the following expression for $W^*(r)$:

$$\frac{\alpha^G p^G f(I^G(r, W^*(r))) - r I^G(r, W^*(r)) - \alpha^G p^G f(\bar{I}(r)) + r \bar{I}(r) \frac{p^G}{\bar{p}}}{[\frac{p^G}{\bar{p}} - 1]r} = W^*(r)$$

Differentiation yields

$$\frac{r[\alpha^G p^G f'(I^G) - r] \frac{I^G \frac{p^B}{p^G} + W[1 - \frac{p^B}{p^G}] - I^B}{\alpha^B p^B f'(I^G) - \frac{p^B}{p^G} r} + \alpha^G p^G [f(\bar{I}) - f(I^G)]}{r\{\frac{p^G}{\bar{p}}r - r\} - [\alpha^G p^G f'(I^G) - r] \frac{dI^G}{dW}} = W^{*'}(r) \quad (50)$$

When $\bar{I} > I^G$ (as at the switching point under our assumptions) the numerator in (50) is positive. The denominator can be signed by noting that, at the switching point,

$$\alpha^G p^G f'(I^G) > \frac{p^G}{\bar{p}}r$$

whereas incentive compatibility implies that

$$\frac{dI^G}{dW} = \frac{[1 - \frac{p^B}{p^G}]r}{\alpha^B p^B f'(I^G) - \frac{p^B}{p^G} r} \geq 1 \Leftrightarrow I^G \geq I^B$$

These two conditions jointly imply that the denominator is negative whenever $\bar{I} > I^G$, which allows us to conclude that

$$\bar{p} > \frac{\alpha^B p^B}{\alpha^G} \Leftrightarrow \bar{I} > I^G \Rightarrow W^{*'}(r) < 0$$

7.4 Equilibrium when r is endogenously determined

Proof. [Lemma 6] We wish to analyze the conditions under which total investment, denoted by $\vartheta(r, W)$, is decreasing in r . Total investment is derived from bank loans. If the equilibrium in the loans market is pooling, we have

$$\vartheta(r, W) = (\lambda^B + \lambda^G)\bar{I}(r)$$

so that $\vartheta'_r(r, W) < 0$, since $\bar{I}'(r) < 0$.

If the equilibrium in the loans market is separating, on the other hand,

$$\vartheta(r, W) = \lambda^B I^B(W, r) + \lambda^G I^G(W, r)$$

which has the following derivative with respect to the interest rate,

$$\vartheta'_r(r, W) = \lambda^B \frac{dI^B}{dr} + \lambda^G \frac{dI^G}{dr} < 0$$

If the separating allocation is such that $I^B > I^G$, then the previous condition is trivially satisfied, since $\frac{dI^B}{dr} < 0$ and $\frac{dI^G}{dr} < 0$. When $I^G > I^B$, on the other hand, we know that $\frac{dI^B}{dr} < 0$ while $\frac{dI^G}{dr} > 0$ and therefore the sign of $\vartheta'_r(r, W)$ is ambiguous. From the incentive compatibility constraint,

$$\frac{dI^G}{dr} = \frac{[I^G \frac{p^B}{p^G} + (1 - \frac{p^B}{p^G})W - I^B]}{\alpha^B p^B f'(I^G) - \frac{p^B}{p^G} r} > 0$$

whereas the absence of distortions in B - *type* loans deliver,

$$\frac{dI^B}{dr} = \frac{1}{\alpha^B p^B f'(I^B)} < 0$$

Thus, what we require is that,

$$\left| \frac{1}{\alpha^B p^B f'(I^B)} \right| \lambda^B > \frac{[I^G \frac{p^B}{p^G} + (1 - \frac{p^B}{p^G})W - I^B]}{\alpha^B p^B f'(I^G) - \frac{p^B}{p^G} r} \lambda^G$$

which is more likely to be satisfied the less concave technology. ■

The intuition behind this condition is as follows: in the case of pooling contracts, it is easy to see that the total amount of credit increases as the interest rate falls. In the case of separating contracts when $I^G < I^B$ this is also true, since the former responds to interest rate changes in the same direction as the latter. The problem, however, arises for the case of separating contracts when $I^G > I^B$: in such situations, as can be seen from (42), a decrease in r expands I^B but contracts I^G . Therefore, the total amount lent to entrepreneurs might contract or expand as the interest rate decreases. Condition (16) says that - as long as the optimal B - *type* investment is sufficiently responsive to changes in r - total credit will move in a direction opposite to that of r .

Proof. [Proposition 7] Consider an economy in which entrepreneurial and consumer levels of wealth are given by W and A , respectively. By Inada conditions, it must be the case that total investment satisfies

$$\begin{aligned} \lim_{r \rightarrow \infty} \vartheta(r, W) &= 0 \\ \lim_{r \rightarrow 0} \vartheta(r, W) &= \infty \end{aligned}$$

regardless of whether equilibrium contracts are pooling or separating. In the case in which $\bar{p} = \frac{\alpha^B p^B}{\alpha^G}$, moreover, $\vartheta(r, W)$ is continuous in r and hence there must exist a level of the latter for which:

$$\vartheta(r, W) = MA + W$$

When $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$, $\vartheta(r, W)$ is discontinuous at r^* for which $W = W^*(r^*)$. At r^*

$$\lambda^B I^B(r^*) + \lambda^G I^G(r^*, W) < \bar{I}(r^*)$$

so that an equilibrium in deterministic contracts may fail to exist if

$$\lambda^B I^B(r^*) + \lambda^G I^G(r^*, W) < MA + W < \bar{I}(r^*) \quad (51)$$

Existence can be restored in such a case by introducing random contracts. A random contract is an array $\{\omega, (I^B, R^B, c^B), (I^G, R^G, c^G), (\bar{I}, \bar{R}, \bar{c})\}$ as shown in [31], where $\omega \in [0, 1]$ and the contract triples are as previously defined. The introduction of random contracts does not change the characteristics of optimal contracts. Whenever the optimal contract is separating (pooling) among the set of deterministic contract, so it is in the set of random contracts. For the switching value of r that yields as optimal both the pooling and the separating contracts, the set of optimal contracts coincides with the latter together with an indeterminate probability ω . Hence, the introduction of random contracts is, from an efficiency point of view, irrelevant. It is, however, relevant from an equilibrium point of view, because the indeterminacy of ω and the law of large numbers provide a natural randomization device that restores market clearing.

Indeed, from (51), there is a unique value of ω for which

$$\omega[\lambda^B I^B(r^*) + \lambda^G I^G(r^*, W)] + (1 - \omega)\bar{I}(r^*) = MA + W$$

Finally, as remarked in [31], this type of optimal random contracts are indeed a Nash equilibrium of the Hellwig three-stage specification. The reason is very simple. At the switching point, both the pooling and the separating contracts yield the same profits to G entrepreneurs, whereas B entrepreneurs prefer the pooling to the separating contracts. Hence, the set of profitable deviations by banks is empty.

Thus, an equilibrium always exists although it may entail randomization between pooling and separating contracts. Regarding uniqueness, this characteristic can be guaranteed if the investment mapping is monotonically decreasing in r , which is the case under (16). In such a scenario, there is a unique level of r that clears the market. ■

7.5 Dynamics

7.5.1 Existence of a Stable Steady state under Adverse Selection

Proof. [Lemma 8] Regardless of the presence of asymmetric information, we know that $f'(k) \rightarrow \infty$ as $k \rightarrow 0$. On the other hand, it is also the case that $f'(k) \rightarrow 0$ as $k \rightarrow \infty$. Finally, for a given initial condition, the capital accumulation path under the separating regime is bounded from above by its full information counterpart. Therefore, this economy must display at least one stable steady state which cannot have a higher capital stock than k^* . ■

7.5.2 Slope of Indifference Mapping $S^{SW}(w_t)$

In the present subsection we show that there is a positive relationship between the levels of entrepreneurial wealth and the expected rental prices of capital that make G entrepreneurs indifferent

between pooling and separating contracts. We start from this indifference condition,

$$q^e \alpha^G p^G f(q^e, r, W^*(r, q^e)) - r I^G(q^e, r, W^*(r, q^e)) = q^e \alpha^G p^G f(\bar{I}(r, q^e)) - r \bar{I}(r, q^e) \frac{p^G}{\bar{p}} - [1 - \frac{p^G}{\bar{p}}] r W^*(r, q^e)$$

which allows us to obtain the following expression for $W^*(r, q^e)$:

$$\frac{q^e \alpha^G p^G f(I^G(q^e, r, W^*(r, q^e)) - r I^G(q^e, r, W^*(r, q^e)) - q^e \alpha^G p^G f(\bar{I}(r, q^e)) + r \bar{I}(r, q^e) \frac{p^G}{\bar{p}}}{[\frac{p^G}{\bar{p}} - 1]r} = W^*(r, q^e)$$

Differentiation yields

$$\frac{r[q^e \alpha^G p^G f'(I^G) - r] \frac{\alpha^B p^B [f(I^B) - f(I^G)]}{q^e \alpha^B p^B f'(I^G) - \frac{p^B}{p^G} r} + \alpha^G p^G [f(I^G) - f(\bar{I})]}{r[\frac{p^G}{\bar{p}} - 1] - [q^e \alpha^G p^G f'(I^G) - r] \frac{dI^G}{dW}} = \frac{dW^*}{dq^e} \quad (52)$$

When $\bar{I} > I^G$ (as at the switching point under our assumptions) the numerator in (52) is negative. The denominator can be signed by noting that, at the switching point,

$$\alpha^G p^G f'(I^G) > \frac{p^G}{\bar{p}} \frac{r}{q^e}$$

whereas incentive compatibility implies that

$$\frac{dI^G}{dW} = \frac{[1 - \frac{p^B}{p^G}]r}{q^e \alpha^B p^B f'(I^G) - \frac{p^B}{p^G} r} \geq 1 \Leftrightarrow I^G \geq I^B$$

These two conditions jointly imply that the denominator is negative whenever $\bar{I} > I^G$, which allows us to conclude that

$$\bar{p} > \frac{\alpha^B p^B}{\alpha^G} \Leftrightarrow \frac{dW^*}{dq^e} > 0$$

7.6 Regime Switching and Cycles

Proof. [Proposition 11] In order to prove the proposition, we start as in the main body of the paper by assuming an arbitrary economy in which $w_{POOL} > w_{SEP}$. We perturb this economy by changing the productivities of capital and labor in the production of the final good and the measure of households. Both of these modifications affect the relationship between w and q at equilibrium without affecting the mapping $S^{SW}(w_t)$, which characterizes the pairs (w_t, q^e) that make G -type entrepreneurs indifferent between the pooling and separating contracts.

In order simplify the presentation, we assume that the final good is produced by a Cobb-Douglas production function of the form

$$Y_t = \theta K_t^{1-\alpha} L_t^\alpha \quad (53)$$

and consider first an economy that is initially in Case 1, so that $w_{POOL} \leq w_1$. We show that we can construct an alternative economy in which $w_{SEP} < w_1 < w_{POOL}$.

We perturb (53) by increasing the productivity of labor α and the total factor productivity parameter θ so as to keep capital demand constant. Since the demand for capital is given by its marginal productivity, note that it is increasing in θ and decreasing in α

$$\begin{aligned} q &= \theta(1-\alpha)\left(\frac{K}{L}\right)^{-\alpha} \\ \frac{\partial q}{\partial \theta} &= (1-\alpha)\left(\frac{L}{K}\right)^\alpha > 0 \\ \frac{\partial q}{\partial \alpha} &= -\theta\left(\frac{K}{L}\right)^{-\alpha}(1-\alpha)\ln\left(\frac{K}{L}\right) - \theta\left(\frac{K}{L}\right)^{-\alpha} < 0 \text{ for } \frac{K}{L} > 1 \end{aligned}$$

Hence, we choose pairs (α_n, θ_n) so that

$$\begin{aligned} \alpha_n &> \alpha \\ \theta_n &> \theta \end{aligned}$$

for all n while also satisfying

$$\begin{aligned} \theta(1-\alpha)\left(\frac{L}{K_{POOL}}\right)^\alpha &= \theta_n(1-\alpha_n)\left(\frac{L}{K_{POOL}}\right)^{\alpha_n} \\ \left[\frac{\theta_n(1-\alpha_n)}{\theta(1-\alpha)}\right]^{\frac{1}{\alpha_n-\alpha}} &= \left(\frac{K_{POOL}}{L}\right) \end{aligned}$$

so that the marginal productivity of capital is preserved at $K = K_{POOL}$, and therefore the equilibrium of the capital market under the pooling regime is not altered. In the labor market, w_{POOL} must necessarily increase, since the price of labor relative to capital must increase and the latter is kept constant. Note additionally, that w_{POOL} can be increased arbitrarily in this manner, since at equilibrium it must be that

$$\frac{w_{POOL}}{q_{POOL}} = \frac{\alpha_n}{(1-\alpha_n)}\left(\frac{K_{POOL}}{L}\right)$$

An economy perturbed in this way will then experience an increase in the steady state value of wages under the pooling regime while keeping constant w_1 , which is unaffected since neither q_{POOL} nor the relationship $S^{SW}(w_t)$ as defined in the main body of the paper are modified.

The only thing left to verify is that w_{POOL} can in this way be increased beyond w_1 while preserving the ordering of steady states by which $w_{POOL} > w_{SEP}$. It is easy to verify that a

perturbation of the economy like the one proposed above can either decrease or increase w_{SEP} . In the former case there is no problem, since the relative ordering of the steady state wages are preserved. In the latter case, though, it must be checked that w_{POOL} increases beyond w_1 before w_{SEP} does. This we argue by contradiction: suppose, to the contrary, that in carrying out the perturbation w_{SEP} increases to the level of w_{POOL} before the latter surpasses w_1 . This cannot be the case, since it would imply that there is an economy with a steady state wages

$$\tilde{w} = w_{SEP} = w_{POOL} < w_1$$

so that both regimes produce the same amount of capital but G – *types* prefer nonetheless to pool. Due to our assumption by which $\bar{p} > \frac{\alpha^B p^B}{\alpha^G}$, however, this can never happen, since a scenario in which both regimes produce the same amount of capital must necessarily imply that the separating contracts are preferred by G – *type* entrepreneurs.

On the other hand, consider an economy that is initially in Case 2, so that $w_{SEP} > w_2$. In this case, we perturb the economy by increasing the measure of households while preserving those of G – and B – *type* entrepreneurs. By doing so, we expand the supply of labor which - at equilibrium - decreases wages and increases the price of capital q by expanding the demand for this factor. Consequently, w_{SEP} and w_{POOL} are contracted whereas q_{SEP} and q_{POOL} increase. Note once again that increasing the measure of households has no effect on the pairs (w_t, q_{t+1}) of switching points, so that the increase in q_{POOL} and q_{SEP} unambiguously increases w_1 and w_2 . By perturbing the economy in this manner, both steady state wages can be reduced below the switching wages: eventually, it must then be that

$$w_{SEP} < w_{POOL} < w_1 < w_2$$

since otherwise there would be wage levels $w < w_1$ for which separating contracts produce higher levels of capital but are nonetheless dominated by their pooling counterparts. Such a perturbation would then transform a Case 2 economy into a Case 1 economy, which as we showed before can itself be perturbed to yield a Case 3 economy. ■