Competition amongst Contests*

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and

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Abstract

This article analyses the optimal allocation of prizes in contests. While existing models on contest design take the set of participants as being exogenously given, in our model contests compete for participants. Our analysis provides a new argument for the widespread occurrence of prizes that reward suboptimal performance. Moreover, we identify a negative relationship between a contest's predictability and its competitiveness. In particular, we show that the equilibrium prize allocation becomes steeper when the correlation between participants' ability and performance decreases. Using data from professional road running, we provide empirical evidence to support this relationship.

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1 Introduction

The world is full of contests. In investment banks, financial analysts compete for promotions; in architectural competitions, architects contend for design contracts; and in sports contests, athletes compete for prize-money. How many hierarchical levels should an investment bank implement? How should an architectural competition be set up? And how should a sports contest distribute its prize-budget across ranks? The answer to these questions depends on the contest designer's objective.

Until now economists have focused exclusively on the contest designer's need to provide participants with *incentives to exert effort*. However, as potential participants often have to choose between several contests, the design of a contest is equally likely to affect the players' *incentives to participate*. For example, an investment banker may decide to work for bank A rather than bank B because it offers a steeper hierarchical structure and, in turn, a fast-tracking career. A marathon runner may enter the New York Marathon instead of the Chicago Marathon because it awards a larger fraction of its prize money to suboptimal performances. For similar reasons, an architect may be reluctant to devote his time and effort on a design proposal for the World Trade Center Site Memorial and instead participate in the design competitions for the London 2012 Olympic Park.

From the viewpoint of the contest designers, attracting entry, especially entry by certain key-players, is of utmost importance. Investment banks spend a considerable amount of resources on the hiring of graduates from top ranked MBA programs. Architectural competitions greatly benefit from the presence of prominent architects. And big-city marathons boost their media interest by securing the participation of elite runners.¹ Referring to the recent contests for European 3G telecom licenses, Paul Klemperer (2002) notes that “a key determinant of success of the European telecom auctions was how well their designs attracted entry [...]”. However, due to time or other resource constraints, players can only participate in one or a few of the many contests available.

¹For example, in 2006 organizers of three major marathons went head to head in their bid for the world record holder, Paula Radcliffe, and the U.S. record holder, Deena Castor (see “Marathons: Top Races are Vying for the Elite Runners”, *International Herald Tribune*, June 12, 2006).

In this paper we therefore consider a model in which several contests compete for the participation of a given set of key-players. We are especially interested in the implications that this competition has for the contests’ optimal design. In particular, we ask how a contest should allocate its prize budget across ranks, in order to attract the largest number of key-players.

In reality one observes significant differences in the distribution of prizes. Some contests choose to have an extremely flat prize structure or even award several identical prizes. For example, the 2005 architectural competition for the Flight 93 Memorial in Western Pennsylvania offered \$25000 to each of the five best designs. In contrast, other contests implement an extremely steep prize structure or even offer their entire prize budget to the winner. This “winner-takes-all” principle is often employed in science and engineering competitions.² Most contests, however, settle for an intermediate prize schedule. For example, the New York Marathon specifies ten prizes, awarding \$80000 to the winner and \$45000 for second place.

Our model offers an explanation for these differences in prize structures. In contrast to earlier articles on contest design we abstract from players’ effort decisions and instead focus on the players’ contest choice. Our explanation begins with the observation that in some contests the outcome depends almost entirely on the participants’ abilities (e.g. chess) whereas in others there exists a sizeable element of luck (e.g. poker). While in the former the winner can be predicted with reasonable accuracy, in the latter the outcome is more random. We show that there exists a negative relationship between a contest’s predictability and its competitiveness. In particular, contests with low predictability tend to choose a steep prize structure while contests with high predictability prefer a flat prize structure.

This result is the consequence of a simple trade-off. Consider, for example, two big-city marathons taking place at the same time (e.g. the London Marathon and the Boston Marathon). Suppose that both marathons aim to attract the world record holder and the current world champion and that these athletes make their contest

²The NASA 2007 Astronaut Glove Competition awards a single first prize of \$250000 to the designer of the best performing glove. Similarly, the DARPA (Defense Advanced Research Projects Agency) 2005 Grand Challenge awarded \$2 million to the fastest driverless car on a 132-mile desert course.

choice simultaneously and independently. On the one hand, if they happen to enter different races, both runners expect to win with high probability and therefore prefer steep prize structures. On the other hand, runners make their choice contingent on their privately known physical shape and are more likely to enter a race with a steep prize structure when they are in a good shape. Hence the world champion expects the world record holder to be in a better shape when he meets him in the contest with the steeper prize structure and vice versa. This effect makes steep prize structures less attractive. Which of the two effects dominates depends on the extent to which the outcome of the contest is influenced by the runners' shape as compared to external or random factors. We show that when random factors are sufficiently strong, both contests will implement the "winner-takes-all" principle, whereas when random factors are sufficiently weak, contests will award several identical prizes. In an intermediate range, multiple but strictly decreasing prizes turn out to be optimal.

Using data from professional road running, we provide empirical evidence for this negative relationship between a contest's predictability and its competitiveness. We argue that the distance of a race is a measure for its predictability. As the distance increases a race becomes less predictable.³ We investigate whether prize structures become steeper as the race distance increases. We find that as the race distance increases, there is a monotonic increase in the ratio between first and second prize. For example, as the race moves from 5km to 42km, the ratio between the first and the second prize increases by 4 percentage points. We find qualitatively similar results when using alternative measures of competitiveness and after controlling for various important factors.

This paper is the first to model how contests compete for participants and to derive the implications for the optimal allocation of prizes. Our analysis provides a new argument for the wide spread occurrence of prizes that award sub-optimal performances. Until now the literature on contest design has mostly considered single contests with an exogenously given set of participants. Authors have tried to determine the prize

³While in a 5km race the prediction of the winner based on past performance turns out to be correct in 43% of the cases, this number reduces to 20% for a marathon. For details see Section 6.

structure that maximizes the participants' aggregate effort.⁴ For example, in their seminal paper Moldovanu and Sela (2001) show that the optimal allocation of prizes depends critically on the shape of participants' cost of effort functions. Multiple prizes become optimal when the costs of effort are sufficiently convex. Additional justification for the use of multiple prizes has been derived from players' risk aversion (Krishna and Morgan (1998)) and players' heterogeneity (Szymanski and Valletti (2005)). On the other hand some papers provide arguments for the use of a single (Clark and Riis (1998)) or large (Rosen (2001)) first prize. In this paper, we show that in the presence of several contests, multiple prizes arise naturally from the contest designers' need to attract participants.

Although some papers endogenize the set of participants, they maintain the focus on a single contest. Taylor (1995) and Fullerton and McAfee (1999), for example, study how the set of participants, and hence the expected winning performance, in a research tournament varies with its entry fee.

Competition for participants has attracted some attention in the literature on auction and mechanism design. McAfee (1993), Peters and Severinov (1997) and Burguet and Sakovics (1999) for example, consider models in which auctions compete for bidders. However, while in our model contests compete via their prize allocation, in these papers, prizes are fixed and auctions compete by using their reservation price. Our insights apply here because in some auctions, designers may influence the "prize structure" by determining the number and the value of the objects on sale. For example, besides a reserve prize, organizers of the European 3G telecom auctions were able to choose the number and the spectrum size of their licenses. In the light of the above quote it is important to understand how this choice affected entry.

The literature on labor tournaments is also relevant here. Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), and Mookherjee (1984) have shown that the introduction of some form of contest among workers could provide optimal incentives to exert effort inside a firm. While Green and Stokey (1983) and

⁴Other issues considered by this literature include the splitting of a contest into various sub-contests (Moldovanu and Sela (2007)) and the seeding of players in elimination tournaments (Groh et. al. (2003)).

Mookherjee (1984) take the set of workers as exogenously given, Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983) assume a competitive labor market in which each firm hires a fixed number of workers. In these papers each worker always meets the same number of opponents, whereas our results are driven by the fact that a player's set of opponents itself depends on the contest's allocation of prizes. Our theory could be applied to this set up, which would allow us to derive the implications of a firm's hierarchical structure on its hiring success in the labor market.

The paper is organized as follows. In Section 2 we describe the theoretical model. Sections 3 and 4 consider contests with low and high levels of predictability, respectively. These sections contain our main result concerning the negative relation between a contest's predictability and competitiveness. In Section 5 we discuss the monotonicity of this relationship. Section 6 tests the theoretical model using data on professional road running. Section 7 concludes. All proofs and empirical tables are contained in the Appendix.

2 The model

We consider two contests, $i \in \{1, 2\}$, and $N \geq 2$ risk neutral players.⁵ Each player can participate in, at most, one of the two contests because of time or other resource constraints. It is the contests' objective to attract as many as possible of the N players. Note that in some situations attracting too many participants might be disadvantageous as players might be discouraged from exerting effort.⁶ It is therefore useful to think of the N players as being "key-players" or "stars" whose participation is desirable due to their exceptional ability or fame.

We focus on the extremely able players and neglect those who would only have a minor chance of winning. Nevertheless, there may exist small ability differences

⁵As it is our aim to explain the occurrence of multiple prizes, risk neutrality is the most conservative assumption. Risk aversion makes players prefer flatter prize structures, giving contests an additional incentive to award several prizes.

⁶Taylor (1995) and Fullerton and McAfee (1999) for example show that in order to maximize the expected winning performance, entry into a research tournament should be restricted. In contrast, Moldovanu, Sela and Shi (2007) provide conditions under which aggregate effort in a status contest increases in the number of players.

within the group of key-players. For example, a famous architect may or may not be having a creative period and an elite marathon runner may or may not be in shape. We therefore assume that a player's performance depends on his ability, a , and that ability may take two values, i.e. $a \in \{a_L, a_H\}$. In order to simplify terminology we will denote these abilities as low and high. However, it is important to bear in mind that from the viewpoint of the contest organizers these ability differences are negligible as key-players are, by definition, much more able than the rest. The probability that a key-player has ability a_H is $h \in (0, 1)$ and abilities are private information.

Apart from possible differences in the allocation of prizes, contests are homogeneous. We assume that contests face the same prize budget. The value of the prize budget is normalized to 1. Contests must choose how to distribute their prize budget across ranks. In particular, contest i chooses a *prize structure*, i.e. a vector of non-negative real numbers $V_i = (V_i^1, V_i^2, \dots, V_i^N)$ such that V_i^n is (weakly) decreasing in n and $\sum_{n=1}^N V_i^n = 1$. V_i^n denotes the prize that contest i pays to the player with the n -th highest performance. The set of feasible prize structures is denoted as $\mathcal{V}$ and we assume that it is feasible to award the entire prize budget to the winner, i.e. $(1, 0, \dots, 0) \in \mathcal{V}$, and to choose N identical prizes, i.e. $(\frac{1}{N}, \frac{1}{N}, \dots, \frac{1}{N}) \in \mathcal{V}$.⁷ In order to compare different prize structures we make the following definition:

Definition 1 *The prize structure V_i is more competitive than the prize structure V_j ($V_i \succ V_j$) if $\sum_{m=1}^n V_i^m \geq \sum_{m=1}^n V_j^m$ for all $n \in \{1, 2, \dots, N\}$ with strict inequality for some n .*

Note that two arbitrary prize structures may not be comparable according to this relation but $(1, 0, \dots, 0)$ is more competitive than all other prize structures and all prize structures are more competitive than $(\frac{1}{N}, \frac{1}{N}, \dots, \frac{1}{N})$.

The timing is as follows. First, contests simultaneously choose their prize structures, V_i , and nature determines each player's ability. Second, players simultaneously decide which contest to enter.⁸ We denote the subgame, which starts after contests have

⁷ $\mathcal{V}$ may be identical to $\{V \in \mathbb{R}_+^N | V^1 \geq V^2 \geq \dots \geq V^N, \sum_{n=1}^N V^n = 1\}$ but may differ in the presence of indivisibilities (see Section 5).

⁸While our results remain unchanged when contests are allowed to choose their prize structure sequentially, the assumption that entry takes place simultaneously is crucial as it rules out coordination.

announced the prize structures V_1 and V_2 , as the (V_1, V_2) entry game. Finally, players perform and then prizes are distributed.

Our objective is to explain the occurrence of multiple prizes as a means of attracting participants, rather than to maximize players' effort. We therefore abstract from the players' choice of effort. We assume that a player's expected payoff in contest i , $u(a, n_L, n_H, V_i; p)$, depends on the player's ability, a , the number of low and high ability opponents, n_L and n_H , and the contest's prize structure, V_i . The parameter $p \in [0, 1]$ denotes the contests' *predictability*. This is a crucial parameter in our model, which represents the correlation between players' ability and performance. Here, p takes into account the fact that in some contests winning depends entirely on ability (e.g. chess), whereas in others there is an element of luck (e.g. poker). We assume that when $p = 0$, ability differences among key-players are negligible and every key-player is equally likely to win. In contrast, when $p = 1$ key-players with ability a_H outperform those with ability a_L with certainty. We assume that $u(a, n_L, n_H, V_i; p)$ is continuous in p .

Note that as participation is assumed to be costless, players prefer to participate in *some* contest rather than to not participate at all. A player's strategy therefore consists of the probabilities, $q_H \in [0, 1]$ and $q_L \in [0, 1]$, of entering contest 1 conditional on having high or low ability. As players are identical ex ante, we restrict our attention to the *symmetric* Bayesian Nash equilibria of the entry game, i.e. we assume that q_H and q_L are the same for all players. A player's ex ante probability of entering contest 1 is $q = hq_H + (1 - h)q_L$. The expected number of participants is $Q_1 = Nq$ in contest 1 and $Q_2 = N(1 - q)$ in contest 2.

3 Low predictability

It is often the case that a contest's outcome does not only depend on the players' abilities but is influenced, to a large extent, by external factors. For example, in the financial market, luck is considered to be an important determinant of a portfolio's profitability. As Nassim Taleb (2005), a legend amongst option traders, wrote: "We tend to think that traders are successful because they are good [...] (but) one can make money in the financial market totally out of randomness." As a consequence, a

financial analyst's career path in an investment bank's promotional contest is marked by uncertainty.

In this section we focus on the case of low values of p and consider contests in which luck and other external factors are relatively important. In this case, possible ability differences amongst the key-players play only a minor role. In the extreme case where $p = 0$ all key-players are equally likely to win, irrespective of whether their ability is a_L or a_H . A player's expected payoff in contest i is independent of his own ability and depends only on the *total* number of key-players $n = n_L + n_H$ who chose to enter the same contest, i.e.

$$u(a_H, n_L, n_H, V_i; 0) = u(a_L, n_L, n_H, V_i; 0) = \frac{\sum_{m=1}^{n+1} V_i^m}{n+1}. \quad (1)$$

Suppose that contest 1 chooses a more competitive prize structure than contest 2, i.e. $V_1 \succ V_2$. In both contests, a player's expected payoff conditional on meeting n key-players is decreasing in n . However, in contest 1 this payoff is at least as large as in contest 2 for all $n \in \{0, 1, \dots, N-1\}$ and strictly greater for some n . In order to offset this advantage of contest 1, the probability to meet key-players has to be smaller in contest 2. In other words, for $p = 0$, it must be the case that contest 1 expects a larger number of key-players. The continuity of $u(a, n_L, n_H, V_i; p)$ in p implies that this result continues to hold when the contests' predictability is sufficiently small. This finding is summarized in the following lemma:

Lemma 1 *Suppose that contest 1 has a more competitive prize structure than contest 2, i.e. $V_1 \succ V_2$. Then there exists a $\underline{p}(V_1, V_2) > 0$ such that for all $p \leq \underline{p}(V_1, V_2)$ and any equilibrium of the (V_1, V_2) entry game, the expected number of key-players is strictly larger in contest 1.*

Lemma 1 shows that for sufficiently low values of predictability, the contest with the more competitive prize structures is more attractive to key-players. The competitive prize structure is preferred as it guarantees a higher payoff in the presence of few key-players. On the other hand, as we will see in the next sections, key-players expect their opponents to be stronger in the contest with the more competitive prize structure.

When the contests' outcome is sufficiently random, this second effect becomes negligible and the contest with the more competitive prize structure is more attractive.

The immediate implication of Lemma 1 is that for sufficiently low values of predictability, contests have an incentive to choose a more competitive prize structure than their rival. In particular contest i has an incentive to choose a $V_i \succ V_j$ such that $p < \underline{p}(V_i, V_j)$. If $V_j \neq (1, 0, \dots, 0)$ and p is sufficiently small then contest i can always find a $V_i \succ V_j$ that satisfies this inequality. Hence for sufficiently small levels of predictability every contest prefers to be the most competitive one and we get the following result:

Proposition 1 *There exists a $\underline{p} \geq 0$ such that for all $p \in [0, \underline{p}]$ in equilibrium both contests choose to be winner-takes-all contests, i.e. $V_1^* = V_2^* = (1, 0, \dots, 0)$. If $\mathcal{V}$ is finite then $\underline{p} > 0$.*

According to Proposition 1, contests award their entire prize budget to the winner when ability and performance are sufficiently uncorrelated. As key-players prefer competitive prize structures, each contest tries to implement a more competitive prize structure than its rival. The resulting Bertrand style competition leads to “winner-takes-all” contests.

Note that the existence of *several* contests is crucial for this result. If there was only one contest and all players had the same outside option irrespective of their ability, then players with a_H would prefer more competitive prize structure whereas players with a_L would prefer less competitive prize structure. As the former expect a higher payoff from participating in the contest than the latter, the contest would try to attract all players by choosing the least competitive prize structure $(\frac{1}{N}, \frac{1}{N}, \dots, \frac{1}{N})$. Our results are driven by the fact that each player's “outside option” depends on the other players' contest choice.

A recent review of financial packages at Wall Street approximated that the average entry level annual payments to an analysts was \$150,000 while a top Managing Director could receive as much as \$8 million (including bonuses), making the “top prize” more than 53 times the “lowest prize”.⁹ So far the literature on contest design has explained

⁹“How much am I worth? M&A banker, leading investment bank, Wall Street”, *Institutional*

the steep hierarchies found in the financial industry by referring to the firms' attempt to provide employees with optimal incentives to exert effort. In the light of Proposition 1 they can be seen as a consequence of investment banks' competition for the best employees. Those firms that offer the steepest hierarchies and the fast track careers attract the most graduates from top ranked MBA programs.

4 High predictability

In many contests the winners can be predicted with reasonable accuracy. For example, examining data from more than 200,000 chess tournament games, Glickman and Jones (1999) find that, given a 200 point difference in the "Elo system", higher ranked players are about three times more likely to win than lower ranked players.¹⁰ Similarly, architectural contests are frequently won by the most highly acclaimed architects. In these examples external or random factors are relatively unimportant and contest outcomes are mainly determined by the participants' abilities.

In this section we therefore turn our attention to the case of high values of predictability. We will see that in this case, the key-players' contest choice and hence its implications for the optimal allocation of prizes, are very different from the case of low predictability.

Intuition suggests that key-players with ability a_H have a stronger propensity to enter a contest with a competitive prize structure than key-players with a_L . A prominent architect is more inclined to participate in the most competitive contests during a creative period and an elite runner is more likely to enter the most competitive races when he is in top-shape. Hence, when predictability is sufficiently high, the fact that more competitive prize structures guarantee higher prizes in the presence of few key-players may be offset by the participation of stronger rather than more numerous key-players.

To see this, consider the case where ability and performance are perfectly correlated, i.e. $p = 1$. Here a key-player with ability a_H is certain to outperform a key-player

Investor Magazine, 2006.

¹⁰The Elo system ranks chess players by calculating a numerical rating between 0 and 3000 based on past performances in competitive chess.

with ability a_L and his expected payoff in contest i is

$$u(a_H, n_L, n_H, V_i; 1) = \frac{\sum_{m=1}^{n_H+1} V_i^m}{n_H + 1}. \quad (2)$$

If $V_1 \succ V_2$, then an argument similar to the one in Section 3 implies that in equilibrium the expected number of key-players with ability a_H must be strictly larger in contest 1, i.e. $q_H^* > \frac{1}{2}$. Their presence, however, discourages the participation of key-players with a_L leading to a reduction in the *total* number of key-players when prize structures are sufficiently similar.

To see this suppose that both contests offer first and second prizes only, i.e. $V_i = (V_i^1, 1 - V_i^1, 0, \dots, 0)$. If the difference between V_1^1 and V_2^1 is sufficiently small then q_H^* has to be close to $\frac{1}{2}$ and $hq_H^* < \frac{1}{2}$. In particular, in the proof of Lemma 2 we show that if we define $\bar{V}(V_1^1, N, h) < V_1^1$ as

$$\bar{V}(V_1^1, N, h) \equiv \begin{cases} (1-h)^{N-1}V_1^1 + \sum_{n=1}^{N-1} \frac{\binom{N-1}{n}}{n+1} h^n (1-h)^{N-1-n} & \text{if } h \leq \frac{1}{2} \\ (\frac{1}{3-2h})^{N-1}V_1^1 + \sum_{n=1}^{N-1} \frac{\binom{N-1}{n}}{n+1} [(\frac{1}{3-2h})^{N-1} - (\frac{2h-1}{3-2h})^n] & \text{if } h > \frac{1}{2} \end{cases} \quad (3)$$

then for all $V_2^1 \in (\bar{V}, V_1^1)$ it holds that high ability players enter contest 1 with probability $q_H^* \in (\frac{1}{2}, \min(\frac{1}{2h}, 1))$ and $hq_H^* < \frac{1}{2}$. If key-players with ability a_L enter contest 1 with probability $\bar{q}_L = \frac{1}{1-h}(\frac{1}{2} - hq_H^*)$ then both contests expect the same number of key-players and the probability to meet opponents is identical in both contests. However, as $q_H^* > \frac{1}{2}$ and $\bar{q}_L < \frac{1}{2}$ the opponents in contest 1 are, on average, stronger than the opponents in contest 2. Moreover, if the fraction h of key-players with ability a_H is sufficiently large then key-players with ability a_L prefer the contest with the less competitive prize structure even when contests are identical with respect to their participation (i.e. when $q_H = q_L = \frac{1}{2}$). Hence if h is sufficiently large then in equilibrium players with a_L will enter contest 1 with probability $q_L^* < \bar{q}_L$ leading to a higher total number of key-players in contest 2. Lemma 2 shows that this intuition continues to hold as long as contests are sufficiently predictable.

Lemma 2 *Suppose that $V_i = (V_i^1, 1 - V_i^1, 0, \dots, 0)$ and contest 1 has a (slightly) more competitive prize structure than contest 2, i.e. $V_1^1 > V_2^1 > \bar{V}(V_1^1, N, h)$. If $h \geq \bar{h}$, then there exists a $\bar{p}(V_1, V_2) < 1$ such that for all $p \geq \bar{p}(V_1, V_2)$ and any equilibrium of the*

(V_1, V_2) entry game, the expected number of key-players is strictly larger in contest 2. For $N \leq 3$, $\bar{h} = 0$ and for $N > 3$, $\bar{h}$ is the solution to $h(2-h)^{N-1} = 1$ in $(0, \frac{1}{N-1})$.

One might argue that although the contest with the less competitive prize structure attracts more key-players, key-players are expected to be (slightly) stronger in the other contest. Here it is crucial that, from the viewpoint of the contest organizers, ability differences across key-players are negligible. Organizers aim to attract key-players no matter whether their ability is a_L or a_H . For example, the organizer of an architectural competition may be especially concerned that his building is associated with the name of a famous architect. Similarly, big-city marathons benefit from the media interest created by the participation of the world champion but whether this runner is in top-shape and runs one minute faster or slower is of secondary interest.

Lemma 2 has an immediate implication for the contest designers' choice of prize structure when contests are restricted to distribute their prize budget between a first and a second prize. For sufficiently high values of predictability, each contest has an incentive to choose a (slightly) less competitive prize structure than its rival. In particular if $(\frac{1}{2}, \frac{1}{2}, 0, \dots, 0) \in \mathcal{V}$ and $V_j^1 > \frac{1}{2}$, contest i can attract more key-players than contest j by choosing a (slightly) smaller first prize V_i^1 such that $V_j^1 > V_i^1 > \bar{V}(V_j^1, N, h)$ and $\bar{p}(V_j, V_i) < p$. In the next section we will see that in the presence of indivisibilities a $V_i \in \mathcal{V}$ that satisfies the first condition may not exist. However, as $\bar{V}(V_j^1, N, h)$ decreases in N with $\lim_{N \rightarrow \infty} \bar{V}(V_j^1, N, h) = 0$, the condition is automatically satisfied when the number of players is sufficiently large. We therefore get the following result:

Proposition 2 *Suppose that $\mathcal{V} \subset \{(V^1, V^2, 0, \dots, 0) | V^1, V^2 \in \mathbb{R}_+, V^1 + V^2 = 1\}$ and $(\frac{1}{2}, \frac{1}{2}, 0, \dots, 0) \in \mathcal{V}$. If N is sufficiently large so that $h(2-h)^{N-1} \geq 1$ and $\bar{V}(1, N, h) \leq \frac{1}{2}$ then there exists a $\bar{p} \leq 1$ such that for all $p \in [\bar{p}, 1]$, in equilibrium both contests choose the least competitive of all feasible prize structures, i.e. $V_1^* = V_2^* = (\frac{1}{2}, \frac{1}{2}, 0, \dots, 0)$. If $\mathcal{V}$ is finite then $\bar{p} < 1$.*

Note that for intermediate values of h , the conditions of Proposition 2 are satisfied for relatively small N . For example, if $h = \frac{1}{2}$ then the statement in Proposition 2 holds for all $N > 3$. Proposition 2 provides an explanation for the wide spread occurrence

of second prizes. According to Proposition 2, contests will offer several identical prizes when the correlation between players' ability and performance is sufficiently strong. Although a flatter prize structure is less attractive for key-players with a_H , the absence of these players encourages the entry of key-players with a_L , leading to an increase in the total number of key-players when prize structures are sufficiently similar. In the resulting Bertrand style competition, each contest has an incentive to (slightly) undercut the first prize of its rival. In equilibrium both contests award two identical prizes. A recent example in which several identical prizes were awarded is the 2005 design competition for the Flight 93 Memorial in Somerset County, Western Pennsylvania. This competition awarded five identical prizes of \$25,000 each to the five best designs.

5 Monotonicity

Our results in Sections 3 and 4 suggest that there exists a negative relationship between a contest's predictability and its competitiveness. Contests in which the correlation between players' ability and performance is low choose to award their entire prize budget to the winner. In contrast, when ability and performance are sufficiently correlated, multiple identical prizes become optimal. In this section we provide an example for which this relationship is monotone, in that equilibrium first prizes decrease monotonically in the contests' predictability. The example fixes the number of key-players $N = 2$ and their composition $h = \frac{1}{2}$ but makes no further assumptions about the relationship between ability and performance. For $N = 2$ contest i 's prize structure can be fully described by its first prize. Hence in the following V_i will denote contest i 's prize structure as well as the corresponding first prize. Moreover, we denote by $\gamma(p)$ the probability with which a player with ability a_H wins against a player with ability a_L .

Example 1 *Suppose that $N = 2$, $h = \frac{1}{2}$ and $\gamma(p)$ is strictly increasing.*

1. *For $V_1 > V_2$ the (V_1, V_2) entry game has a unique equilibrium $(q_L^*, q_H^*) \in [0, 1]^2$. $q_H^* > q_L^*$ for all p and Q_1 is monotonically decreasing in p . In particular, if we define $\underline{p}(V_1, V_2) \equiv \gamma^{-1}(\frac{2V_1 - V_2 - \frac{1}{2}}{2V_1 - 1})$ and $\bar{p}(V_1, V_2) \equiv \gamma^{-1}(\min(1, \frac{V_1 - \frac{1}{2}}{2V_2 - 1}))$ then the*

following holds: If $p \in [0, \underline{p}(V_1, V_2))$ then $q_H^* = 1$, $q_L^* \in (0, 1)$, and $Q_1 > Q_2$. If $p \in [\underline{p}(V_1, V_2), \bar{p}(V_1, V_2)]$ then $q_H^* = 1$, $q_L^* = 0$, and $Q_1 = Q_2$. Finally, if $p \in (\bar{p}(V_1, V_2), 1]$ then $q_H^* \in (\frac{1}{2}, 1)$, $q_L^* \in [0, 1 - q_H^*)$, and $Q_1 < Q_2$.

2. Suppose that $\mathcal{V} = \{(\frac{1}{2}, \frac{1}{2}), (\frac{1}{2} + \frac{1}{M}, \frac{1}{2} - \frac{1}{M}), \dots, (1, 0)\}$ for M integer and even. Two prize structures with $V_1 \geq V_2$ form an equilibrium if and only if

$$\underline{p}(1, V_2) \leq p \leq \bar{p}(V_1, V_1 - \frac{1}{M}). \quad (4)$$

Equilibrium first prizes weakly decrease in the contests' predictability, i.e. if $V_1 = V_2 = V'$ is an equilibrium for $p = p'$ and $V_1 = V_2 = V''$ is an equilibrium for $p = p'' > p'$ then $V' \geq V''$.

The first part of Example 1 is concerned with the key-players' contest choice. It shows that the attractiveness of the more competitive contest decreases monotonically in the contests' predictability. Figure 1 provides a graphical representation of this result. For low levels of predictability, key-players with ability a_H enter contest 1 with certainty and key-players with ability a_L will mix. The contest with the steeper prize structure has a higher expected number of participants. For medium levels of predictability, both contests are equally attractive to participants. There is a complete separation of abilities since key-players with a_H will enter the more competitive contest, while key-players with a_L enter the less competitive contest.¹¹ Finally, if the difference between the contests' prize structures is sufficiently small then there exists a range of high levels of predictability for which the less competitive contest is more attractive. In this range both types of players enter contest 2 with positive probability. Note that the key-players' ex ante probability of choosing the contest with the steeper prize structure, $q = \frac{q_L + q_H}{2}$ decreases *monotonically* in the contests' predictability, p .

In order to understand why this relationship is monotone remember that from the viewpoint of the players, steep prize structures offer an advantage and a disadvantage.

¹¹This sorting of abilities is similar to a result in Lazear (1986), which shows that firms might choose fixed salaries and piece rates in order to screen between workers of low and high productivity. However Lazear ignores the possibility of compensation based on relative performance, which is the focus here.

Figure 1: Characterization of the entry game equilibrium in dependence of the predictability parameter p for the case $N = 2$ and $h = \frac{1}{2}$. The equilibrium probabilities with which high and low ability players enter the more competitive contest 1 are given by q_H and q_L respectively. Contest 1 attracts more participants than contest 2 if and only if $\frac{q_L+q_H}{2} > \frac{1}{2}$. Note that the attractiveness of contest 1 decreases monotonically in p .

While the advantage of leaving more to be divided amongst the few is independent of p the disadvantage of attracting stronger opponents increases monotonically in p .

In order to see whether the monotonicity of the players' contest choice carries over to the contest designers' allocation of prizes, the second part of Example 1 assumes that the set of feasible prize structures is given by the finite set $\mathcal{V} = \{(\frac{1}{2}, \frac{1}{2}), (\frac{1}{2} + \frac{1}{M}, \frac{1}{2} - \frac{1}{M}), \dots, (1, 0)\}$. This is, for example, the case when contests distribute M identical items (e.g. stock options) between a first and a second prize. The example shows that equilibrium first prizes decrease monotonically in p . Table 1 highlights this result. Note that for some values of p there are multiple equilibria. When multiple equilibria exist, some are symmetric and some are asymmetric. In the symmetric equilibria, contests choose the same prize structure and players enter each contest with equal probability irrespective of their ability. In the asymmetric equilibria, one contest chooses to be more competitive than the other and there is complete separation of abilities. The

p	$[0, 0.1)$	$[0.1, \frac{1}{4}]$	$(\frac{1}{4}, \frac{1}{3}]$	$[0.4, 0.5]$	$[0.6, 0.8)$	$[0.8, 1)$	1
$\{V_i, V_j\}$	$\{1, 1\}$	$\{1, 1\}$ $\{.9, .9\}$	$\{.9, .9\}$	$\{.8, .8\}$	$\{.7, .7\}$	$\{.7, .7\}$ $\{.6, .6\}$	$\{.7, .7\}$ $\{.6, .6\}$ $\{.5, .5\}$
$\{V_i, V_j\}$		$\{1, .9\}$				$\{.7, .6\}$	$\{.7, .6\}$ $\{.7, .5\}$ $\{.6, .5\}$

Table 1: Equilibrium first prizes in dependence of the contests' predictability p for the parameters specified in Example 1 with $M = 10$ and $\gamma(p) = \frac{1}{2}(1 + p)$. Symmetric and asymmetric equilibria are depicted separately. For $p \in [0.1, \frac{1}{4}] \cup [0.8, 1]$ there are multiple equilibria while for $p \in (\frac{1}{3}, 0.4) \cup (0.5, 0.6)$ no equilibrium in pure strategies exists.

coexistence of contests with different degrees of competitiveness and the sorting of players according to abilities is frequently observed in reality.

To understand the equilibrium condition (4), consider the deviations that make the deviating contest the *most* competitive. Amongst all of these deviations, the one in which contest 1 deviates to $V_1 = 1$ is profitable for the widest range of p . Hence, if p is such that $\underline{p}(1, V_2) \leq p$ then no such deviation is profitable. In other words, if predictability is large enough then even the prospect of winning the highest possible first prize is not sufficient to compensate players for the fact that their opponent is expected to be stronger in the contest with the steeper prize structure. Now consider the deviations that make the deviating contest the *least* competitive. Amongst all of these deviations, the one in which contest 2 deviates to $V_1 - \frac{1}{M}$ is profitable for the widest range of p . Hence, if $p \leq \bar{p}(V_1, V_1 - \frac{1}{M})$ then no such deviation is profitable. This implies that if predictability is small enough then the fact that the opponent is expected to be weaker in the contest with the flatter prize structure is not sufficient to compensate for even the smallest possible first prize differential $\frac{1}{M}$. Hence the prize structures $V_1, V_2 \in \mathcal{V}$ with $V_1 \geq V_2$ constitute an equilibrium if and only if (4) holds. Since $\underline{p}(1, V_2)$ decreases in V_2 and $\bar{p}(V_1, V_1 - \frac{1}{M})$ decreases in V_1 , equilibrium first prizes weakly decrease in p .

Example 1 shows how multiple prizes arise naturally from the contests’ desire to attract as many key-players as possible. When ability matters, sorting of players by ability becomes an important issue and key-players are attracted by less competitive contests in which they expect to meet weaker opponents. As a consequence, contest designers have an incentive to increase their second prize. The higher the contests’ predictability the stronger is the designer’s incentive to raise the second prize.

6 Empirical Framework

The theory outlined in the previous sections implies that there is a negative relationship between a contest’s predictability and its competitiveness. A similar relationship has been shown to exist in the labor tournament models of Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983). However, while our theory is based on contests’ competition for the best participants, these models focus on the maximization of players’ aggregate effort. Given that firms need to attract able workers *and* provide incentives to exert effort, both aspects can be expected to be present in labor tournament data. Indeed, using firm level data, Eriksson (1999) finds that the dispersion of pay between job levels is greater in firms which operate in noisy environments. In order to abstract from the competing aspects of these models, we use sports data, i.e. professional road running, where the provision of incentives to exert effort is less of an issue than it is in firm level data. Given their dependence on media interest and sponsor support, sports contests typically strive to attract the most famous athletes. Sports data therefore provides the perfect framework to test our theory. Running data works particularly well as running contests are organised at a disaggregate or “firm” level instead of being governed by a federation as it is for example the case for tennis and golf.

Sports contests tend to be invariably rank ordered and the measurement of individual performance is generally straight forward, making sports data increasingly fashionable to test contest theory. Nevertheless, the empirical literature on contest design is scarce and the few papers that do exist test, whether prize levels and prize differentials have incentive effects. For example, Ehrenberg and Bognanno (1990a, b) use individual player and aggregate event data from US and European Professional Golf

Associations to test whether prizes affect players' performance. For a recent review of the literature that uses sports data to test contest theory, see Frick (2003).

There are two papers that share our focus on professional road running. Both papers seek to test the hypothesis that prize structures affect finishing times. Maloney and McCormick (2000) use 115 foot races with different distances in the US and find that the average prize and prize spread have negative effects on the finishing times. Lynch and Zax (2000) use 135 races and also find that finishing times are faster in races offering higher prize money. However, Lynch and Zax conclude that the effect is not due to the provision of stronger incentives but rather a result of sorting of runners according to abilities. Once fixed effects are controlled for, the incentive effect disappears. This finding supports our projection that in professional road running, the provision of incentives to exert effort is less important than the attraction of the most able runners.

In order to provide empirical evidence for the negative relationship between a contest's predictability and its competitiveness, we have collected a dataset containing 368 road running contests. Road running contests differ in their race course but are (almost) identical with regard to their organisational set-up. We use the distance of a race as a measure of race predictability and argue that longer races are less predictable than shorter races.¹²

6.1 Race distance as a measure of predictability

There are three strands of support for the assertion that longer races are less predictable. Firstly, the longer the race, the stronger the influence of external factors (e.g. weather conditions, race course profile, nutrition) on the runners' performance. This was evident during the 2004 Olympic Games in Athens. In the women's marathon the highly acclaimed world recorder holder, Paula Radcliffe, was predicted to win. However, after a consistent lead, at the 23rd mile mark, Paula stopped and sat crying on the side path suffering the symptoms of heat exhaustion.

¹²To combat the concern that very short races may be quite unpredictable as they are often decided by millisecond differences, we restrict our analysis to distances of 5km or more.

Secondly, the longer the race, the less accurate is the estimate of a runner's ability based on past performance as longer races are run less frequently. For example, although it is possible to run a 5km race each week, elite runners typically restrict themselves to two marathons per year (see Noakes (1985)).

Finally, there exists statistical evidence showing that longer races are less predictable than shorter races. This evidence has been kindly provided to us by Ken Young, a statistician at the "Association of Road Racing Statisticians" (www.arrs.net). Using a data set containing more than 500,000 performances, Ken Young predicts the outcome of several hundred road running contests of varying distances between 1999 and 2003. As an example, Table 2 in the Appendix reports his results for the Men's races in 1999.¹³

Two distinct methods were used to predict the winner of a given race. A regression based handicapping (HA) evaluation attempts to predict each runner's finishing time based on past performance. The predicted time was assumed to be normally distributed for each runner and the numerical integration yielded the probability that each runner would win the race. The second method was a Point Level (PL) evaluation based on a rating system similar to the Elo system in chess or the ATP ranking in tennis, in which runners take points from runners they beat and lose points to runners they are beaten by.

Averaging over 274 Men's races with distances between 5km and 42km, the PL prediction of the winner was correct in 43% of the "Short" races (distance ≤ 10 km), 41% of the "Medium" races ($10\text{km} < \text{distance} < 42\text{km}$) and 20% of the Long races (distance $\geq 42\text{km}$). For the HA prediction the numbers are 45%, 46%, and 21% respectively. Hence while Short and Medium distance races are similar in terms of predictability, Long distance races seem to be much less predictable.

6.2 Data Description

The empirical investigation is done using data on professional road running from the Road Race Management Directory (2004). This Directory provides a detailed account

¹³The complete set of results is available on <http://www.econ.upf.edu/~azmat/>.

of the prize structures, summaries, invitation guidelines, and contacts for almost 500 races. It is an important source of information for elite runners planning their race season. With the exception of a few, most of the races took place in the United States. The event listings are arranged in chronological order beginning in April 2004 and extend through to April 30th 2005. In our analysis we only include races that have at least \$600 in prize money and a race distance greater or equal to 5km, leaving us with 368 races. The Directory provides us with information on the event name, event date, city, state and previous year's number of participants. The prize money information includes the total amount of prize money as well as the prize money breakdown. We focus on the Men's races by including only the Men's prize money distributions.

The Directory contains further information that may influence runners' race selection. In particular, it includes data on whether a race was a championship, took place on a cross country or mountain course, and the race's winning performance in the previous year. In order to make finishing times in races over different distances comparable with each other, we use the Riegel formula (see Riegel (1981)) to calculate 10km equivalent finishing times.¹⁴

Finally, given that the weather conditions play a role in the outcome of an outdoor race, we collect information on the weather using an internet site called Weatherbase (www.weatherbase.com). We can get information on the average temperature and average rainfall in the month that the race takes place.¹⁵ Table 3 presents the summary statistics for three race distance categories: "Short" (distance ≤ 10 km); "Medium" ($10\text{km} < \text{distance} < 42\text{km}$) and "Long" (distance $\geq 42\text{km}$). In general, races tend to be clustered, the most frequent being 5km, 10km, 16km, 21km and 42km. Most runners specialize and run either Short or Long distance races, while Medium distance races are run by both types.

From the summary statistics in Table 3 we see that there are some obvious differences between the three distance categories. In particular, the mean total prize

¹⁴This formula predicts an athlete's finishing time t in a race of distance d on the basis of his finishing time T in a race of distance D as $t = T(\frac{d}{D})^{1.06}$. It is used by the IAAF to construct scoring tables of equivalent athletic performances.

¹⁵We also collected data on average wind speed. However, the data was incomplete. Our results remain the same with and without conditioning on the average wind speed.

money (in US\$) increases as the distance increases (\$2,990, \$5,664 and \$23,207, respectively).¹⁶ The average number of participants also increases with distance (3,359, 5,268 and 5,324, respectively). It is important to note that although the “size” of these contests increases with distance, typically the number of elite runners is similar.¹⁷ In addition, we do not worry about congestion affects in the populated races because elite runners will run separately (and typically before) the non-elite runners.

There is consistency in the weather variables when we look across the race types. In addition, there is a similar probability that the race has a championship status and the average Riegel measure of performance is almost identical. This is reassuring as it implies that the “quality” of runners is independent of the race distance.

6.3 Analysis

To obtain estimates for the differences in prize structure, we estimate the following compensation equations using 368 men’s races:

$$C_i = \alpha + \beta D_i + \varepsilon_i. \quad (5)$$

C_i represents the competitiveness of the prize structure and D_i denotes the distance (and acts as our measure of predictability) for race i . We use various measures of competitiveness C : (1) a concentration index (C. I.), similar to the Herfindahl-Hirschman index, calculated from the top three prizes, i.e. $C = \frac{(1st)^2 + (2nd)^2 + (3rd)^2}{(1st + 2nd + 3rd)^2}$, (2) the ratio between first and second prize, (3) the ratio between first and third prize and (4) the ratio between first prize and total prize money. We expect these measures to increase with the race distance.

For the distance D we use a continuous measure, i.e. a km by km increase in distance, as well as a comparison between Short, Medium and Long distance races. As

¹⁶We use the sum of the top 10 Men’s prizes as the “total prize money”. This variable is more important for the race choice of elite Men’s runners than the race’s total prize budget as prize money that is to be distributed to Women’s or Age-group runners is not accessible to them. For comparison of prize money across countries, we convert all prizes into US dollars using monthly historical exchange rates for 2004–2005 (www.gocurrency.com).

¹⁷Our participation data contains elite and non-elite runners. Unfortunately the number of elite runners was unavailable. Our participation variable therefore only gives a rough measure for the popularity of the event amongst elite runners.

mentioned earlier, races tend to be clustered and so it is more informative to look at how the prize structure changes when we compare each group. In doing so, we can estimate the percentage point change in competitiveness when going from Short to Medium or Long races.

We report the results for all four measures of competitiveness, using the two different distance measures in Table 4. Overall, the results support the hypothesis that as the distance increases, the prize structure becomes steeper. In particular, using our concentration index we observe that as the distance of a race increases by 1km, there is a 0.1% increase in competitiveness. This implies, for example, that the prize structure of a marathon is almost 4% more concentrated towards the first prize than the prize structure of a 5km race. Similarly, we find that when the race changes from being Short to Long, there is a 3.2% increase in the concentration index. The coefficient of moving from Short to Medium is positive but insignificant. This is reassuring as in Ken Young’s analysis these races had a similar degree of predictability.

When we look at the other measures of competitiveness, we observe very similar patterns. In particular, we find that as the distance increases, the gap between the first prize and the second or third prize widens. When the distance increases by 1km, there is a 0.1% rise in the ratio between the first and the second or third prize. When we look across different race types, we see that the ratio between the first and second prize increases by 3.0%, while the ratio between the first and the third prize increases by 2.5% when moving from Short to Long. The proportion of total prize money that goes to the winner also increases with the distance but results are not significant.

Next, we extend the analysis of looking at the simple correlation to account for various factors that may affect runners’ race selection and hence the prize structure. In particular, we may be concerned that the popularity of a certain race in the world of running may be important. For example, if the race is a championship race or if it offers a fast race course (where records can be established) then the race may be attractive for elite runners, irrespective of its prize structure. In addition, weather conditions may play a role. We control for these factors by estimating the following equation:

$$C_i = \alpha + \beta D_i + \delta X_i + \varepsilon_i. \quad (6)$$

X includes average temperature, average rainfall, an indicator identifying whether the race was a championship, the number of race participants, total prize money, the 10km Riegel equivalent of the previous edition's winning time and an indicator for whether the race is a cross country or a mountain race. It is very reassuring to see that the results remain very similar to the results without controls. In fact, as we can see in Table 5, the coefficients for all of the prize structure measures and both measures of distance are almost identical with and without controls.

When we look at the effect that the control variables have on competitiveness, it is only the average rainfall in the month of the race that has a consistently significant negative effect on the spread of prizes. However, neither the significance nor the size of the coefficients have been affected by including controls.

7 Conclusion

How should a contest allocate its prize budget across ranks? This has been a dominant question in the literature on contest design. In order to answer it, one has to specify the contest designer's objective. So far the literature has focused exclusively on the contest designer's aim to provide participants with incentives to exert effort. However, in many contests attracting the best participants may be equally or even more important. In this paper, we have considered the optimal allocation of prizes in the light of this second objective. While the literature has struggled to explain the wide spread occurrence of multiple prizes using the provision of incentives, in our model multiple prizes arise naturally from the contests' desire to attract key-players. We therefore consider our theory as complementary to the one that focuses exclusively on the provision of incentives. As most contest designers aim to provide both, incentives to exert effort *and* incentives to participate, an interesting extension would be a theory that combines both aspects. For example, one might consider a model in which players choose contests and exert effort, while contest designers aim to maximize the expected winning performance. As a contest's expected winning performance increases with players' effort *and* with the number of participants, the contest designer cares for both. We leave this issue for future research.

Appendix 1: Proofs

Proof of Lemma 1

Consider the case $p = 0$ and suppose high and low abilities enter contest 1 with probability q_H and q_L respectively. The ex ante probability that a player enters contest 1 is $q \equiv hq_H + (1 - h)q_L$. Expected payoffs in the two contests are independent of ability, i.e.

$$U_1(a_L) = U_1(a_H) = \sum_{n=0}^{N-1} q^n (1 - q)^{N-1-n} \binom{N-1}{n} \frac{\sum_{m=1}^{n+1} V_1^m}{n+1} \quad (7)$$

$$U_2(a_L) = U_2(a_H) = \sum_{n=0}^{N-1} q^{N-1-n} (1 - q)^n \binom{N-1}{n} \frac{\sum_{m=1}^{n+1} V_2^m}{n+1}. \quad (8)$$

Suppose that $q \leq \frac{1}{2}$. Then the probability to meet less than n opponents is larger in contest 1 than in contest 2. As $\frac{1}{n+1} \sum_{m=1}^{n+1} V_i^m$ is decreasing in n and $\sum_{m=1}^{n+1} V_1^m \geq \sum_{m=1}^{n+1} V_2^m$ for all $n \in \{0, 1, \dots, N-1\}$ with strict inequality for some n it follows that $U_1 > U_2$. Hence both types strictly prefer contest 1 which implies that there cannot exist an equilibrium with $q \leq \frac{1}{2}$. This implies that $Q_1 = Nq > N(1 - q) = Q_2$ in every equilibrium of the (V_1, V_2) entry game. As the inequality $U_1 > U_2$ is strict and expected payoffs are continuous in p , there exists a $\underline{p}(V_1, V_2) > 0$ such that this result continues to hold for all $p \leq \underline{p}(V_1, V_2)$. ■

Proof of Proposition 1

According to Lemma 1, for all $V_1 \succ V_2$ there exists a $\underline{p}(V_1, V_2) > 0$ such that for all $p \leq \underline{p}(V_1, V_2)$, it holds that $Q_1 > Q_2$ in every equilibrium of the (V_1, V_2) entry game. If $\mathcal{V}$ is finite define

$$\underline{p} \equiv \min\{\underline{p}(V_1, V_2) | V_1, V_2 \in \mathcal{V}, V_1 \succ V_2\}. \quad (9)$$

Otherwise let $\underline{p} = 0$. Suppose that $p \in [0, \underline{p}]$. It then holds that $Q_1 > \frac{N}{2} > Q_2$ for all $V_1 \succ V_2$ and every equilibrium of the (V_1, V_2) entry game. Hence every contest has an incentive to choose a more competitive prize structure than its rival. It follows that $V_1^* = V_2^* = (1, 0, \dots, 0)$ is the unique candidate for an equilibrium prize structure. It is indeed part of an equilibrium because $q_L = q_H = \frac{1}{2}$ constitutes an equilibrium of the corresponding entry game for all p implying $Q_1 = Q_2 = \frac{N}{2}$ and $Q_i < \frac{N}{2} < Q_j$ for all V_i such that $(1, 0, \dots, 0) = V_j \succ V_i$. ■

Proof of Lemma 2

We consider the case $p = 1$ and then use the continuity of $u(a, n_L, n_H, V; p)$ in p . Suppose that high and low abilities enter contest 1 with probability q_H and q_L respectively. For

$p = 1$, $u(a_H, n_L, 0, V; 1) = V^1$ and $u(a_H, n_L, n_H, V; 1) = \frac{1}{n_H+1}$ for all $n_H > 0$ imply that the expected payoff of a high ability player who enters contest i is

$$U_i^H(q_H) = \pi_i(0)V_i^1 + \sum_{n=1}^{N-1} \pi_i(n) \frac{1}{n+1} \quad (10)$$

where $\pi_i(n)$ for $n \in \{0, \dots, N-1\}$ denotes the probability of meeting n high ability opponents in contest i , i.e. $\pi_1(n) = \binom{N-1}{n} (hq_H)^n (1 - hq_H)^{N-1-n}$ and $\pi_2(n) = \binom{N-1}{n} (h(1 - q_H))^n (1 - h(1 - q_H))^{N-1-n}$. We consider the cases $h \leq \frac{1}{2}$ and $h > \frac{1}{2}$ separately:

Case $h \leq \frac{1}{2}$: For $q_H = 1$ a high ability player's expected payoff in contest 1 is

$$U_1^H(1) = (1 - h)^{N-1} V_1^1 + \sum_{n=1}^{N-1} \binom{N-1}{n} h^n (1 - h)^{N-1-n} \frac{1}{n+1} \equiv \bar{V} \quad (11)$$

while in contest 2 he wins $U_2^H(1) = V_2^1$ with certainty. Defining $\Delta U^H(q_H) \equiv U_1^H(q_H) - U_2^H(q_H)$ we thus have $\Delta U^H(1) = \bar{V} - V_2^1 < 0$ and $\Delta U^H(\frac{1}{2}) = (1 - \frac{h}{2})^{N-1} (V_1^1 - V_2^1) > 0$. As ΔU^H strictly decreases in q_H there exists a unique $\frac{1}{2} < q_H^* < 1$ defined by $\Delta U^H(q_H^*) = 0$ that makes high abilities indifferent between entering contest 1 and entering contest 2.

Case $h > \frac{1}{2}$: For $Q_1 < Q_2$ to hold it is necessary that $hq_H < \frac{1}{2}$. For $q_H = \frac{1}{2h}$ players with a_H expect $U_1^H(\frac{1}{2h}) = \frac{1}{2^{N-1}} V_1^1 + \sum_{n=1}^{N-1} \binom{N-1}{n} \frac{1}{2^{N-1}} \frac{1}{n+1}$ and $U_2^H(\frac{1}{2h}) = (\frac{3}{2} - h)^{N-1} V_2^1 + \sum_{n=1}^{N-1} \binom{N-1}{n} (h - \frac{1}{2})^n (\frac{3}{2} - h)^{N-1-n} \frac{1}{n+1}$. Define $\bar{V}$ as the V_2^1 that makes these payoffs identical, i.e.

$$\bar{V} \equiv \left(\frac{1}{3-2h}\right)^{N-1} V_1^1 + \sum_{n=1}^{N-1} \frac{\binom{N-1}{n}}{n+1} \left[\left(\frac{1}{3-2h}\right)^{N-1} - \left(\frac{2h-1}{3-2h}\right)^n\right]. \quad (12)$$

As ΔU^H strictly decreases in V_2^1 it follows that for all $V_2^1 > \bar{V}$, $\Delta U^H(\frac{1}{2h}) < 0$. As in the case above there therefore exists a unique $\frac{1}{2} < q_H^* < \frac{1}{2h}$ defined by $\Delta U^H(q_H^*) = 0$ that makes high abilities indifferent between entering contest 1 and entering contest 2.

We have therefore shown that for all $h \in (0, 1)$ in equilibrium high ability players mix and $hq_H^* < \frac{1}{2}$. We now consider low ability players. A low ability player's expected payoff is zero when he meets more than one high ability opponent. If he meets exactly one high ability opponent then $u(a_L, n_L, 1, V; 1) = \frac{1-V^1}{n_L+1}$. For $n_H = 0$ we have $u(a_L, 0, 0, V; 1) = V^1$ and $u(a_L, n_L, 0, V; 1) = \frac{1}{n_L+1}$ for $n_L \geq 1$. We now argue that for all $q_L \geq \bar{q}_L \equiv \frac{1}{1-h}(\frac{1}{2} - hq_H^*)$ low ability players have to strictly prefer contest 2. This then implies that in equilibrium it has to hold that $(1 - h)q_L^* + hq_H^* < \frac{1}{2}$ which implies $Q_2 > Q_1$.

As $\Delta U^L(q_L, q_H^*) \equiv U_1^L(q_L, q_H^*) - U_2^L(q_L, q_H^*)$ strictly decreases in q_L it is sufficient to show that $\Delta U^L(\bar{q}_L, q_H^*) < 0$. To see this first note that for $q_H = q_H^*$ and $q_L = \bar{q}_L$, the probability to

meet $n \in \{0, 1, \dots, N-1\}$ opponents is identical in both contests. However, as $q_H^* > \frac{1}{2}$ and $\bar{q}_L < \frac{1}{2}$ the opponents in contest 1 are, on average, more able than in contest 2. It therefore has to hold that $\Delta U^L(\bar{q}_L, q_H^*) < \Delta U^L(\frac{1}{2}, \frac{1}{2})$. We find

$$\Delta U^L(\frac{1}{2}, \frac{1}{2}) = \frac{(V_1^1 - V_2^1)}{2^{N-1}} \frac{1 - h(2-h)^{N-1}}{1-h}. \quad (13)$$

Hence if $h(2-h)^{N-1} \geq 1$ then the above claim is true. Define $\bar{h}(N)$ as the $h \in (0, \frac{1}{N-1})$ that makes this inequality binding. We have therefore shown that $Q_2 > Q_1$ if $h > \bar{h}(N)$.

Note that $h > \bar{h}$ is a sufficient but not a necessary condition for $Q_2 > Q_1$. For $N = 3$ and $q_L = \frac{1}{1-h}(\frac{1}{2} - hq_H)$ we find $\Delta U(q_H) \equiv \Delta U^H - \Delta U^L = \frac{1}{4}(3 - 2hq_H)(V_1^1 - V_2^1) + \frac{1}{2}h(2q_H - 1)(\frac{2}{3} - V_2^1)$. Moreover, $\Delta U^H(q_H^*) = 0$ implies that $V_1^1 - V_2^1 = \frac{h(1-2q_H^*)(1-\frac{2}{3}h-(2-h)V_2^1)}{(1-hq_H^*)^2}$ which leads

$$\Delta U(q_H^*) = (2q_H^* - 1) \left(\frac{h}{2}(\frac{2}{3} - V_2^1) + \frac{1}{4}((2-h)V_2^1 - 1 + \frac{2}{3}h) \frac{3 - 2hq_H^*}{(1 - hq_H^*)^2} \right) \quad (14)$$

$$> (2q_H^* - 1) \left(\frac{h}{2}(\frac{2}{3} - V_2^1) + \frac{1}{4}((2-h)V_2^1 - 1 + \frac{2}{3}h) \right) \quad (15)$$

$$> (2q_H^* - 1) \min(\frac{1}{12}h + \frac{1}{4}, \frac{3}{24}h) > 0. \quad (16)$$

Hence for $N = 3$ it has to hold that $Q_2 > Q_1$ for all $h \in (0, 1)$. A similar analysis shows that the same is true for $N = 2$. As all of the above inequalities are strict and expected payoffs are continuous in p , there exists a $\bar{p}(V_1, V_2) < 1$ such that our result continues to hold for all $p > \bar{p}(V_1, V_2)$. ■

Proof of Proposition 2

As $\bar{V}(V_j^1, N, h) \leq \bar{V}(1, N, h)$ and $\bar{V}(1, N, h)$ is strictly decreasing in N with $\lim_{N \rightarrow \infty} \bar{V}(1, N, h) < \frac{1}{2}$, there exists a N' such that $\bar{V}(V_j^1, N, h) < \frac{1}{2}$ for all $N > N'$ and all $V_j \in \mathcal{V}$. Hence if $N > N'$ then the threshold $\bar{V}(V_j^1, N, h)$ is no longer binding and can be neglected.

Lemma 2 requires that $h(2-h)^{N-1} \geq 1$ which is equivalent to $N > N'' \equiv 1 - \frac{\ln h}{\ln(2-h)}$. Let $\bar{N} = \max(N', N'')$ and suppose that $N > \bar{N}$. Lemma 2 then implies that for all $V_i, V_j \in \mathcal{V}$ with $V_i^1 < V_j^1$ there exist a $\bar{p}(V_j, V_i)$ such that $Q_i > Q_j$ for all $p \geq \bar{p}(V_j, V_i)$ and every equilibrium of the (V_j, V_i) entry game. If $\mathcal{V}$ is finite define

$$\bar{p} \equiv \max\{\bar{p}(V_1, V_2) | V_1, V_2 \in \mathcal{V}, V_1^1 > V_2^1\}. \quad (17)$$

Otherwise let $\bar{p} = 1$. Suppose that $p \in [\bar{p}, 1]$. If $V_j \in \mathcal{V}$ is such that $V_j^1 > \frac{1}{2}$ then for all $V_i \in \mathcal{V}$ with $V_i^1 < V_j^1$ and every equilibrium of the (V_j, V_i) entry game it holds that $Q_i > \frac{N}{2} > Q_j$. Every contest has an incentive to choose a strictly smaller first prize than its rival. Hence

$V_1^* = V_2^* = (\frac{1}{2}, \frac{1}{2}, 0, \dots, 0)$ is the unique candidate for an equilibrium prize structure. It is indeed part of an equilibrium as $q_L = q_H = \frac{1}{2}$ constitutes an equilibrium of the corresponding entry game for all p implying $Q_1 = \frac{N}{2} = Q_2$ and $Q_i < \frac{N}{2} < Q_j$ for $V_j = V_j^*$ and all $V_i \in \mathcal{V}$ such that $V_i^1 > V_j^1$. ■

Proof of Example 1

1. Contest choice

For $N = 2$ and $h = \frac{1}{2}$ expected payoffs in contest 1 are $U_1^L = (1 - \frac{Q_1}{2})V_1 + \frac{q_L}{4} + \frac{q_H}{2}u_1^L$ and $U_1^H = (1 - \frac{Q_1}{2})V_1 + \frac{q_L}{2}u_1^H + \frac{q_H}{4}$ for players with ability a_L and a_H respectively. In contest 2 players expect $U_2^L = (1 - \frac{Q_2}{2})V_2 + \frac{1-q_L}{4} + \frac{1-q_H}{2}u_2^L$ and $U_2^H = (1 - \frac{Q_2}{2})V_2 + \frac{1-q_L}{2}u_2^H + \frac{1-q_H}{4}$ where $u_i^L = 1 - u_i^H = \gamma(p)(1 - V_i) + (1 - \gamma(p))V_i$. Define $\Delta U^L \equiv U_1^L - U_2^L$, $\Delta U^H \equiv U_1^H - U_2^H$ and $\Delta U \equiv \Delta U^H - \Delta U^L$. We find that

$$\Delta U = \frac{Q_1}{2}[u_1^H - \frac{1}{2}] - \frac{Q_2}{2}[u_2^H - \frac{1}{2}]. \quad (18)$$

Consider the case $V_1 = V_2$ first. Suppose that in equilibrium $Q_1 \neq Q_2$ and without loss of generality assume that $Q_1 > Q_2$. It has to hold that $q_H < q_L$ as otherwise all players would strictly prefer to enter contest 2. Moreover it follows from $Q_1 > 1$ that $q_H > 0$. Hence high abilities have to be indifferent. But as $\Delta U > 0$ this implies that $q_L = 0$ contradicting $Q_1 > Q_2$. Hence for $V_1 = V_2$, $Q_1 = Q_2$ has to hold in every equilibrium.

Now consider the case $V_1 > V_2$. As $V_1 > V_2$ implies that $u_1^H > u_2^H$ it holds that $\Delta U > 0$ whenever $Q_1 \geq Q_2$. This implies that there cannot exist an equilibrium in which $q_H < q_L = 1$. Also note that both abilities entering the same contest i with certainty cannot be an equilibrium as low abilities expected payoff in contest i is strictly smaller than $\frac{1}{2}$ whereas entering the other contest j would lead $V_j \geq \frac{1}{2}$. Consider the remaining cases.

Case $q_L = 0, q_H = 1$: We have $\Delta U^L = \frac{1}{2}(V_1 - V_2 + u_1^L - \frac{1}{2})$ and $\Delta U^H = \frac{1}{2}(V_1 - V_2 + u_2^L - \frac{1}{2})$. Note that ΔU^L strictly decreases in $p \in (0, 1)$. For $p = 0$ it holds that $\Delta U^L > 0$ and for $p = 1$ we have $\Delta U^L \leq 0$ with strict inequality if $V_2 > \frac{1}{2}$. Hence, there exists a unique $\underline{p}(V_1, V_2)$ that solves $\Delta U^L = 0$. It is given by

$$\underline{p}(V_1, V_2) = \gamma^{-1} \left(\frac{2V_1 - V_2 - \frac{1}{2}}{2V_1 - 1} \right). \quad (19)$$

For $V_2 = \frac{1}{2}$, $\underline{p}(V_1, V_2) = 1$ and for $V_2 > \frac{1}{2}$, $\underline{p}(V_1, V_2) \in (0, 1)$. ΔU^H strictly decreases in $p \in (0, 1)$ and for $p = 0$ it holds that $\Delta U^H > 0$. For $p = 1$, $\Delta U^H < 0$ if and only if $V_2 > \frac{1}{2}V_1 + \frac{1}{4}$. Defining

$$\bar{p}(V_1, V_2) = \begin{cases} \gamma^{-1} \left(\frac{V_1 - \frac{1}{2}}{2V_2 - 1} \right) & \text{if } V_2 > \frac{1}{2}V_1 + \frac{1}{4} \\ 1 & \text{if } V_2 \leq \frac{1}{2}V_1 + \frac{1}{4} \end{cases} \quad (20)$$

we therefore find that an equilibrium in which high abilities enter contest 1 and low abilities enter contest 2 exists if and only if $p \in [\underline{p}(V_1, V_2), \bar{p}(V_1, V_2)]$. As $\Delta U^H > \Delta U^L$ for all $p \in (0, 1]$ it holds that $\underline{p}(V_1, V_2) < \bar{p}(V_1, V_2)$ if $V_2 > \frac{1}{2}$ and $\underline{p}(V_1, V_2) = \bar{p}(V_1, V_2) = 1$ otherwise.

Case $q_L \in (0, 1), q_H = 1$: In this case $\Delta U^L = \frac{1}{2}(1 - V_1 - V_2)q_L + \frac{1}{2}(V_1 - V_2 + u_1^L - \frac{1}{2})$. Note that ΔU^L is strictly decreasing in q_L and strictly negative for $q_L = 1$. Above we have shown that for $q_L = 0$, ΔU^L is strictly positive if and only if $p < \underline{p}(V_1, V_2)$. Hence there exists a unique $q_L^* \in (0, 1)$ that solves $\Delta U^L = 0$ if and only if $p < \underline{p}(V_1, V_2)$, i.e.

$$q_L^* = \frac{V_1 - V_2 + u_1^L - \frac{1}{2}}{V_1 + V_2 - 1}. \quad (21)$$

q_L^* strictly decreases in p with $\lim_{p \rightarrow 0} q_L^* = \frac{V_1 - V_2}{V_1 + V_2 - 1}$ and $\lim_{p \rightarrow \underline{p}} q_L^* = 0$. As $Q_1 = q_L^* + 1 > Q_2$, it follows from (18) that high abilities have to strictly prefer contest 1 given that low abilities are indifferent. Hence we have shown that there exists an equilibrium in which high abilities enter contest 1 and low abilities mix if and only if $p \in [0, \underline{p}(V_1, V_2))$.

Case $q_L \in (0, 1), q_H \in (0, 1)$: The system of equations $\Delta U^L = 0, \Delta U^H = 0$ has a unique solution:

$$q_L^* = \frac{(2V_2 - 1)\gamma(p) + \frac{1}{2} - 2V_1 + V_2}{(V_1 + V_2 - 1)(2\gamma(p) - 1)} \quad (22)$$

$$q_H^* = \frac{(2V_2 - 1)\gamma(p) + 1 + 2V_1 - 3V_2}{(V_1 + V_2 - 1)(2\gamma(p) - 1)}. \quad (23)$$

Note that $q_H^* > \frac{1}{2}$ and $q_L^* < 1 - q_H^*$ for all p . Moreover $q_H^* < 1$ if and only if $p > \bar{p}(V_1, V_2)$. Finally $q_L^* > 0$ if and only if $\gamma(p) > \frac{2V_1 - V_2 - \frac{1}{2}}{2V_2 - 1}$. Defining

$$\bar{\bar{p}}(V_1, V_2) = \begin{cases} \gamma^{-1}\left(\frac{2V_1 - V_2 - \frac{1}{2}}{2V_2 - 1}\right) & \text{if } V_2 > \frac{2}{3}V_1 + \frac{1}{6} \\ 1 & \text{if } V_2 \leq \frac{2}{3}V_1 + \frac{1}{6} \end{cases} \quad (24)$$

we have therefore shown that an equilibrium in which both abilities mix exists if and only if $p \in (\bar{\bar{p}}(V_1, V_2), 1]$. Note that $\bar{p} < \bar{\bar{p}}(V_1, V_2)$ if $V_2 > \frac{1}{2}V_1 + \frac{1}{4}$ and $\bar{p}(V_1, V_2) = \bar{\bar{p}}(V_1, V_2) = 1$ otherwise. Also note that $Q_1 = q_L^* + q_H^* = \frac{2V_2 - 1}{V_1 + V_2 - 1}$ is independent of p and strictly smaller than Q_2 .

Case $q_L = 0, q_H \in (0, 1)$: We have $\Delta U^L = \frac{1}{2}(u_1^L + u_2^L - V_1 - V_2)q_H + V_1 - \frac{1}{4} - \frac{1}{2}u_2^L$ and $\Delta U^H = \frac{1}{2}(1 - V_1 - V_2)q_H + V_1 - \frac{1}{4} - \frac{1}{2}u_2^H$. For $q_H = \frac{1}{2}$, it holds that $\Delta U^H = \frac{1}{4}(V_1 - V_2) + \frac{1}{2}(V_1 - u_2^H) > 0$. We have shown above that for $q_H = 1$, $\Delta U^H < 0$ if and only if $p > \bar{p}(V_1, V_2)$. As ΔU^H is strictly decreasing in q_H it follows that $\Delta U^H > 0$ for all $q_H \in (0, 1)$ if $p \leq \bar{p}(V_1, V_2)$. For $p > \bar{p}(V_1, V_2)$ there exists a unique $q_H^* \in (\frac{1}{2}, 1)$ such that $\Delta U^H = 0$:

$$q_H^* = \frac{2V_1 + V_2 - \frac{3}{2} - (2V_2 - 1)\gamma(p)}{V_1 + V_2 - 1}. \quad (25)$$

q_H^* is strictly decreasing in p with $\lim_{p \rightarrow \bar{p}} q_H^* = 1$. For $q_H = q_H^*$ we find $\Delta U^L = \frac{1}{4}(2\gamma(p) - 1)((4V_2 - 2)\gamma(p) + 1 - 4V_1 + 2V_2)$. Hence for $q_H = q_H^*$, $\Delta U^L \leq 0$ if and only if $p \leq \bar{p}(V_1, V_2)$. We have therefore shown that an equilibrium in which low abilities enter contest 2 and high abilities mix exists if and only if $p \in (\bar{p}(V_1, V_2), \bar{\bar{p}}(V_1, V_2)]$.

2. Prize allocation

Let V_1^*, V_2^* be equilibrium prize structures and without loss of generality assume $V_1^* \geq V_2^*$. It has to hold that $Q_1^* = Q_2^*$. Otherwise the less attractive contest would have an incentive to imitate the prize allocation of the more attractive contest. Hence either $V_1^* = V_2^*$ or $V_1^* > V_2^*$ and $\underline{p}(V_1^*, V_2^*) \leq p \leq \bar{p}(V_1^*, V_2^*)$. We now discuss the profitability of possible deviations.

Suppose that $V_1^* < 1$ and consider a deviation of contest 2 to a prize structure $V_2' > V_1^*$. This deviation is profitable if and only if $p < \underline{p}(V_2', V_1^*)$. As $\underline{p}(V_1, V_2)$ increases in its first argument, contest 2 has an incentive to deviate to some $V_2' > V_1^*$ if and only if $p < \underline{p}(1, V_1^*)$. Similarly contest 1 has an incentive to deviate to some $V_1' > V_1^*$ if and only if $p < \underline{p}(1, V_2^*)$. Note that $\underline{p}(1, V_1^*) \leq \underline{p}(1, V_2^*)$. Hence no contest has an incentive to deviate to a first prize $V' > V_1^*$ if and only if $p \geq \underline{p}(1, V_2^*)$.

Now suppose that $V_1^* > \frac{1}{2}$ and $V_2^* < V_1^* - \frac{1}{M}$ and consider a deviation of contest 2 to a prize structure V_2' such that $V_2^* < V_2' < V_1^*$. Such a deviation is profitable if and only if $p > \bar{p}(V_1^*, V_2')$. As $\bar{p}(V_1, V_2)$ is decreasing in its second argument such a profitable deviation exists if and only if $p > \bar{p}(V_1^*, V_1^* - \frac{1}{M})$. Similarly if $V_2^* > \frac{1}{2}$ then contest 1 has an incentive to deviate to some $V_1' < V_2^*$ if and only if $p > \bar{p}(V_2^*, V_2^* - \frac{1}{M})$. Note that $\bar{p}(V_2^*, V_2^* - \frac{1}{M}) \geq \bar{p}(V_1^*, V_1^* - \frac{1}{M})$. Hence no contest has an incentive to deviate to a first prize slightly smaller than its rival's if and only if $p \leq \bar{p}(V_1^*, V_1^* - \frac{1}{M})$. Finally note that contest 2 never has an incentive to deviate to prize structures $V_2' < V_2^*$ and contest 1 never has an incentive to deviate to prize structures $V_1' < V_1^*$.

We have therefore shown that the prize allocation $\frac{1}{2} < V_1^* = V_2^* = V^* < 1$ constitutes an equilibrium if and only if $\underline{p}(1, V^*) \leq p \leq \bar{p}(V^*, V^* - \frac{1}{M})$. $V_1^* = V_2^* = \frac{1}{2}$ is an equilibrium if and only if $p = 1$. $V_1^* = V_2^* = 1$ is an equilibrium if and only if $p \leq \bar{p}(1, 1 - \frac{1}{M}) = \gamma^{-1}(\frac{1}{2 - \frac{1}{M}})$.

Note that $\lim_{M \rightarrow \infty} \bar{p}(1, 1 - \frac{1}{M}) = 0$. As $\underline{p}(1, 1) = 0$ and $\bar{p}(\frac{1}{2}, \frac{1}{2} - \frac{1}{M}) = 1$ we can summarize this finding by saying that $V_1^* = V_2^* = V^*$ is an equilibrium if and only if $\underline{p}(1, V^*) \leq p \leq \bar{p}(V^*, V^* - \frac{1}{M})$.

If $V_2^* < V_1^* - \frac{1}{M}$ then contest 2's incentive to deviate to a first prize $V_1^* - \frac{1}{M}$ is stronger than contest 1's incentive to deviate to a first prize $V_2^* - \frac{1}{M}$. Hence $V_2^* < V_1^* - \frac{1}{M}$ is an equilibrium if and only if $\underline{p}(1, V_2^*) \leq p \leq \bar{p}(V_1^*, V_1^* - \frac{1}{M})$. Finally for $V_1^* = V^*$ and $V_2^* = V^* - \frac{1}{M}$ it has to hold that $p \leq \bar{p}(V_1^*, V_2^*)$ in order to guarantee that $Q_1 = Q_2$. Hence $V_1^* = V^*$ and $V_2^* = V^* - \frac{1}{M}$ is an equilibrium if and only if $\underline{p}(1, V^* - \frac{1}{M}) \leq p \leq \bar{p}(V^*, V^* - \frac{1}{M})$.

In summary, we have therefore shown that an arbitrary (V_1^*, V_2^*) constitutes an equilibrium prize structure if and only if $\underline{p}(1, V_2^*) \leq p \leq \bar{p}(V_1^*, V_1^* - \frac{1}{M})$. Monotonicity follows from the fact that $\underline{p}(1, V_2^*)$ decreases in V_2^* and $\bar{p}(V_1^*, V_1^* - \frac{1}{M})$ decreases in V_1^* . ■

Appendix 2: Empirical Tables

Table 2: Ken Young’s prediction of Men’s winner (1999)

Date	Race Name	Distance (km)	HA Prob	HA WP	PL CI	PL WP
3/5/1999	IAAF World Indoor Champs (JPN)	3.0	80	1	796	1
3/5/1999	NCAA Indoor Champs (IN/USA)	5.0	70	2	398	1
3/6/1999	Gate River Run (FL/USA)	15.0	78	1	434	4
3/6/1999	NCAA Indoor Champs (IN/USA)	3.0	78	1	458	4
3/14/1999	Los Angeles (CA/USA)	42.2	54	2	650	4
3/27/1999	Azalea Trail (AL/USA)	10.0	97	1	432	1
4/11/1999	Cherry Blossom (DC/USA)	16.1	43	3	677	4
4/17/1999	Stramilano (ITA)	21.1	80	1	864	1
4/18/1999	Rotterdam (HOL)	42.2	71	2	709	1
4/19/1999	Boston (MA/USA)	42.2	37	2	727	4
4/25/1999	Sallie Mae (DC/USA)	10.0	66	2	728	1
5/2/1999	Pittsburgh (PA/USA)	42.2	47	1	355	4
5/16/1999	Volvo Midland Run (NJ/USA)	16.1	59	4	376	6
5/16/1999	Bay to Breakers (CA/USA)	12.0	50	3	676	1
5/31/1999	Bolder Boulder (CO/USA)	10.0	25	9	673	13
6/2/1999	NCAA Champs (ID/USA)	10.0	35	5	379	5
6/4/1999	NCAA Champs (ID/USA)	5.0	78	1	456	1
6/12/1999	Stockholm (SWE)	42.2	47	2	433	1
6/19/1999	Grandma’s (MN/USA)	42.2	40	2	392	2
6/27/1999	Fairfield (CT/USA)	21.1	60	6	572	4
7/4/1999	Peachtree (GA/USA)	10.0	68	1	831	2
7/4/1999	Golden Gala (ITA) 5000m	5.0	52	1	1003	1
7/17/1999	Crazy 8’s (TN/USA)	8.0	60	5	714	2
7/25/1999	Wharf to Wharf (CA/USA)	9.7	75	1	673	3
7/31/1999	Quad-Cities Bix (IA/USA)	11.3	88	1	767	1
8/15/1999	Falmouth (MA/USA)	11.3	72	2	845	2
8/21/1999	Parkersburg (WV/USA)	21.1	57	1	338	1
8/24/1999	IAAF World Champs (ESP)	10.0	74	2	960	1
8/28/1999	IAAF World Champs (ESP)	5.0	68	1	992	2
8/28/1999	IAAF World Champs (ESP)	42.2	7	5	699	18
9/3/1999	Ivo Van Damme (BEL)	10.0	42	13	843	12
9/26/1999	Berlin (GER)	42.2	57	1	586	1
10/24/1999	Chicago (IL/USA)	42.2	66	1	752	1
11/7/1999	New York City (NY/USA)	42.2	57	1	702	9
12/5/1999	California International (CA/USA)	42.2	12	8	378	11

Data kindly provided by Ken Young, Association of Road Racing Statisticians. For the handicapping (HA) evaluation, “HA Prob” denotes the probability with which the predicted winner was expected to win and “HA WP” reports the placing he actually obtained. Using a Point Level (PL) system the average rating for the five highest ranked runners in the race was compared to the average rating for the ten highest ranked runners in the world at the time of the race in order to construct the competition index (CI). The higher the index the better the quality of the field. The column “PL WP” reports the actual placing obtained by the highest ranked runner.

Table 3: Descriptive statistics

Short (Distance $\leq 10\text{km}$)					
Variable	Observations	Mean	Std. Dev.	Min	Max
Rain (cm)	175	7.29	3.19	0	17
Temperature ($^{\circ}\text{C}$)	175	21.44	6.65	3	33
Championship	175	0.07	0.26	0	1
Total (US \$)	175	2,989.68	6,389.31	125	60,000
Size	175	3,358.98	7,324.30	18	55,000
Riegel 2003 (Sec)	175	1,821.77	96.91	1,647	2,276
Trail	175	0.01	0.11	0	1
Medium (10km < Distance < 42km)					
Variable	Observations	Mean	Std. Dev.	Min	Max
Rain (cm)	97	7.06	2.92	0	16
Temperature ($^{\circ}\text{C}$)	97	20.27	6.00	0	32
Championship	97	0.12	0.33	0	1
Total (US \$)	97	5,664.21	10,011.01	175	70,000
Size	97	5,267.56	10,671.53	100	80,000
Riegel 2003 (Sec)	97	1,849.14	203.44	1,653	3,056
Trail	97	0.05	0.22	0	1
Long (Distance $\geq 42\text{km}$)					
Variable	Observations	Mean	Std. Dev.	Min	Max
Rain (cm)	96	6.49	3.25	0	16
Temperature ($^{\circ}\text{C}$)	95	18.44	5.13	9	33
Championship	96	0.11	0.32	0	1
Total (US \$)	96	23,206.75	46,878.09	225	270,000
Size	96	6,277.22	8,707.80	100	46,000
Riegel 2003 (Sec)	96	1,883.97	180.92	1,629	2,628
Trail	96	0.03	0.17	0	1

Table 4: Prize structure without controls

	C. I.	1:2	1:3	1:Total
Distance (km)	0.0008	0.0008	0.0007	0.0006
	[0.0003]**	[0.0002]**	[0.0002]**	[0.0004]
Constant	0.394	0.6241	0.7321	0.4511
	[0.0075]**	[0.0058]**	[0.0063]**	[0.0102]**
Observations	368	368	368	368

	C. I.	1:2	1:3	1:Total
Medium	0.0119	0.0115	0.0112	0.0162
	[0.0113]	[0.0086]	[0.0094]	[0.0153]
Long	0.0316	0.0299	0.025	0.0245
	[0.0113]**	[0.0087]**	[0.0094]**	[0.0153]
Constant	0.3989	0.6292	0.7358	0.4531
	[0.0067]**	[0.0052]**	[0.0056]**	[0.0091]**
Observations	368	368	368	368

Standard errors in parentheses. (*) and (**) denote significance at the 95 and 99 percent level. Omitted group is Short Distance.

Table 5: Prize structure with controls

	C. I.	C. I.	1:2	1:2	1:3	1:3	1:Total	1:Total
Dist.	0.0008	0.0007	0.0008	0.0008	0.0007	0.0006	0.0006	0.0004
	[0.0003]**	[0.0003]*	[0.0002]**	[0.0003]**	[0.0002]**	[0.0003]*	[0.0004]	[0.0004]
Rain		-0.0059		-0.0037		-0.0037		-0.008
		[0.0015]**		[0.0011]**		[0.0012]**		[0.0019]**
Temp.		0.0011		0.0007		0.0005		0.0005
		[0.0007]		[0.0006]		[0.0006]		[0.0010]
Champ.		0.0006		-0.0034		-0.0039		-0.0206
		[0.0156]		[0.0121]		[0.0131]		[0.0203]
Prize		0		0		0		0
		[0.0000]		[0.0000]		[0.0000]		[0.0000] [†]
Size		0		0		0		0
		[0.0000]		[0.0000]		[0.0000]		[0.0000]
Riegel		0.0001		0		0.0001		0.0002
		[0.0000]		[0.0000]		[0.0000] [†]		[0.0001]**
Trail		-0.0148		-0.0067		-0.0398		-0.0991
		[0.0351]		[0.0272]		[0.0294]		[0.0456]*
Const.	0.394	0.302	0.6241	0.6191	0.7321	0.6413	0.4511	0.0984
	[0.0075]**	[0.0726]**	[0.0058]**	[0.0562]**	[0.0063]**	[0.0609]**	[0.0102]**	[0.0943]
Obs.	368	368	368	368	368	368	368	368

	C. I.	C. I.	1:2	1:2	1:3	1:3	1:Total	1:Total
Medium	0.0119	0.011	0.0115	0.0109	0.0112	0.0112	0.0162	0.0163
	[0.0113]	[0.0112]	[0.0086]	[0.0087]	[0.0094]	[0.0094]	[0.0153]	[0.0145]
Long	0.0316	0.0291	0.0299	0.0289	0.025	0.0233	0.0245	0.0183
	[0.0113]**	[0.0122]*	[0.0087]**	[0.0094]**	[0.0094]**	[0.0102]*	[0.0153]	[0.0158]
Rain		-0.006		-0.0038		-0.0038		-0.0081
		[0.0015]**		[0.0011]**		[0.0012]**		[0.0019]**
Temp.		0.0011		0.0007		0.0005		0.0006
		[0.0007]		[0.0006]		[0.0006]		[0.0010]
Champ.		0		-0.0037		-0.0043		-0.0215
		[0.0156]		[0.0121]		[0.0131]		[0.0203]
Prize		0		0		0		0
		[0.0000]		[0.0000]		[0.0000]		[0.0000] [†]
Size		0		0		0		0
		[0.0000]		[0.0000]		[0.0000]		[0.0000]
Riegel		0.0001		0		0.0001		0.0002
		[0.0000] [†]		[0.0000]		[0.0000]*		[0.0000]**
Trail		-0.0163		-0.0092		-0.0424		-0.1021
		[0.0351]		[0.0273]		[0.0295]		[0.0456]*
Const.	0.3989	0.3021	0.6292	0.6168	0.7358	0.6389	0.4531	0.096
	[0.0067]**	[0.0726]**	[0.0052]**	[0.0564]**	[0.0056]**	[0.0610]**	[0.0091]**	[0.0944]
Obs.	368	368	368	368	368	368	368	368

Standard errors in parentheses. ([†]), (*) and (**) denote significance at the 90, 95 and 99 percent level. Omitted group is Short Distance.

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